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Theoretical calculations for neutrino-induced charged current reactions in ^{12}C and recent experimental results

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(Received 28 November 1988; revised manuscript received 19 June 1989)

Theoretical calculations are presented for the reaction $\nu_e + {}^{12}\mathrm{C} \to {}^{12}\mathrm{N}_{\mathrm{g.s.}} + e^-$ for E_ν from threshold to 135 MeV, for the reaction $\nu_\mu + {}^{12}\mathrm{C} \to {}^{12}\mathrm{N}_{\mathrm{g.s.}} + \mu^-$, and the corresponding antineutrino reaction for E_ν from threshold to 160 MeV. Use is made of updated form factors based on more recent data for $e^- + {}^{12}\mathrm{C} \to {}^{12}\mathrm{C}^* + e'^-$ and $\gamma + {}^{12}\mathrm{C} \to {}^{12}\mathrm{C}^*$. The recent neutrino reaction experiments are discussed in light of these calculations.

I. INTRODUCTION

Recently two experiments involving neutrino-induced charged current reactions have been performed on ^{12}C, namely (Refs. 1 and 2) $\nu_e + {}^{12}\mathrm{C} \to {}^{12}\mathrm{N}_{\mathrm{g.s.}} + e^-$ and $\nu_\mu + {}^{12}\mathrm{C} \to {}^{12}\mathrm{N}_{\mathrm{g.s.}} + \mu^-$. The former experiment was for a total cross section averaged over the electron neutrino spectrum from muon decay. This spectrum extends to a maximal energy of 52.8 MeV and has a mean energy of 31.7 MeV. Over 100 events meeting all tests were observed and a total weighted cross section of $\langle \sigma \rangle = [1.32 \pm 0.13(\mathrm{stat}) \pm 0.15(\mathrm{sys})] \times 10^{-41}$ cm^2 was observed. This number is in reasonable agreement with a theoretical calculation by Mintz,[3] of $\langle \sigma \rangle = 0.9 \times 10^{-41}$ cm^2, with an associated error at 15–20% and with a calculation by Donnelly[4] of $\langle \sigma \rangle = 0.94 \times 10^{-41}$ cm^2. The updated result calculated by Mintz and Purkaviani here yields a result of $\langle \sigma \rangle = (0.90 \pm 0.1) \times 10^{-41}$ cm^2. Thus although the experimental value obtained is somewhat larger than the theoretical calculation, the results overlap when error is taken into account and therefore are in general agreement.

The second experiment for the reaction $\nu_\mu + {}^{12}\mathrm{C} \to {}^{12}\mathrm{N}_{\mathrm{g.s.}} + \mu^-$ covers a wider q^2 range than the first. Neutrinos were produced by decaying pions which had resulted from the reaction of 800 MeV protons with a water target. Neutrinos with energy up to a few hundred MeV were obtained. Originally the experiment was meant to study inclusive neutrino interactions $\nu_\mu({}^{12}\mathrm{C}, X)\mu^-$ but a few events passing all tests for $\nu_\mu({}^{12}\mathrm{C}, {}^{12}\mathrm{N}_{\mathrm{g.s.}})\mu^-$ were obtained. These appear to require a cross section 2.5–3 times calculated[3,4] rates. One important difference between the two experiments is the range of q^2 available which is less than $-m_\pi^2$ for the first experiment and of the order of $-6m_\pi^2$ for the second.

Clearly, this discrepancy between the calculated and the experimental results needs investigation, and in this paper we shall examine various possibilities which might lead to a greatly enhanced cross section. Among the possibilities that should be considered are enhancement of the axial current form factor, meson exchange contributions, a large pseudoscalar contribution, and second class currents.

In Sec. II of this paper we construct matrix elements for the transition in view of the possible enhancements. In Sec. III we obtain the cross sections under various assumptions. In Sec. IV we discuss the results and consider the possibilities for further experimental tests.

II. MATRIX ELEMENTS

The transition matrix elements for the reaction $\nu_l({}^{12}\mathrm{C}, {}^{12}\mathrm{N}_{\mathrm{g.s.}})l$ where l refers to the charged lepton, may be written as

$$\mathcal{M} = \frac{[G\cos(\theta_c)]}{\sqrt{2}} \bar{u}_l \gamma^\mu (1-\gamma_5) u_{\nu_l} \langle {}^{12}\mathrm{N}_{\mathrm{g.s.}} | J_\mu(0) | {}^{12}\mathrm{C} \rangle \tag{1}$$

to lowest order in G. Because the range of q^2 considered here is still very small compared to M_W^2, the mass of the intermediate vector boson squared, the form given by Eq. (1), is appropriate.

The matrix element of weak hadronic current $\langle {}^{12}\mathrm{N}_{\mathrm{g.s.}} | J_\mu(0) | {}^{12}\mathrm{C} \rangle$ may be written as

$$\langle {}^{12}\mathrm{N}_{\mathrm{g.s.}}|V_\mu(0)|{}^{12}\mathrm{C}\rangle=-i\sqrt{2}m_i\epsilon_{\mu\delta\beta\gamma}q^\delta\xi^\beta Q^\gamma\frac{F_M(q^2)}{(2m_i2m_p)} \tag{2}$$

and

$$\langle {}^{12}\mathrm{N}_{\mathrm{g.s.}}|A_\mu(0)|{}^{12}\mathrm{C}\rangle=\sqrt{2}m_i\left[\xi_\mu F_A(q^2)+q_\mu\xi\cdot q\frac{F_p(q^2)}{m_\pi^2}-Q_\mu\xi\cdot q\frac{F_E(q^2)}{(2m_i2m_p)}\right], \tag{3}$$

where $q_\mu=P_{f\mu}-P_{i\mu}$ is the four momentum transfer, $Q_\mu=P_{f\mu}+P_{i\mu}$, and $P_{i\mu}$ and $P_{f\mu}$ are the ^{12}C and ^{12}N four momenta, respectively, and m_f, m_i, m_π, and m_μ are the ^{12}N, ^{12}C, pion and muon masses, while ξ_μ is the ^{12}N polarization vector.

We note here that we have made no assumptions regarding the first and second class current contributions. In general

$$F_i=F_i^{(\mathrm{I})}+F_i^{(\mathrm{II})},\quad \text{where } i=M,A,P,E\ . \tag{4}$$

As is clear,[5] the physics of the problem is contained in the form factors F_M, F_A, F_P, and F_E, which must be determined.

The form factor $F_M(q^2)$ can be determined via the conserved vector hypothesis (CVC) to be

$$F_M(q^2)=\sqrt{2}\mu(q^2) \tag{5}$$

if the commutation rule $[I_j,V_k]=i\epsilon_{jkl}V_l$ is assumed, where $\mu(q^2)$ is the electromagnetic form factor describing the matrix element of the electromagnetic current

$$\langle {}^{12}\mathrm{C}^*|J_\mu^{\mathrm{em}}(0)|{}^{12}\mathrm{C}\rangle=-i\sqrt{2}m_i\epsilon_{\mu\delta\beta\gamma}q^\delta\xi^\beta Q^\gamma\frac{\mu(q^2)}{(2m_i2m_p)}\ . \tag{6}$$

The form factor $\mu(q^2)$ can be obtained from data for the reactions (Refs. 6 and 7) $\gamma+{}^{12}\mathrm{C}\to{}^{12}\mathrm{C}^*$ and $e^-+{}^{12}\mathrm{C}\to{}^{12}C^*+e'^-$.

The form of CVC used above implies that (Refs. 8) $F_M^{(\mathrm{II})}=0$. For the range of q^2 up to $q^2\simeq-m_\mu^2$, i.e., appropriate for muon capture, there is no evidence for second class currents; furthermore, because the cross section for the reaction of interest does not depend strongly on (Ref. 9) $F_M(q^2)$, we shall make use of Eq. (5) to obtain $F_M(q^2)$.

In the past decade, additional data[6] has become available over a wide range of q^2 for the process $e^-+{}^{12}\mathrm{C}\to{}^{12}\mathrm{C}^*+e'^-$ and another result has been reported for the reaction $\gamma+{}^{12}\mathrm{C}\to{}^{12}\mathrm{C}^*$. This more recent data leads to a somewhat different fit to the data for $\mu(q^2)$. We find that over a range of q^2 from 0 to $-3.7m_\pi^2$, we obtain a best fit to the data with

$$F_M(q^2)=\frac{4.04\cos^2(-q^2/3.12m_\pi^2)}{(1-q^2/2.86m_\pi^2)^2}\quad \text{for } |q^2|\le 3.7m_\pi^2\ , \tag{7}$$

where the dipole factor represents a fit to the lowest energy data, up to $q^2\simeq-0.3m_\pi^2$. The cosine function is simply a convenient way of parametrizing corrections at the q^4 and higher orders.

The fit represented by Eq. (7) leads to a value of $F_M(0)=4.04$, which is well in accord[10] with values for $F_M(0)$ directly obtained from beta decay measurements

$$F_M(0)=3.9\pm0.72\ ,\quad ({}^{12}B_{\mathrm{g.s.}}\leftrightarrow{}^{12}\mathrm{C})\ , \tag{8}$$

and

$$F_M(0)=3.88\pm.62,\quad ({}^{12}N_{\mathrm{g.s.}}\leftrightarrow{}^{12}\mathrm{C})\ . \tag{9}$$

As can be seen in Fig. 1, Eq. (7) fits the present data very well over a very large range of q^2 and at low q^2 departs only very slightly from a dipole fit. We therefore shall use Eq. (7) for $F_M(q^2)$. We plot $\mu(q^2)=F_M(q^2)/\sqrt{2}$ given by Eq. (7) in Fig. 1.

At $|q^2|\le m_\mu^2$, i.e., in the q^2 range of muon capture, the weak transitions $^{12}\mathrm{C}\leftrightarrow{}^{12}\mathrm{N}_{\mathrm{g.s.}}$ and $^{12}\mathrm{C}\leftrightarrow{}^{12}\mathrm{B}_{\mathrm{g.s.}}$ are dominated by the value of the axial current form factor,[11] $F_A(q^2)$. The value of $F_A(0)$ is known from beta decay data. No direct measurements for $F_A(q^2)$ are available but an argument due to Kim and Primakoff,[11] based on a nucleons-only impulse approximation but extended[12] to include some pion-exchange current corrections, leads to

$$\frac{F_A(q^2)}{F_A(0)}\approx\frac{F_M(q^2)}{F_M(0)}\ , \tag{10}$$

where

$$F_A(0)=1.03\ . \tag{11}$$

This relationship is appropriate to the nonrelativistic case and thus is not expected to hold over an indefinitely large q^2 range. It has, however, been tested[13] for the reaction $\mu^-+{}^{12}\mathrm{C}\to{}^{12}\mathrm{B}_{\mathrm{g.s.}}+\nu_\mu$ and is consistent with the measured partial and polarized muon-capture rates.

We note from Fig. 1 that $F_M(q^2)$ falls rapidly in the region for which $|q^2|\le 3.7m_\pi^2$. If this is a diffractive effect, we would expect similar decreases in $F_A(q^2)$ and $F_E(q^2)$, which reflect the size and shape of the nucleus. Thus we provisionally accept Eq. (10) recognizing that at very large q^2 values, we expect departures from it. For essentially the same reasons, we make use of $F_E(q^2)$ given[8] by

$$\frac{F_E(q^2)}{F_E(0)}\approx\frac{F_M(q^2)}{F_M(0)}\ , \tag{12}$$

where

$$F_E(0)=3.75\ . \tag{13}$$

The form factor $F_P(q^2)$ presents a more complicated situation. Arguments based upon partially conserved axial-vector current (PCAC) lead to a form which may be written as

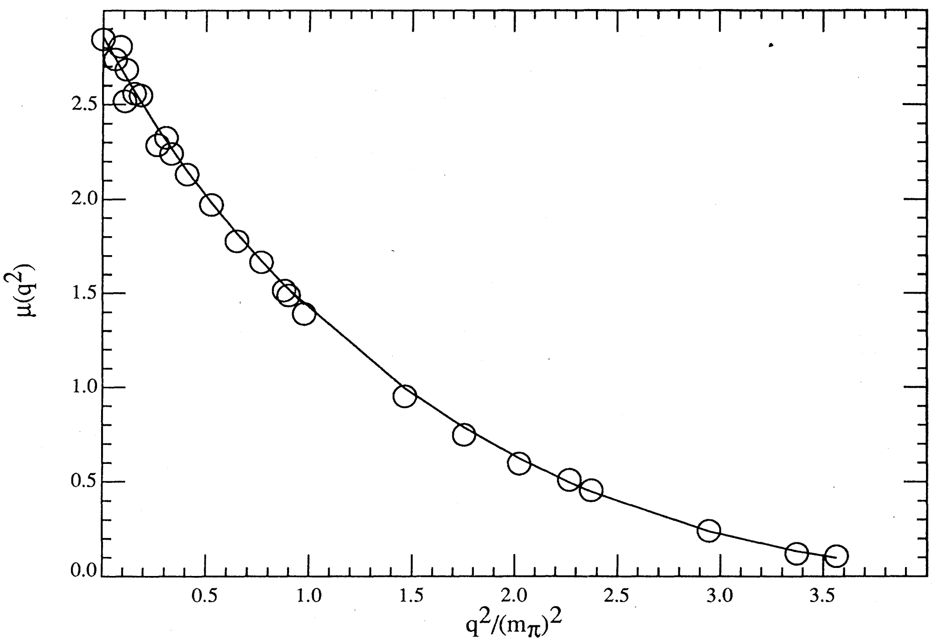

FIG. 1. Plot of electromagnetic form factor as a function of momentum transfer. The circles are the experimental data points. The fit is given by Eq. (7).

$$F_P(q^2)=\frac{m_\pi^2 F_A(q^2)}{m_\pi^2-q^2}[1+\xi(q^2)] \ . \tag{14}$$

There have been estimates for $\xi(q^2)$ for $q^2 \simeq -m_\mu^2$ appropriate for muon capture, yielding $\xi(q^2)$ is the range of $-0.44 \sim -0.29$. A direct measurement of F_P (Ref. 14) yields a ratio of

$$\frac{F_A(q^2)}{F_P(q^2)}=-1.09\pm.31 \ , \tag{15}$$

leading to values of $\xi(q^2)$ from near 0 to almost one, and therefore a range of muon-capture rates from 5700–6109 sec^{-1}, which is in accord with experimental results in the 5700–6300 sec^{-1} range.[15–18] It would be very useful to have a very small error experimental capture rate.

The point that should be stressed here is that we expect the trend of F_A, F_M, and F_E to be generally downward as q^2 increases. The behavior is dipole for $|q^2| \le m_\mu^2$ but departs increasingly from a dipole form as q^2 increases. This is as would be expected from unitarity. For the $F_M(q^2)$ form factor, conserved vector current (CVC) implies that it behaves as $\mu(q^2)$, the electromagnetic form factor. This form factor has a sharp minimum at $q^2 \approx -3.7m_\pi^2$ and consequently the same behavior for $F_M(q^2)$ is predicted. A test of this would be very interesting.

Thus it seems that if there is an anomalously large cross section for the muon neutrino reaction, the most likely cause in the energy range discussed is a large pseudoscalar term. Because of possible anomalous threshold effects, F_P might be enhanced in the $m_\pi^2 \le |q^2| \le 9m_\pi^2$ region. We expect, however, that the trend of the other form factors F_M, F_A, and F_E as mentioned to be downward whether or not second class currents are present. Even larger than expected meson exchange corrections would not alter the general conclusion, particularly with regard to the total cross sections. These terms are important[19] for large q_0^2, whereas the total cross sections are not so strongly dependent on the large q_0^2 region. It is for these reasons that we make use of Eqs. (7), (10), (11), (12), and (13) for F_M, F_A, and F_E on an *ad hoc* basis in the range of $|q^2| \le 3.7m_\pi^2$ of interest here. We shall use $F_P(q^2)$ given by Eq. (14) with a number of different assumptions concerning $\xi(q^2)$.

III. CROSS SECTIONS AND CAPTURE RATES

The matrix element, $|M|^2$ for the process $\nu_\mu + {}^{12}\text{C} \to {}^{12}\text{N}_{\text{g.s.}} + \mu^-$ is essentially identical to that for $\nu_e + {}^{12}\text{C} \to {}^{12}\text{N}_{\text{g.s.}} + e^-$. It has been given previously.[5] The matrix element squared for the muon-capture process, $\mu^- + {}^{12}\text{C} \to {}^{12}\text{B}_{\text{g.s.}} + \nu_\mu$ is also very similar, and need not be discussed separately.

In Fig. 2, we plot the muon-capture rate as a function of $\xi(q^2)=\alpha(-q^2/0.74m_\mu^2)$, where $q^2=-0.74m_\mu^2$ over a large range of possible values of α. This figure effectively gives the variation of Γ with ξ. In Fig. 3, we plot the electron neutrino cross section for values of E_{ν_e} from threshold to 135 MeV. This final value of energy, which can be seen from Fig. 4, is chosen such that $|q^2| \le 3.7m_\pi^2$, the limit of validity of the fit for $F_M(q^2)$.

In Fig. 5 we plot[20] the total cross section for the muon neutrino and antineutrino reactions, again limited by the restriction that $|q^2| \le 3.7m_\pi^2$. The contributions from the

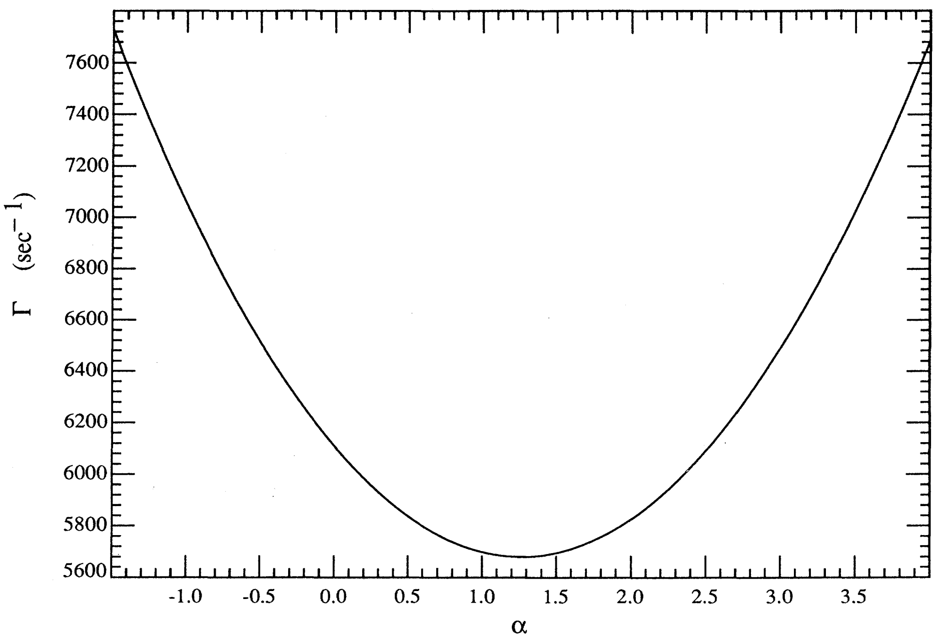

FIG. 2. Plot of muon-capture rate as a function of $\xi(q^2)=\alpha(-q^2/0.74m_\mu^2)$, over a range of α.

individual form factors are shown. These are obtained by setting in turn all form factors but one to zero.

In Fig. 6, we plot the effect of changing the pseudoscalar form factor on the cross section. Because F_P has been measured only at $q^2=-0.74m_\mu^2$, and because of the large errors associated with the measurement, we have made a number of different assumptions concerning the variation of the pseudoscalar form factor with q^2. Because muon-capture measurements provide a limiting range for only the low q^2 variation and because for $m_\pi^2 \leq |q^2| \leq 9m_\pi^2$ for

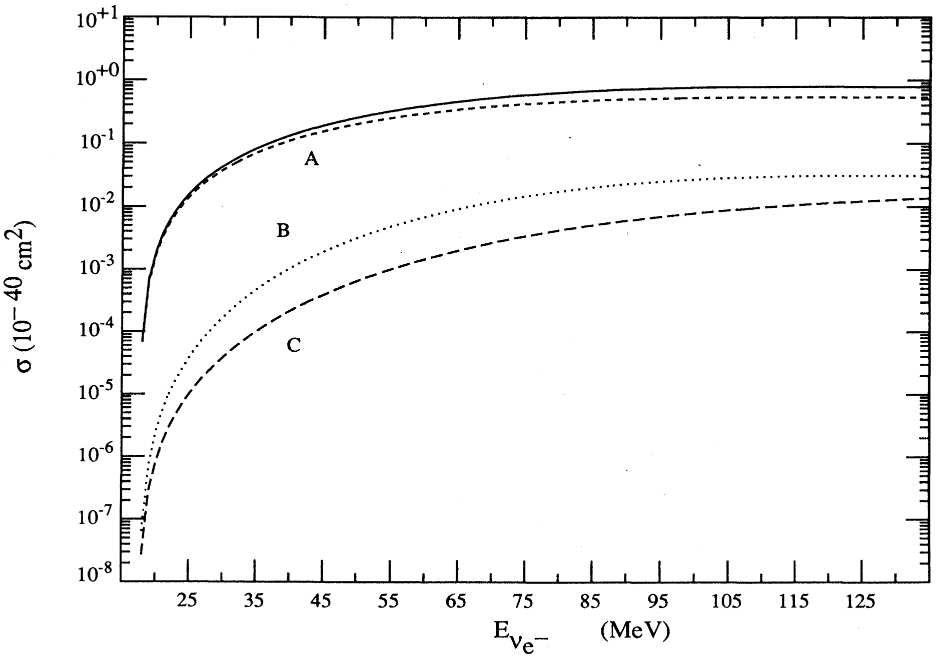

FIG. 3. Plot of the total cross section for the reaction $\nu_e+{}^{12}\mathrm{C}\rightarrow{}^{12}\mathrm{N}_{\mathrm{g.s.}}+e^-$ as a function of incoming neutrino energy. Curves (A), (B), and (C) refer to the contribution from F_A, F_M, and F_E, respectively.

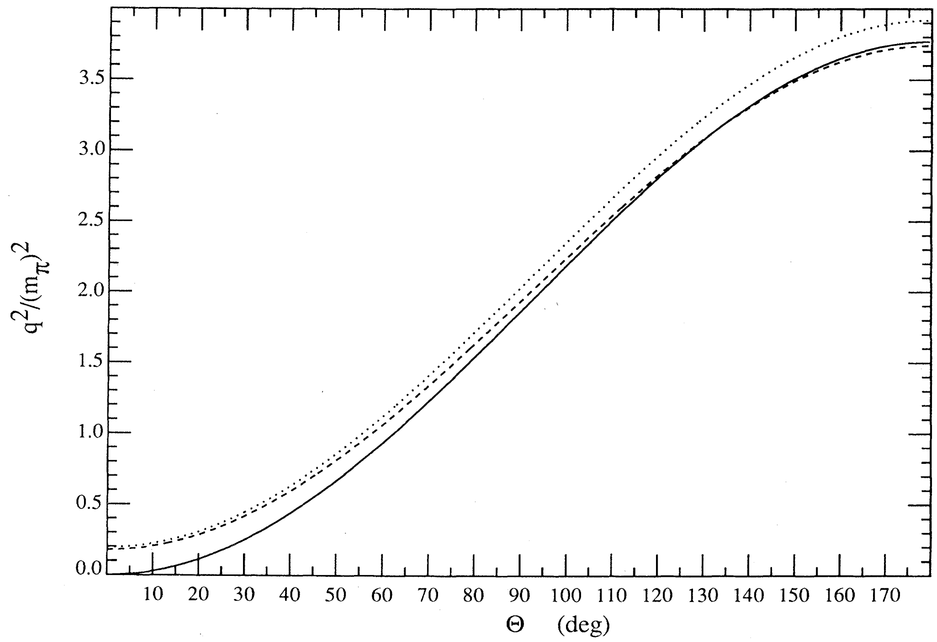

FIG. 4. Plot of momentum transfer as a function of outgoing lepton laboratory angle. Solid, dotted, and dashed curves refer to $E_{\nu_e}=140$ MeV, $E_{\nu_\mu}=165$ MeV, and $E_{\bar{\nu}_\mu}=160$ MeV, respectively.

the muon neutrino reaction, anomalous threshold states may contribute to F_P. In Fig. 6 for $E_{\nu_\mu}=140$ MeV we show σ/σ_0, where σ_0 is the value of cross section for $\xi(q^2)=0$ and σ is the value of cross section for $\xi(q^2)=\alpha(-q^2/0.74m_\mu^2)$ and $\xi(q^2)=\alpha(-q^2/0.74m_\mu^2)^2$ for positive and negative values of α. We also calculate the flux[21] averaged cross section for the muon neutrino experiment for the case of $\alpha=0$, and for α equal to 0.4 and -0.15, respectively, with $\xi(q^2)$ having the same q^2 dependence as mentioned earlier. These results are:

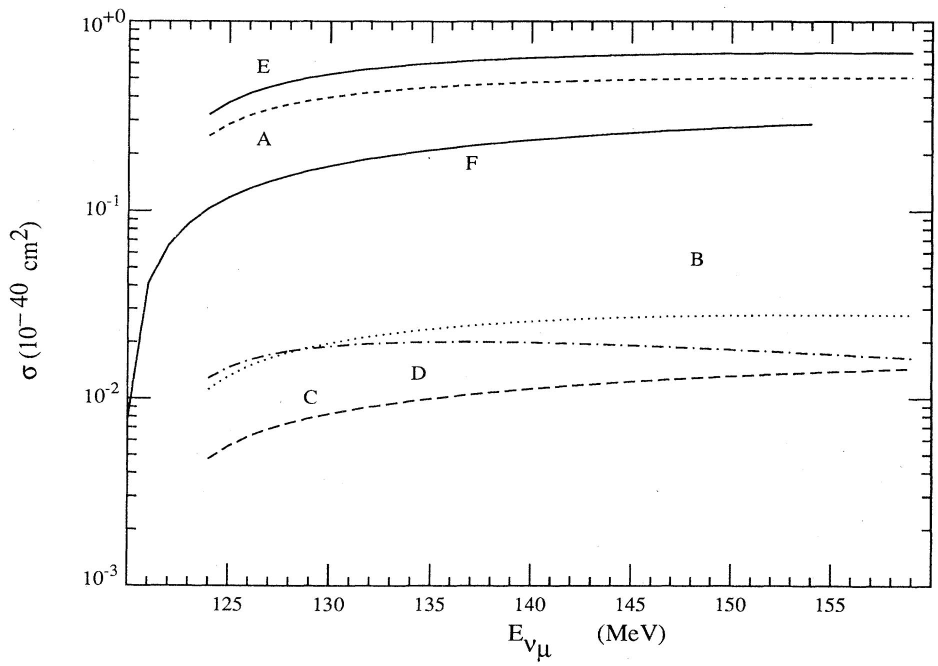

FIG. 5. Plot of the total cross section for ν_μ and $\bar{\nu}_\mu$ reaction on ^{12}C as a function of incoming neutrino energy. Curves (A), (B), (C), and (D) refer to the contribution from F_A, F_M, F_E, and F_P, respectively, for these reactions. Curve F refers to the reaction $\bar{\nu}_\mu+{}^{12}C\rightarrow{}^{12}B_{g.s.}+\mu^+$.

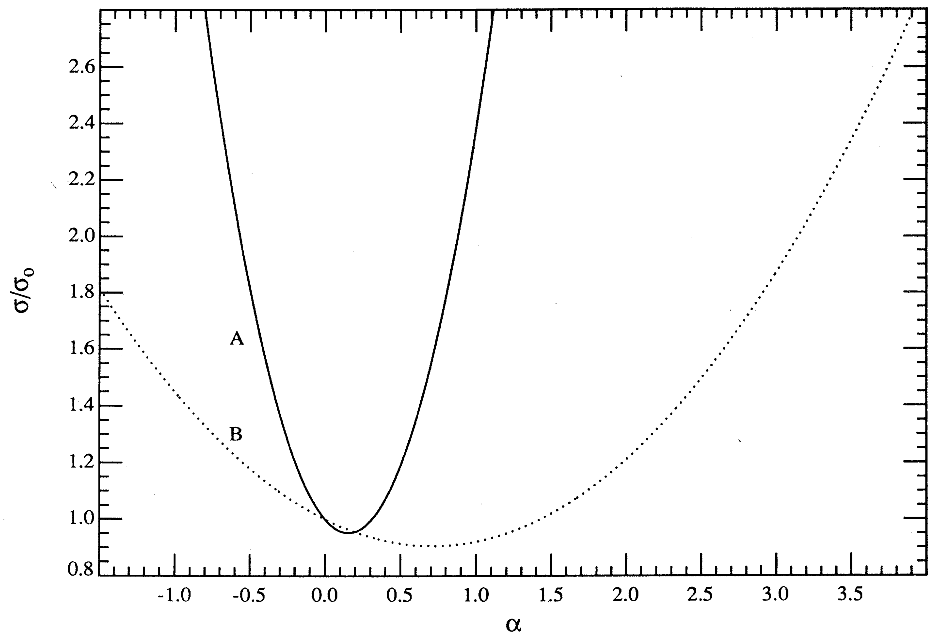

FIG. 6. Plot of the ratio of the cross section σ/σ_0 for the reaction $\nu_\mu + {}^{12}\mathrm{C} \rightarrow {}^{12}\mathrm{N}_{\mathrm{g.s.}} + \mu^-$ at $E_{\nu_\mu} = 140$ MeV. The total cross section, σ is given as a function of $F_P(q^2)$ for various assumptions concerning $\xi(q^2)$. The cross section σ_0 refers to a value of $\xi=0$. Curves (A) and (B) refer to $\xi(q^2)=\alpha(-q^2/0.74m_\mu^2)^2$, and $\xi(q^2)=\alpha(-q^2/0.74m_\mu^2)$, respectively.

$$\bar{\sigma} = \begin{cases} 0.63\times 10^{-40}\ \mathrm{cm}^2, & \text{if } \xi=0\ , \\ 0.75\times 10^{-40}\ \mathrm{cm}^2, & \text{if } \xi=-0.15(-q^2/0.74m_\mu^2)\ , \\ 0.95\times 10^{-40}\ \mathrm{cm}^2, & \text{if } \xi=-0.15(-q^2/0.74m_\mu^2)^2\ , \\ 0.66\times 10^{-40}\ \mathrm{cm}^2, & \text{if } \xi=+0.4(-q^2)/0.74m_\mu^2)\ , \\ 1.55\times 10^{-40}\ \mathrm{cm}^2, & \text{if } \xi=+0.4(-q^2/0.74m_\mu^2)^2\ . \end{cases} \tag{16}$$

IV. DISCUSSION

Two recent neutrino experiments for the ${}^{12}\mathrm{C}\leftrightarrow{}^{12}\mathrm{N}_{\mathrm{g.s}}$ transitions have been performed. One, the electron neutrino experiment, seems very well in accord with predictions, whereas the muon neutrino result is at present more difficult to explain. The differences in the circumstances of the two experiments are greater than they appear. The electron neutrino is a low energy, low q^2 experiment. Because the outgoing lepton is an electron, and because all F_P terms are proportional to the lepton mass squared m_e^2, they therefore make almost no contribution to the cross section. In this reaction the total cross section is sensitive only to F_A and therefore to the scaling assumption, Eq. (10). We may therefore take the good agreement between the calculated and observed results as being a further verification of Eq. (10) (Ref. 22) at low q^2.

It would, although not likely at present, clearly be desirable to have electron neutrino experiments over a much larger range of incident neutrino energies. From Fig. 3, the total cross section is really a measure of F_A and hence, a relatively clear test of Eq. (10). Should Eq. (10) continue to hold over a wider range of q^2, it would imply that neither second class current contributions nor meson exchange currents are very large under these conditions. The result, Eq. (10), was derived by neglecting second class current contributions and most meson exchange corrections.

The situation for the muon neutrino experiment is at present much less clear as there is presently a reevaluation[23] in progress which is near its completion. However, the very fact of it having been performed is encouraging. The energy range under which it was performed with $|q^2|\leq 6m_\pi^2$ is a particularly good one because the pseudoscalar contribution is maximized[24] in this range. The possibility of a larger than expected pseudoscalar contribution exists for $m_\pi^2 \leq |q^2| \leq 9m_\pi^2$ because of possible contributions from anomalous threshold states which involve the virtual breakup of the nucleus. From Fig. 6 and the results given in Eq. (16), it is clear that a large pseudoscalar term coming on in the anomalous threshold region could yield a result of the order observed, but there is no evidence for it except this one experiment. Muon capture takes place at a q^2 which is too small, i.e., $-0.74m_\mu^2$, to expect a large effect from these states and so further experiments in the q^2 range of the present experiments would be very welcome. From Fig. 2 and Eq. (16) it can be seen that a precise experimental result for muon capture would point the theory in the right direction as far as the correction term $\xi(q^2)$ is concerned. This is particularly true because it still remains the highest $|q^2|$ weak process which is available for the ${}^{12}\mathrm{C}\leftrightarrow{}^{12}\mathrm{B}_{\mathrm{g.s.}}$ transition. The current situation is that Γ, the partial capture rate for the reaction $\mu^- + {}^{12}\mathrm{C} \rightarrow {}^{12}\mathrm{B}_{\mathrm{g.s.}} + \nu_\mu$ is not well known. Experiments[15–18] place it from $5.7\times10^3\ \mathrm{sec}^{-1}$ to $6.3\times10^3\ \mathrm{sec}^{-1}$. Because Γ is particularly sensitive to $F_A(q^2)$, an accurate measurement could be combined with polarized muon-capture results to obtain more accurate ratios for $F_P(q^2)/F_A(q^2)$ and $F_A(q^2)/F_M(q^2)$. These would be of great aid in evaluating the corresponding muon neutrino-induced reactions.

We should also comment on what might be gained from antineutrino experiments in this range of q^2. In the cross section, assuming first class currents, only the sign of the $F_M F_A$ term changes as a result of using antineutrinos as the projectile [$(+)$ for neutrinos and $(-)$ for antineutrinos]. Accurate measurements of $\sigma(\nu_l + {}^{12}\mathrm{C} \rightarrow {}^{12}\mathrm{N}_{\mathrm{g.s.}} + l)$ and $\sigma(\bar{\nu}_l + {}^{12}\mathrm{C} \rightarrow {}^{12}\mathrm{B}_{\mathrm{g.s.}} + l)$ would thus enable the term $F_A F_M$ to be determined and from Eq. (5) (i.e., the assumption of strong CVC), F_A would be known.

In conclusion, we see that a substantially enhanced cross section is possible for the reaction $\nu_\mu + {}^{12}\mathrm{C} \rightarrow {}^{12}\mathrm{N}_{\mathrm{g.s.}} + \mu^-$ as a result of an enhanced pseudoscalar form factor and that this can be done consistently with the present values for muon-capture rates and with the results for electron neutrino reactions on ${}^{12}\mathrm{C}$. In particular we have examined in detail the behavior of σ as a function of F_P for $\xi(q^2)$ increasing as q^2 and q^4. However it is clear that any similarly increasing form should have similar results. Anomalous threshold states might provide an explanation for such an increase. Clearly a detailed study of possible contributions from these states would be desirable. Finally additional experimental work on the $\nu_\mu + {}^{12}\mathrm{C} \rightarrow {}^{12}\mathrm{N}_{\mathrm{g.s}} + \mu^-$ reaction in a similar energy range to the present experiment would be very welcome.[25]

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[20]We note that the earlier calculation referred to in Ref. 3 made use of a dipole form factor over the entire q^2 range. The present calculation has a more rapidly falling form factor based on presently available electron scattering results. This has resulted in a substantially lower cross section at higher neutrino energies without substantially affecting the low-energy region. The current discrepancy between the experimental results mentioned and theoretical results refer to the present rather than the earlier calculation. We note that other phenomenologically based calculations have experienced the same adjustment as ours for the same reasons.

[21]T. Dombeck *et al.*, Phys. Lett. B **194**, 591 (1987).

[22]It appears that the group of Ref. 1 has additional data that leads to a cross section for $\nu_e + {}^{12}\mathrm{C} \rightarrow {}^{12}\mathrm{N}_{\mathrm{g.s.}} + e^-$ of the order of $(1.05\pm0.15)\times10^{-41}\ \mathrm{cm}^2$, which is substantially closer to the result presented here (R. L. Burman, private communication). A number for the same reaction is given as

$$[1.03\pm0.12(\mathrm{stat})\pm0.10(\mathrm{sys})]\times10^{-41}\ \mathrm{cm}^2$$

by D. A. Krakauer in his thesis as yet unpublished (D. A. Krakauer, private communication).

[23]The experimental group is reevaluating their neutrino flux. However, they have now reduced their disagreements to the 20% level in the flux which will not affect their conclusions significantly. They intend to publish a cross section in the near future (T. Dombeck, private communication).

[24]See Fig. 5, curve *D*. In general the contribution of F_P to the total cross section begins to fall beyond the 140 MeV incident neutrino range. Thus the range presented here is an ideal one to study this form factor.

[25]Members of the muon neutrino reaction group are considering the possibility of further experiments, possibly at TRIUMF; T. Dombeck, private communication.