Magnetic form factors of the trinucleons

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The magnetic form factors of ^{3}H and ^{3}He are calculated with the Monte Carlo method from variational ground-state wave functions obtained for the Argonne and Urbana two- and three-nucleon interactions. The electromagnetic current operator contains one- and two-body terms that are constructed so as to satisfy the continuity equation with the two-nucleon potential in the Hamiltonian. The results obtained with the Argonne two-nucleon interaction are in overall agreement with the empirical values. It appears that the remaining theoretical uncertainty, in the calculation of these form factors from a given interaction model, is dominated by that in the electromagnetic form factors of the nucleon. It is found that the isovector magnetic form factors are rather sensitive to the details of the isospin-dependent tensor force, and they are much better reproduced with the Argonne than the Urbana potential. The isoscalar magnetic form factors appear to be sensitive to the spin-orbit interactions, and are better reproduced with the Urbana potential. The Argonne potential has a stronger $\tau_1 \cdot \tau_2$ tensor force, while the Urbana one has a shorter-range spin-orbit interaction.

I. INTRODUCTION

The theoretical investigation of the structure of the bound hydrogen and helium isotopes differs from the study of larger nuclei in that the wave functions of these nuclei can be treated exactly in the numerical sense.[1-4] The uncertainty limits in the calculations are thus set solely by the available computer power. This fact makes the electromagnetic form factors of these few-body systems the observables of choice for testing the quality of models for the nucleon-nucleon interaction. The situation is, however, complicated by the fact that the electromagnetic current operator contains irreducible two-body exchange current components, which through the continuity equation also depend on the potential model. Consistent gauge invariant calculations of the electromagnetic form factors thus, in general, require that the same potential model be used to generate both the wave functions and the exchange current operators.

The importance of the exchange current corrections is particularly large in the case of the magnetic form factors of the three-body nuclei ^{3}H and ^{3}He.[5-8] Because of a destructive interference in the matrix elements for transitions between the *S*- and *D*-state components of the wave functions, the impulse approximation predictions[9] for these form factors have distinct minima around 2.5 fm^{-1} in disagreement with the experimental data.[10-12] The situation is closely related to that of the backward cross section for electrodisintegration of the deuteron,[13] which is also dominated by exchange current contributions for values of momentum transfer above 2.5 fm^{-1}.

In previous studies of the exchange current contributions to the magnetic form factors of the bound trinucleons, the model for the exchange current operator has been based on simple meson exchange mechanisms.[5-8] Such models for the exchange current operator are in a qualitative sense consistent with the models for the realistic nucleon-nucleon potentials that are used to construct the nuclear wave functions, since those too are typically based on boson exchange mechanisms. The usual *ad hoc* treatment of the short-range part of the exchange current operators does, however, imply that the continuity equation is satisfied only approximately (at best). In this investigation we shall try to improve on this situation by using the methods proposed in Refs. 14–16 to construct the exchange current operators from the potential model used to calculate the wave function.

We use the variational ground-state wave functions[17] obtained with the Argonne[18] and Urbana[19] two-body potentials and the Urbana model VII three-nucleon interaction. Most of the form factors in the region of momentum transfer around 3 fm^{-1} are due to exchange current contributions. The dominant term of the current in this region is the isovector exchange current operator associated with the isospin-dependent component of the nucleon-nucleon tensor interaction.[15] The small exchange currents associated with the velocity-dependent components of the nucleon-nucleon interaction are hardly visible, unless one considers the isoscalar combination of the magnetic form factors of the trinucleons.[20]

In addition to the above exchange currents required by the continuity equation, we also consider contributions of purely transverse $N\Delta\gamma$, $\rho\pi\gamma$, and $\omega\pi\gamma$ exchange currents. These transverse currents are not constrained by the nuclear potentials, but their contribution is small except at the highest values of momentum transfer considered. The three-body exchange currents associated with the small three-nucleon interaction are neglected.

The matrix elements of the current operator are evaluated with the Monte Carlo method.[1,21] This method is particularly convenient because the calculation of the matrix elements of two- (or more) body operators is in practice no more complicated than that of a one-body operator, and can be done exactly. The method can be used with any wave function, such as the Faddeev ground state;[22,23] here we have used the variational Monte Carlo wave functions[17] because they are convenient and presumably accurate enough for the present work. Nevertheless, we hope that this work will be repeated with the more accurate Faddeev wave functions.

The magnetic form factors are calculated with several different parametrizations of the electromagnetic form factors of the nucleon. It should be emphasized here that the modern semiempirical parametrizations of the electric and magnetic form factors of the nucleon[24–26] are not in good agreement with each other even at relatively low values of momentum transfer. The theoretical uncertainty in the calculation of the magnetic form factors, starting from a given interaction model, is dominated by that in these parametrizations. Within the uncertainty limits, the predicted values of the magnetic form factors of the trinucleons are in good agreement with the experimental data when the Argonne[18] interaction is used, but not with the Urbana[19] model. The main difference in these models is between their isospin-dependent tensor forces.

The previous study of the magnetic form factors of the trinucleons, which is closest in spirit to the present work, is that of Ref. 8. In that paper the form factors were calculated using interactions based on the Paris potential,[27] and components containing up to one Δ particle in the ground-state wave function were treated explicitly. The exchange current operators used in Ref. 8 were constructed from meson exchange mechanisms whereas we use currents that are, as far as possible, derived from the interaction model. Our results differ from those of Ref. 8, in which reasonable agreement with data was obtained only when the Dirac $F(q)$ form factors were used in the main exchange current operator, whereas we obtain good agreement with the Sachs $G(q)$ form factors. The continuity equation is exactly satisfied when the longitudinal currents are calculated from the interactions, and the Sachs form factors are used for both one- and two-body currents.

This paper falls into four sections. In Sec. II we describe the form of the electromagnetic current operator. A short description of the Monte Carlo method used to evaluate the matrix elements of the current operators is given in Sec. III. In Sec. IV we present the numerical results for the magnetic form factors using different parametrizations of the electromagnetic form factors of the nucleon, and a concluding discussion. A number of useful formulas are given in the Appendix.

II. ELECTROMAGNETIC CURRENT OPERATOR

A. General structure of the current operator

The electromagnetic current operator is written as a sum of single-nucleon and two-body exchange current operators. The impulse approximation for the magnetic form factors includes only the contribution of the single-nucleon current of the usual form

$$\mathbf{j}(\mathbf{q})=\frac{1}{4m_N}[G_E^S(q)+G_E^V(q)\tau_z](\mathbf{p}'+\mathbf{p}) +\frac{i}{4m_N}[G_M^S(q)+G_M^V(q)\tau_z]\boldsymbol{\sigma}\times\mathbf{q}\ . \tag{2.1}$$

Here $\mathbf{q}$ is the momentum transfer to the nucleon and $\mathbf{p}$ and $\mathbf{p}'$ the initial and final nucleon momenta. The nucleon mass is denoted by m_N and the electric and magnetic form factors are normalized so as $G_E^S(0)=G_E^V(0)=1$, $G_M^S(0)=0.88$ and $G_M^V=4.706$.

The exchange current operator is separated into a sum of a model-independent and a model-dependent part. The model-independent component is constructed from the nucleon-nucleon interaction by the methods developed in Refs. 14–16. The model-dependent part of the exchange current operator, which is purely transverse and therefore not constrained by the form of the nucleon-nucleon interaction via the continuity equation, is constructed from the commonly considered meson exchange current mechanisms that have the longest range.

B. Model-independent exchange current operator

The continuity equation, which links the two-body exchange current operator to the nucleon-nucleon interaction, has the form

$$\nabla\cdot\mathbf{j}_{\rm ex}(\mathbf{x};\mathbf{r}_1,\mathbf{r}_2)+i[v(\mathbf{r}_1,\mathbf{r}_2),\rho(\mathbf{x})]=0\ , \tag{2.2}$$

where $\rho(\mathbf{x})$ is the charge density. The Urbana and Argonne potential models have the explicit form

$$\begin{aligned}v(\mathbf{r}_1-\mathbf{r}_2)={}&v^c(r_{12})+v^\sigma(r_{12})\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+v^t(r_{12})S_{12}+[v^\tau(r_{12})+v^{\sigma\tau}(r_{12})\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+v^{t\tau}(r_{12})S_{12}]\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\\&+[v^{\rm SO}(r_{12})+v^{\rm SO\tau}(r_{12})\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2]\mathbf{L}\cdot\mathbf{S}+[v^{\rm SO2}(r_{12})+v^{\rm SO2\tau}(r_{12})\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2]\tfrac{1}{2}(\boldsymbol{\sigma}_1\cdot\mathbf{L}\boldsymbol{\sigma}_2\cdot\mathbf{L}+\boldsymbol{\sigma}_2\cdot\mathbf{L}\boldsymbol{\sigma}_1\cdot\mathbf{L})\\&+\{v^{\rm LL}(r_{12})+v^{\rm LL\sigma}(r_{12})\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+[v^{\rm LL\tau}(r_{12})+v^{\rm LL\sigma\tau}(r_{12})\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2]\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\}L^2\ .\end{aligned} \tag{2.3}$$

In Refs. 18 and 19 these interactions are given using the velocity-dependent operators $\mathbf{L}\cdot\mathbf{S}$, $(\mathbf{L}\cdot\mathbf{S})^2$, and L^2; they can be easily cast in the above form by noting that

$$(\mathbf{L}\cdot\mathbf{S})^2=\tfrac{1}{2}L^2-\tfrac{1}{2}\mathbf{L}\cdot\mathbf{S}+\tfrac{1}{4}(\boldsymbol{\sigma}_1\cdot\mathbf{L}\boldsymbol{\sigma}_2\cdot\mathbf{L}+\boldsymbol{\sigma}_2\cdot\mathbf{L}\boldsymbol{\sigma}_1\cdot\mathbf{L}) . \tag{2.4}$$

Only the first three terms v^c, v^σ, and v^t commute with $\rho(\mathbf{x})$; all the rest generate two-body currents.

The most important term in $\mathbf{j}_{\text{ex}}$ is the isovector exchange current associated with the isospin-dependent static components $v^{t\tau}$, $v^{\sigma\tau}$, and v^τ of the nucleon-nucleon interaction. These components can actually be fairly well described as being due to one-pion and one-rho-meson exchange. The expressions for the one-pion and the one-rho-meson exchange current operators are well known:[6,15]

$$\mathbf{j}_\pi(\mathbf{k}_1,\mathbf{k}_2)=-3i(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_z\left[v_\pi(k_2)\boldsymbol{\sigma}_1(\boldsymbol{\sigma}_2\cdot\mathbf{k}_2)-v_\pi(k_1)\boldsymbol{\sigma}_2(\boldsymbol{\sigma}_1\cdot\mathbf{k}_1)\right.$$
$$\left.-\frac{\mathbf{k}_1-\mathbf{k}_2}{k_1^2-k_2^2}(\boldsymbol{\sigma}_1\cdot\mathbf{k}_1)(\boldsymbol{\sigma}_2\cdot\mathbf{k}_2)[v_\pi(k_2)-v_\pi(k_1)]\right] , \tag{2.5}$$

$$\mathbf{j}_\rho(\mathbf{k}_1,\mathbf{k}_2)=-3i(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_z\left[v_\rho(k_2)\boldsymbol{\sigma}_1\times(\boldsymbol{\sigma}_2\times\mathbf{k}_2)-v_\rho(k_1)\boldsymbol{\sigma}_2\times(\boldsymbol{\sigma}_1\times\mathbf{k}_1)\right.$$
$$+\frac{v_\rho(k_2)-v_\rho(k_1)}{k_1^2-k_2^2}[(\mathbf{k}_1-\mathbf{k}_2)(\boldsymbol{\sigma}_1\times\mathbf{k}_1)\cdot(\boldsymbol{\sigma}_2\times\mathbf{k}_2)$$
$$+(\boldsymbol{\sigma}_1\times\mathbf{k}_1)\boldsymbol{\sigma}_2\cdot(\mathbf{k}_1\times\mathbf{k}_2)+(\boldsymbol{\sigma}_2\times\mathbf{k}_2)\boldsymbol{\sigma}_1\cdot(\mathbf{k}_1\times\mathbf{k}_2)]$$
$$\left.-\frac{1}{3}\frac{\mathbf{k}_1-\mathbf{k}_2}{k_1^2-k_2^2}[v_\rho^{\text{S}}(k_2)-v_\rho^{\text{S}}(k_1)]\right] . \tag{2.6}$$

Here $\mathbf{k}_1$ and $\mathbf{k}_2$ are the fractional momenta delivered to nucleons 1 and 2. The functions $v_\pi(k)$, $v_\rho(k)$, and $v_\rho^{\text{S}}(k)$ are, respectively, the tensor components of the pion and rho-meson and central component of the rho-meson exchange interaction

$$v_\pi(k)=\frac{1}{3}\left[\frac{f_\pi}{m_\pi}\right]^2\frac{1}{m_\pi^2+k^2} , \tag{2.7}$$

$$v_\rho(k)=-\frac{1}{3}\left[\frac{g_\rho}{2m_N}\right]^2\frac{(1+\kappa)^2}{m_\rho^2+k^2} , \tag{2.8}$$

$$v_\rho^S(k)=g_\rho\frac{1}{m_\rho^2+k^2} . \tag{2.9}$$

Here m_π and m_ρ are the meson masses, f_π is the pseudovector πNN, and g_ρ and κ are the vector and tensor ρNN coupling constants. The currents (2.5) and (2.6) satisfy the continuity equation when the v^τ, $v^{\sigma\tau}$, and $v^{t\tau}$ interactions are given exactly by π and ρ meson exchange, which is generally not the case.

The method of obtaining current operators $\mathbf{j}_{\text{PS}}$ and $\mathbf{j}_{\text{V}}$, which satisfy the continuity equation for any given v^τ, $v^{\sigma\tau}$, and $v^{t\tau}$ potentials, is outlined in Ref. 15. In this method these potentials are attributed to exchanges of families of pion-like pseudoscalar (PS) mesons and ρ-like vector (V) mesons. The sum of all $T=1$ PS- and V-exchange terms is then obtained as

$$V_{\text{PS}}(k)=\tfrac{1}{3}[2v^{t\tau}(k)-v^{\sigma\tau}(k)] , \tag{2.10}$$

$$V_{\text{V}}(k)=\tfrac{1}{3}[v^{t\tau}(k)+v^{\sigma\tau}(k)] , \tag{2.11}$$

$$V_{\text{V}}^{\text{S}}(k)=v^\tau(k) , \tag{2.12}$$

where

$$v^{\sigma\tau}(k)=\frac{4\pi}{k^2}\int_0^\infty dr\, r^2[j_0(kr)-1]v^{\sigma\tau}(r) , \tag{2.13}$$

$$v^{t\tau}(k)=\frac{4\pi}{k^2}\int_0^\infty dr\, r^2 j_2(kr)v^{t\tau}(r) , \tag{2.14}$$

$$v^\tau(k)=4\pi\int_0^\infty dr\, r^2 j_0(kr)v^\tau(r) . \tag{2.15}$$

The current operators $\mathbf{j}_{\text{PS}}$ and $\mathbf{j}_{\text{V}}$ obtained by using $V_{\text{PS}}(k)$, $V_{\text{V}}(k)$ and $V_{\text{V}}^{\text{S}}(k)$ in place of $v_\pi(k)$, $v_\rho(k)$, and $v_\rho^{\text{S}}(k)$ in Eqs. (2.5) and (2.6) satisfy the continuity equation with the empirical potentials $v^{\sigma\tau}$, $v^{t\tau}$, and v^τ in the model interaction used to fit the nucleon-nucleon scattering data and calculate ground-state wave functions. In this way consistency between the interaction model and the exchange current operator has been achieved. It is worth noting that this method of constructing the isovector exchange current operator has recently been given a more formal justification that does not rely on the explicit boson exchange analogy.[28]

The utility of this approach was proven for the case of the parametrized Paris model of the N-N interaction.[14,15,29] Here we use the Argonne and Urbana models[18,19] to construct these current operators. It is, in fact, interesting to analyze the generalized pseudoscalar and vector exchange components of these interaction models. The contributions of the PS and V exchanges to the $S_{ij}\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j$ interaction between nucleons are compared in Figs. 1 and 2 with the single point pion and rho-meson $S_{ij}\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j$ potentials, respectively. It is evident from these figures that the PS and V exchange tensor interactions in these models are quite similar to those due to π and ρ exchanges, respectively ($\cdots$). ($f_\pi^2/4\pi=0.081$, $g_\rho^2/4\pi=0.55$, and $\kappa=6.6$).

In the last term of the generalized ρ-meson exchange current operator (2.6) we use the Fourier transform of the isospin-dependent central potential $v^\tau(r)$ in place of the $v_\rho^S(k)$. The $v^\tau(r)$ in the Urbana and Argonne models does not closely resemble the ρ-meson exchange interaction (Fig. 3).

The exchange current operators (2.5) and (2.6) have to

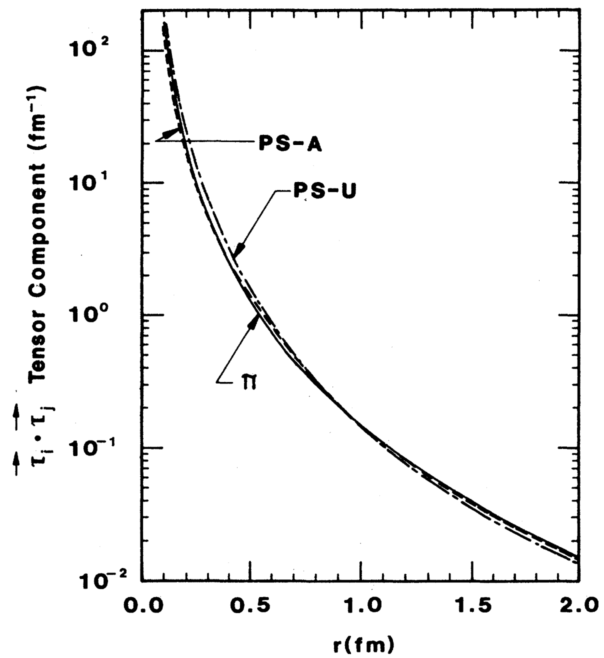

FIG. 1. Pseudoscalar exchange part of the isospin-dependent tensor component of the Argonne and Urbana v_{14} potentials (PS-A and PS-U, respectively), and the bare one-pion-exchange tensor interaction.

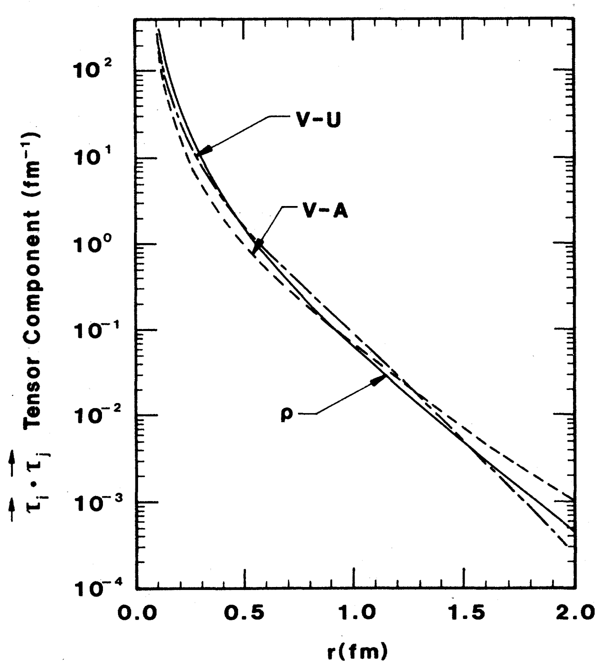

FIG. 2. Vector exchange part of the isospin-dependent tensor component of the Argonne and Urbana v_{14} potentials (V-A and V-U, respectively), and the bare rho-meson-exchange tensor interaction (in absolute value).

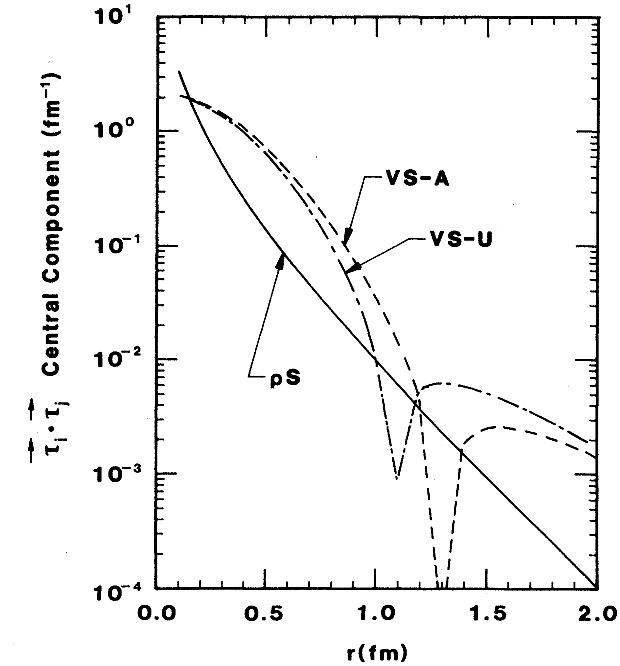

FIG. 3. Isospin-dependent central component of the Argonne and Urbana v_{14} potentials (VS-A and VS-U, respectively), and the bare rho-meson-exchange central interaction.

be multiplied by a nucleon electromagnetic form factor, which by the continuity equation (2.2) should be the Fourier transform of the isovector component of the nucleon charge density. The nonrelativistic continuity equation requires that it should be the Sachs form factor $G_E^V(q)$ used in the one-body current. In the context of Dirac theory, $G_E^V(q)$ includes a relativistic correction proportional to the Pauli form factor $F_2^V(q)$

$$G_E^V(q)=F_1^V(q)-\frac{q^2}{4m_N}F_2^V(q)\ . \qquad (2.16)$$

Since the relativistic correction arises from the purely transverse Pauli term in the electromagnetic current operator of the nucleon, it drops out of the relativistic continuity equation $\partial_\mu j^\mu=0$, suggesting that $F_1^V(q)$ should be used to multiply the exchange current contributions (2.5) and (2.6). However, in an approach that uses Dirac spinors to describe nucleons, there are additional exchange current contributions from meson exchange diagrams with intermediate nucleon-antinucleon pairs that are coupled to the electromagnetic field by the Pauli coupling term. Their contribution essentially provides the relativistic correction in Eq. (2.16), and thus, even in this approach, $G_E^V(q)$ should be used to multiply the contributions (2.5) and (2.6). This argument was first obtained in Ref. 30, but because of an overall sign mistake, with the opposite conclusion. We shall return to a discussion of this form factor difference, which is numerically significant, in Sec. IV.

The isovector exchange current operators (2.5) and (2.6) are the ones of greatest importance for the magnetic form factors of the three-nucleon bound states.[5] The contribution of the other current operators to be considered below is typically of an order of magnitude smaller than that given by the exchange current operators (2.5) and (2.6).

The explicitly velocity-dependent components of the nucleon-nucleon interaction also require the presence of exchange current operators. The most important velocity-dependent component is the spin-orbit interaction. The question of the proper form for the exchange current operator to be associated with the spin-orbit interaction is an old problem of photonuclear theory.[31,32] One part of this exchange current operator can be constructed directly by minimal substitution in the spin-orbit interaction, but this procedure still leaves an incompletely determined nonlocal current operator, associated with the isospin-dependent spin-orbit interaction, which is proportional to $(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_z$. The first part, while satisfying the continuity equation by construction, does not, however, agree with the form of the corresponding operators that are given by meson exchange diagrams.[14]

We shall use here the form for the exchange current operator associated with the spin-orbit force which was derived in Ref. 14. This current operator is constructed by minimal substitution in the interaction, but only after the interaction has been cast in a form valid in a general reference frame. This requires the separation of the spin-orbit interaction into two components v_a and v_b, which are *a priori* not known separately as only their sum gives the usual (known) spin-orbit interaction. For interaction models, the forms of which are consistent with that of a relativistic boson exchange amplitude, a method was found in Ref. 14 to obtain the spin-orbit potential components v_a and v_b as linear combinations of the (known) five spin components of the potential:

$$v_a(k)=\frac{5}{8}v^{\rm SO}(k)-\frac{7}{32m_N^2}v^c(k)+v^\sigma(k)+v^t(k)-\frac{m_N^2}{2}v^{\rm SO2}(k)\ , \tag{2.17}$$

$$v_b(k)=\frac{3}{8}v^{\rm SO}(k)+\frac{7}{32m_N^2}v^c(k)-v^\sigma(k)-v^t(k)+\frac{m_N^2}{2}v^{\rm SO2}(k)\ . \tag{2.18}$$

The current operator associated with the $\mathbf{L}\cdot\mathbf{S}$ interaction is then given by

$$\mathbf{j}_{\rm SO}(\mathbf{k}_1,\mathbf{k}_2)=-\frac{i}{4}\{(1+\tau_{2,z})[v_a(k_1)\boldsymbol{\sigma}_2\times(\mathbf{k}_1-\mathbf{q})+v_b(k_1)\boldsymbol{\sigma}_1\times\mathbf{k}_1]+(1+\tau_{1,z})[v_a(k_2)\boldsymbol{\sigma}_1\times(\mathbf{k}_2-\mathbf{q})+v_b(k_2)\boldsymbol{\sigma}_2\times\mathbf{k}_2]\}\ . \tag{2.19}$$

The current operator associated with the $\mathbf{L}\cdot\mathbf{S}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$ interaction is also obtained from Eq. (2.19) with the replacements

$$(1+\tau_{2,z})\rightarrow(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2+\tau_{1,z})\ ,\qquad (1+\tau_{1,z})\rightarrow(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2+\tau_{2,z})\ , \tag{2.20}$$

$$v_{a,b}\rightarrow v_{a,b}^\tau\ , \tag{2.21}$$

and the $v_{a,b}^\tau$ are obtained from v^τ, $v^{t\tau}$, $v^{\sigma\tau}$, $v^{\rm SO\tau}$, and $v^{\rm SO2\tau}$.

In addition to the linearly velocity-dependent spin-orbit interaction, all realistic models of the nucleon-nucleon interaction contain quadratically velocity-dependent terms.[16] These are, on the one hand, relativistic corrections to the central and spin-spin interactions, which are proportional to $p^2/2m_N$,[27,33] and, on the other hand, quadratic spin-orbit interactions. To construct the associated exchange current operators from these interaction components in a way that is consistent with boson exchange mechanisms is, in general, difficult or impossible, because of the many approximations typically used to simplify the structure of these interaction components.[16] Moreover, the Urbana and Argonne potentials contain a term proportional to L^2, which does not appear in any natural way in boson exchange models. Therefore we construct the exchange current operators associated with the terms of second order in $\mathbf{L}$ in these potentials by direct minimal substitution

$$\mathbf{p}_i\rightarrow\mathbf{p}_i-\tfrac{1}{2}[G_E^S(q)+G_E^V(q)\tau_{i,z}]\,\mathbf{A}(\mathbf{r}_i) \tag{2.22}$$

into these interaction components. The terms linear in the vector potential $\mathbf{A}$ are written as

$$-\int d\mathbf{x}\,\mathbf{j}(\mathbf{x})\cdot\mathbf{A}(\mathbf{x})\ , \tag{2.23}$$

and $\mathbf{j}(\mathbf{x})$ is identified as the exchange current operator associated with the L^2 and $(\mathbf{L}\cdot\mathbf{S})^2$ components. We should point out that the Argonne and Urbana potentials also contain the isospin-dependent terms $L^2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$ and $(\mathbf{L}\cdot\mathbf{S})^2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$. The minimal substitution in these components is performed after symmetrizing as follows:

$$L^2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2=\tfrac{1}{2}(L^2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2+\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_2L^2)\ . \tag{2.24}$$

However, as is shown in Sec. IV the numerical effect on the magnetic structure of the three-body nuclei of the exchange current operators associated with the isospin-independent and isospin-dependent L^2 and $(\mathbf{L}\cdot\mathbf{S})^2$ components of the Argonne and Urbana potentials is very small.

C. Model-dependent exchange current operators

In addition to the model-independent exchange current operators considered above, we shall also take into account the corrections from the most important transverse model-dependent exchange current operators. These are the pion and ρ-meson exchange current operators associated with excitation of intermediate Δ_{33} resonances (Fig. 4) and the $\rho\pi\gamma$ and $\omega\pi\gamma$ exchange current operators illustrated by the Feynman diagrams in Fig. 5.

The pion exchange current operator that is associated with excitation of intermediate Δ_{33} resonances is[34]

$$\mathbf{j}_{\pi\Delta}(\mathbf{k}_1,\mathbf{k}_2)=i\frac{4f_\pi^2}{25m_N m_\pi^2(m_\Delta-m_N)}G_M^V(q)\left[4\tau_{1,z}\frac{(\boldsymbol{\sigma}_1\cdot\mathbf{k}_1)\mathbf{k}_1}{m_\pi^2+k_1^2}+4\tau_{2,z}\frac{(\boldsymbol{\sigma}_2\cdot\mathbf{k}_2)\mathbf{k}_2}{m_\pi^2+k_2^2}\right.$$
$$\left.-(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_z\left[\frac{(\boldsymbol{\sigma}_1\times\mathbf{k}_2)(\boldsymbol{\sigma}_2\cdot\mathbf{k}_2)}{m_\pi^2+k_2^2}-\frac{(\boldsymbol{\sigma}_2\times\mathbf{k}_1)(\boldsymbol{\sigma}_1\cdot\mathbf{k}_1)}{m_\pi^2+k_1^2}\right]\right]\times\mathbf{q}\,. \tag{2.25}$$

Here m_Δ is the Δ_{33} mass and $G_M^V(q)$ the isovector magnetic form factor. In Eq. (2.25) the $\pi N\Delta$ coupling has been expressed in terms of the πNN coupling constant f_π by using the static quark model.[35] Similarly, the $\gamma N\Delta$ transition form factor has been expressed in terms of the isovector magnetic form factor with the same justification.

Although in early work on exchange current corrections to nuclear magnetic transition rates[36] the Δ_{33} pion exchange current operator was thought to be as important as the model-independent pion exchange current operator (2.5), it has since then been realized that its effect is strongly reduced by the corresponding ρ-meson exchange contribution.[6] This is associated with the general cancelation between the pion and ρ-meson exchange tensor interactions.[37] The ρ-meson exchange current operator is[34]

$$\mathbf{j}_{\rho\Delta}(\mathbf{k}_1,\mathbf{k}_2)=-i\frac{g_\rho^2(1+\kappa)^2}{25m_N^3(m_\Delta-m_N)}G_M^V(q)\left[4\tau_{1,z}\frac{(\boldsymbol{\sigma}_1\times\mathbf{k}_1)\times\mathbf{k}_1}{m_\rho^2+k_1^2}+4\tau_{2,z}\frac{(\boldsymbol{\sigma}_2\times\mathbf{k}_2)\times\mathbf{k}_2}{m_\rho^2+k_2^2}\right.$$
$$\left.-(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_z\left[\frac{\boldsymbol{\sigma}_1\times[(\boldsymbol{\sigma}_2\times\mathbf{k}_2)\times\mathbf{k}_2]}{m_\rho^2+k_2^2}-\frac{(\boldsymbol{\sigma}_2\times[(\boldsymbol{\sigma}_1\times\mathbf{k}_1)\times\mathbf{k}_1]}{m_\rho^2+k_1^2}\right]\right]\times\mathbf{q}\,. \tag{2.26}$$

In this expression the $\rho N\Delta$ and $\gamma N\Delta$ coupling constants have again been eliminated in favor of the corresponding ρNN and γNN coupling constants with the help of the static quark model. Finally, we shall introduce form factors at the baryon-meson vertices in the expressions (2.25) and (2.26) to take into account the effect of the finite size of the baryons and mesons. These form factors are introduced by the replacements

$$\frac{1}{m_\pi^2+k^2}\to\frac{1}{m_\pi^2+k^2}\left[\frac{\Lambda_\pi^2-m_\pi^2}{\Lambda_\pi^2+k^2}\right]^2,$$
$$\frac{1}{m_\rho^2+k^2}\to\frac{1}{m_\rho^2+k^2}\left[\frac{\Lambda_\rho^2-m_\rho^2}{\Lambda_\rho^2+k^2}\right]^2, \tag{2.27}$$

in the meson propagators. The cutoff masses Λ_π and Λ_ρ do, of course, represent arbitrary parameters, but we shall here use the values $\Lambda_\pi=1200$ MeV and $\Lambda_\rho=2000$ MeV suggested by studies of the reaction[38] $\pi^+d\to pp$. It is important to note that once both the pion and ρ-meson exchange current operators (2.25) and (2.26) are taken into account, the resulting nuclear matrix elements are not very sensitive to the choice of cutoff mass scales Λ_π and Λ_ρ. If the ρ-meson exchange current (2.26) is dropped as was done, for example, in Ref. 39, the results are, in contrast, extremely sensitive to the cutoff mass value.

Within the usual meson exchange framework the $\rho\pi\gamma$ and $\omega\pi\gamma$ exchange current operators are completely model dependent and, being purely transverse, they are unrelated to the nucleon-nucleon interaction. Within the framework of the recently developed topological soliton or Skyrme[40] model framework the former one of these exchange current operators becomes linked to the chiral anomaly and thus also attains a degree of model independence.[41,42] Although at low values of momentum transfer both the $\rho\pi\gamma$ and $\omega\pi\gamma$ exchange current operators contribute only very small corrections to the magnetic form factors, the $\rho\pi\gamma$ exchange current operator gives a very important correction to the magnetic form factor of the deuteron.[16,43,44]

The expression for the $\rho\pi\gamma$ exchange current operator is

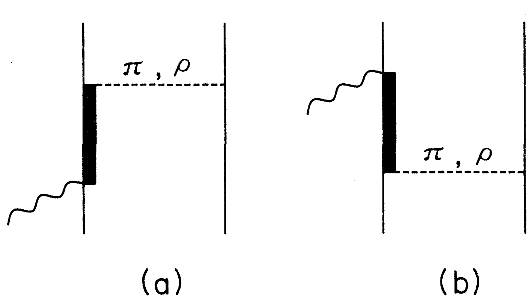

FIG. 4. Pion and rho-meson-exchange current operators that involve excitation of intermediate Δ_{33} resonances.

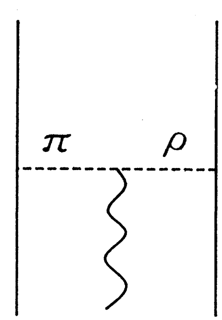

FIG. 5. $\rho\pi\gamma$ exchange current mechanism. The $\omega\pi\gamma$ exchange current operator is similar with the ω meson in place of the ρ meson.

$$\mathbf{j}_{\rho\pi}(\mathbf{k}_1,\mathbf{k}_2)=i\frac{f_{\pi}g_{\rho}g_{\rho\pi\gamma}}{m_{\pi}m_{\rho}}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 G_{\omega}(q)\mathbf{k}_1\times\mathbf{k}_2 \times\left[\frac{\boldsymbol{\sigma}_1\cdot\mathbf{k}_1}{(k_1^2+m_{\pi}^2)(k_2^2+m_{\rho}^2)}-\frac{\boldsymbol{\sigma}_2\cdot\mathbf{k}_2}{(k_1^2+m_{\rho}^2)(k_2^2+m_{\pi}^2)}\right]. \tag{2.28}$$

Here $G_{\omega}(q)$ is an appropriate electromagnetic form factor, which according to the usual vector-meson-dominance model, would be

$$G_{\omega}(q)=\frac{1}{1+q^2/m_{\omega}^2}. \tag{2.29}$$

$g_{\rho\pi\gamma}$ is the coupling constant for the $\rho\pi\gamma$ vertex for which we shall use the value $g_{\rho\pi\gamma}=0.4$.[45] The exchange current operator that corresponds to the $\omega\pi\gamma$ exchange current mechanism in Fig. 5 is

$$\mathbf{j}_{\omega\pi}=i\frac{f_{\pi}g_{\omega}g_{\omega\pi\gamma}}{m_{\omega}m_{\pi}}G_{\rho}(q)\mathbf{k}_1\times\mathbf{k}_2 \times\left[\frac{\boldsymbol{\sigma}_1\cdot\mathbf{k}_1}{(k_1^2+m_{\pi}^2)(k_2^2+m_{\omega}^2)}\tau_{1,z}-\frac{\boldsymbol{\sigma}_2\cdot\mathbf{k}_2}{(k_1^2+m_{\omega}^2)(k_2^2+m_{\pi}^2)}\tau_{2,z}\right], \tag{2.30}$$

where $G_{\rho}(q)$ is the electromagnetic (e.m.) form factor associated with the $\omega\pi\gamma$ vertex, which again, according to the vector-meson-dominance model, is

$$G_{\rho}(q)=\frac{1}{1+q^2/m_{\rho}^2}. \tag{2.31}$$

For the $\omega\pi\gamma$ coupling strength we use the value $g_{\omega\pi\gamma}=0.68$.[46]

III. MONTE CARLO CALCULATION

A convenient expression to calculate the transverse elastic form factor $F_T(q)$ is obtained if the coordinate system is oriented so that the momentum transfer $\mathbf{q}$ lies in the x direction, and the z axis is taken as the quantization axis for the magnetic quantum number.[9,21] It is then found that

$$F_T(q)=\sqrt{2}\langle 0_R|j_y(q\hat{\mathbf{x}})|0\rangle, \tag{3.1}$$

where $|0_R\rangle$ denotes the ground state recoiling with momentum $\mathbf{q}=q\hat{\mathbf{x}}$ ($\hat{\mathbf{x}}$ is the unit vector in the x direction), and $j_y(q\hat{\mathbf{x}})$ is the y component of the current operator in momentum space

$$\mathbf{j}(\mathbf{q})=\int d\mathbf{x}\, e^{i\mathbf{q}\cdot\mathbf{x}}\mathbf{j}(\mathbf{x}) =\sum_{i=1}^{A}\mathbf{j}^{(1)}(\mathbf{q};\mathbf{r}_i\mathbf{p}_i\boldsymbol{\sigma}_i\tau_{i,z}) +\sum_{i<j\le 1}^{A}\mathbf{j}^{(2)}(\mathbf{q};\mathbf{r}_i\mathbf{r}_j\mathbf{p}_i\mathbf{p}_j\boldsymbol{\sigma}_i\boldsymbol{\sigma}_j\boldsymbol{\tau}_i\boldsymbol{\tau}_j). \tag{3.2}$$

The operators $\mathbf{j}^{(1)}$ and $\mathbf{j}^{(2)}$ are associated with the one- and two-body parts of $\mathbf{j}$

$$\mathbf{j}^{(1)}(\mathbf{q};\mathbf{r}_i\mathbf{p}_i\boldsymbol{\sigma}_i\tau_{i,z}) =\frac{1}{2}[G_E^S(q)+G_E^V(q)\tau_{i,z}]\frac{1}{2m_N}\{\mathbf{p}_i,e^{i\mathbf{q}\cdot\mathbf{r}_i}\} -\frac{i}{4m_N}[G_M^S(q)+G_M^V(q)\tau_{i,z}]\mathbf{q}\times\boldsymbol{\sigma}_i e^{i\mathbf{q}\cdot\mathbf{r}_i}, \tag{3.3}$$

$$\mathbf{j}^{(2)}(\mathbf{q};\mathbf{r}_i\mathbf{r}_j\mathbf{p}_i\mathbf{p}_j\boldsymbol{\sigma}_i\boldsymbol{\sigma}_j\boldsymbol{\tau}_i\boldsymbol{\tau}_j) =\int\frac{d\mathbf{k}_1}{(2\pi)^3}\frac{d\mathbf{k}_2}{(2\pi)^3}e^{i\mathbf{k}_1\cdot\mathbf{r}_i}e^{i\mathbf{k}_2\cdot\mathbf{r}_j} \times(2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2-\mathbf{q})\mathbf{j}^{(2)}(\mathbf{k}_1,\mathbf{k}_2) \tag{3.4}$$

and $\mathbf{j}^{(2)}(\mathbf{k}_1,\mathbf{k}_2)$ are the momentum-space expressions of the two-body current operators given in Sec. II. The expressions for $\mathbf{j}^{(2)}(\mathbf{q})$ are listed in the Appendix.

The expectation value (3.1) is computed, without any approximation, by Monte Carlo integration.[1] The ground-state wave function is written as a vector in the spin-isospin space of the A nucleons for any given spatial configuration $\mathbf{R}\equiv\{\mathbf{r}_1,\mathbf{r}_2\mathbf{r}_3\}$. For the given $\mathbf{R}$, we calculate the state vector $j_y(q\hat{\mathbf{x}})|0\rangle$ by performing exactly the spin-isospin algebra with the methods described in Ref. 1. The momentum-dependent terms in $j_y(q\hat{\mathbf{x}})$ are calculated numerically; for example,

$$\nabla_{i,\alpha}\psi(\mathbf{R})=\frac{1}{2\delta_{i,\alpha}}[\psi(\mathbf{R}+\boldsymbol{\delta}_{i,\alpha})-\psi(\mathbf{R}-\boldsymbol{\delta}_{i,\alpha})], \tag{3.5}$$

where $\delta_{i,\alpha}$ is a small increment in the $r_{i,\alpha}$ component of $\mathbf{R}$. The $\mathbf{R}$ integration is performed with the Monte Carlo method by sampling the $\mathbf{R}$ configurations according to the Metropolis algorithm.

In the limit of $q\to 0$, $F_T(q)$ behaves like

$$F_T(q)\simeq\frac{1}{\sqrt{2}}\frac{q}{m_N}\mu, \tag{3.6}$$

where μ is the magnetic moment of the nucleus in nuclear magnetons (nm), and the magnetic form factor is then defined as

$$F_M(q)=\sqrt{2}\frac{m_N}{\mu}\frac{F_T(q)}{q}. \tag{3.7}$$

IV. MAGNETIC FORM FACTORS OF ^{3}H AND ^{3}He

The calculated magnetic form factors of ^{3}H and ^{3}He are compared with the experimental data[10–12] in Figs. 6 and 7. The ground-state wave functions are calculated using the Argonne two-nucleon and Urbana model VII three-nucleon interactions, with the variational Monte Carlo method. It should be pointed out that these wave functions give binding energies, charge radii and D- to S-state ratios in the d-p and d-n channels, which are quite close to the empirical values.[17] Further tests of their accuracy have been carried out by direct comparison with results obtained with exact Faddeev[3] or Green's function Monte Carlo[4] wave functions, as, for example, those for the two-body correlation functions[23,47] and the longitudi-

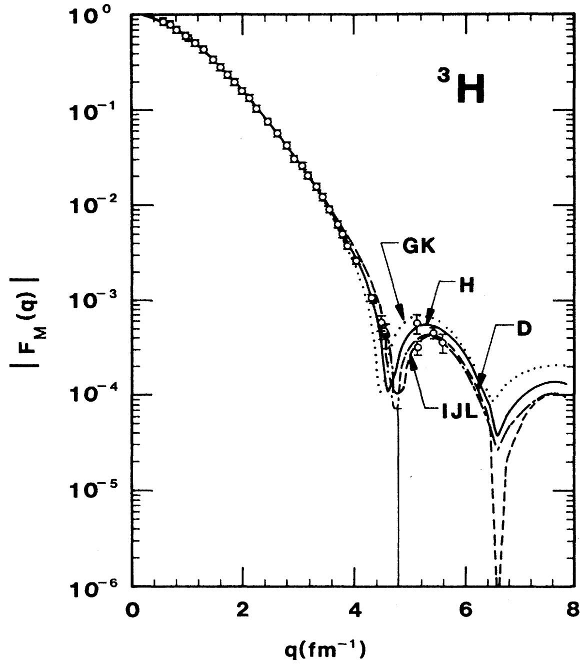

FIG. 6. Magnetic form factor of ^{3}H as a function of the four-momentum transfer. The calculated form factor includes the exchange current contributions; the curves labeled H, GK, IJL, and D are obtained, respectively, with the Höhler, Gari-Krümpelmann, Iachello-Jackson-Lande, and dipole parametrizations of the nucleon electromagnetic Sachs form factors.

nal energy-weighted sum rule.[48] The exchange current operator used in these calculations is obtained from the Argonne v_{ij} as discussed in Sec. II. The four theoretical curves show results obtained with the nucleon form factor parametrizations of Höhler (H), Iachello, Jackson, and Landè (IJL), Gari and Krümpelmann (GK), and the dipole fit (D). All the theory curves follow the experimental data quite well, the results obtained with the IJL five-parameter dipole parametrization of the nucleon form factors being particularly close to the data.

As mentioned in Sec. II, we have used the Sachs form factors $G_E^V(q)$ and $G_E^S(q)$ in the isovector and isoscalar exchange current operators associated with the isospin- and momentum-dependent terms in the N-N interaction. Much poorer agreement with the data is obtained if one uses the Dirac F_1 form factors in these exchange current operators as shown in Figs. 8 and 9. It is important to note the difference between these results and those obtained in previous calculations, which typically have not come close to the data unless using the Dirac F_1 form factor for π and ρ exchange currents. These earlier calculations have relied on simple meson exchange current operators that are not fully consistent with the interaction used to calculate the ground-state wave function. One of the differences between the present and earlier results for the exchange current contributions to the magnetic form factors is that the pion-like PS component of the isospin-dependent tensor potential is stronger for the

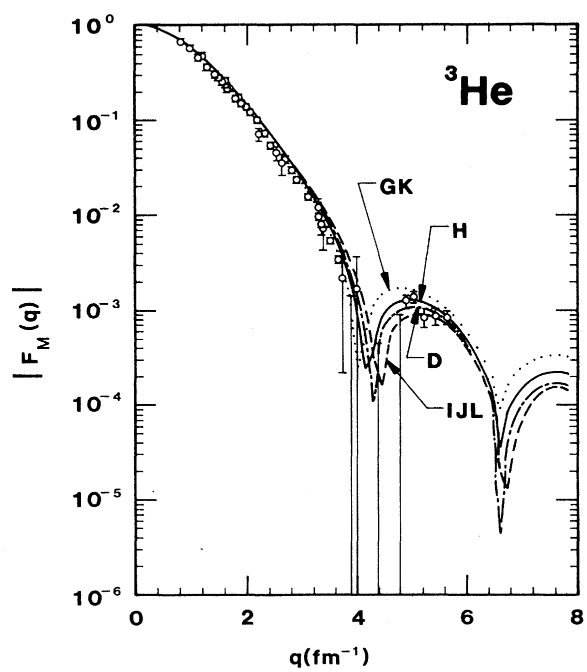

FIG. 7. Magnetic form factor of ^{3}He as a function of the four-momentum transfer. The calculated form factor includes the exchange current contributions; the curves are labeled as in Fig. 6.

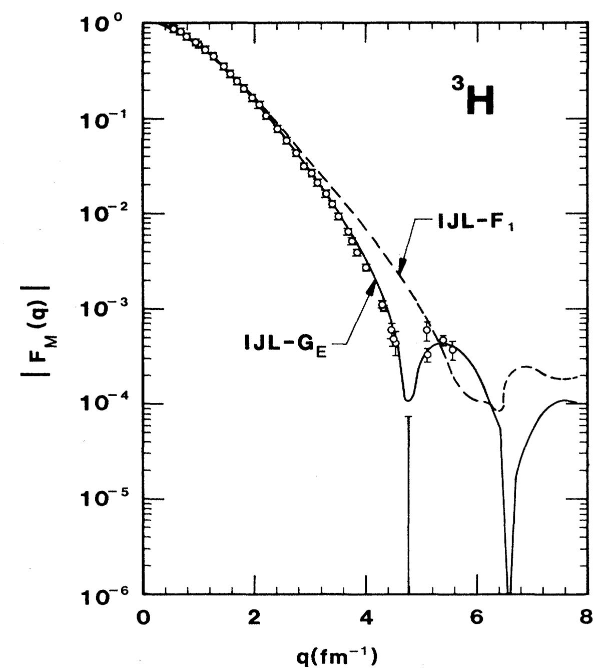

FIG. 8. Calculated magnetic form factor of ^{3}H, as a function of the four-momentum transfer, obtained with the Iachello-Jackson-Lande parametrizations of the electromagnetic form factors of the nucleon. The curve labeled F_1 is obtained by using the Dirac in place of the Sachs form factors in the nuclear current operator.

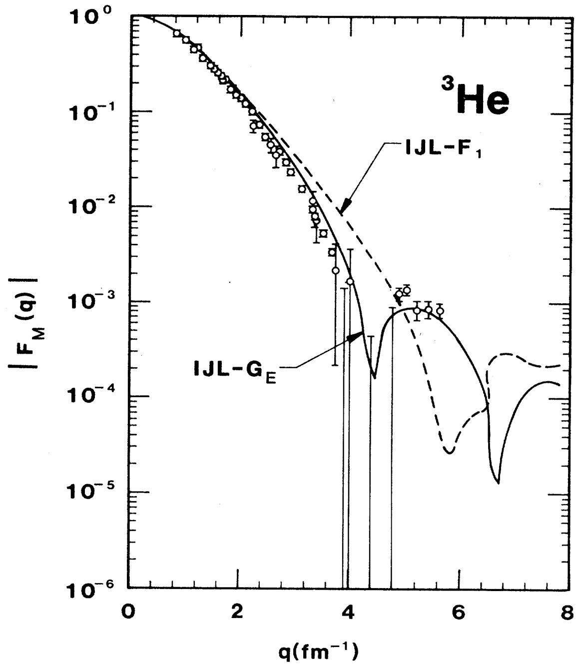

FIG. 9. Same as in Fig. 8, but of ^{3}He.

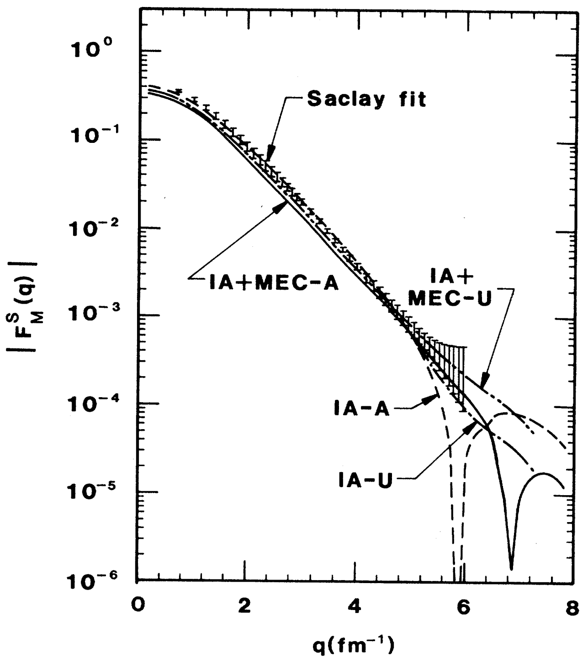

FIG. 11. Same as in Fig. 10, but of the isoscalar combination of the ^{3}H and ^{3}He magnetic form factors.

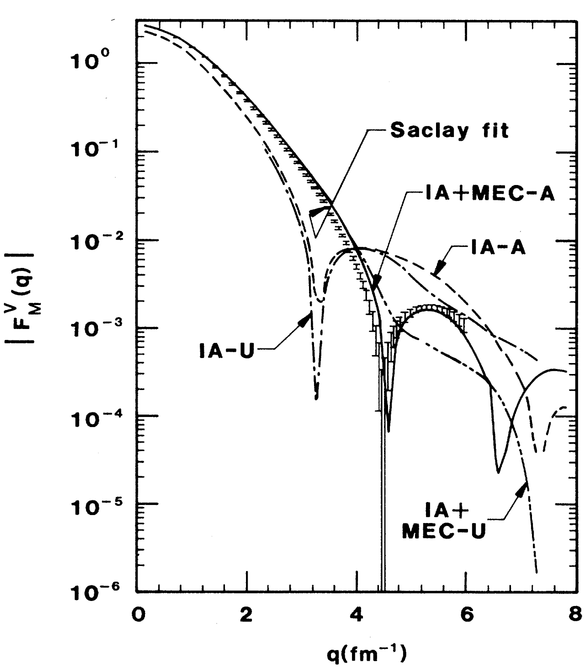

FIG. 10. Isovector combination of the ^{3}H and ^{3}He magnetic form factors, as a function of the four-momentum transfer, obtained with the Argonne and Urbana v_{14} potentials, and the Iachello-Jackson-Lande parametrization of the electromagnetic form factors of the nucleon. Both the impulse approximation (IA), and impulse approximation and exchange current contribution (IA+MEC) results are displayed.

potential models used here (Fig. 1) than for the Paris potential.

Figures 10 and 11 compare the calculated isovector and isoscalar form factors with the experimental data.

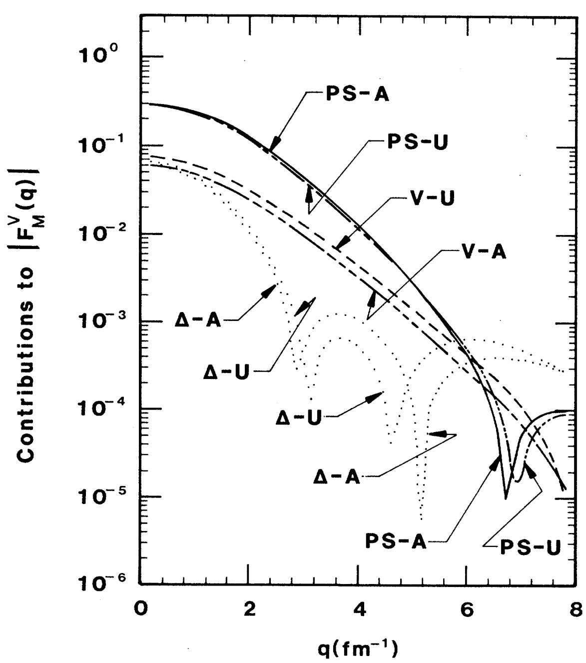

FIG. 12. Main isovector exchange current contributions, as function of the four-momentum transfer, obtained with the Argonne and Urbana v_{14} potentials. The Iachello-Jackson-Lande parametrizations of the nucleon electromagnetic form factors are used.

These figures also include the results obtained using ground-state wave functions and exchange currents obtained from the Urbana v_{ij} and Urbana model VII V_{ijk}. It appears that the isovector form factors obtained from the Urbana v_{ij} are not in good agreement with the data. The $\tau_1 \cdot \tau_2$ tensor potential $v^{t\tau}(r)$ in the Urbana v_{ij} is much weaker than that in the Argonne v_{ij}. For example, the Urbana, Paris, and Argonne deuterons have 5.2, 5.8, and 6.1 % D states. The dominant isovector exchange current contributions obtained from the Urbana and Argonne v_{ij} are compared in Fig. 12. They are quite similar; the main difference between these models is in their single-nucleon (IA) current contributions shown in Fig. 10.

All the contributions to the isovector and isoscalar form factors obtained with the Argonne interaction are shown in Figs. 13 and 14. In the region of the diffraction minimum (4–6 fm^{-1}) the pion-like and ρ-like exchange currents are dominant, while the Δ and $\omega\pi\gamma$ terms become relatively more important at larger momentum transfers. The contributions of the currents associated with the velocity-dependent interactions to the isovector form factors are entirely negligible.

In contrast, the currents associated with the velocity-

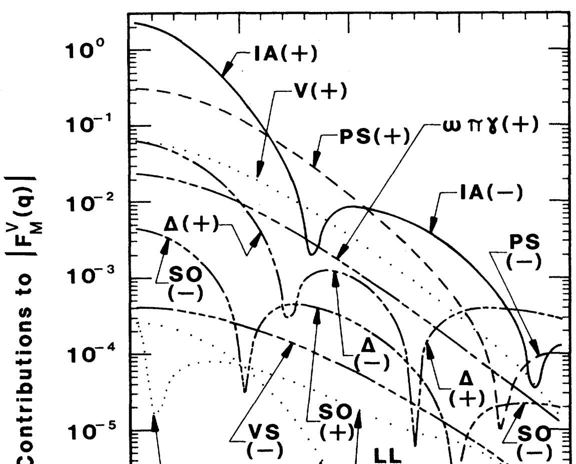

FIG. 13. Individual contributions to the isovector combination of the ^{3}H and ^{3}He magnetic form factors, as functions of the four-momentum transfer, obtained with the Iachello-Jackson-Lande parametrizations of the nucleon electromagnetic form factors. The contributions due to the single-nucleon current (IA) and the exchange currents associated with the pseudoscalar (PS) and vector (V) parts of the isospin-dependent tensor component, the central isospin-dependent component (VS), the spin-orbit (SO), L^2 (LL) and quadratic spin-orbit (SO2) components of the Argonne v_{14} interaction, the Δ_{33} resonance (Δ), and the $\omega\pi\gamma$ mechanisms are displayed.

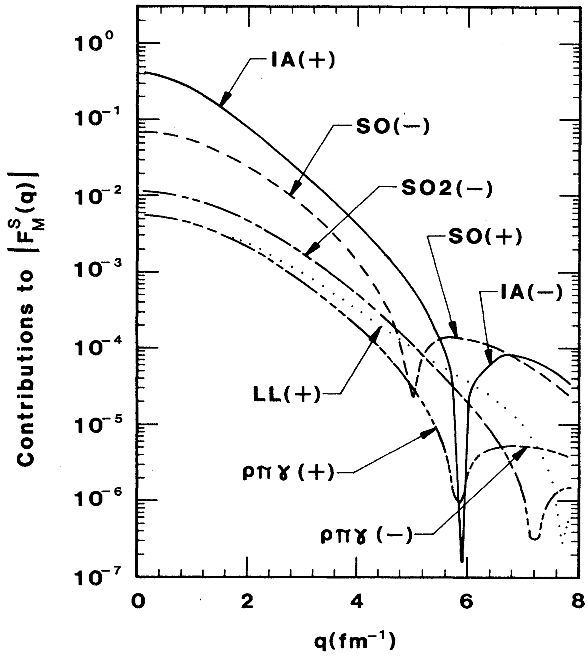

FIG. 14. Individual contributions to the isoscalar combination of the ^{3}H and ^{3}He magnetic form factors, as functions of the four-momentum transfer, obtained with the Iachello-Jackson-Lande parametrizations of the nucleon electromagnetic form factors. The contributions due to the single-nucleon current (IA) and the exchange currents associated with the spin-orbit (SO), L^2 (LL) and quadractic spin-orbit (SO2) components of the Argonne v_{14} interaction, and the $\rho\pi\gamma$ mechanism are displayed.

TABLE I. Contributions from the different components of the nuclear electromagnetic current operator to the isoscalar and isovector combinations of the magnetic moments of the trinucleons.

	Isoscalar		Isovector	
	Argonne	Urbana	Argonne	Urbana
IA	+0.406	+0.405	−2.189	−2.192
PS			−0.295	−0.281
V			−0.0593	−0.0746
VS			+0.0004	+0.000 01
SO	−0.0696	−0.0431	+0.0043	+0.0020
LL	+0.0052	+0.0049	+0.0003	−0.0002
SO2	−0.0116	−0.0105	−0.0003	+0.0004
$\rho\pi\gamma$	+0.0056	+0.0051		
$\omega\pi\gamma$			−0.0232	−0.0229
Δ			−0.0624	−0.0668
Total	+0.336	+0.361	−2.624	−2.635
Experiment	+0.426		−2.553	

	Argonne	Urbana	Experiment
^{3}He	−2.288	−2.274	−2.127
^{3}H	+2.960	+3.00	+2.979

dependent interactions, particularly the spin-orbit, give the leading correction to the isoscalar form factor (Fig. 14). These corrections are small (Fig. 11), but they spoil the agreement between the IA and experiment at small q. This problem may be specific to the spin-orbit components of the potential models considered here, as these exchange current contributions are much smaller if evaluated with the Paris potential.

The contributions of the various terms in the current operator to the isoscalar and isovector magnetic moments are compared in Table I. The predicted isovector magnetic moments are very close to the empirical value -2.553 nm for both potential models (-2.624 nm for the Argonne and -2.635 nm for the Urbana potential). On the other hand, the predicted isoscalar magnetic moment is close to the empirical value 0.426 nm only for the Urbana potential (0.361 nm). The considerably lower value 0.336 nm obtained with the Argonne potential is a consequence of the large negative exchange current contribution due to the spin-orbit component of that potential. It is worth noting that with the Paris potential we obtain a much smaller value (-0.0069 nm) for the exchange current contribution associated with the spin-orbit interaction, in agreement with the corresponding estimate for the magnetic moment of the deuteron found in Ref. 14.

It is interesting to compare the present exchange current contributions to the magnetic moments with the results obtained in Ref. 49 by solving the Faddeev equations with the Reid soft core potential.[50] In Ref. 49 the exchange current operators were based on simple meson exchange diagrams with no short-range form factors. The "model-independent" spin exchange current mechanisms were found to contribute -0.241 nm to μ^V in Ref. 49. The values found here for the corresponding contribution from the pseudoscalar exchange current operator (2.5) constructed from the Argonne and Urbana potentials are considerably larger in magnitude (-0.295 nm and -0.281 nm). This is due to the fact that the pseudoscalar exchange part of the tensor components of the two potential models considered here is stronger than the bare pion exchange tensor potential at intermediate range (Fig. 1).

The contribution to μ^V of the vector-meson-like exchange current operator (2.6) found here (-0.0593 nm and -0.0746 nm) represent 20% enhancements of the contribution of the pseudoscalar exchange current operator (2.5). This is in reasonable agreement with the result found in Ref. 15 for the corresponding enhancement of 14%, due to vector-meson exchange, of the pseudoscalar exchange current contribution to the amplitude for radiative np capture. The contributions to μ^V of the exchange currents associated with the explicitly velocity-dependent interaction components are small.

The contributions to μ^V from the model-dependent exchange current operators in Table I differ considerably from those obtained in Ref. 49. The reason that we obtain a two times larger contribution from the $\omega\pi\gamma$ exchange current mechanism (-0.023 nm) than found in Ref. 49 (-0.012 nm) is due to our use of a larger value for the ωNN coupling constant g_ω. Here g_ω is taken to be 14.6, which is the value used in the Bonn potential, whereas in Ref. 49 the SU(3) value of 6.82 was used. One may, however, argue that the smaller value for g_ω may be more reasonable since the ω exchange interaction in boson exchange models takes into account effective multimeson exchange (e.g., $\pi\rho$ exchange) in addition to the simple single-omega-meson interaction, and that this is reflected in an unrealistically large value for the coupling constant.

The contribution to μ^V in Table I from the exchange current operators associated with intermediate Δ_{33} resonances (-0.062 nm) is much smaller than that found in Ref. 49 (-0.166 nm). The reason for the present small value is the large canceling effect of the ρ-meson exchange current operator (2.26), which was not considered in Ref. 49.

The contribution to the isoscalar magnetic moment μ^S from the $\rho\pi\gamma$ exchange current operator (2.28) found here (0.0056 nm and 0.0052 nm for the Argonne and Urbana potentials, respectively) is somewhat smaller than that found in Ref. 49 (0.01 nm). The exchange current contributions associated with the velocity-dependent potentials have not been considered in previous work. It is interesting to note that especially the contribution due to the spin-orbit interaction is very sensitive to the potential model.

The predicted total values with the Argonne (Urbana) potential model for the magnetic moments of ^{3}He and ^{3}H are -2.29 nm (-2.28 nm) and $+2.96$ nm ($+3.00$ nm), respectively. These are close to the empirical values, -2.13 nm and $+2.98$ nm, and we regard them as satisfactory.

ACKNOWLEDGMENTS

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APPENDIX

In this Appendix we list the configuration space expressions of the exchange current operators given in momentum space in Sec. II. They are defined as

$$\mathbf{j}^{(2)}(\mathbf{q})=\int d\mathbf{x}\, e^{i\mathbf{q}\cdot\mathbf{x}}\int \frac{d\mathbf{k}_1}{(2\pi)^3}\frac{d\mathbf{k}_2}{(2\pi)^3}e^{i\mathbf{k}_1\cdot(\mathbf{r}_1-\mathbf{x})} \times e^{i\mathbf{k}_2\cdot(\mathbf{r}_2-\mathbf{x})}\mathbf{j}^{(2)}(\mathbf{k}_1,\mathbf{k}_2)\ . \tag{A1}$$

We find that the *transverse components* of $\mathbf{j}_{\rm PS}$, $\mathbf{j}_{\rm V}$, and $\mathbf{j}_{\rm VS}$ are given by

$$\mathbf{j}_{\rm PS}(\mathbf{q})=3(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_z\Bigg[e^{i\mathbf{q}\cdot\mathbf{r}_1}g_{\rm PS}(r)\boldsymbol{\sigma}_1(\boldsymbol{\sigma}_2\cdot\hat{\mathbf{r}})+e^{i\mathbf{q}\cdot\mathbf{r}_2}g_{\rm PS}(r)\boldsymbol{\sigma}_2(\boldsymbol{\sigma}_1\cdot\hat{\mathbf{r}})$$
$$+e^{i\mathbf{q}\cdot\mathbf{R}}\Bigg[\frac{1}{r^2}G_{{\rm PS},1}(\mathbf{r})[\boldsymbol{\sigma}_1(\boldsymbol{\sigma}_2\cdot\hat{\mathbf{r}})+\boldsymbol{\sigma}_2(\boldsymbol{\sigma}_1\cdot\hat{\mathbf{r}})+\hat{\mathbf{r}}(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)]+i\frac{1}{r}G_{{\rm PS},2}(\mathbf{r})\boldsymbol{\sigma}_1(\boldsymbol{\sigma}_2\cdot\mathbf{q})-i\frac{1}{r}G_{{\rm PS},3}(\mathbf{r})\boldsymbol{\sigma}_2(\boldsymbol{\sigma}_1\cdot\mathbf{q})$$
$$-i\frac{1}{r}G_{{\rm PS},4}(\mathbf{r})\hat{\mathbf{r}}(\boldsymbol{\sigma}_1\cdot\hat{\mathbf{r}})(\boldsymbol{\sigma}_2\cdot\mathbf{q})+i\frac{1}{r}G_{{\rm PS},5}(\mathbf{r})\hat{\mathbf{r}}(\boldsymbol{\sigma}_1\cdot\mathbf{q})(\boldsymbol{\sigma}_2\cdot\hat{\mathbf{r}})$$
$$-G_{{\rm PS},6}(\mathbf{r})\hat{\mathbf{r}}(\boldsymbol{\sigma}_1\cdot\mathbf{q})(\boldsymbol{\sigma}_2\cdot\mathbf{q})-\frac{1}{r^2}G_{{\rm PS},7}(\mathbf{r})\hat{\mathbf{r}}(\boldsymbol{\sigma}_1\cdot\hat{\mathbf{r}})(\boldsymbol{\sigma}_2\cdot\hat{\mathbf{r}})\Bigg]\Bigg]\;, \tag{A2}$$

$$\mathbf{j}_{\rm V}(\mathbf{q})=3(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_z\Bigg[e^{i\mathbf{q}\cdot\mathbf{r}_1}g_{\rm V}(r)\boldsymbol{\sigma}_1\times(\boldsymbol{\sigma}_2\times\hat{\mathbf{r}})+e^{i\mathbf{q}\cdot\mathbf{r}_2}g_{\rm V}(r)\boldsymbol{\sigma}_2\times(\boldsymbol{\sigma}_1\times\hat{\mathbf{r}})$$
$$-e^{i\mathbf{q}\cdot\mathbf{R}}\Bigg[\frac{1}{r^2}G_{{\rm V},1}(\mathbf{r})[(\boldsymbol{\sigma}_2\times\hat{\mathbf{r}})\times\boldsymbol{\sigma}_1+(\boldsymbol{\sigma}_1\times\hat{\mathbf{r}})\times\boldsymbol{\sigma}_2+2\hat{\mathbf{r}}(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)]$$
$$+i\frac{1}{r}G_{{\rm V},2}(\mathbf{r})(\boldsymbol{\sigma}_2\times\mathbf{q})\times\boldsymbol{\sigma}_1-i\frac{1}{r}G_{{\rm V},3}(\mathbf{r})(\boldsymbol{\sigma}_1\times\mathbf{q})\times\boldsymbol{\sigma}_2$$
$$-i\frac{1}{r}G_{{\rm V},4}(\mathbf{r})\hat{\mathbf{r}}(\boldsymbol{\sigma}_1\times\hat{\mathbf{r}})\cdot(\boldsymbol{\sigma}_2\times\mathbf{q})+i\frac{1}{r}G_{{\rm V},5}(\mathbf{r})\hat{\mathbf{r}}(\boldsymbol{\sigma}_1\times\mathbf{q})\cdot(\boldsymbol{\sigma}_2\times\hat{\mathbf{r}})$$
$$-G_{{\rm V},6}(\mathbf{r})\hat{\mathbf{r}}(\boldsymbol{\sigma}_1\times\mathbf{q})\cdot(\boldsymbol{\sigma}_2\times\mathbf{q})-\frac{1}{r^2}G_{{\rm V},7}(\mathbf{r})\hat{\mathbf{r}}(\boldsymbol{\sigma}_1\times\hat{\mathbf{r}})\cdot(\boldsymbol{\sigma}_2\times\hat{\mathbf{r}})\Bigg]$$
$$+\frac{1}{2}e^{i\mathbf{q}\cdot\mathbf{R}}\Bigg[-G_{{\rm V},2}(\mathbf{r})(\boldsymbol{\sigma}_2\times\mathbf{q})\boldsymbol{\sigma}_1\cdot(\mathbf{q}\times\hat{\mathbf{r}})-G_{{\rm V},3}(\mathbf{r})(\boldsymbol{\sigma}_1\times\mathbf{q})\boldsymbol{\sigma}_2\cdot(\mathbf{q}\times\hat{\mathbf{r}})$$
$$-i\frac{1}{r}[G_{{\rm V},4}(\mathbf{r})+G_{{\rm V},5}(\mathbf{r})][(\boldsymbol{\sigma}_2\times\hat{\mathbf{r}})\boldsymbol{\sigma}_1\cdot(\hat{\mathbf{r}}\times\mathbf{q})-(\boldsymbol{\sigma}_1\times\hat{\mathbf{r}})\boldsymbol{\sigma}_2\cdot(\hat{\mathbf{r}}\times\mathbf{q})]$$
$$-i\frac{1}{r}[G_{{\rm V},2}(\mathbf{r})+G_{{\rm V},3}(\mathbf{r})][\boldsymbol{\sigma}_2\times(\boldsymbol{\sigma}_1\times\mathbf{q})-\boldsymbol{\sigma}_1\times(\boldsymbol{\sigma}_2\times\mathbf{q})]\Bigg]\Bigg]\;, \tag{A3}$$

$$\mathbf{j}_{\rm VS}(\mathbf{q})=-(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_z e^{i\mathbf{q}\cdot\mathbf{R}}G_{\rm VS}(\mathbf{r})\hat{\mathbf{r}}\;, \tag{A4}$$

where

$$\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2,\quad \hat{\mathbf{r}}=\frac{\mathbf{r}}{r},\quad \mathbf{R}=\tfrac{1}{2}(\mathbf{r}_1+\mathbf{r}_2)\;. \tag{A5}$$

We have defined

$$g_{\rm PS}(r)=-\frac{1}{3r^2}\left[\int_r^\infty dr' r'^2 v^{\sigma\tau}(r')+2r^3\int_r^\infty dr' v^{t\tau}(r')/r'\right]\;, \tag{A6}$$

$$g_{\rm V}(r)=\frac{1}{3r^2}\left[\int_r^\infty dr' r'^2 v^{\sigma\tau}(r')-r^3\int_r^\infty dr' v^{t\tau}(r')/r'\right]\;, \tag{A7}$$

$$G_{a,1}(\mathbf{r})=\int_{-1/2}^{+1/2}dx\, e^{-ix\mathbf{q}\cdot\mathbf{r}}\left[E_a(x;r)-r\frac{d}{dr}E_a(x;r)\right]\;, \tag{A8}$$

$$G_{a,2}(\mathbf{r})=\int_{-1/2}^{+1/2}dx\, e^{-ix\mathbf{q}\cdot\mathbf{r}}(\tfrac{1}{2}+x)E_a(x;r)\;, \tag{A9}$$

$$G_{a,3}(\mathbf{r})=G_{a,2}(-\mathbf{r})\;, \tag{A10}$$

$$G_{a,4}(\mathbf{r})=\int_{-1/2}^{+1/2}dx\, e^{-ix\mathbf{q}\cdot\mathbf{r}}(\tfrac{1}{2}+x)\times\left[E_a(x;r)-r\frac{d}{dr}E_a(x;r)\right]\;, \tag{A11}$$

$$G_{a,5}(\mathbf{r})=G_{a,4}(-\mathbf{r})\;, \tag{A12}$$

$$G_{a,6}(\mathbf{r})=\int_{-1/2}^{+1/2}dx\, e^{-ix\mathbf{q}\cdot\mathbf{r}}(\tfrac{1}{4}-x^2)E_a(x;r)\;, \tag{A13}$$

$$G_{a,7}(\mathbf{r})=\int_{-1/2}^{+1/2}dx\, e^{-ix\mathbf{q}\cdot\mathbf{r}}\left[3E_a(x;r)-3r\frac{d}{dr}E_a(x;r)+r^2\frac{d^2}{dr^2}E_a(x;r)\right]\;, \tag{A14}$$

$$G_{\rm VS}(\mathbf{r})=\int_{-1/2}^{+1/2}dx\, e^{-ix\mathbf{q}\cdot\mathbf{r}}E_{\rm VS}(x;r)\;, \tag{A15}$$

where $a=$PS and V, and the $\mathbf{q}$ dependence of the $G_{a,i}$, $i=1,\ldots,7$, $G_{\rm VS}$, and E_a is not explicitly shown. $E_a(x;r)$ $a=$PS, V, and VS are defined as

$$E_a(x;r)=\sum_{i=1}^{12}\frac{A_i^a}{4\pi}e^{-rL_i(x)}\;, \tag{A16}$$

$$L_i(x)=\left[m_i^2+\frac{q^2}{4}(1-4x^2)\right]^{1/2}\;. \tag{A17}$$

The coefficients A_i^a and m_i are determined in the following way. The terms involving the G_a functions in Eqs. (A2), (A3), and (A4) arise because of the terms proportional to $[v_a(k_1)-v_a(k_2)]/(k_2^2-k_1^2)$ in Eqs. (2.5) and (2.6). The configuration-space expression of such terms requires the evaluation of

$$K_a(\mathbf{q})=\int d\mathbf{x}\, e^{i\mathbf{q}\cdot\mathbf{x}}\int \frac{d\mathbf{k}_1}{(2\pi)^3}\frac{d\mathbf{k}_2}{(2\pi)^3}\frac{[v_a(k_1)-v_a(k_2)]}{k_2^2-k_1^2} \times e^{i\mathbf{k}_1(\mathbf{r}_1-\mathbf{x})}e^{i\mathbf{k}_2(\mathbf{r}_2-\mathbf{x})} . \tag{A18}$$

In order to perform the above integrals efficiently, we expand

$$v_a(k)=\sum_{i=1}^{12}\frac{A_i^a}{k^2+m_i^2}, \quad a=\text{PS, V,VS} . \tag{A19}$$

The mass parameters are those of the Paris potential,[27] while the A_i^a coefficients are determined by fitting the expressions (2.10)–(2.12). We then find

$$K_a(\mathbf{q})=\sum_i A_i^a K_i^a(\mathbf{q}) , \tag{A20}$$

$$K_i^a(\mathbf{q})=e^{i\mathbf{q}\cdot\mathbf{R}}\int\frac{d\mathbf{p}}{(2\pi)^3}e^{i\mathbf{p}\cdot\mathbf{r}}\frac{1}{\left[\left[\frac{\mathbf{q}}{2}+\mathbf{p}\right]^2+m_i^2\right]\left[\left[\frac{\mathbf{q}}{2}-\mathbf{p}\right]^2+m_i^2\right]} . \tag{A21}$$

By using Feynman's parametrization

$$\frac{1}{D_1D_2}=\int_0^1 dy\frac{1}{[(D_1-D_2)y+D_2]^2} , \tag{A22}$$

K_i^a can be written as

$$K_i^a(\mathbf{q})=e^{i\mathbf{q}\cdot\mathbf{R}}\int_{-1/2}^{+1/2}dx\, e^{-ix\mathbf{q}\cdot\mathbf{r}}\frac{1}{8\pi L_i(x)}e^{-rL_i(x)} , \tag{A23}$$

with $L_i(x)$ defined as in Eq. (A17).

The current associated with the isospin-independent and isospin-dependent spin-orbit components of the potential is written in momentum space as

$$\mathbf{j}_{\rm SO}(\mathbf{k}_1,\mathbf{k}_2)=-\frac{i}{2}\{[P(2)v_a(k_1)+Q(1)v_a^\tau(k_1)]\boldsymbol{\sigma}_2\times(\mathbf{k}_1-\mathbf{q})+[P(2)v_b(k_1)+Q(1)v_b^\tau(k_1)]\boldsymbol{\sigma}_1\times\mathbf{k}_1+1\rightleftarrows 2\} , \tag{A24}$$

$$P(i)=\tfrac{1}{2}(1+\tau_{i,z}),\quad Q(i)=\tfrac{1}{2}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2+\tau_{i,z}) , \tag{A25}$$

by combining Eqs. (2.17), (2.18), (2.20), and (2.21). The r-space expression is then found to be

$$\begin{aligned}\mathbf{j}_{\rm SO}(\mathbf{q})=\tfrac{1}{2}\Bigg[&e^{i\mathbf{q}\cdot\mathbf{r}_1}\left[i(\boldsymbol{\sigma}_1\times\mathbf{q})H_a(r;12)+(\boldsymbol{\sigma}_1\times\hat{\mathbf{r}})\frac{d}{dr}H_a(r;12)+(\boldsymbol{\sigma}_2\times\hat{\mathbf{r}})\frac{d}{dr}H_b(r;12)\right]\\&+e^{i\mathbf{q}\cdot\mathbf{r}_2}\left[i(\boldsymbol{\sigma}_2\times\mathbf{q})H_a(r;21)-(\boldsymbol{\sigma}_2\times\hat{\mathbf{r}})\frac{d}{dr}H_a(r;21)-(\boldsymbol{\sigma}_1\times\hat{\mathbf{r}})\frac{d}{dr}H_b(r;21)\right]\Bigg] ,\end{aligned} \tag{A26}$$

$$H_{a,b}(r;ij)=P(i)g_{a,b}(r)+Q(j)g_{a,b}^\tau(r) , \tag{A27}$$

$$g_p(r)\equiv\frac{1}{2\pi^2}\int_0^\infty dk\, k^2 j_0(kr)v_p(k),\quad p=a,b,a\tau,b\tau , \tag{A28}$$

For example, $g_a(r)$ is expressed as

$$g_a(r)=\frac{5}{8}g_{\rm SO}(r)-\frac{7}{32m_N^2}v^c(r)+g^\sigma(r)+g^t(r) -\frac{m_N^2}{2}g^{\rm SO2}(r) , \tag{A29}$$

where

$$g^{\rm SO}(r)=-\int_r^\infty dr'r'v^{\rm SO}(r') , \tag{A30}$$

$$g^\sigma(r)=\int_r^\infty dr'r'^2v^\sigma(r')\left[\frac{1}{r'}-\frac{1}{r}\right] , \tag{A31}$$

$$g^t(r)=\frac{1}{2}\int_r^\infty dr'r'v^t(r')\left[1-\frac{r^2}{r'^2}\right] , \tag{A32}$$

$$g^{\rm SO2}(r)=-\frac{1}{2}\int_r^\infty dr'r'^3v^{\rm SO2}(r')\left[1-\frac{r^2}{r'^2}\right] . \tag{A33}$$

Derivatives of the $g_p, p=a$, b, $a\tau$, and $b\tau$, easily follow from the equations above. In obtaining Eqs. (A30)–(A33), as well as Eqs. (A6) and (A7), we have used

$$\frac{2}{\pi}\int_0^\infty dk\, j_0(kr)j_0(kr')=\begin{cases}\dfrac{1}{r} & r>r'\\[2mm] \dfrac{1}{r'} & r<r' ,\end{cases} \tag{A34}$$

$$\frac{2}{\pi}\int_0^\infty dk\, j_0(kr)j_2(kr')=\begin{cases}0 & r>r'\\ \dfrac{1}{2r'}\left[1-\dfrac{r^2}{r'^2}\right] & r<r'\,.\end{cases} \tag{A35}$$

As already mentioned in Sec. II, the exchange currents associated with the L^2 and quadratic spin-orbit components $\frac{1}{2}(\mathbf{L}\cdot\boldsymbol{\sigma}_1\mathbf{L}\cdot\boldsymbol{\sigma}_2+\mathbf{L}\cdot\boldsymbol{\sigma}_2\mathbf{L}\cdot\boldsymbol{\sigma}_1)$ are constructed by minimal substitution

$$\mathbf{p}_i\rightarrow\mathbf{p}_i-P(i)\,\mathbf{A}(\mathbf{r}_i)\,, \tag{A36}$$

where $\mathbf{p}_i$ is the nucleon momentum and $\mathbf{A}$ the vector potential. This procedure generates in the L^2 and quadratic spin-orbit potentials two additional terms, linear and quadratic in $\mathbf{A}$. The term linear in $\mathbf{A}$ is written in the form

$$\text{term linear in }\mathbf{A}=-\int d\mathbf{x}\,\mathbf{j}(\mathbf{x})\cdot\mathbf{A}(\mathbf{x})\,, \tag{A37}$$

where $\mathbf{j}(\mathbf{x})$ is the current density operator, and

$$\mathbf{j}(\mathbf{q})=\int d\mathbf{x}\,e^{i\mathbf{q}\cdot\mathbf{x}}\mathbf{j}(\mathbf{x})\,. \tag{A38}$$

We find after some straightforward algebraic manipulations

$$\begin{aligned}\mathbf{j}_{\rm LL}(\mathbf{q})=&[v^{\rm LL}(r)+v^{{\rm LL}\sigma}(r)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2][D_-(\mathbf{q})(i\mathbf{r}-\mathbf{r}\times\mathbf{L})-\tfrac{1}{4}D_+(\mathbf{q})\mathbf{r}\times(\mathbf{r}\times\mathbf{q})]\\&+[v^{{\rm LL}\tau}(r)+v^{{\rm LL}\sigma\tau}(r)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2][D'_-(\mathbf{q})(i\mathbf{r}-\mathbf{r}\times\mathbf{L})-\tfrac{1}{4}D'_+(\mathbf{q})\mathbf{r}\times(\mathbf{r}\times\mathbf{q})]\,,\end{aligned} \tag{A39}$$

$$\begin{aligned}\mathbf{j}_{\rm SO2}(\mathbf{q})=&\tfrac{1}{8}v^{\rm SO2}(r)D_+(\mathbf{q})\boldsymbol{\sigma}_1\cdot(\mathbf{r}\times\mathbf{q})\boldsymbol{\sigma}_2\times\mathbf{r}+\tfrac{1}{4}v^{\rm SO2}(r)D_-(\mathbf{q})\{\boldsymbol{\sigma}_1\cdot\mathbf{L},\boldsymbol{\sigma}_2\times\mathbf{r}\}\\&+\tfrac{1}{8}v^{\rm SO2}(r)D_+(\mathbf{q})\boldsymbol{\sigma}_2\cdot(\mathbf{r}\times\mathbf{q})\boldsymbol{\sigma}_1\times\mathbf{r}+\tfrac{1}{4}v^{\rm SO2}(r)D_-(\mathbf{q})\{\boldsymbol{\sigma}_2\cdot\mathbf{L},\boldsymbol{\sigma}_1\times\mathbf{r}\}\\&+\tfrac{1}{8}v^{{\rm SO2}\tau}(r)D'_+(\mathbf{q})\boldsymbol{\sigma}_1\cdot(\mathbf{r}\times\mathbf{q})\boldsymbol{\sigma}_2\times\mathbf{r}+\tfrac{1}{4}v^{{\rm SO2}\tau}(r)D'_-(\mathbf{q})\{\boldsymbol{\sigma}_1\cdot\mathbf{L},\boldsymbol{\sigma}_2\times\mathbf{r}\}\\&+\tfrac{1}{8}v^{{\rm SO2}\tau}(r)D'_+(\mathbf{q})\boldsymbol{\sigma}_2\cdot(\mathbf{r}\times\mathbf{q})\boldsymbol{\sigma}_1\times\mathbf{r}+\tfrac{1}{4}v^{{\rm SO2}\tau}(r)D'_-(\mathbf{q})\{\boldsymbol{\sigma}_2\cdot\mathbf{L},\boldsymbol{\sigma}_1\times\mathbf{r}\}\,,\end{aligned} \tag{A40}$$

where

$$D_\pm(\mathbf{q})\equiv P(1)e^{i\mathbf{q}\cdot\mathbf{r}_1}\pm P(2)e^{i\mathbf{q}\cdot\mathbf{r}_2}\,, \tag{A41}$$

$$D'_\pm(\mathbf{q})\equiv Q(2)e^{i\mathbf{q}\cdot\mathbf{r}_1}\pm Q(1)e^{i\mathbf{q}\cdot\mathbf{r}_2}\,, \tag{A42}$$

We finally list the expressions for the $\mathbf{j}_{\pi\Delta}$, $\mathbf{j}_{\rho\Delta}$, and $\mathbf{j}_{\rho\pi}$ currents (that for $\mathbf{j}_{\omega\pi}$ is similar to that of $\mathbf{j}_{\rho\pi}$):

$$\begin{aligned}\mathbf{j}_{\pi\Delta}(\mathbf{q})=-i\frac{4}{25}\frac{f_\pi^2}{m_Nm_\pi^2}\frac{1}{m_\Delta-m_N}\{&4\tau_{1,z}e^{i\mathbf{q}\cdot\mathbf{r}_2}[g_{\pi\Delta}(r)\boldsymbol{\sigma}_1\times\mathbf{q}+g'_{\pi\Delta}(r)(\boldsymbol{\sigma}_1\cdot\hat{\mathbf{r}})\hat{\mathbf{r}}\times\mathbf{q}]\\&+4\tau_{2,z}e^{i\mathbf{q}\cdot\mathbf{r}_1}[g_{\pi\Delta}(r)\boldsymbol{\sigma}_2\times\mathbf{q}+g'_{\pi\Delta}(r)(\boldsymbol{\sigma}_2\cdot\hat{\mathbf{r}})\hat{\mathbf{r}}\times\mathbf{q}]\\&-(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_ze^{i\mathbf{q}\cdot\mathbf{r}_1}[g_{\pi\Delta}(r)(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)\times\mathbf{q}+g'_{\pi\Delta}(r)(\boldsymbol{\sigma}_2\cdot\hat{\mathbf{r}})(\boldsymbol{\sigma}_1\times\hat{\mathbf{r}})\times\mathbf{q}]\\&+(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_ze^{i\mathbf{q}\cdot\mathbf{r}_2}[g_{\pi\Delta}(r)(\boldsymbol{\sigma}_2\times\boldsymbol{\sigma}_1)\times\mathbf{q}+g'_{\pi\Delta}(r)(\boldsymbol{\sigma}_1\cdot\hat{\mathbf{r}})(\boldsymbol{\sigma}_2\times\hat{\mathbf{r}})\times\mathbf{q}]\}\,,\end{aligned} \tag{A43}$$

$$\begin{aligned}\mathbf{j}_{\rho\Delta}(\mathbf{q})=i\frac{g_\rho^2(1+\kappa)^2}{25m_N^3}\frac{1}{m_\Delta-m_N}\{&4\tau_{1,z}e^{i\mathbf{q}\cdot\mathbf{r}_2}[g_{\rho\Delta}(r)\boldsymbol{\sigma}_1\times\mathbf{q}+g'_{\rho\Delta}(r)(\boldsymbol{\sigma}_1\cdot\hat{\mathbf{r}})\hat{\mathbf{r}}\times\mathbf{q}]\\&+4\tau_{2,z}e^{i\mathbf{q}\cdot\mathbf{r}_1}[g_{\rho\Delta}(r)\boldsymbol{\sigma}_2\times\mathbf{q}+g'_{\rho\Delta}(\boldsymbol{\sigma}_2\cdot\hat{\mathbf{r}})\hat{\mathbf{r}}\times\mathbf{q}]\\&-(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_ze^{i\mathbf{q}\cdot\mathbf{r}_1}[g_{\rho\Delta}(r)(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)\times\mathbf{q}+g'_{\rho\Delta}(r)(\boldsymbol{\sigma}_2\cdot\hat{\mathbf{r}})(\boldsymbol{\sigma}_1\times\hat{\mathbf{r}})\times\mathbf{q}]\\&+(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_ze^{i\mathbf{q}\cdot\mathbf{r}_2}[g_{\rho\Delta}(r)(\boldsymbol{\sigma}_2\times\boldsymbol{\sigma}_1)\times\mathbf{q}+g'_{\rho\Delta}(r)(\boldsymbol{\sigma}_1\cdot\hat{\mathbf{r}})(\boldsymbol{\sigma}_2\times\hat{\mathbf{r}})\times\mathbf{q}]\\&+m_\rho^2[4\tau_{1,z}e^{i\mathbf{q}\cdot\mathbf{r}_2}f_{\rho\Delta}(r)\boldsymbol{\sigma}_1\times\mathbf{q}+4\tau_{2,z}e^{i\mathbf{q}\cdot\mathbf{r}_1}f_{\rho\Delta}(r)\boldsymbol{\sigma}_2\times\mathbf{q}\\&\quad-(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_ze^{i\mathbf{q}\cdot\mathbf{r}_1}f_{\rho\Delta}(r)(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)\times\mathbf{q}\\&\quad+(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_ze^{i\mathbf{q}\cdot\mathbf{r}_2}f_{\rho\Delta}(r)(\boldsymbol{\sigma}_2\times\boldsymbol{\sigma}_1)\times\mathbf{q}]\}\,,\end{aligned} \tag{A44}$$

where

$$f_{a\Delta}(r)\equiv\frac{1}{4\pi r}\left\{e^{-m_ar}-e^{-\Lambda_ar}-\frac{1}{2}\left[1-\left[\frac{m_a}{\Lambda_a}\right]^2\right]\Lambda_are^{-\Lambda_ar}\right\},\quad a=\pi,\rho \tag{A45}$$

$$g_{a\Delta}(r) \equiv \frac{1}{r}\frac{d}{dr}f_{a\Delta}(r), \quad g'_{a\Delta} \equiv r\frac{d}{dr}g_{a\Delta}(r) , \tag{A46}$$

$$\mathbf{j}_{\rho\pi}(\mathbf{q}) = \frac{1}{2} i f_\pi \frac{g_\rho g_{\rho\pi\gamma}}{m_\pi m_\rho} \tau_1 \cdot \tau_2 e^{i\mathbf{q}\cdot\mathbf{R}} \left[-G_{\rho\pi,1}(\mathbf{r})\frac{1}{r}\mathbf{q}\times\boldsymbol{\sigma}_1 - iG_{\rho\pi,2}(\mathbf{r})\mathbf{q}\times\hat{\mathbf{r}}(\mathbf{q}\cdot\boldsymbol{\sigma}_1) + G_{\rho\pi,3}(\mathbf{r})\frac{1}{r}\mathbf{q}\times\hat{\mathbf{r}}(\hat{\mathbf{r}}\cdot\boldsymbol{\sigma}_1) \right.$$
$$\left. - G'_{\rho\pi,1}(\mathbf{r})\frac{1}{r}\mathbf{q}\times\boldsymbol{\sigma}_2 + iG'_{\rho\pi,2}(\mathbf{r})\mathbf{q}\times\hat{\mathbf{r}}(\mathbf{q}\cdot\boldsymbol{\sigma}_2) + G'_{\rho\pi,3}(\mathbf{r})\frac{1}{r}\mathbf{q}\times\hat{\mathbf{r}}(\hat{\mathbf{r}}\cdot\boldsymbol{\sigma}_2) \right] , \tag{A47}$$

$$G_{\rho\pi,1}(\mathbf{r}) = \int_{-1/2}^{+1/2} dx\, e^{-ix\mathbf{q}\cdot\mathbf{r}} E_{\rho\pi}(x;r) , \tag{A48}$$

$$G_{\rho\pi,2}(\mathbf{r}) = \int_{-1/2}^{+1/2} dx\, e^{-ix\mathbf{q}\cdot\mathbf{r}} (\tfrac{1}{2}-x) E_{\rho\pi}(x;r) , \tag{A49}$$

$$G_{\rho\pi,3}(\mathbf{r}) = \int_{-1/2}^{+1/2} dx\, e^{-ix\mathbf{q}\cdot\mathbf{r}} [1 + r\Lambda_{\rho\pi}(x)] E_{\rho\pi}(x;r) , \tag{A50}$$

$$E_{\rho\pi}(x;r) = \frac{1}{4\pi} e^{-\Lambda_{\rho\pi}(x) r} , \tag{A51}$$

$$\Lambda_{\rho\pi}(x) \equiv \left[\frac{q^2}{4}(1-4x^2) + m_\pi^2(\tfrac{1}{2}+x) + m_\rho^2(\tfrac{1}{2}-x) \right]^{1/2} . \tag{A52}$$

The functions $G'_{\rho\pi}$ are obtained from $G_{\rho\pi}$ by the replacement

$$\Lambda_{\rho\pi}(x) \to \Lambda'_{\rho\pi}(x) = \Lambda_{\rho\pi}(-x) . \tag{A53}$$

We note that Eq. (A47) has been obtained for the case of bare πNN and ρNN vertices. The effect of monopole form factors at these vertices is easily included by noting that

$$\frac{1}{k_1^2+m_\pi^2}\frac{\Lambda_\pi^2-m_\pi^2}{k_1^2+\Lambda_\pi^2}\frac{1}{k_2^2+m_\rho^2}\frac{\Lambda_\rho^2-m_\rho^2}{k_2^2+\Lambda_\rho^2} = \left[\frac{1}{k_1^2+m_\pi^2} - \frac{1}{k_1^2+\Lambda_\pi^2} \right] \left[\frac{1}{k_2^2+m_\rho^2} - \frac{1}{k_2^2+\Lambda_\rho^2} \right] . \tag{A54}$$

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