Spin-one Kemmer-Duffin-Petiau equations and intermediate-energy deuteron-nucleus scattering

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We present a description of the spin-one Kemmer-Duffin-Petiau equations. An effective second-order equation is obtained for various types of interactions. An argument is given for the use of one particular form of interaction for deuteron-nucleus scattering. This provides the basis for a generalization of the standard Watanabe model in which Dirac scalar and vector nucleon-nucleus potentials are used as input. Parameter-free calculations are performed using both phenomenological and microscopic nucleon-nucleus potentials, and results are compared with deuteron-nucleus elastic scattering data at 400 and 700 MeV. Qualitative agreement is generally obtained. A good description of the forward-angle vector analyzing power data is achieved.

I. INTRODUCTION

In recent years, relativistic approaches have proven quite successful in describing intermediate-energy proton-nucleus scattering data.[1-13] Particularly striking was the superiority of the Dirac equation using both phenomenological analyses[1-4,8-10] and the parameter-free relativistic impulse approximation,[5-7,11-13] in reproducing the spin observables compared with standard nonrelativistic approaches. Such success encourages the application of relativistic wave equations to other probes of nuclei. An interesting candidate for such an analysis is the deuteron. Its loosely bound structure suggests that deuteron-nucleus scattering can be essentially described in terms of free nucleon-nucleus scattering. Moreover, there now exist data from Saclay[14-15] for deuteron-nucleus scattering at intermediate energies where spin observables as well as differential cross sections have been measured.

The Saclay data have been analyzed by Yahiro, Kameyama, Iseri, Kamimura, and Kawai[16,17] in terms of the nonrelativistic Watanabe model.[18] In the Watanabe model, the deuteron-nucleus optical potential is obtained by folding the sum of the neutron and proton optical potentials, evaluated at half the incident deuteron energy, over the internal deuteron density. The Watanabe model, which is not successful at low energy due to its neglect of virtual deuteron breakup,[18,19] is expected to be more accurate at intermediate energies where the breakup effect is less important.[16,17] The authors of Ref. 16 assume that the differences between standard Woods-Saxon optical potentials and the effective central and spin-orbit potentials arising from a Dirac equation approach are representative of the ambiguity in the nonrelativistic nucleon-nucleus optical model. These authors use a Schrödinger equation, with relativistic kinematics, to calculate the deuteron-nucleus elastic scattering observables. They obtained good agreement with the 400-MeV $d+{}^{58}$Ni data at forward angles, and found that the Dirac-based optical potentials gave better agreement with the deuteron-nucleus data than the use of standard Woods-Saxon nucleon-nucleus optical potentials, although each set gave equally good fits to proton-nucleus data. Similar results were obtained when comparing calculations with deuteron-nucleus data at 700 MeV.[14,15] It is therefore of interest to independently analyze these data using a relativistic treatment throughout employing a relativistic wave equation to describe the scattered deuteron.

The idea of treating deuteron-nucleus scattering relativistically is not without precedent. Shepard, Rost, and Murdock[20] used the Breit equation as a basis for a phenomenological treatment and obtained reasonable agreement with $d+{}^{58}$Ni data at 80 MeV.[21] Santos and collaborators[22,23] have studied the problem employing both the Proca and Weinberg equations. They argue that such an approach yields a deuteron spin-orbit potential that is too small to reproduce the actual spin observables,[22] although they find qualitative agreement with the Saclay data when parameters in their model are adjusted.[23] More recently, there have been some investigations concerning the effects of the deuteron internal structure in a relativistic formalism. Santos and Amorim[24] studied the above problem within a Bethe-Salpeter framework, while

Kameyama and Yahiro[25] have used a (v/c) expansion.

In the present approach, a one-body wave equation is used to describe the deuteron-nucleus system, the interactions are determined by the usual Watanabe assumptions, and, without varying any parameters, a comparison is made with the deuteron-nucleus data. This parameter-free approach provides a test for the validity of a one-body relativistic description and emphasizes the predictive power of our model. Although one might question the appropriateness of using a one-body equation for a particle as extended as the deuteron, possible justification for this procedure has been given by Bleszynski, Bleszynski, and Jaroszewicz.[26]

We will use the Kemmer-Duffin-Petiau (KDP) formalism[27–29] as the basis for a relativistic treatment of deuteron-nucleus scattering.[30] The KDP formalism yields a first-order, multicomponent relativistic wave equation for spin-1 particles that is similar in structure to the Dirac equation. An appealing property of the KDP formalism is that its structure suggests an underlying direct product of two spin-$\frac{1}{2}$ Dirac fields.[31] The KDP formalism also yields a relativistic spin-zero equation which has been applied with reasonable success to pion-nucleus and kaon-nucleus scattering at intermediate energies.[32] In this work a relativistic impulse approximation was used to obtain the meson-nucleus optical potentials. The KDP approach gave qualitatively similar results to those of a standard first-order Kerman, McManus, and Thaler[33] (KMT) treatment. It should be remembered, however, that the most dramatic differencess between the Dirac and Schrödinger analyses of proton-nucleus scattering occur in the spin observables. It is the proven ability of relativistic equations to reproduce proton-nucleus spin observables, together with the underlying two-body structure inherent in the KDP formalism which encourages us to use it for describing deuteron-nucleus scattering.

In this paper we show how the KDP formalism can be applied to deuteron-nucleus scattering and compare our results with intermediate-energy deuteron-nucleus data. Also included is a discussion of the relation between our work and the nonrelativistic models of Yahiro *et al.*[16,17] and Ray.[34] Sec. II contains a brief description of the spin-1 KDP formalism and Sec. III shows how interactions can be added to the formalism. Section IV discusses the second-order KDP equation. Section V shows how the spin-1 KDP equations can be applied to deuteron-nucleus scattering. In Sec. VI the predictions of the KDP approach for the deuteron-nucleus scattering observables are compared with the Saclay data. The paper concludes with a summary in Sec. VII.

II. THE SPIN-ONE KDP FORMALISM

The free-particle KDP equation is of the form[27]

$$(i\beta^{\mu}\partial_{\mu}-m)\psi=0 , \tag{2.1}$$

where the β^{μ} obey the following algebra:

$$\beta^{\mu}\beta^{\nu}\beta^{\lambda}+\beta^{\lambda}\beta^{\nu}\beta^{\mu}=g^{\mu\nu}\beta^{\lambda}+g^{\lambda\nu}\beta^{\mu} . \tag{2.2}$$

A representation of the KDP algebra defined by Eq. (2.2) can be given in terms of 16×16 matrices as

$$\beta^{\mu}=\tfrac{1}{2}(I_1\otimes\gamma_2^{\mu}+\gamma_1^{\mu}\otimes I_2) , \tag{2.3}$$

where I is the 4×4 identity matrix, the γ^{μ} are the Dirac matrices, and $\otimes$ indicates a direct product. The form of Eq. (2.3) is suggestive of a system of two spin-$\frac{1}{2}$ particles. The representation of the β^{μ} given in Eq. (2.3) is reducible and can be decomposed into three irreducible representations: a trivial one-dimensional representation in which all of the β^{μ} are equal to zero, a five-dimensional representation which yields a set of spin-zero wave equations, and a ten-dimensional representation which gives a set of spin-one equations. Hereafter, we shall only be concerned with the ten-dimensional spin-one KDP equations.

A set of 10×10 matrices which satisfies the algebra of Eq. (2.2) can be written in block-diagonal form as

$$\beta^{\mu}=\begin{bmatrix}0 & a^{\mu}\\ b^{\mu} & 0\end{bmatrix} \tag{2.4}$$

where the a^{μ} are 4×6 matrices and the b^{μ} are 6×4 matrices. Written out explicitly, the a^{μ} are

$$a^0=\begin{bmatrix}0&0&0&0&0&0\\1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\end{bmatrix} , \tag{2.5a}$$

$$a^1=\begin{bmatrix}1&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&-1\\0&0&0&0&1&0\end{bmatrix} , \tag{2.5b}$$

$$a^2=\begin{bmatrix}0&1&0&0&0&0\\0&0&0&0&0&1\\0&0&0&0&0&0\\0&0&0&-1&0&0\end{bmatrix} , \tag{2.5c}$$

$$a^3=\begin{bmatrix}0&0&1&0&0&0\\0&0&0&0&-1&0\\0&0&0&1&0&0\\0&0&0&0&0&0\end{bmatrix} , \tag{2.5d}$$

and

$$b^{\mu}=g^{\mu\mu}a_{\mu}^{T} \quad \text{(no summation)} ,$$

where

$$g^{\mu\nu}=\mathrm{diag}(1,-1,-1,-1) . \tag{2.6}$$

If the ten-component wave function of Eq. (2.1) is written in the form

$$\psi=\begin{bmatrix}A^{\mu}\\ F^{\mu\nu}\end{bmatrix} , \tag{2.7}$$

where A^{μ} is a four-vector and $F^{\mu\nu}$ are the six indepen-

dent components of an antisymmetric second-rank tensor then, for $m\neq 0$, it can be shown that each component of A^μ satisfies a Klein-Gordon equation

$$(\Box - m^2)A^\mu = 0 , \tag{2.8}$$

where

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu . \tag{2.9}$$

It can be shown that the set of Eqs. (2.8) and (2.9) is equivalent to the more familiar Proca equations,[35] which are usually written as Eq. (2.8) together with the subsidiary condition, $\partial_\mu A^\mu = 0$. Thus, for the free-particle case, the spin-one KDP formalism and the Proca equations are equivalent.

Alternatively, one can use Eqs. (2.4)–(2.6) and write the spin-one KDP equation in a form that resembles Maxwell's equations, in which case one obtains

$$i\nabla\cdot E = m\varphi , \tag{2.10a}$$

$$i\partial_t \mathbf{E} - i\nabla\times\mathbf{B} = m\,\mathbf{A} , \tag{2.10b}$$

$$i\partial_t \mathbf{A} - i\nabla\varphi = m\mathbf{E} , \tag{2.10c}$$

and

$$-i\nabla\times\mathbf{A} = m\mathbf{B} . \tag{2.10d}$$

The ten-component wave function ψ is given by

$$\psi = \begin{bmatrix} \varphi \\ \mathbf{A} \\ \mathbf{E} \\ -\mathbf{B} \end{bmatrix} \tag{2.11}$$

in obvious analogy with classical electrodynamics. We shall use the form (2.10) in the next section when introducing interactions into the formalism.

Although we have established the equivalence of the spin-one KDP and Proca formalisms for a free particle, such equivalence will not necessarily hold when interactions are included. Thus the results of a calculation may be dependent on which wave equation is used. Below we develop the KDP equations for the interacting particle case. A similar treatment of the Proca equations and the Weinberg equations has been given previously by Santos and Van Dam.[22] An investigation of the differences between various spin-one formalisms and the implication for deuteron-nucleus scattering is currently underway and the results will be presented in a subsequent publication.[36]

III. INTERACTIONS

In order to describe an actual scattering situation, an interaction U must be introduced into the KDP formalism. The KDP equation then becomes

$$(i\beta^\mu\partial_\mu - m - U)\psi = 0 . \tag{3.1}$$

As is the case with the Dirac equation, the interaction U can be written as a sum of terms, each of which has a specific Lorentz character.[37] For example, one can have Lorentz tensors of second, third, and fourth rank. They will not be considered here since they may produce noncausal effects[38] in the KDP formalism. In addition, pseudoscalar and pseudovector terms also exist, but they are eliminated by the imposition of parity invariance. We are thus left with only Lorentz scalar and vector interactions to consider. As in the spin-0 KDP case, there are two scalars and two vectors. In addition to the ten-dimensional identity matrix, it can be shown that the projection matrix $P = \mathrm{diag}(1,1,1,1,0,0,0,0,0,0)$ transforms as a Lorentz scalar. The two Lorentz vectors may be written as β^μ and $P\beta^\mu$. Assuming rotational invariance and parity conservation, the form for U is taken to be

$$U(r) = S(r) + PS_p(r) + \beta^\mu V_\mu(r) + \beta^\mu P V_{p\mu}(r) . \tag{3.2}$$

In the case of elastic scattering, as is shown in Appendix A, the spacelike parts of the vectors can be transformed away by defining new phase-equivalent wave functions in exactly the same manner as for the Dirac equation.[1,3] In this case Eq. (3.2) becomes

$$U(r) = S(r) + PS_p(r) + \beta^0 V(r) + \beta^0 P V_p(r) . \tag{3.3}$$

It should be noted that although any combination of the interactions in Eq. (3.3) could be acceptable for the nuclear interactions, the electromagnetic interactions must be included as a Lorentz vector of the form $\beta^\mu A_\mu$, to ensure gauge invariance. (This corresponds to the usual "minimal substitution" prescription.) As is shown in Appendix B the KDP formalism for a spherically symmetric static Coulomb interaction yields the same equations as those derived by Corben and Schwinger[39] from the Proca equation. This establishes the equivalence of the KDP and Proca equations in this case.

For the more general interaction, given by Eq. (3.3), the time-dependent "Maxwell-like" form of the KDP equations is

$$i\nabla\cdot\mathbf{E} = m_1\varphi , \tag{3.4a}$$

$$\omega_2\mathbf{E} - i\nabla\times\mathbf{B} = m_1\,\mathbf{A} , \tag{3.4b}$$

$$\omega_1\,\mathbf{A} - i\nabla\varphi = m_2\mathbf{E} , \tag{3.4c}$$

$$-i\nabla\times\mathbf{A} = m_2\mathbf{B} , \tag{3.4d}$$

where

$$m_1(r) = m + S(r) + S_p(r) , \tag{3.5a}$$

$$m_2(r) = m + S(r) , \tag{3.5b}$$

$$\omega_1(r) = E - V(r) - V_p(r) , \tag{3.5c}$$

and

$$\omega_2(r) = E - V(r) . \tag{3.5d}$$

In order to obtain the elastic scattering observables from Eq. (3.4), it is only necessary to know the behavior of the wave function in the asymptotic region. Since this is the free-particle regime where the Proca and KDP formalisms are equivalent, the observables are fully determined by the four-vector A^μ, or more specifically the three independent components $\mathbf{A}$.[40] Therefore it is sufficient to calculate $\mathbf{A}$ in order to obtain the elastic scattering observables. Such an equation can be obtained by first eliminating Eqs. (3.4a) and (3.4d) from the set (3.4), and then eliminating $\mathbf{E}$ in favor of $\mathbf{A}$ giving

$$\omega_1\mathbf{A}+\nabla\left[\frac{1}{m_1}\nabla\cdot\left[\frac{m_1}{\omega_2}\mathbf{A}\right]\right]+\nabla\left\{\frac{1}{m_1}\nabla\cdot\left[\frac{1}{\omega_2}\nabla\times\left[\frac{1}{m_2}\nabla\times\mathbf{A}\right]\right]\right\}-\frac{m_1m_2}{\omega_2}\mathbf{A}-\frac{m_2}{\omega_2}\nabla\times\left[\frac{1}{m_2}(\nabla\times\mathbf{A})\right]=0\ . \quad (3.6)$$

Equation (3.6) will be used as the starting point for our investigations. Through the use of various spin and vector identities, it can be manipulated into the form of a second-order Schrödinger-like equation, discussed in the next section. Although one can of course work solely with the first-order Eq. (3.4), we will consider a second-order equation since "effective" central, spin-orbit, and tensor potentials can be defined in analogy with the Dirac case. This facilitates comparisons with standard Schrödinger-based analyses. In addition, existing codes for deuteron-nucleus scattering can be used to perform numerical calculations.

IV. THE SECOND-ORDER KDP EQUATION

In deriving an effective Schrödinger-like equation from Eq. (3.6) we will, as was done in Ref. 32, restrict ourselves to four special cases of the interaction U. The cases we consider correspond to choosing one scalar and one vector term each from Eq. (3.3). Thus, we have

case I:

$$U=PS(r)+\beta^0PV(r)\ , \quad (4.1a)$$

case II:

$$U=S(r)+\beta^0PV(r)\ , \quad (4.1b)$$

case III:

$$U=PS(r)+\beta^0V(r)\ , \quad (4.1c)$$

and

case IV:

$$U=S(r)+\beta^0V(r)\ , \quad (4.1d)$$

where each case is distinguished by whether or not the projection operator P appears in an interaction. We shall present an argument in the next section that the case-IV potential is the appropriate one to use for deuteron-nucleus scattering.

The second-order equation for each of the four cases is obtained from Eq. (3.6) by setting the appropriate quantities equal to zero in Eq. (3.5). Inspection of Eq. (3.6) reveals the third term to have the most complicated structure. However, for cases I and II (P in vector), where $\omega_2=$const, this term is identically zero because it is the gradient of a curl. In cases III and IV, this term is no longer zero, but will be neglected as it is of order m^{-3} and vanishes in the nonrelativistic limit. Therefore Eq. (3.6) is simplified to

$$m_2\nabla\times\left[\frac{1}{m_2}\nabla\times\mathbf{A}\right]-\omega_2\nabla\left[\frac{1}{m_1}\nabla\cdot\left[\frac{m_1}{\omega_2}\mathbf{A}\right]\right]+(m_1m_2-\omega_1\omega_2)\mathbf{A}\approx0\ , \quad (4.2)$$

where the above relationship holds exactly for cases I and II.

The manipulation of Eq. (4.2) into Schrödinger-like form is straightforward, but laborious. The details are given in Appendices C and D. In each case, the following effective equation is obtained:

$$\left[-\frac{1}{2E}\nabla^2+\frac{1}{2E}\hat{U}\right]\mathbf{A}=\frac{k^2}{2E}\mathbf{A}\ , \quad (4.3)$$

where

$$\hat{U}=U_c+U_{\text{s.o.}}(\mathbf{L}\cdot\mathbf{S})+U_D(\mathbf{r}\cdot\nabla)+U_{RR}^T(\mathbf{S}\cdot\mathbf{r})^2+iU_{PR}^T(\mathbf{S}\cdot\mathbf{p})(\mathbf{S}\cdot\mathbf{r}) \quad (4.4)$$

and $k^2=E^2-m^2$. As in the Dirac case, the effective second-order equation contains a central potential $[(1/2E)U_c]$, a spin-orbit potential $[(1/2E)U_{\text{s.o.}}]$, and a Darwin term $[(1/2E)U_D]$. In addition, there occur two of the possible three tensor potentials that can occur for spin-one scattering.[41] It was noted by Satchler[41] that the inclusion of a potential proportional to $(\mathbf{S}\cdot\mathbf{p})(\mathbf{S}\cdot\mathbf{r})$, as appears in Eq. (4.4), leads to an S matrix which is asymmetric. This formal property, however, should not cause any practical difficulties.

We also note that the effective potential in Eq. (4.4) is in a slightly different form than given in Ref. 30. In particular, the tensor potentials in Ref. 30 were given in Satchler invariant form,[41] whereas in the above equation they are not. If Eq. (4.3) with Eq. (4.4) were to be solved exactly, such a distinction would be immaterial. However, since the tensor potentials will be neglected in our actual calculations, the different ways of writing the tensor term constitute, in effect, different approximations owing to the presence of $(\mathbf{L}\cdot\mathbf{S})$ terms in them. The form used above maximizes the contributions of both the scalar and vector potentials to the $(\mathbf{L}\cdot\mathbf{S})$ term for the case of deuteron-nucleus scattering and yields slightly better agreement with the data. Whether it represents a better approximation to the exact KDP first-order equation (3.1) is not yet known. Obviously, the way to resolve such a question and also include the effects of all tensor potentials would be to solve the first-order Eq. (3.1) rather than Eq. (4.3). The appropriate codes for doing so are now being written.

In giving the specific forms of the effective potentials that occur in Eq. (4.4) for each case, it will be convenient to define the following quantities:

$$\Lambda=\frac{1}{r}\frac{\partial}{\partial r}\ln(m+S)\ , \quad (4.5)$$

$$\Sigma=\frac{1}{r}\frac{\partial}{\partial r}\ln(E-V_c)\ , \quad (4.6)$$

and

$$\Omega=\frac{1}{r}\frac{\partial}{\partial r}\ln(E-V_c-V) . \tag{4.7}$$

Using the above definitions, and the results in Appendices C and D the effective potentials for each case may be obtained. For the sake of generality, each of the expressions below explicitly contains a Coulomb potential, in addition to the potentials of Eq. (4.1). They are the following. Case I:

$$U_c^{(\mathrm{I})}=mS+2EV_c-V_c^2+(E-V_c)V-3\Lambda-r\frac{\partial\Lambda}{\partial r}+5\Sigma+r\frac{\partial\Sigma}{\partial r}+r^2\Sigma\Lambda-r^2\Sigma^2 , \tag{4.8a}$$

$$U_{\mathrm{s.o.}}^{(\mathrm{I})}=\Sigma-\Lambda , \tag{4.8b}$$

$$U_D^{(\mathrm{I})}=2\Sigma-\Lambda , \tag{4.8c}$$

$$U_{RR}^{T(\mathrm{I})}=\frac{1}{r}\frac{\partial\Lambda}{\partial r}-\frac{1}{r}\frac{\partial\Sigma}{\partial r}-\Sigma\Lambda+\Sigma^2 , \tag{4.8d}$$

$$U_{PR}^{T(\mathrm{I})}=\Lambda-2\Sigma . \tag{4.8e}$$

Case II:

$$U_c^{(\mathrm{II})}=2mS+S^2+2EV_c-V_c^2+(E-V_c)V-5\Lambda-r\frac{\partial\Lambda}{\partial r}+5\Sigma+r\frac{\partial\Sigma}{\partial r}+r^2\Sigma\Lambda-r^2\Sigma^2 , \tag{4.9a}$$

$$U_{\mathrm{s.o.}}^{(\mathrm{II})}=\Sigma-2\Lambda , \tag{4.9b}$$

$$U_D^{(\mathrm{II})}=2\Sigma-\Lambda , \tag{4.9c}$$

$$U_{RR}^{T(\mathrm{II})}=\frac{1}{r}\frac{\partial\Lambda}{\partial r}-\frac{1}{r}\frac{\partial\Sigma}{\partial r}-\Sigma\Lambda+\Sigma^2 , \tag{4.9d}$$

$$U_{PR}^{T(\mathrm{II})}=2\Lambda-2\Sigma . \tag{4.9e}$$

Case III:

$$U_c^{(\mathrm{III})}=mS+2E(V_c+V)-(V_c+V)^2-3\Lambda-r\frac{\partial\Lambda}{\partial r}+5\Omega+r\frac{\partial\Omega}{\partial r}+r^2\Omega\Lambda-r^2\Omega^2 , \tag{4.10a}$$

$$U_{\mathrm{s.o.}}^{(\mathrm{III})}=\Omega-\Lambda , \tag{4.10b}$$

$$U_D^{(\mathrm{III})}=2\Omega-\Lambda , \tag{4.10c}$$

$$U_{RR}^{T(\mathrm{III})}=\frac{1}{r}\frac{\partial\Lambda}{\partial r}-\frac{1}{r}\frac{\partial\Omega}{\partial r}-\Omega\Lambda+\Omega^2 , \tag{4.10d}$$

$$U_{PR}^{T(\mathrm{III})}=\Lambda-2\Omega . \tag{4.10e}$$

Case IV:

$$U_c^{(\mathrm{IV})}=2mS+S^2+2E(V_c+V)-(V_c+V)^2-5\Lambda-r\frac{\partial\Lambda}{\partial r}+5\Omega+r\frac{\partial\Omega}{\partial r}+r^2\Omega\Lambda-r^2\Omega^2 , \tag{4.11a}$$

$$U_{\mathrm{s.o.}}^{(\mathrm{IV})}=\Omega-2\Lambda , \tag{4.11b}$$

$$U_D^{(\mathrm{IV})}=2\Omega-\Lambda , \tag{4.11c}$$

$$U_{RR}^{T(\mathrm{IV})}=\frac{1}{r}\frac{\partial\Lambda}{\partial r}-\frac{1}{r}\frac{\partial\Omega}{\partial r}-\Omega\Lambda+\Omega^2 , \tag{4.11d}$$

$$U_{PR}^{T(\mathrm{IV})}=2\Lambda-2\Omega . \tag{4.11e}$$

The flexibility of the KDP formalism is illustrated by Eqs. (4.8)–(4.11). As can be seen, radically different effective equations result depending on the initial choices of vector and scalar interaction. For example, some of the central potentials contain terms quadratic in the interaction. In some cases, the nuclear vector interaction gives a contribution to the spin-orbit potential, while in others it does not. This variety occurs even though we have restricted our study to four special cases, whereas one has the freedom to choose any combination of the interactions that appear in Eq. (3.3).

The appropriate choice for U presumably depends on the particular system that is being studied. In the spin-zero case, for example, it was found that pion-nucleus data could best be described by one particular choice of the interaction, while kaon-nucleus data could be reasonably described with several different types of interactions.[32] For reasons discussed in the following section, we will use the case-IV interaction (P in neither) for deuteron-nucleus scattering. In the remainder of the paper we will only use the effective interaction given in Eq. (4.11), and omit the superscript IV.

V. DEUTERON-NUCLEUS SCATTERING

As noted earlier, the deuteron is a weakly bound neutron-proton system. This fact suggests that the neutron and proton interact almost independently with the target, therefore it is reasonable to describe the process in terms of nucleon-nucleus scattering. Such a picture is borne out in nonrelativistic analyses[42–44] and indeed, has been used extensively for deuteron-nucleus scattering.[16–18,44] We shall show below that the structure of the KDP equation is particularly amenable to the above picture of deuteron-nucleus scattering, and that the physics dictates a particular choice for the interaction. In order to have a consistent relativistic approach the input nucleon-nucleus potentials are taken to be the usual Lorentz scalar and vector combination familiar from Dirac analyses.

In discussing the deuteron-nucleus optical model investigated in this work, it is useful to recall the 16-dimensional reducible representation for β^μ given in Eq. (2.2). This structure is suggestive of a system of two noninteracting spin-$\frac{1}{2}$ particles. We can construct a 16-dimensional KDP equation using Eq. (2.2) and an interaction V of the form

$$V=U_1\otimes I_2+I_1\otimes U_2 , \tag{5.1}$$

which describes two Dirac particles interacting with the external fields U_1 and U_2. The U_i are taken to be the Dirac nucleon-nucleus optical potentials evaluated at half the incident deuteron energy and integrated over the finite coordinate space matter distribution of the deuteron, in analogy with the nonrelativistic models. Using the standard scalar-vector Dirac optical model we thus get

$$U_i(E)=I_iS^N(E/2)+\gamma_i^\mu V_\mu^N(E/2) , \tag{5.2}$$

where the N superscript denotes a nucleon-nucleus potential and S^N (V^N) is the scalar (vector) part of that potential. If it is assumed that $U_1=U_2$, then the 16-dimensional KDP equation is

$$\{\tfrac{1}{2}(I_1\otimes\gamma_2^\mu+\gamma_1^\mu\otimes I_2)[p_\mu-2V_\mu^N(E/2)] -(I_1\otimes I_2)[m+2S^N(E/2)]\}\psi=0 . \quad (5.3)$$

In the above equation twice the nucleon vector potential is substracted from the derivative term and twice the nucleon scalar potential appears as a "correction" to the mass. This indicates that the case-IV potential (P in neither), given by Eq. (4.1d), is an appropriate choice. With this in mind, the deuteron-nucleus optical potential we use is

$$U^D(E)=2S^N(E/2)+2V^N(E/2)\beta^0 , \quad (5.4a)$$

where

$$S^N(E/2,r)=\int d^3r'\rho_m(r')U_S\left[E/2,\left|\mathbf{r}+\frac{\mathbf{r}'}{2}\right|\right] , \quad (5.4b)$$

$\mathbf{r}$ is the relative deuteron-nucleus coordinate, $\mathbf{r}'$ is the relative neutron-proton coordinate in the deuteron, and

$$\rho_m(r')=\frac{1}{4\pi r'^2}[u(r')^2+w(r')^2] . \quad (5.4c)$$

The latter quantity defined in Eq. (5.4c) is the deuteron matter density where $u(r')$ and $w(r')$ are the $l=0$ and 2 radial wave functions of the deuteron, respectively.[45] In Eq. (5.4b) $U_S(E/2,r)$ is the scalar part of the actual proton-nucleus optical potential. An analogous expression is used to obtain $V^N(E/2)$.

We will present a comparison between the effective central and spin-orbit potentials in our model and those that arise from a nonrelativistic Watanabe approach. First, however, we discuss the effective potentials that arise from the model.

As with the Dirac equation, the central potential given in Eq. (4.11a) contains a cross term between the nuclear and Coulomb potential as well as terms quadratic in the scalar and vector potentials. These latter terms are responsible for the "wine-bottle" shape which appears to be necessary to describe proton-nucleus scattering around 200 MeV.[1] There is considerable evidence[15–17,34] that the intermediate-energy deuteron-nucleus data also prefer a wine-bottle shape rather than a Woods-Saxon form. The effective central potential obtained from the case-IV interaction displays the most pronounced wine-bottle shape in comparison with those of the other three cases.

The spin-orbit potential is surface peaked, with the scalar contribution multiplied by a factor of 2. This is an important feature of this model. The nonlocal Darwin term is roughly the same magnitude as in the proton-nucleus case. The term proportional to $(\mathbf{S}\cdot\mathbf{r})^2$ is about the same size as the corresponding term from the Watanabe model.[46] It should be noted that a nonzero tensor term always occurs in the present model. The tensor term U_{PR}^T arising from the KDP treatment is a feature of this approach which has not been previously considered.

We now compare our results with those that would be obtained from a Schrödinger-based Watanabe model. The Watanabe model,[18] described in the Introduction, gives an approximate relationship between the deuteron-nucleus and nucleon-nucleus optical potentials and is expected to be reasonable at intermediate energies.[16,17] Within such a model it can be shown that

$$U_{\text{cent}}^D(E)\approx 2U_{\text{cent}}^N(E/2) \quad (5.5a)$$

and

$$U_{\text{s.o.}}^D(E)\approx\tfrac{1}{2}U_{\text{s.o.}}^N(E/2) . \quad (5.5b)$$

The above equations are exact in the Watanabe model if a "point" deuteron, one in which the density is replaced by a delta function, is considered.

When the assumption in Eq. (5.4) is made for the KDP deuteron-nucleus potential it is easily seen that the effective central potential is roughly twice the effective central nucleon-nucleus potential obtained from the Dirac equation at half the incident deuteron energy, in agreement with Eq. (5.5a). It is much harder, however, to analytically compare the effective spin-orbit potentials. The approximation $E\approx m$ and $S^N\approx -V^N$ that are made in Refs. 22 and 23 are inappropriate at the energies we are considering and yield misleading results. Actual calculations show that the KDP effective spin-orbit potential is roughly 50–60 % the magnitude of the appropriate nucleon-nucleus potential for $E=400$–700 MeV, in approximate agreement with Eq. (5.5b). (The deuteron spin-orbit potential used in Ref. 30 is about 35–40 % of the appropriate nucleon potential at medium energy.) Thus it appears that the use of a Watanabe model in a relativistic wave equation does not yield a spin-orbit term that is too small. This fact, in turn, yields good agreement with the data, as will be seen in the following section.

VI. RESULTS

In this section we present calculations of deuteron-nucleus scattering observables based on the KDP equation and compare the results with experiment.[14] These calculations are "parameter-free" in the sense that no parameters are varied to fit the deuteron-nucleus data. The input nucleon-nucleus potentials used are based on both phenomenological and theoretical relativistic models.

The nuclear parts of the neutron and proton potentials are assumed to be identical and a Coulomb potential is added to the nuclear proton potential. The Darwin term in Eq. (4.11c), which makes only a small contribution to proton-nucleus scattering, is neglected. The tensor terms are also not included in these calculations as it has been shown[21] that tensor terms affect the observables σ, A_y, and A_{yy} only minimally for $d+{}^{58}$Ni at 80 MeV. Of course this may not be true at the higher energies and we

intend to consider the effect of including these terms in a later paper.

We first consider the case of deuterons elastically scattering from ^{58}Ni at 400 MeV. One difficulty with the analysis concerns the lack of proton elastic scattering data from ^{58}Ni at 200 MeV. However, there exists $p+^{40}$Ca data at this energy. As in Ref. 16, we assume the input $p+^{58}$Ni potentials can be obtained from phenomenological $p+^{40}$Ca potentials by "rescaling" the radius parameters in these potentials by the factor $A^{1/3}$. We use the parametrization given in Ref. 16. The resulting deuteron-nucleus scalar and vector potentials, shown in Fig. 1, are large and of opposite sign. Just as in the case of the Dirac equation, they "cancel" and produce an effective central potential of reasonable size.

The effective KDP central and spin-orbit potentials are shown in Fig. 2 along with those of Yahiro *et al.*[16] The effective central potential has the Coulomb potential V_C subtracted before plotting. In general the shapes of each component of the KDP effective potential and that of Ref. 16 are similar, however the absolute magnitudes are different. In particular, the KDP approach produces a wine-bottle shape for the real central potential, which, as noted above, is an important ingredient to the success of the model in Ref. 16. The absolute magnitude obtained here is somewhat larger than that in Ref. 16. The absorptive central potential has a characteristic volume-type geometry and is smaller in magnitude than that in Ref. 16. The spin-orbit potentials are surface peaked and are slightly smaller in absolute magnitude than the potentials in Ref. 16.

The results of the calculation and a comparison with experimental data[14] are shown in Fig. 3. The overall magnitude and general rate of decrease with increasing scattering angle for the differential cross-section data are reproduced. The angular position of the first and each subsequent minimum is, however, predicted at too small an angle. Virtual deuteron breakup effects for this case were shown by Yahiro *et al.*[16,17] to be most significant in the differential cross section at angles greater than 20° c.m. Such effects might improve the KDP prediction by increasing the differential cross section beyond 20° c.m.

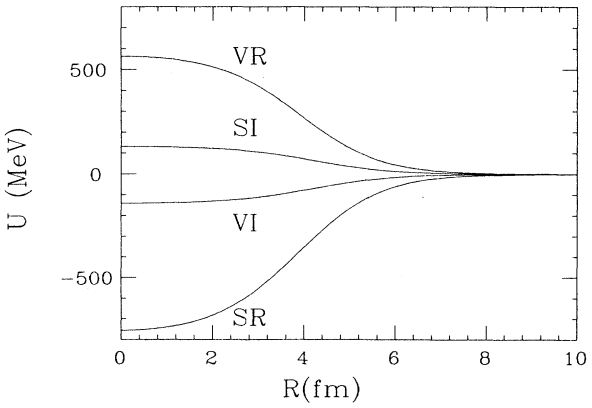

FIG. 1. The KDP scalar and vector potentials for $d+^{58}$Ni elastic scattering at 400 MeV.

The damping of the diffractive, oscillatory pattern in the data between 12° and 20° c.m. is similar to that observed in proton-nucleus elastic scattering data at similar incident energy per nucleon and similar momentum transfer.[47,48] This damping phenomenon in nucleon-nucleus scattering results from a complicated interplay between the separate energy dependences of the real and imaginary parts of both the spin-independent and spin-orbital optical potentials.[48] It is interesting that these 200 MeV per nucleon deuteron-nucleus scattering data display a similar structure, suggesting a dynamical origin analogous to that for nucleon-nucleus scattering. Both the KDP calculations and the nonrelativistic Watanabe predictions of Yahiro *et al.*[16] display similar damping at intermediate angles although both fail to reproduce the $d+^{58}$Ni data in detail. The predicted damping is clearly a consequence of the proton-nucleus input potentials which produce this behavior in $p+^{40}$Ca elastic scattering at 200 MeV. More detailed analysis must await the availability of 200-MeV $p+^{58}$Ni elastic scattering data whereby the $A^{1/3}$ scaling of the $p+^{40}$Ca optical potentials can be avoided. Alternatively, one could use a global optical potential; such potentials are being developed.[49]

The agreement with the A_y data is good over the for-

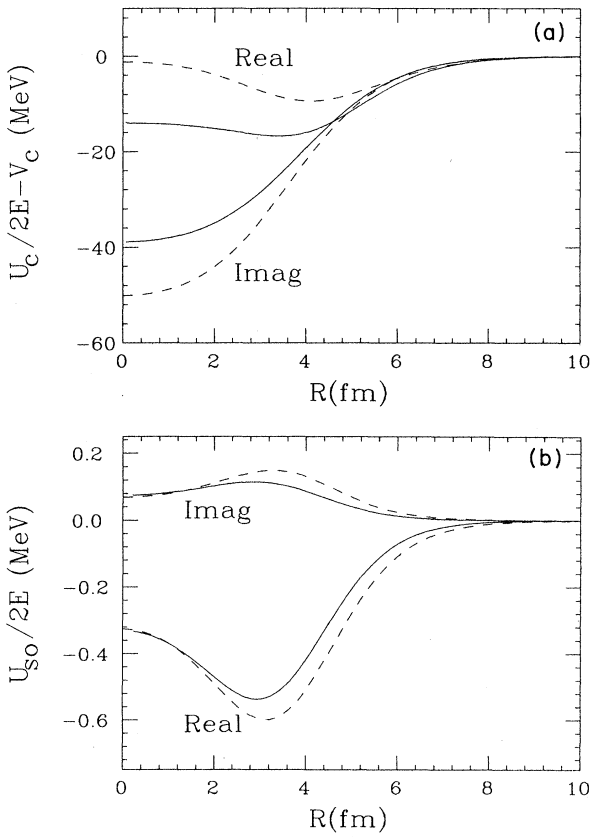

FIG. 2. Effective central and spin-orbit potentials for $d+^{58}$Ni elastic scattering at 400 MeV. The solid lines represent the KDP model predictions and the dashed curves the model prediction of Ref. 16.

ward angular range of the data. The calculated structure of the tensor polarization A_{yy} agrees qualitatively with the data, however, the predicted overall magnitude is too small. It is possible that the inclusion of tensor potentials may affect this observable. It should be noted that Greben[50] found it impossible to fit this observable within a relativistic Watanabe model for $d+^{16}$O scattering, although he did not consider effective Schrödinger potentials deduced from a Dirac approach, as was done by Yahiro *et al.*[16]

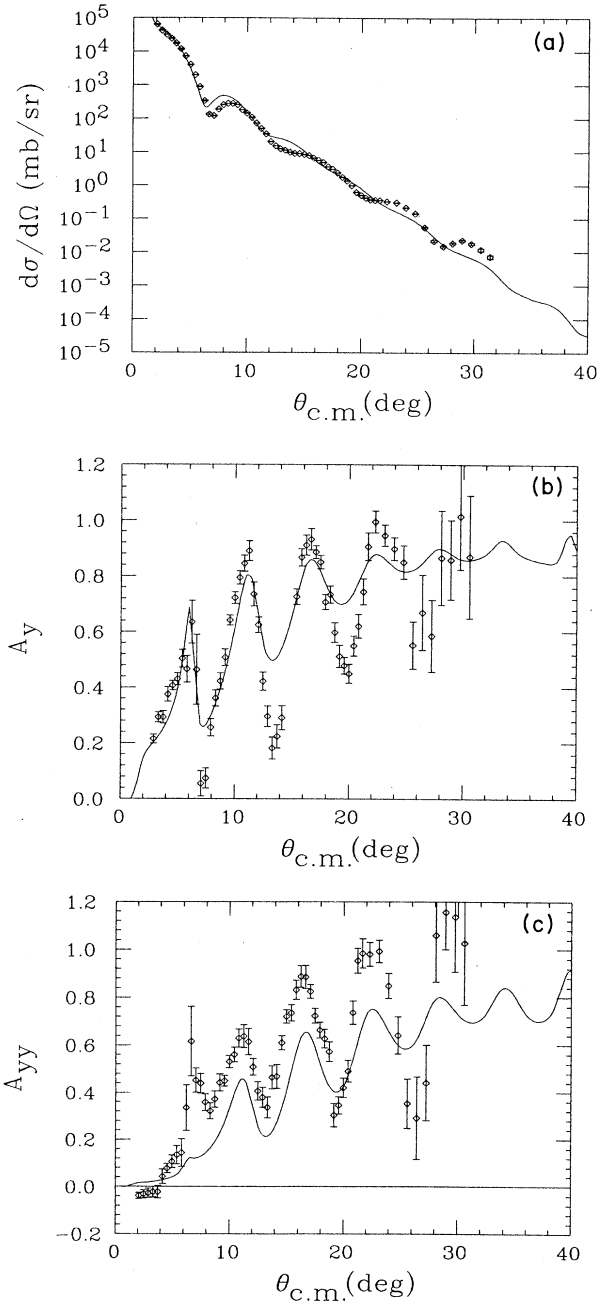

FIG. 3. Elastic scattering observables for $d+^{58}$Ni at 400 MeV. The experimental data are from Ref. 14 and the solid curves are the predictions of the KDP model.

We now turn to an examination of the $d+^{40}$Ca data at 700 MeV. For this analysis, nucleon-nucleus optical potentials at 350 MeV are needed. Since $p+^{40}$Ca data exist at 362 MeV,[51] phenomenological potentials are available at nearly the correct energy with no "rescaling" necessary as was the case for the ^{58}Ni target. In addition, the Murdock-Horowitz parameter-free relativistic nucleon-nucleus optical potential[12] can be employed. This model is similar to the relativistic impulse approximation[5,6] (RIA) model except that pseudovector coupling is substituted for pseudoscalar coupling, exchange terms are explicitly treated, and the effects of the nuclear medium are included. In the following, both types of input proton-nucleus optical potentials will be used in the relativistic KDP model. The results will be compared with those obtained from a nonrelativistic Watanabe model based on the Schrödinger equation.[34]

As in the previous case, the deuteron-nucleus scalar and vector potentials are large and have opposite signs. The effective potentials, however, are again of reasonable magnitude. A comparison of the KDP-based effective potentials, calculated as in Fig. 2, and those obtained from a Schrödinger-based Watanabe model with input Dirac effective potentials is shown in Fig. 4. The nucleon-nucleus potential parameters of Ref. 51 have

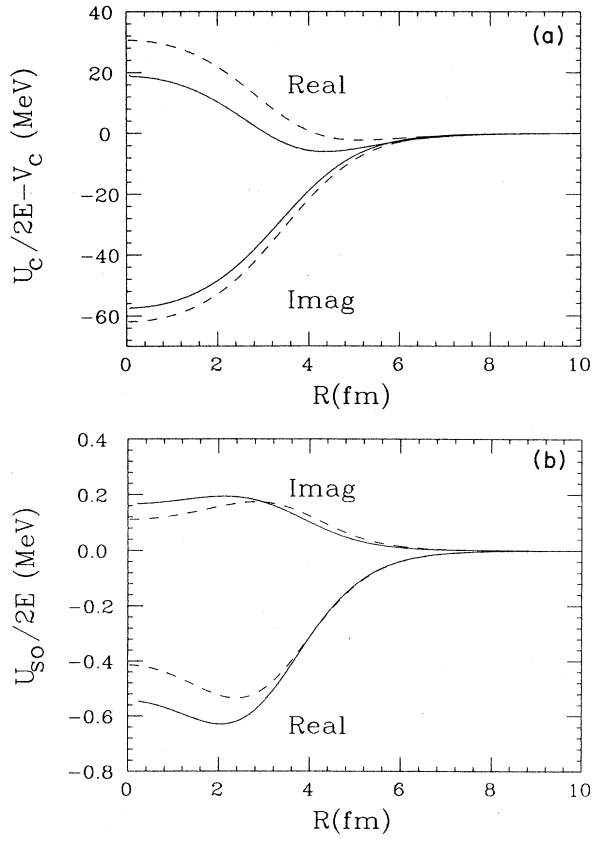

FIG. 4. Effective central and spin-orbit potentials for $d+^{40}$Ca elastic scattering at 700 MeV. The solid lines represent the KDP model predictions and the dashed curves the nonrelativistic model predictions of Ref. 34.

been used. The two models are seen to give similar effective potentials. Again, the main features of the KDP effective potentials are a wine-bottle-shaped central potential and a surface-peaked spin-orbit potential with comparable magnitude to the Schrödinger-based Watanabe model.

The scattering observables calculated with the KDP-based Watanabe model for various input nucleon-nucleus potentials are compared with data in Fig. 5. It is seen that the calculation based on proton-nucleus Dirac phenomenology agrees qualitatively with the observed differential cross section at forward angles, while it is below the data at larger angles. Virtual deuteron breakup effects might improve the agreement.[16] The KDP differential cross-section prediction based on the Murdock-Horowitz $p+^{40}$Ca optical potential is grossly lacking in sharp diffractive structure and generally lies above the data. This failure is due, for the most part, to inadequacies in the input $p+^{40}$Ca optical potential which poorly describes the $p+^{40}$Ca, 362-MeV differential cross-section data.

Both KDP predictions, shown in Fig. 5(b), for the 700-MeV $d+^{40}$Ca vector analyzing power are fairly good. For both $p+^{40}$Ca input optical potentials, a good description of the 362-MeV $p+^{40}$Ca analyzing power data is provided. The principal deficiency with the proton-nucleus Dirac phenomenology-based prediction is that the minima are too shallow, corresponding to insufficiently deep diffractive minima in the differential cross-section predictions. The Murdock-Horowitz-based KDP predictions display the same problem with even shallower minima, reflecting the washed out structure in the differential cross-section predictions. Otherwise, both calculations produce very similar $d+^{40}$Ca A_y predictions except for the position of the second maximum which occurs at 8° c.m. (9° c.m.) for the Dirac phenomenology-(Murdock-Horowitz) based KDP prediction. This shift in angular position of the second maximum in A_y also appears in the 362-MeV $p+^{40}$Ca A_y calculations. In both cases the results using the Dirac phenomenology are in better agreement with the data.

Both calculations yield a qualitative description of the structure of A_{yy} and have the common feature that the minima are too deep and that the overall magnitude is too small. Again, the disagreement may be the result of our neglect of the tensor potentials.

In Fig. 6 we compare the KDP results shown by the solid curves from Fig. 5 with the results of a Schrödinger equation Watanabe model calculation. The latter calculation uses Schrödinger equivalent central and spin-orbit optical potentials obtained from the Dirac equation fit to the 362-MeV $p+^{40}$Ca data. The resulting potentials are folded with the deuteron matter density as in Eq. (5.4b) and as described in Ref. 34. The general structure of the measured differential cross section is reproduced, however, the predicted overall magnitude is too small and the angular position of each diffractive minimum occurs at successively too small an angle, unlike that for the corresponding KDP prediction. Virtual deuteron breakup[16,17] and medium effects[34] might alleviate some of this discrepancy with the nonrelativistic model predictions. The A_y data are described very well overall while the A_{yy} data are described only qualitatively.

Since the Schrödinger equation calculation and the KDP equation calculation are both based on the same proton-nucleus optical potential which provides an excellent description of the 362-MeV $p+^{40}$Ca elastic scatter-

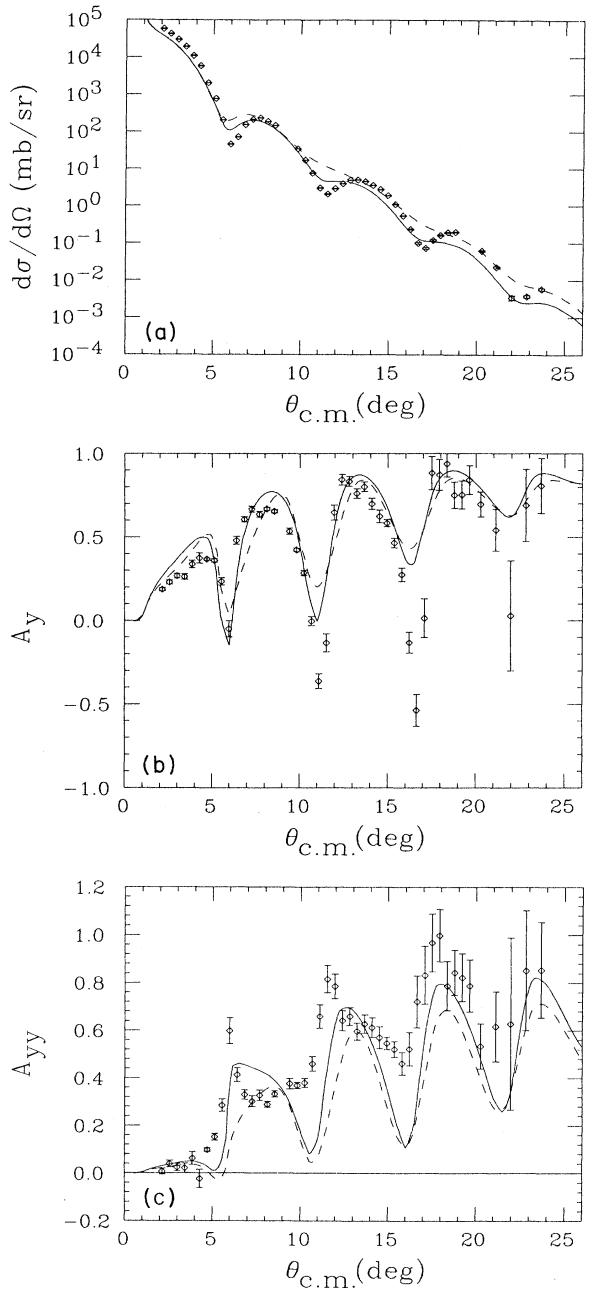

FIG. 5. Elastic scattering observables for $d+^{40}$Ca at 700 MeV. The experimental data are from Ref. 14 and the two sets of curves represent KDP model calculations with different input nucleon-nucleus potentials. The solid curves present calculations which use Dirac phenomenological potentials while the dashed curves show calculations employing the Murdock-Horowitz potentials.[12]

ing data, comparison of these two sets of predicted observables provides a measure of the differences between the KDP and the traditional Schrödinger equation approaches. The differences are significant with both models displaying advantages and disadvantages, but neither of these first-order calculations provides a quantitative fit to the data.

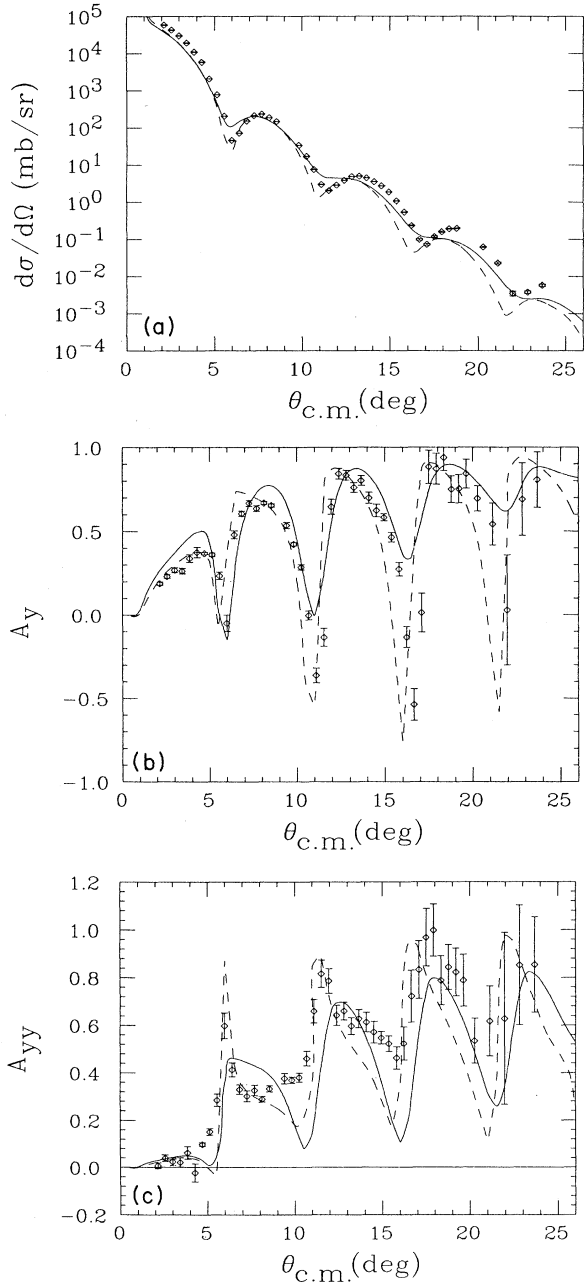

FIG. 6. Elastic scattering observables for $d+^{40}$Ca at 700 MeV. The data are from Ref. 14. The dashed curves are from a Schrödinger equation Watanabe model calculation using Schrödinger equivalent, Dirac phenomenological nucleon-nucleus potentials. The corresponding KDP calculations (solid curves) are the same as the solid curves in Fig. 5.

VII. SUMMARY

In this paper we have developed the spin-one KDP formalism and applied it to intermediate-energy deuteron-nucleus elastic scattering. The formalism, which is equivalent to the more familiar Proca equation in the free-particle case gives, in general, different results when interactions are considered. The KDP equation is particularly flexible in regard to interactions, as two distinct Lorentz scalars and vectors exist. We have performed an approximate reduction of the first-order KDP equation to an effective second-order form for various choices of the interaction. A heuristic argument was given as to why the "two-body" nature of the deuteron implies a specific form of the interaction to be used for deuteron-nucleus scattering.

A relativistic generalization of the Watanabe model was used to determine the KDP deuteron-nucleus scalar and vector potentials in terms of the nucleon-nucleus scalar and vector potentials obtained from a Dirac equation approach. No further parameters were varied in an attempt to optimize the fit to the deuteron-nucleus data. This was done in order to test the consistency of a relativistic approach to nuclear scattering and also to investigate the predictive power of the model. Various effects were omitted from our treatment; these include the tensor potentials that occur in the effective second-order KDP equation, and virtual deuteron breakup. The tensor potentials were neglected in order to make it possible to perform calculations with existing computer codes. A straightforward way of including them would be to solve the first-order equation (3.1). The breakup effect involves the complexities of the relativistic few-body problem and is more difficult to deal with.

We have seen that a generalization of the Watanabe model based on a one-body spin-1 relativistic wave equation provides a reasonable starting point for describing intermediate-energy deuteron-nucleus scattering. This conclusion is based on a direct comparison of the predicted scattering observables with experimental data. It would be interesting to see how the above results are modified if Eq. (3.1) is solved exactly or if one includes a relativistic structure model. The present treatment is a "first-order" model of relativistic deuteron-nucleus scattering which possesses predictive power and provides evidence for the consistency of the relativistic approach.

ACKNOWLEDGMENTS

The authors thank F. Santos, E. Stephenson, and P. Schwandt for helpful conversations, and C. Horowitz and D. Murdock for providing their proton-nucleus optical potentials. This work was supported in part by the National Science Foundation under Grant Phy8600702 and the U.S. Department of Energy.

APPENDIX A

It is demonstrated that, for a spherically symmetric interaction, the spacelike parts of the vector potentials do not affect the elastic scattering.

We write Eq. (3.1) in time-independent form

$$(\beta^0 E + i\boldsymbol{\beta}\cdot\nabla - m - U)\psi = 0 \ , \tag{A1}$$

and assume that U in Eq. (3.2) has spherical symmetry and therefore has the form

$$U(r) = S(r) + PS_p(r) + \beta^0 V^0(r) - \boldsymbol{\beta}\cdot\hat{r} V^r(r) + \beta^0 P V_p^0(r) - \boldsymbol{\beta}\cdot\hat{r} P V_p^r(r) \ . \tag{A2}$$

We define the new wave function χ by

$$\psi = \exp\left[-i\int_r^\infty [V^r(r') + PV_p^r(r')]dr'\right]\chi \ . \tag{A3}$$

It is clear that for short-ranged interactions χ approaches ψ as $r\to\infty$. Thus χ will yield the same scattering observables as ψ. An equation for χ can be derived by substituting (A3) into (A1). The result is

$$\{\beta^0[E - V(r) - PV_p(r)] + i\boldsymbol{\beta}\cdot\nabla - [m + S(r) + PS_p(r)]\}\chi = 0 \ , \tag{A4}$$

where the 0 superscripts have been dropped from the timelike vector potentials.

The "phase-equivalent" wave function χ obeys an equation which is identical to that obeyed by ψ except that the spacelike parts of the vector potentials do not appear. Hence, one need only consider the simpler interaction given in Eq. (3.3) for the case of elastic scattering.

APPENDIX B

We show that for a spherically symmetric static Coulomb potential, the KDP and Proca equations yield identical results.

We work with Eqs. (3.4) and (3.5) with $m_1 = m_2 = m$ and $\omega_1 = \omega_2 \equiv \omega$ where $\omega = E - V_C$ with V_C being the static Coulomb potential. Eliminating **E** and **B** from Eq. (3.4) gives

$$i\nabla\cdot(\omega\mathbf{A}) + \nabla^2\varphi = m^2\varphi \ , \tag{B1}$$

and

$$\omega^2\mathbf{A} - i\omega\nabla\varphi - \nabla\times\nabla\times\mathbf{A} = m^2\mathbf{A} \ . \tag{B2}$$

By taking the gradient of Eq. (B2) and substituting from Eq. (B1), the following useful auxiliary equation is obtained:

$$\nabla\cdot\mathbf{A} + i\omega\varphi = f \ , \tag{B3}$$

where

$$f \equiv \frac{1}{m^2}[\nabla\omega\cdot(\omega\mathbf{A}) - i\nabla\omega\cdot\nabla\varphi] \ .$$

We now use the vector identity

$$\nabla\times\nabla\times\mathbf{A} = \nabla(\nabla\cdot\mathbf{A}) - \nabla^2\mathbf{A} \tag{B4}$$

in Eq. (B2) and substitute the result in Eq. (B3) to get

$$(\nabla^2 + \omega^2 - m^2)\mathbf{A} = -i(\nabla\omega)\varphi + \nabla f \ . \tag{B5}$$

Inserting Eq. (B3) into Eq. (B1) yields the equation

$$(\nabla^2 + \omega^2 - m^2)\varphi = -i\nabla\omega\cdot\mathbf{A} - i\omega f \ . \tag{B6}$$

When the explicit expression for ω is substituted into Eqs. (B3), (B5), and (B6) they become, assuming spherical symmetry,

$$\nabla\cdot\mathbf{A} + i\omega\varphi = -\frac{1}{m^2}\frac{dV_C}{dr}\left[(E - V_C)\frac{\mathbf{r}}{r}\cdot\mathbf{A} - i\frac{\partial\varphi}{\partial r}\right] \ , \tag{B7}$$

$$(\nabla^2 + \omega^2 - m^2)\mathbf{A} = \frac{i}{r}\frac{dV_C}{dr}\mathbf{r}\varphi - \frac{1}{m^2}\nabla\left[\frac{dV_C}{dr}\left[(E - V_C)\frac{\mathbf{r}}{r}\cdot\mathbf{A} - i\frac{\partial\varphi}{\partial r}\right]\right] \ , \tag{B8}$$

and

$$(\nabla^2 + \omega^2 - m^2)\mathbf{A} = i\frac{dV_C}{dr}\frac{\mathbf{r}}{r}\cdot\mathbf{A} + i\frac{\omega}{m^2}\frac{dV_C}{dr}\left[(E - V_C)\frac{\mathbf{r}}{r}\cdot\mathbf{A} - i\frac{\partial\varphi}{\partial r}\right] \ , \tag{B9}$$

which are identical to the equations derived by Corben and Schwinger[39] starting from the Proca formalism. (In order to make contact with the conventions of Ref. 39, the replacements $\varphi\to -i\varphi$ and $V_C\to -V_C$ must be made.)

APPENDIX C

We derive some relations which will be used in Appendix D to determine the structure of Eq. (4.2). In order to write Eq. (4.2) in terms of spin operators, the following representation of the spin-one matrices will be used:

$$[S_k]_{ij} = -i\epsilon_{ijk} \ , \tag{C1}$$

where ϵ_{ijk} is the totally antisymmetric Levi-Cevita symbol. One can also represent vector cross products in terms of ϵ_{ijk} as

$$(\mathbf{C}\times\mathbf{D})_i = \epsilon_{ijk} C_j D_k \ . \tag{C2}$$

We first wish to rewrite the expression $\mathbf{r}(\mathbf{r}\cdot\mathbf{A})$ in terms of spin-one matrices. This can be done by expanding the expression $(\mathbf{S}\cdot\mathbf{r})(\mathbf{S}\cdot\mathbf{r})\mathbf{A}$ to obtain

$$[(\mathbf{S}\cdot\mathbf{r})(\mathbf{S}\cdot\mathbf{r})\mathbf{A}]_i = \epsilon_{ijk}\epsilon_{jlm} r_k r_m A_l \ , \tag{C3}$$

where Eq. (C1) has been used. Employing the well-known identity

$$\epsilon_{rmn}\epsilon_{rst} = \delta_{ms}\delta_{nt} - \delta_{mt}\delta_{ns} \ , \tag{C4}$$

where δ_{ij} is the Kronecker delta, Eq. (C3) becomes

$$[(\mathbf{S}\cdot\mathbf{r})(\mathbf{S}\cdot\mathbf{r})\mathbf{A}]_i = r_k r_k A_i - r_i r_k A_k \ . \tag{C5}$$

Converting Eq. (C5) back to vector notation and comparing with Eq. (C3) yields the identity

$$\mathbf{r}(\mathbf{r}\cdot\mathbf{A}) = r^2\mathbf{A} - (\mathbf{S}\cdot\mathbf{r})(\mathbf{S}\cdot\mathbf{r})\mathbf{A} \tag{C6}$$

which will be used in the following appendix.

It will be convenient to rewrite the vector expression $\nabla\times(\mathbf{r}\times\mathbf{A})$. Expanding into components gives

$$[\nabla\times(\mathbf{r}\times\mathbf{A})]_i=\epsilon_{ijk}\nabla_j\epsilon_{klm}r_l A_m \ . \tag{C7}$$

The above equation can be expressed in terms of spin-one matrices with the help of Eq. (C1) where the right-hand side of Eq. (C7) becomes

$$-i[S_j]_{ik}P_j[S_l]_{km}r_l A_m \ . \tag{C8}$$

Combining Eqs. (C7) and (C8) results in the following identity:

$$\nabla\times(\mathbf{r}\times\mathbf{A})=-i(\mathbf{S}\cdot\mathbf{p})(\mathbf{S}\cdot\mathbf{r})\mathbf{A} \ , \tag{C9}$$

which will be used below.

The expression $(\mathbf{r}\times\nabla)\times\mathbf{A}$ also appears when manipulating Eq. (4.2) and is easily seen to give a spin-orbit type of contribution. Using the definition of the angular momentum vector, we get

$$(\mathbf{r}\times\nabla)\times\mathbf{A}=i\mathbf{L}\times\mathbf{A} \ . \tag{C10}$$

The expression on the right-hand side of the above equation is given in terms of components as

$$i\epsilon_{ijk}L_j A_k \ . \tag{C11}$$

If Eq. (C1) is substituted into the above expression the following identity is seen to hold:

$$(\mathbf{r}\times\nabla)\times\mathbf{A}=(\mathbf{L}\cdot\mathbf{S})\mathbf{A} \ . \tag{C12}$$

It is also necessary to rewrite $\mathbf{r}(\nabla\cdot\mathbf{A})$. With the help of the following vector identity:

$$\mathbf{C}(\nabla\cdot\mathbf{D})=(\mathbf{C}\cdot\nabla)\mathbf{D}+\nabla\times(\mathbf{C}\times\mathbf{D})-(\mathbf{D}\cdot\nabla)\mathbf{C}+\mathbf{C}(\nabla\cdot\mathbf{D}) \ , \tag{C13}$$

and using the special properties of the vector $\mathbf{r}$ we obtain

$$\mathbf{r}(\nabla\cdot\mathbf{A})=2\mathbf{A}+(\mathbf{r}\cdot\nabla)\mathbf{A}+\nabla\times(\mathbf{r}\times\mathbf{A}) \ . \tag{C14}$$

On applying Eq. (C9), the above equation becomes

$$\mathbf{r}(\nabla\cdot\mathbf{A})=2\mathbf{A}+(\mathbf{r}\cdot\nabla)\mathbf{A}-i(\mathbf{S}\cdot\mathbf{p})(\mathbf{S}\cdot\mathbf{r})\mathbf{A} \ , \tag{C15}$$

which is a useful result for Appendix D.

We will also need to reexpress $\mathbf{r}\times(\nabla\times\mathbf{A})$ in terms of spin-one matrices. Written in terms of components it is

$$\epsilon_{ijk}r_j\epsilon_{klm}\nabla_l A_m \ . \tag{C16}$$

Using the permutation properties of the Levi-Cevita symbol expression (C16) becomes

$$\epsilon_{kij}\epsilon_{klm}r_j\nabla_l A_m \ . \tag{C17}$$

If the identity (C4) is used to compare the factor $\epsilon_{kij}\epsilon_{klm}$ with $\epsilon_{ikm}\epsilon_{kjl}$, it can be seen that expression (C17) is equivalent to

$$(\epsilon_{ikm}\epsilon_{kjl}+\delta_{ml}\delta_{ij}-\delta_{im}\delta_{jl})r_j\nabla_l A_m \ , \tag{C18}$$

or in vector notation,

$$(\mathbf{r}\times\nabla)\times\mathbf{A}+\mathbf{r}(\nabla\cdot\mathbf{A})-(\mathbf{r}\cdot\nabla)\mathbf{A} \ . \tag{C19}$$

Combining Eqs. (C12), (C15), and (C16) we obtain

$$\mathbf{r}\times(\nabla\times\mathbf{A})=2\mathbf{A}+(\mathbf{L}\cdot\mathbf{S})\mathbf{A}-i(\mathbf{S}\cdot\mathbf{p})(\mathbf{S}\cdot\mathbf{r})\mathbf{A} \ . \tag{C20}$$

Finally, we wish to rewrite the vector expression $\nabla(\mathbf{r}\cdot\mathbf{A})$. Using the identity

$$\nabla(\mathbf{C}\cdot\mathbf{D})=(\mathbf{D}\cdot\nabla)\mathbf{C}+(\mathbf{C}\cdot\nabla)\mathbf{D}+\mathbf{D}\times(\nabla\times\mathbf{C})+\mathbf{C}\times(\nabla\times\mathbf{D}) \ , \tag{C21}$$

and again the properties of the $\mathbf{r}$ vector, it is found that

$$\nabla(\mathbf{r}\cdot\mathbf{A})=3\mathbf{A}+(\mathbf{r}\cdot\nabla)\mathbf{A}+(\mathbf{r}\times\nabla)\times\mathbf{A}+\nabla\times(\mathbf{r}\times\mathbf{A}) \ . \tag{C22}$$

Substituting from Eqs. (C9) and (C12) yields

$$\nabla(\mathbf{r}\cdot\mathbf{A})=3\mathbf{A}+(\mathbf{r}\cdot\nabla)\mathbf{A}+(\mathbf{L}\cdot\mathbf{S})\mathbf{A}-i(\mathbf{S}\cdot\mathbf{p})(\mathbf{S}\cdot\mathbf{r})\mathbf{A} \ . \tag{C23}$$

The relevant equations for Appendix D are (C6), (C15), (C20), and (C23).

APPENDIX D

In this appendix Eqs. (4.3) and (4.4) are derived from Eq. (4.2) and the effective potentials given in Eqs. (4.8)–(4.11) are obtained for the various cases.

The left-hand side of Eq. (4.2) is seen to consist of three terms. The structure of the third term is trivial and will not be explicitly worked out here. The first term consists of two pieces:

$$m_2\nabla\times\left[\frac{1}{m_2}\nabla\times\mathbf{A}\right]=\nabla\times\nabla\times\mathbf{A}+\left[m_2\nabla\frac{1}{m_2}\right](\nabla\times\mathbf{A}) \ . \tag{D1}$$

For convenience, we make the definitions

$$F_1\equiv\nabla\times\nabla\times\mathbf{A} \ , \tag{D2}$$

and

$$F_2\equiv\left[m_2\nabla\frac{1}{m_2}\right]\times(\nabla\times\mathbf{A}) \ . \tag{D3}$$

While F_2 is nonzero only for cases II and IV, the F_1 term is present in every case. Using the vector identity Eq. (B4), it is seen that

$$F_1=\nabla(\nabla\cdot\mathbf{A})-\nabla^2\mathbf{A} \ . \tag{D4}$$

For a spherically symmetric potential, the F_2 term can be rewritten in the form

$$F_2=-\Lambda[\mathbf{r}\times(\nabla\times\mathbf{A})] \ , \tag{D5}$$

with Λ defined by Eq. (4.5). After substituting from Eq. (C20), the above equation now reads

$$F_2=-\Lambda[2+(\mathbf{L}\cdot\mathbf{S})-i(\mathbf{S}\cdot\mathbf{p})(\mathbf{S}\cdot\mathbf{r})]\mathbf{A} \ . \tag{D6}$$

It is also useful to split the second term from the left-hand side of Eq. (4.2) into two terms as

$$-\omega_2\nabla\left[\frac{1}{m_1}\cdot\nabla\left[\frac{m_1}{\omega_2}\mathbf{A}\right]\right]=G_1+G_2\ , \tag{D7}$$

where

$$G_1\equiv-\nabla\left[\frac{1}{m_1}\nabla\cdot(m_1\mathbf{A})\right]\ , \tag{D8}$$

and

$$G_2\equiv-\omega_2\left[\nabla\frac{1}{\omega_2}\right]\frac{1}{m_1}\nabla\cdot(m_1\mathbf{A})-\omega_2\nabla\left[\left[\nabla\frac{1}{\omega_2}\right]\cdot\mathbf{A}\right]\ . \tag{D9}$$

The term G_1 contributes in each of the four cases, while G_2 yields a nonzero contribution from the Coulomb potential for cases I and II and for Coulomb plus the nuclear vector potential for cases III and IV. For cases III and IV, assuming spherical symmetry, we have

$$G_1=-\nabla(\nabla\cdot\mathbf{A})-\frac{1}{r}\frac{\partial\Lambda}{\partial r}\mathbf{r}(\mathbf{r}\cdot\mathbf{A})-\Lambda\nabla(\mathbf{r}\cdot\mathbf{A})\ , \tag{D10}$$

and

$$G_2=\Omega[\mathbf{r}(\nabla\cdot\mathbf{A})+\nabla(\mathbf{r}\cdot\mathbf{A})]+\left[\frac{1}{4}\frac{\partial\Omega}{\partial r}+\Omega\Lambda-\Omega^2\right]\mathbf{r}(\mathbf{r}\cdot\mathbf{A})\ , \tag{D11}$$

where Ω is defined in Eq. (4.7). Using identities (C6) and (C23) in Eq. (D10) yields

$$G_1=-\nabla(\nabla\cdot\mathbf{A})-\Lambda[3+(\mathbf{r}\cdot\nabla)+(\mathbf{L}\cdot\mathbf{S})-i(\mathbf{S}\cdot\mathbf{p})(\mathbf{S}\cdot\mathbf{r})]\mathbf{A}+\frac{1}{r}\frac{\partial\Lambda}{\partial r}[(\mathbf{S}\cdot\mathbf{r})^2-r^2]\mathbf{A}\ , \tag{D12}$$

while employing identities (C6), (C15), and (C23) in Eq. (D11) gives

$$G_2=\Omega[5+2(\mathbf{r}\cdot\nabla)+(\mathbf{L}\cdot\mathbf{S})-2i(\mathbf{S}\cdot\mathbf{p})(\mathbf{S}\cdot\mathbf{r})]\mathbf{A}+\left[\frac{1}{r}\frac{\partial\Omega}{\partial r}+\Omega\Lambda-\Omega^2\right][r^2-(\mathbf{S}\cdot\mathbf{r})^2]\mathbf{A}\ . \tag{D13}$$

For cases I and II replace Ω by Σ in Eqs. (D11) and (D13).

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