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Model calculations using boson mappings

P. Navrátil and J. Dobeš

Institute of Nuclear Physics, Czechoslovak Academy of Sciences, CS 250 68 Řež, Czechoslovakia

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The shell-model fermion problem for systems of four protons and four neutrons in single-j shells is transformed into a boson problem using the generalized Dyson boson mapping. Exact boson calculations are performed. The spectrum of physical and nonphysical boson states and the effect of truncation in the boson space are studied. The necessity of transformation from the Dyson boson description to the seniority boson description is demonstrated within the truncated space.

I. INTRODUCTION

In recent years, there has been continuing activity in study of the methods of transforming the fermion problem into the boson one.[1–5] Especially in nuclear physics, these methods have become widespread in connection with the description of nuclear collective states which exhibit many features of bosonlike behavior. The boson mapping is then believed to link the microscopic fermion shell-model approach and the phenomenological boson models, like the interacting boson model (IBM).[6]

Two steps are involved in the passage from the fermion shell model to the collective boson picture—a bosonization and a truncation. The bosonization is a mapping of the fermion space and operators onto the boson ones. The truncation is a restriction of the whole original space to a subspace of physically relevant degrees of freedom.

Frequently, the bosonization is achieved by the requirement of preserving the commutation relations of the original bifermion SO($2k$) algebra (k is the dimension of the fermion space). As the dimension of the ideal boson space, equal to $k(k-1)/2$, is greater than the dimension of the fermion space, a problem of mixing and identifying nonphysical boson states arises. Fortunately, when the SO($2k$) algebra is preserved in the boson mapping, the mapped boson Hamiltonian does not connect the physical and nonphysical states.[7] This is not true when a truncation step is introduced. The operators in the truncated boson space need not fulfill the algebraic properties of the original fermion operators. As a consequence, a mixing between the physical and nonphysical boson states occurs.

There are, however, cases in which the boson mapping works well even in the truncation space. These occur when the Hamiltonian is writtten in terms of a subalgebra of the SO($2k$) algebra. In the truncated space, one gets a separation of the physical and nonphysical subspaces as far as the commutation relations of the particular subalgebra are obeyed.

A very frequent example is the quasispin (or seniority) SU(2) algebra. That can be reproduced in the space of bosons with $l=0$. On the other hand, the monopole pairing Hamiltonian for single-j shell or several degenerate-j shells is expressed through the elements of the quasispin algebra. The monopole pairing interaction explains an essential part of the interaction between like nucleons in nuclei very well. It can thus serve as a starting point for introducing $l=0$ s bosons. In fact, the above features are exploited in the well-known microscopic scheme of Otsuka, Arima, and Iachello (OAI)[8] linking the shell model and the IBM.

Using the seniority scheme, the OAI approach is developed further to also define $l=2$ d bosons conveniently. With the OAI s and d bosons, one obtains reasonable results in the truncated s,d-boson space, even for Hamiltonians different from the pure monopole pairing interaction.[9] We remark that the OAI mapping scheme has been rederived several times by various techniques, always relying on the properties of quasispin algebra.[10–12]

It is evident that the boson images of the same fermion operator could differ in different boson mapping methods. The truncation of the boson space might then work well within particular mapping schemes, whereas it could be completely misleading in others.

To prospect and illustrate the previously discussed aspects of the boson mapping techniques, we study a simple model example in this paper. Namely, we choose a system of four protons and four neutrons in the single-j shells with the monopole pairing interaction between like nucleons and with the quadrupole-quadrupole interaction between unlike nucleons. We believe that this exactly solvable system simulates some aspects of the realistic situation in nuclei reasonably well. In the bosonization step, the generalized Dyson boson mapping (DBM)[3,5] is used. This mapping has an advantage of being finite whereas its non-Hermicity does not represent any considerable obstacle. The seniority mapping in the truncation boson space is also studied.

The paper is organized as follows: In Sec. II, the model is defined and results of calculations using the generalized DBM are given. Discussion of the seniority boson description is presented in Sec. III. Section IV contains concluding remarks.

II. THE GENERALIZED DYSON BOSON MAPPING

Let us define the bifermion operators in the angular momentum coupled representation for a single-j level

$$A^{\dagger}_{\lambda\mu}=1/\sqrt{2}(a^{\dagger}_j a^{\dagger}_j)^{(\lambda)}_{\mu}, \quad \lambda \text{ even}, \tag{1a}$$

$$A_{\lambda\mu}=(A^{\dagger}_{\lambda\mu})^{\dagger}, \tag{1b}$$

$$U_{\lambda\mu}=(a^{\dagger}_j \tilde{a}_j)^{(\lambda)}_{\mu}, \tag{1c}$$

where $a^{\dagger}_{jm}$ is the fermion creation operator, $\tilde{a}_{jm}=(-)^{j-m}a_{j,-m}$, and the angular momentum coupling is denoted by $(\ )^{(\lambda)}_{\mu}$. The generalized DBM for the operators (10) is given by

$$A^{\dagger}_{\lambda\mu}\rightarrow B^{\dagger}_{\lambda\mu}-2\sum_{\lambda'\lambda''\lambda'''\kappa}\hat{\lambda}'\hat{\lambda}''\hat{\lambda}'''\hat{\kappa}\begin{Bmatrix} j & j & \lambda''' \\ j & j & \lambda \\ \lambda' & \lambda'' & \kappa \end{Bmatrix} \times[(B^{\dagger}_{\lambda'}B^{\dagger}_{\lambda''})^{(\kappa)}\tilde{B}_{\lambda'''}]^{(\lambda)}_{\mu}$$

$$A_{\lambda\mu}\rightarrow B_{\lambda\mu}, \tag{2}$$

$$U_{\lambda\mu}\rightarrow 2\sum \hat{\lambda}'\hat{\lambda}''(-1)^{\lambda}\begin{Bmatrix} \lambda' & \lambda'' & \lambda \\ j & j & j \end{Bmatrix}(B^{\dagger}_{\lambda'}\tilde{B}_{\lambda''})^{(\lambda)}_{\mu}.$$

In Eq. (2), $B^{\dagger}_{\lambda\mu}$ and $B_{\lambda\mu}$ are the boson creation and annihilation operators, respectively, obeying the usual boson commutation relations, $\tilde{B}_{\lambda\mu}=(-1)^{\lambda-\mu}B_{\lambda\mu}$ and $\hat{\lambda}=\sqrt{2\lambda+1}$. In addition to the mapping (2) of the bifermion operators, a correspondence between the fermion and boson vacua $|0\rangle_F\rightarrow|0)_B$ has to be stated to define DBM completely.

In the following, we study a simple model system of four protons and four neutrons, each kind of nucleon is in a single-j shell. We take $j_\pi=j_\nu$. The proton and neutron shells are assumed to differ (for example, by parity) so that the isospin degree of freedom is not considered. The Hamiltonian consists of the monopole pairing interaction between like nucleons and the quadrupole-quadrupole interaction between protons and neutrons

$$H=G_\pi S^{\dagger}_\pi S_\pi+G_\nu S^{\dagger}_\nu S_\nu+F_{\pi\nu}Q_\pi\cdot Q_\nu. \tag{3}$$

In the above,

$$S^{\dagger}_\rho=\sqrt{\Omega_\rho}A^{\dagger}_{\rho,00},$$

$$S_\rho=(S^{\dagger}_\rho)^{\dagger},$$

$$Q_{\rho,\mu}=-1/\sqrt{5}\langle j_\rho\|r^2Y^{(2)}\|j_\rho\rangle U_{\rho,2\mu},$$

$$\Omega_\rho=j_\rho+\tfrac{1}{2},$$

with ρ denoting protons ($\rho=\pi$) or neutrons ($\rho=\nu$). For the pairing strengths we choose $G_\pi=-0.13$ MeV, and $G_\nu=-0.10$ MeV, whereas a relation

$$k_{\pi\nu}=F_{\pi\nu}\langle r^2_\pi\rangle\langle r^2_\nu\rangle 5/(4\pi)=-1.0 \text{ MeV}$$

is assumed for the quadrupole-quadrupole force. These parameters are quite reasonable for heavy nuclei.

Using the mapping (2), we obtain the Dyson boson image H_D of the Hamiltonian (3). Clearly, H_D is non-Hermitian as the mapping (2) is nonunitary. A nonsymmetric matrix thus has to be diagonalized in the ideal boson space. The ideal boson space is overcomplete with both the physical and nonphysical states included. Fortunately, as it has been stated in the Introduction, the physical and nonphysical states are separated by diagonalizing the Dyson Hamiltonian in the full ideal boson basis.[13] The respective character of the eigenstates is determined using the Majoranalike operator $\hat{S}$[3,14] which is the DBM image of the fermion identical zero operator written as

$$\hat{0}_F=\hat{n}^2_F-\hat{n}_F-2\sum_{\rho\kappa}\hat{\kappa}(A^{\dagger}_{\rho,\kappa}\tilde{A}_{\rho,\kappa})^{(0)}_0.$$

Diagonal matrix elements of $\hat{S}$ are zero for the physical eigenstates and are greater than zero for the nonphysical eigenstates. Alternatively, adding the operator $\lambda\hat{S}$ to the Dyson Hamiltonian H_D, all nonphysical states are shifted up (in our case by 12λ or 24λ MeV).

We have considered two cases in the calculations of the model system of four protons and four neutrons: (I) $j_\pi=\frac{7}{2}^+$, $j_\nu=\frac{7}{2}^-$, and (II) $j_\pi=\frac{9}{2}^+$, $j_\nu=\frac{9}{2}^-$. Calculated spectra are shown in Figs. 1 and 2, respectively.

Exact fermion calculations are shown in columns (a) of Figs. 1 and 2. In columns (b), the results obtianed by diagonalizing the Dyson boson Hamiltonian in the full ideal boson space are presented. The energies of 0^+ and 2^+ states in Fig. 1 and of 0^+ states in Fig. 2 are only displayed. Here, we were limited by dimensions of the Hamiltonian matrices in the ideal boson space. These dimensions are shown in the upper part of column (b). The physical subspace forms only the minor fraction of the full boson space. The exact boson calculations are more difficult to perform than the exact fermion calculations.

The remarkable feature of the spectra is the fact that the physical states are spread among the the spurious

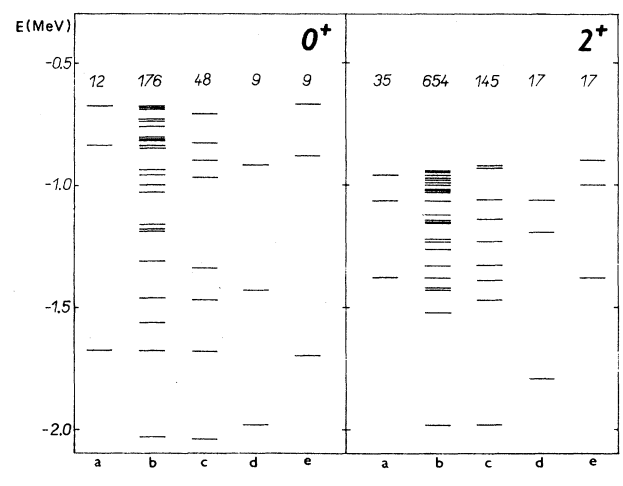

FIG. 1. Energy spectra of the Hamiltonian H_D for the $j_\pi=\frac{7}{2}^+, j_\nu=\frac{7}{2}^-$ case with parameters $k_{\pi\nu}=F_{\pi\nu}\langle r^2_\pi\rangle\langle r^2_\nu\rangle 5/4\pi=-1.0$ MeV, $G_\pi=-0.13$ MeV and $G_\nu=-0.10$ Mev are shown. Results of exact fermion calculations are shown in column (a). Dyson boson calculations in the full space, (s,d,g) space, and (s,d) space are presented in columns (b), (c), and (d), respectively. Results using the seniority boson Hamiltonian are displayed in column (e). The numbers show dimensions of respective spaces. For details see the text.

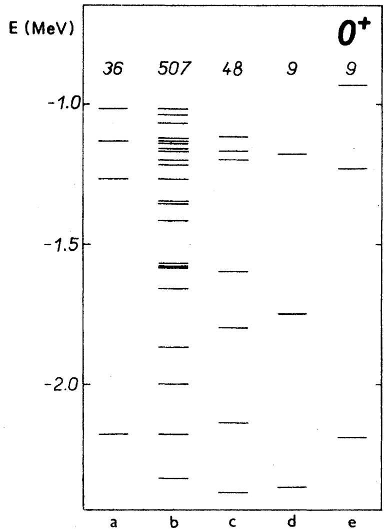

FIG. 2. The same as in Fig. 1 for the $j_\pi=\frac{9}{2}^+, j_\nu=\frac{9}{2}^-$ case and identical Hamiltonian parameters.

states. The lowest states in the spectra are the nonphysical states. This effect was observed in earlier investigations[15,16] and is connected mainly with the particle-hole boson image (1c) used in the mapping of the quadrupole-quadrupole interaction. Switching off $Q\cdot Q$ term, the nonphysical states are pushed up and the lowest states in the boson spectra become physical. Similar conclusions would be reached when the $Q\cdot Q$ interaction is rewritten in the pairing form and mapped using Eqs. (1a) and (1b), thus avoiding the use of Eq. (1c). This was demonstrated in Ref. 15 for the $Q\cdot Q$ interaction between identical nucleons. However, in the case of the proton-neutron quadrupole-quadrupole interaction, that procedure implies a rather unusual introduction of combined proton-neutron bosons with a consequent enlargement of the ideal boson space.

On the other hand, the procedure of rewriting the fermion Hamiltonian in the multipole-multipole form and then mapping by Eq. (1c) results in a Hermitian boson Hamiltonian in the spectrum of which the lowest states are the spurious ones. This statement has been exemplified in Ref. 15 and we have verified it in a number of calculations with simple model systems. We consider this statement to, generally, be very likely true (see also a recent paper by Dukelsky and Pittel).[17]

In recent papers by Kuchta,[18] the Dyson mapping of the fermion Hamiltonian in the multipole-multipole form has been advocated with a subsequent use of the mean-field boson techniques. The ground state of the boson system is searched as a boson condensate by a variational procedure. In view of the above mentioned, the spurious ground state is thus very probably approached. Of course, an unexpected quality of Kuchta's results remains to be understood.

Results of calculations in the boson space truncated to the $l=0,2,4$ (s,d,g) bosons and $l=0,2$ (s,d) bosons are shown in columns (c) and (d), respectively, of Figs. 1 and 2. In the upper part of columns, the dimensions of corresponding spaces are given. Diagonalization of the Dyson Hamiltonian in the truncated space mixes physical and nonphysical states. In Table I, the mean value of the projector onto the physical space is shown for the lowest two states of both (s,d,g) and (s,d) calculations. This mean value, being close to zero for nearly spurious states, can determine an approximate character of eigenstates. The lowest states in truncated calculations reasonably approximate the lowest spurious states of full calculations. In the (s,d,g) case, even the lowest physical state can be related by energy to the exact one, despite its large nonphysical component. In the (s,d) case, however, the mixing of physical and nonphysical states is quite strong, and the identification of physical energies and states is hardly possible. Thus, calculations with the Dyson Hamiltonian in the truncated (s,d) space do not provide any relation to the original fermion system.

III. THE SENIORITY BOSON MAPPING

Switching off in the Hamiltonian H [Eq. (3)], the quadrupole-quadrupole interaction, we are left with the monopole pairing interaction (MPI) between like nucleons. For the MPI, a boson mapping is possible based on the seniority SU(2) algebra, a subalgebra of the full bifermion SO(2k) algebra. The bifermion SU(2) algebra has the Dyson boson realization

$$\begin{aligned} &S^\dagger \rightarrow s^\dagger(\Omega - v - n_s)\ , \\ &S \rightarrow s\ , \\ &\tfrac{1}{2}(\hat{n}_F - \Omega) \rightarrow \tfrac{1}{2}(v + 2n_s - \Omega)\ . \end{aligned} \tag{4}$$

If an identification of the seniority quantum number v is assumed $v=2\sum' B_J^\dagger \cdot \tilde{B}_J$, where the prime means summation over $J\neq 0$, the boson Hamiltonians $H_{\text{sen}}^{\text{pair}}$ and H_D^{pair}, coming from the mapping of the MPI by the seniority mapping (4) and the DBM (2), respectively, have identical spectra. The nonphysical states degenerate with the physical states are present in the spectrum for $v\geq 4$. Even in the truncated boson space, energies of both Ham-

TABLE I. The mean value of projector operator onto the physical space in truncated Dyson boson calculations.

$j=\frac{7}{2},\frac{7}{2}$	0_1^+	0_2^+	2_1^+	2_2^+	$j=\frac{9}{2},\frac{9}{2}$	0_1^+	0_2^+
(s,d,g)	0.0006	0.30	0.0004	0.09	(s,d,g)	0.01	0.17
(s,d)	0.025	0.10	0.018	0.09	(s,d)	0.04	0.07

iltonian $H_{\rm sen}^{\rm pair}$ and $H_D^{\rm pair}$ correspond exactly to a subset of the full space energies. For example, the exact ground-states (g.s.) energy is reproduced with the s bosons only. The g.s. ket wave function of the $H_D^{\rm pair}$ includes, however, bosons of all multipolarities and a truncation to the s-boson space only gives a small portion of $|{\rm g.s.})_D$. On the other hand, the g.s. wave function of $H_{\rm sen}^{\rm pair}$ is written as $|{\rm g.s.})_{\rm sen}=|s^N)$. This suggests that truncation of boson basis could be more acceptable in the seniority boson mapping then in the DBM.

Using an idea developed in Ref. 11, one can now pursue the seniority boson mapping further. Restricting to the (s,d) space, we rewrite the images (4) of the pair operators as

$$S^\dagger \to s^\dagger(\Omega - N - n_d)\ ,\qquad S\to s\ , \tag{5}$$

where $N=n_s+n_d$ is the total boson number operator. Fixing, by (5), the mapping for the operators $S^\dagger$ and S, we want to find boson images of the quadrupole pair operators $D^\dagger$ and D, and of the quadrupole operator Q so as to reproduce the commutation relations of $S^\dagger$ and S with the operators $D^\dagger$, D, and Q. This task is fully solved by the mapping

$$\begin{aligned}
D_\mu^\dagger &= \sqrt{\Omega}A_{2\mu}^\dagger \to d_\mu^\dagger \frac{(\Omega-N-n_d-1)(\Omega-N-n_d)}{\Omega-2n_d-1} - s^\dagger s^\dagger \tilde d_\mu + s^\dagger (d^\dagger \tilde d)_\mu^{(2)} \frac{\Omega-N-n_d}{\Omega-2n_d} 10\sqrt{2\Omega}\begin{Bmatrix}2&2&2\\ j&j&j\end{Bmatrix}\ ,\\
\tilde D_\mu &\to \tilde d_\mu - \frac{1}{\Omega-2n_d+1} s s d_\mu^\dagger + s(d^\dagger\tilde d)_\mu^{(2)} \frac{1}{\Omega-2n_d} 10\sqrt{2\Omega}\begin{Bmatrix}2&2&2\\ j&j&j\end{Bmatrix}\ ,\\
Q_\mu &\to Q_{{\rm sen},\mu} = 1/\sqrt{5}\langle j\|r^2Y^{(2)}\|j\rangle \left[\sqrt{(2/\Omega)}\left[s^\dagger \tilde d_\mu + \frac{\Omega+1-N-n_d}{\Omega+1-2n_d} d_\mu^\dagger s\right] - 10\begin{Bmatrix}2&2&2\\ j&j&j\end{Bmatrix}\frac{\Omega-2N}{\Omega-2n_d}(d^\dagger\tilde d)_\mu^{(2)}\right]\ .
\end{aligned} \tag{6}$$

In Ref. 11, the seniority boson mapping based on the Holstein-Primakoff realization of the SU(2) algebra has been developed. In the derivation of (6), we proceed in complete analogy with Ref. 11, starting from the Dyson realization of SU(2).

We should point out that the mapping (6) only represents the leading terms of the full boson expansion. The commutation relations $[D^\dagger,\tilde D]$, $[Q,D^\dagger]$, and $[Q,\tilde D]$ are not reproduced exactly with (6). This mapping is the simplest possible one in which the $J=2,v=2$ state is represented by the one-d-boson state $|ds^{N-1})$.

An expression for the quadrupole operator identical to (6) has been derived by Geyer[12] using a slightly different procedure. Since the boson Hamiltonian $H_D^{\rm pair}$ and $H_{\rm sen}^{\rm pair}$ have identical spectra, they have to be connected by a similarity transformation

$$H_{\rm sen}^{\rm pair} = Z H_D^{\rm pair} Z^{-1}\ .$$

This similarity transformation is then used in Ref. 12 to relate the Dyson boson operators and the seniority boson operators. In practice, the leading terms only is the transformation matrix Z can be constructed and used.

Due to the nonunitary character of the DBM, the Hamiltonian $H_D^{\rm pair}$ is non-Hermitian. Therefore, the eigenbra vector $(\psi_i|$ is not the Hermitian conjugate of the eigenket vector $|\psi_i)$ and the normalization condition $(\psi_i|\psi_k)=\delta_{ik}$ does not determine a separate normalization of $(\psi_i|$ and $|\psi_i)$. This implies arbitrariness in the Dyson boson basis and consequently in the $Q_{\rm sen}$. This arbitrariness is employed in the well-known Hermitization procedure[18] to get the operator reproducing the Hermitian conjugation properties of the original fermion one in the ideal boson space. An application of this to $Q_{\rm sen}$ gives, for $n_d=1$, the well-known OAI result[8]

$$Q_{{\rm OAI},\mu} = 1/\sqrt{5}\langle j\|r^2Y^{(2)}\|j\rangle \left[\sqrt{(2/\Omega)}\sqrt{(\Omega-N)/(\Omega-1)}(s^\dagger\tilde d_\mu + d_\mu^\dagger s) - 10\begin{Bmatrix}2&2&2\\ j&j&j\end{Bmatrix}\frac{\Omega-2N}{\Omega-2}(d^\dagger\tilde d)_\mu^{(2)}\right]\ . \tag{7}$$

In Ref. 8, the expression (7) has been obtained with the Marumori bosonization technique by requiring an equality of fermion and boson matrix elements for the lowest seniority states. Since the boson images of the $J=0,v=0$ and $J=2,v=2$ states are identical in the OAI method and in the seniority mapping (6), we must recover the OAI result.

Using Eqs. (5) and (7), the seniority boson image of the Hamiltonian (1) is obtained in the (s,d) space. The resulting energies are compared with those obtained in the previous section in column (e) of Figs. 1 and 2. The energies of physical ground states and 2_1^+ state are nicely reproduced in this calculation. There are no spurious states below them. The quadrupole transition matrix element between the ground state and 2_1^+ state is reproduced as well (see Table II).

To show some features of the seniority boson mapping, we have studied the dependence of energies and quadrupole matrix elements of the $j_\pi, j_\nu=\frac{7}{2}$ system on the quadrupole-quadrupole interaction strength $k_{\pi\nu}$. The

TABLE II. Quadrupole matrix elements between the ground state and the first and the third 2^+ states, respectively. The quadrupole matrix elements g.s.$\rightarrow 2_2^+$ are equal to zero in both calculations. Note that for $k_{\pi\nu}=0$, the state denoted as the 2_3^+ state is below the state denoted as the 2_2^+ state.

	0.0		−0.5		−1.0		−1.5		−2.0	
$k_{\pi\nu}$	a	b	a	b	a	b	a	b	a	b
Q_π(g.s.$\rightarrow 2_1^+$)	0.0	0.0	0.71	0.73	0.85	0.89	0.90	0.95	0.92	0.96
Q_ν(g.s.$\rightarrow 2_1^+$)	1.13	1.13	1.01	1.03	0.99	1.03	0.98	1.03	0.97	1.02
Q_π(g.s.$\rightarrow 2_3^+$)	1.13	1.13	−0.85	−0.87	−0.66	−0.70	−0.54	−0.61	−0.47	−0.55
Q_ν(g.s.$\rightarrow 2_3^+$)	0.0	0.0	0.47	0.50	0.45	0.49	0.40	0.47	0.37	0.45

[a]Exact.
[b]Seniority boson.

$k_{\pi\nu}$ has been varied from 0 (the pure monopole pairing interaction) to the value of -2.0 MeV. The latter case represents a rather strong quadrupole-quadrupole interaction with a considerable deviation from the seniority scheme.

The exact fermion energies are compared with the (s,d) seniority boson ones in Fig. 3. The ground state and the lowest three 2^+ states are shown. The quadrupole transition matrix elements between the ground state and the 2^+ states are given in Table II. The characteristics of the ground state and 2_1^+ state are very well described in the boson picture. Even the energies of the 2_2^+ and 2_3^+ states are reasonably reproduced. The identification of these two states in the boson calculation with the exact ones is clearly deduced from the quadrupole matrix elements.

For $k_{\pi\nu}=0$, the states denoted as 2_1^+ and 2_3^+ in Fig. 3, are simply given as $\sim|S_\pi^2 D_\nu S_\nu\rangle$ and $|S_\pi D_\pi S_\nu^2\rangle$, respectively. Their energy splitting comes only from the slightly different proton and neutron pairing strengths. With the increasing quadrupole-quadrupole pairing strength, these two states become mixed and the 2_1^+ state symmetric and 2_3^+ state antisymmetric in the proton-neutron variables develop. The splitting between 2_1^+ and 2_3^+ states increases as well.

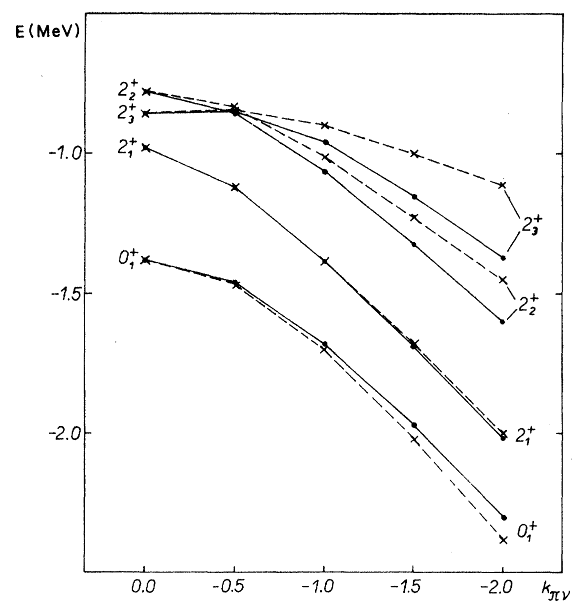

FIG. 3. The dependence of energies on the $k_{\pi\nu}$ strength for the $j_\pi, j_\nu=\frac{7}{2}$ system. The full and dashed lines connect the exact and seniority boson values, respectively.

We stress that there is no spurious state of a given spin in the (s,d) space seniority boson calculations below the states shown in Fig. 3. The second 0^+ state is, however, spurious for $k_{\pi\nu}=0$. The mixing of spurious and physical 0^+ states is, therefore, strong for the higher 0^+ states in calculations with $k_{\pi\nu}\neq 0$. To describe properties of 4^+ states correctly, the inclusion of the $l=4$ g boson is, of course, required.

IV. CONCLUSION

In the simple exactly solvable fermion model, we have investigated some aspects of the boson mappings. In the Dyson boson mapping, two features appear, which make this mapping rather unpracticable: (a) The lowest states of the Dyson boson Hamiltonian are the spurious ones. This prevents use of the mean-field techniques to obtain information on the physical states. (b) When the Dyson Hamiltonian is used in the truncated boson space, unreasonable results are obtained. A substantial mixing between unphysical and physical states occurs and no identification of the resulting eigenstates with the physical states is possible.

On the other hand, the seniority boson mapping, which agrees with the OAI bosonization procedure, seems to be more useful. It is based on the SU(2) seniority scheme but it works rather well even when moving from this limit. The calculations in the truncated (s,d) boson space reproduce properties of the 0^+ g.s. and the lowest 2^+ states.

Behavior of spurious states is of considerable importance in connection with the practical applicability of boson mappings. In the case of the pure monopole pairing interaction, both in the Dyson and seniority boson mappings, the lowest states are physical and correspond to the $v=0,2$ fermion states. If the quadrupole-quadrupole interaction is switched on, the spurious states in the Dyson mapping move down and become the lowest states in the boson space. In the seniority boson calculations in the truncated (s,d) boson space, the spurious states are not present in the low part of the spectra and the lowest states correspond to the physical ones. The latter propertiy, if it is of a general validity, implies the seniority boson mapping to be a useful tool in linking the fermion shell model and phenomenological boson models.

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