PHYSICAL REVIEW C VOLUME 40, NUMBER 5 NOVEMBER 1989

Calculating deuteron photodisintegration amplitudes using nonsingular scattering equations

Chr. von Ferber* and W. Sandhas
Physikalisches Institut, Universität Bonn, D-5300 Bonn 1, Federal Republic of Germany

H. Haberzettl
Department of Physics, The George Washington University, Washington, D.C. 20052
(Received 12 July 1989)

We derive a momentum-space formulation of the deuteron photodisintegration in terms of nonsingular scattering integral equations. The two-body T matrix describing the nuclear final-state interactions enters only on shell, thus offering the possibility of determining it entirely in terms of experimental phase shifts. The formalism suggests a lowest-order approximation which goes beyond the usual plane-wave approximation and which contains the full information of the on-shell final-state interaction. Test calculations using the Reid soft-core potential are reported. It is found that the numerical solution of photodisintegration observables is greatly facilitated by the proposed formalism. Moreover, the lowest-order approximation is seen to be superior to the usual plane-wave approximation, providing good results even in those cases where the usual plane-wave approximation vanishes. Extensions to more complex few-nucleon systems are discussed.

I. INTRODUCTION

The exact calculation of observables for the photodisintegration of the deuteron traditionally is done in position space.[1-3] One usually determines the corresponding matrix elements by first solving the Schrödinger equation for the coordinate-space scattering wave function describing the final-state interaction of the two outgoing nucleons and by a subsequent integration over the respective plane-wave approximation [cf. Eq. (1)]. It has been known for some time that it is also possible to formulate the problem in terms of an integral equation in momentum space,[4] in much the same way in which nuclear transition matrix elements are obtained from the (momentum-space) Lippmann-Schwinger equation for the T matrix. Although, to our knowledge, this procedure has never been used in any numerical investigation of the photodisintegration of the deuteron, it is the standard method for the exact calculation of photodisintegration amplitudes within Faddeev-type integral equation approaches to the three-and four-nucleon problems.[4-6]

In this paper, we will use the integral equation approach to calculate observables for the photodisintegration of the deuteron. In doing so, we will go one step beyond the above-mentioned well-known formulation in terms of an analog of the usual Lippmann-Schwinger equation and rewrite the problem with the use of a nonsingular scattering equation.[7-9] We will show that this procedure leads to equations which are easier to solve numerically because the customary singularities of a scattering problem no longer occur in the integral equation itself, but only in simple quadratures to be performed afterwards. In addition, it will be found that this approach quite naturally leads to lowest-order approximations of the photodisintegration amplitudes which, in numerical applications, prove to provide much better results than the usually employed plane-wave approximations. (A more detailed account than given here can be found in Ref. 10.)

The formulation presented here is based on the W-matrix approach of Ref. 7. (The approach of Ref. 7 is a generalization of a procedure by Kowalski,[8] based on a general subtraction scheme by Coester;[9] all of these momentum-space methods are closely related to a coordinate space formulation of the two-body scattering problem by Sasakawa.[11])

The formal derivation of the set of nonsingular equations for photodisintegration amplitudes is performed in Sec. II A. As already mentioned, as a natural consequence of this approach, we will propose an approximation of these amplitudes which goes well beyond the usual plane-wave approximation, with relatively little extra work, and in which the *on-shell* nuclear final-state amplitudes play a prominent role. In Sec. II B, we then provide explicit expressions for all resulting equations for the case of $E1$ transitions. The results of numerical test calculations employing the Reid soft-core potential[12] are reported in Sec. III; they show that the formalism developed here is not only of numerical advantage in general, but also that our improved plane-wave approximation definitely is superior to the usual one. A summary and our conclusions are given in Sec. IV. Amongst other things, we will argue that the formalism developed here in detail for the deuteron may easily be generalized to the photodisintegration of more complex nuclei, in particular few-nucleon systems like ^{3}H, ^{3}He, or ^{4}He, and that the improved plane-wave approximation proposed here may be a particularly useful tool for such systems. Moreover, in the Appendix, we list the expressions relating some observables to the amplitudes defined in Sec. II. Throughout this paper we use natural units, with the nucleon mass being equal to unity, i.e., 41.47 MeV fm^2 = 1.

II. FORMALISM

All of the following considerations in subsection A are to be understood in partial-wave decomposed form. However, in order to keep the notational overhead at a minimum, we will omit all partial-wave, angular momentum, and spin indices, etc., in the following derivation. Explicit expressions for specific applications can be found very easily along the lines given below; those used for our numerical test calculations are given in subsection B.

A. Formal derivation

The definition of the photodisintegration amplitude reads

$$M(p) \equiv \langle p|M\rangle = {}^{(-)}\langle p|\mathcal{H}_{\rm em}|\Psi\rangle \ , \tag{1}$$

where the initial state $|\Psi\rangle$ represents the deuteron wave function, $\mathcal{H}_{\rm em}$ is the electromagnetic interaction Hamiltonian, and $|p\rangle^{(-)}$ the outgoing final, purely nuclear, scattering state with energy $E=p^2$. Using the identity

$${}^{(-)}\langle p| \equiv \langle p|[1+T(E+i0)G_0(E+i0)] \ , \tag{2}$$

the nuclear final-state interaction between the outgoing nucleons is made explicit via the corresponding transition matrix $T(E+i0)$; the operator

$$G_0(E+i0)=(E+i0-H_0)^{-1}$$

here is the resolvent of the kinetic energy Hamiltonian H_0. With the help of the Lippmann-Schwinger (LS) equation for the T matrix,

$$T(E+i0)=V+VG_0(E+i0)T(E+i0) \ , \tag{3}$$

one then very easily derives an integral equation for the (off-shell) amplitude M, which can be written in abstract form as

$$|M\rangle = |B\rangle + VG_0(E+i0)|M\rangle \ . \tag{4}$$

As can be seen, the only difference between this integral equation and the nuclear LS equation is that the nuclear potential V in the inhomogeneity of the LS equation (3) is now replaced by

$$|B\rangle = \mathcal{H}_{\rm em}|\Psi\rangle \ , \tag{5a}$$

where

$$B(p) \equiv \langle p|B\rangle = \langle p|\mathcal{H}_{\rm em}|\Psi\rangle \tag{5b}$$

is the corresponding plane-wave approximation (PWA) of the photodisintegration process. By analogy with Eq. (3), as we shall sometimes refer to Eq. (5) as the Born approximation of the photodisintegration amplitude.

Obviously, the kernels, and therefore also the singularity structures, of the nuclear LS equation (3) and the integral equation (4) for the (off-shell) photodisintegration amplitude M are identical. In the nuclear case there exists a well-known procedure which allows one to remove the scattering cut for energies $E>0$ from the kernel of Eq. (3),

$$\langle p|VG_0(E+i0)|q\rangle = \frac{V(p,q)}{E+i0-q^2} \ , \tag{6}$$

and to rewrite the LS equation into an equivalent set of equations involving only simple quadratures and a *nonsingular* integral equation. Quite clearly, the scattering problem then becomes numerically much simpler and easier to solve. The similarity between the LS equation (3) and the integral equation (4) now suggests proceeding in the same manner also for the calculation of photodisintegration amplitudes.

First, let us recapitulate those features of nonsingular scattering equations which are of relevance for the present work. The particular variant to be used here is referred to as the W-matrix approach and is explained in detail in Ref. 7. The basic step leading from the singular LS equation (3) to a nonsingular scattering equation is a splitting of the potential according to

$$V(p,q)=V(p,k)+[V(p,q)-V(p,k)] \ , \tag{7}$$

where the parameter k is chosen as $k^2=E$. [We note that for the present purpose, we need not consider the case of negative energies; the distinctions made in Ref. 7 as to how to choose k at negative energies, therefore, play no role here. Also, Eq. (7) is, strictly speaking, true only for s waves; for higher partial waves, see Eq. (21) in subsection B (and Ref. 7).] Employing the splitting (7) in Eq. (3), one finds that the scattering problem can be solved in terms of the nonsingular integral equation,[7]

$$W(E)=V+(V-V^{\rm OS})G_0(E+i0)W(E) \ , \tag{8}$$

where the singular kernel (6) of the LS equation is replaced by the subtracted, nonsingular kernel

$$\langle p|(V-V^{\rm OS})G_0(E+i0)|q\rangle = \frac{V(p,q)-V(p,k)}{E-q^2} \ , \tag{9}$$

in which $E+i0$ can be replaced by E without any problems because the denominator singularity at $E=q^2$ is canceled now by the corresponding zero of the difference $V(p,q)-V(p,k)$. Here, and in the following, the superscript OS denotes the half-on-shell value of the corresponding matrix element [i.e., $V^{\rm OS}$ stands for $V(p,k)$]. The solution of Eq. (8), referred to as the W matrix, is real and nonsingular. Given $W(E)$, the fully off-shell T matrix can be calculated via simple quadratures.[7] For the present purpose, however, it is sufficient to consider the relation between $W(E)$ and the half-on-shell T matrix $T(p,k;k^2+i0)$, viz.,[7]

$$T(p,k;k^2+i0)=\frac{W(p,k;k^2)}{F(k^2+i0)} \ , \tag{10}$$

where

$$F(k^2+i0)=1-\int_0^\infty dq\, q^2 \frac{W(q,k;k^2)}{k^2+i0-q^2} \tag{11}$$

is a momentum-space representation of the well-known Jost function. (Again, since in this paper we deal only with positive energies, the distinction made in Ref. 7 between the positive and negative energy regimes are of no relevance here.) Equations (10) and (11) show that the scattering problem is solved completely in terms of the (real) solution of the nonsingular W-matrix equation (8) and that the scattering cut enters the T matrix explicitly

via the simple quadrature in the Jost function (11).

Taking over this procedure for the calculation of photodisintegration amplitudes and employing now the splitting (7) in the integral equation (4), one very easily finds, with the help of Eqs. (8) and (10) that Eq. (4) can be replaced by the following equivalent set of two equations:

$$|\chi\rangle = |B\rangle + (V - V^{\mathrm{OS}}) G_0(E+i0)|\chi\rangle \ , \tag{12}$$

$$|M\rangle = |\chi\rangle + T^{\mathrm{OS}}(E+i0) \int G_0(E+i0)|\chi\rangle \ , \tag{13}$$

where $T^{\mathrm{OS}}(E+i0)$ is a shorthand notation for the half-on-shell T matrix (10) and

$$\int G_0(E+i0)|\chi\rangle \equiv \int_0^\infty dq\, q^2 \frac{\chi(q)}{E+i0-q^2} \ . \tag{14}$$

The integral equation (12) represents the analog of the W-matrix equation (8). In other words, we again find that the calculation of photodisintegration processes simply requires replacement of the nuclear potential V in the inhomogeneity of (8) by the plane-wave approximation B; the kernels of (8) and (12) are identical. The (off-shell) photodisintegration amplitude now is determined by a simple quadrature over the solution χ of the auxiliary equation (12) and by the half-on-shell nuclear T matrix. Note in this context that the main numerical work in calculating the photodisintegration amplitude M according to (13) is the inversion of the nonsingular kernel

$$(V - V^{\mathrm{OS}}) G_0(E+i0) \ ,$$

and that this allows one to simultaneously determine both χ, according to (12), and T^{OS}, via (8) according to (10); in other words, the explicit occurrence of the T matrix in (13) does not introduce any additional numerical complications.

Equations (12) and (13) summarize the main result of this paper. Obviously, owing to the fact that the kernel of (12) is nonsingular, the numerical solution of the set of equations (12) and (13) is much simpler than the original integral equation (4). Moreover, since the amplitude M is no longer determined directly via an integral equation but by a simple quadrature, it is now possible to consider only the on-shell restriction of Eq. (13), i.e.,

$$M(k) \equiv \langle k|M\rangle = \chi(k) + t(k) \int G_0(k^2+i0)|\chi\rangle \ , \tag{15}$$

where

$$t(k) \equiv T(k,k;k^2+i0)$$

is the nuclear on-shell T matrix. The important point here is that Eq. (15) offers one a very simple way of improving on the usual plane-wave approximation: Instead of solving Eq. (12), one may restrict oneself to the Born approximation, i.e., use B as a lowest-order approximation of χ, and consider

$$M_B(k) = B(k) + t(k) \int G_0(k^2+i0)|B\rangle \tag{16}$$

as the lowest-order approximation of the set of equations (12) and (13). This evidently constitutes an improvement over the usual plane-wave approximation, $B(k)$, of photodisintegration amplitudes, because in addition to the latter, it also contains the on-shell information of the final-state interaction via $t(k)$ and some integrated off-shell contributions from the PWA under the integral sign. Given the usual PWA B and the on-shell amplitude $t(k)$ approximation (16) is relatively easy to calculate. Moreover, Eq. (12) allows one to systematically improve on this lowest-order approximation. [In the latter context, we mention that for uncoupled and for weakly coupled partial waves, Eq. (12) may be solved by iteration; see Ref. 7.]

In Sec. III, we will report on some test calculations performed with the Reid soft-core potential[12] which show that the results obtained with Eq. (16) indeed provide a very definite improvement over the usual PWA, leading to extremely good agreement with the corresponding exact results even in those cases where the normal PWA vanishes (cf. Figs. 8 and 9). We shall refer to Eq. (16) as the improved PWA.

Let us add some remarks concerning the practical usefulness of Eq. (16). Obviously, since the calculation of $t(k)$ and of the full amplitude M both require the inversion of the same kernel (9), and despite the fact that M_B is a much better approximation of M than the PWA B, the usefulness of (16) would be somewhat limited if one still had to calculate the nuclear on-shell amplitude $t(k)$, for then one might as well go all the way and determine the full amplitude M with very little additional work. The practical usefulness of Eq. (16), in our opinion, therefore, stems from the fact that the required nuclear amplitude in (16) is *completely on shell;* in other words, one can write $t(k)$ entirely in terms of the *experimental* phase shifts for the associated nuclear final-state scattering process, and thus altogether avoid the numerical solution of an integral equation. As explained in the summarizing assessment in Sec. IV, this may be particularly important for few-nucleon systems more complex than the deuteron, and it is in this respect that we consider the improved PWA a relatively simple—and possibly very useful—tool for the approximate calculation of photodisintegration amplitudes.

B. Some explicit formulas

The explicit form of the integral equation (4) for the deuteron photodisintegration reads

$$M_l^J(p) = B_l^J(p) + \sum_{l'} \int_0^\infty dq\, q^2 \frac{V_{ll'}^J(p,q)}{k^2+i0-q^2} M_{l'}^J(q) \ ; \tag{17}$$

here, J denotes the total angular momentum and l and l' are partial-wave indices [for uncoupled partial waves, the potential simplifies to $V_{ll'}^J(p,q) = \delta_{ll'} V_l^J(p,q)$, of course]. In the following, we take into account meson exchange currents via Siegert's theorem and we consider only $E1$ transitions; i.e., we have but $J=0,1,2$, in (17) (and all potentials belong to isospin $T=1$, of course). Only uncoupled p waves contribute for $J=0,1$; for $J=2$ the p wave is coupled to an f wave. The three Born amplitudes belonging to $l=1$ are given as

$$B_1^J(p) = \alpha_{10}^J \frac{\partial}{\partial p} \psi_0(p) + \alpha_{12}^J \left[\frac{\partial}{\partial p} + \frac{3}{p} \right] \psi_2(p) \ , \tag{18}$$

and the f-wave Born term for $J=2$ reads

$$B_3^2(p)=\alpha_{32}^2\left[\frac{\partial}{\partial p}-\frac{2}{p}\right]\psi_2(p)\ . \tag{19}$$

The seven nonvanishing coefficients $\alpha_{ll'}^J$ are as follows:

$$\begin{aligned} J=0:&\ \alpha_{10}^0=-\sqrt{1/3},\ \alpha_{12}^0=\sqrt{2/3};\\ J=1:&\ \alpha_{10}^1=1,\ \alpha_{12}^1=\sqrt{1/2};\\ J=2:&\ \alpha_0^2=-\sqrt{5/3},\ \alpha_{12}^2=\sqrt{1/30};\\ &\ \alpha_{32}^2=-\sqrt{9/5}. \end{aligned}$$

The functions ψ_0 and ψ_2 in (18) and (19) are the s- and d-wave components, respectively, of the deuteron wave function; they are normalized in the usual way,

$$\int_0^\infty dp\, p^2(|\psi_0(p)|^2+|\psi_2(p)|^2)=1\ . \tag{20}$$

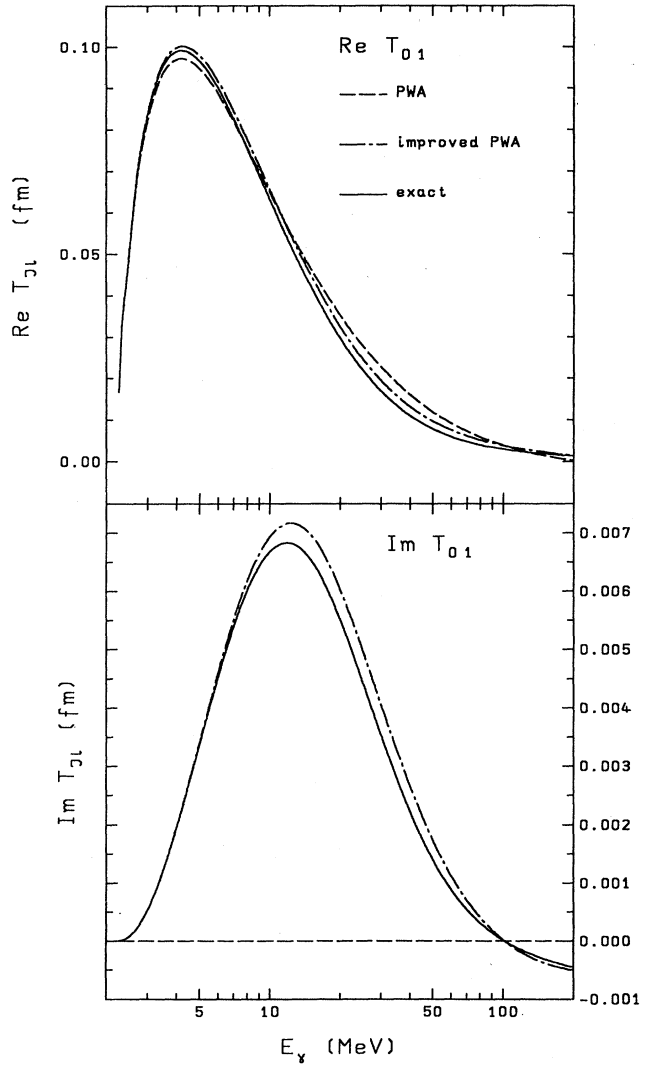

FIG. 1. Real and imaginary parts of the $E1$ amplitude $\mathcal{T}_{01}^{E1}$ vs photon lab energy [cf. Eq. (A3)]. Exact results are given by the solid lines; the usual PWA and the improved PWA are represented by dashed and dash-dotted lines, respectively.

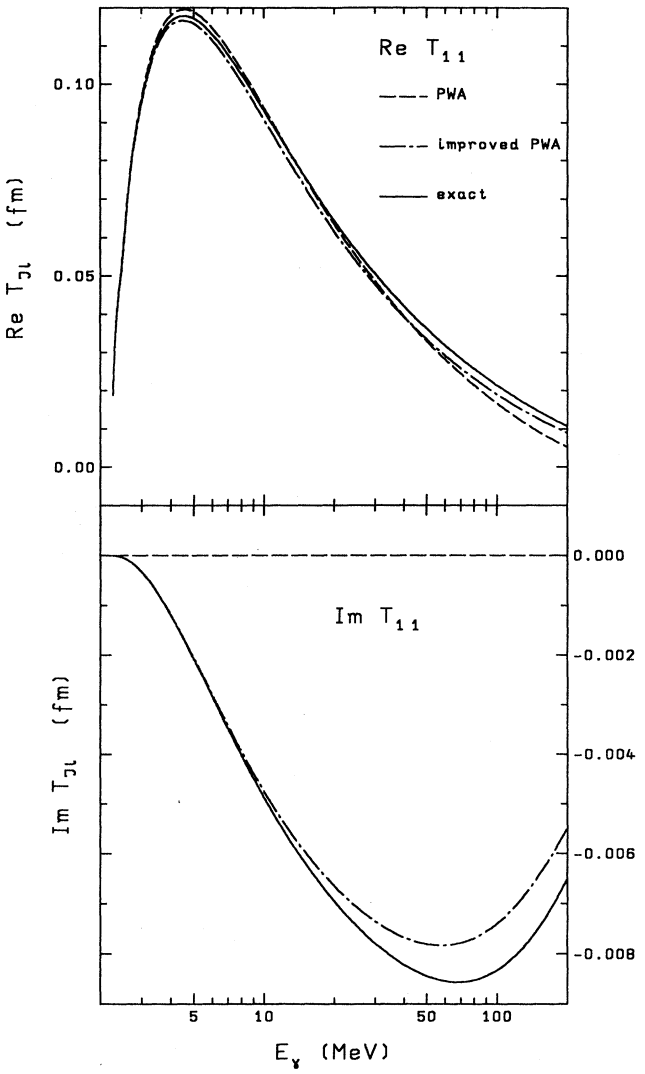

FIG. 2. Same as Fig. 1, for $\mathcal{T}_{11}^{E1}$.

The potential splitting (7), which leads to Eqs. (12) and (13), is given explicitly as

$$V_{ll'}^J(p,q)=U_{ll'}^J(p,k)q^{l'}+[U_{ll'}^J(p,q)-U_{ll'}^J(p,k)]q^{l'}\ , \tag{21}$$

where the function

$$U_{ll'}^J(p,q)=V_{ll'}^J(p,q)q^{-l'}\ , \tag{22}$$

up to the factor $q^{-l'}$ which compensates for the $q^{l'}$-threshold behavior of $V_{ll'}^J(p,q)$ for vanishing q, is identical to the potential. (This factor is introduced to make all entities well defined for all values of the on-shell momentum k; see Ref. 7.) The set of equations equivalent to Eq. (17) thus becomes

$$\chi_l^J(p)=B_l^J(p)+\sum_{l'}\int_0^\infty dq\, q^2\frac{U_{ll'}^J(p,q)-U_{ll'}^J(p,k)}{k^2-q^2}q^{l'}\chi_{l'}^J(q) \tag{23}$$

and

$$M_l^J(k)=\chi_l^J(k)+\sum_{l'} t_{ll'}^J(k)k^{-l'}\int_0^\infty dq\, q^2\frac{q^{l'}\chi_{l'}^J(q)}{k^2+i0-q^2}\ . \tag{24}$$

For the latter equation, we have written only the on-shell version because this is all we need. The on-shell two-body T matrix is given according to Eq. (10) as

$$t_{ll'}^J(k)=\sum_{l''} W_{ll''}^J(k,k;k^2)[F^J(k^2+i0)]_{l''l'}^{-1}k^{l'}\ , \tag{25}$$

where $W_{ll'}^J(k,k;k^2)$ is determined via the half-on-shell restriction of Eq. (8),

$$W_{ll'}^J(p,k;k^2)=U_{ll'}^J(p,k)+\sum_{l''}\int_0^\infty dq\, q^2\frac{U_{ll''}^J(p,q)-U_{ll''}^J(p,k)}{k^2-q^2}\times q^{l''}W_{l''l'}^J(q,k;k^2)\ , \tag{26}$$

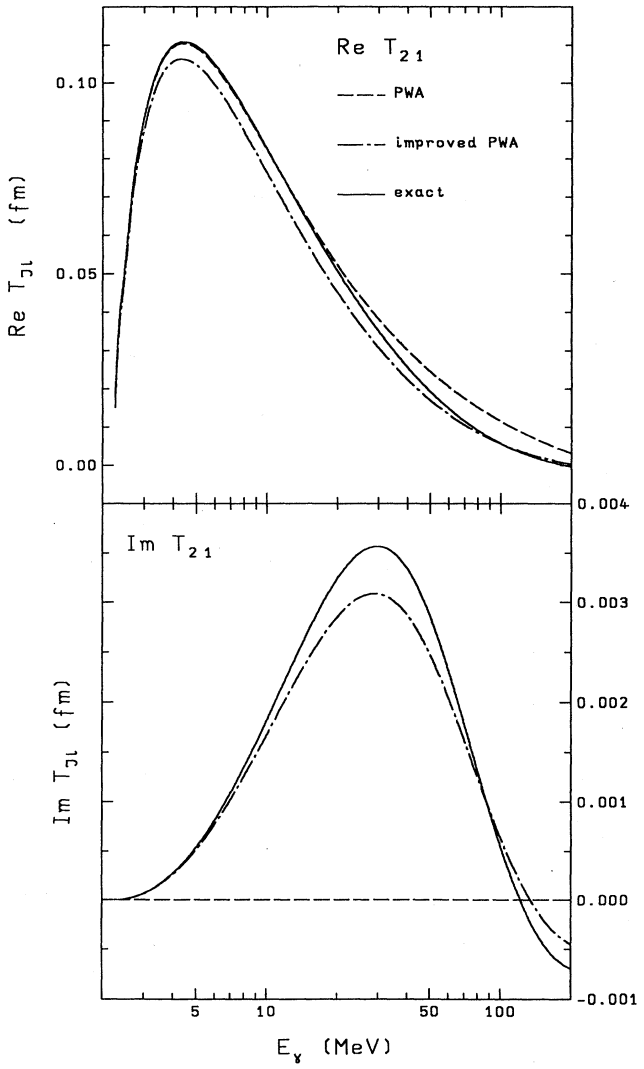

FIG. 3. Same as Fig. 1, for $\mathcal{T}_{21}^{E1}$.

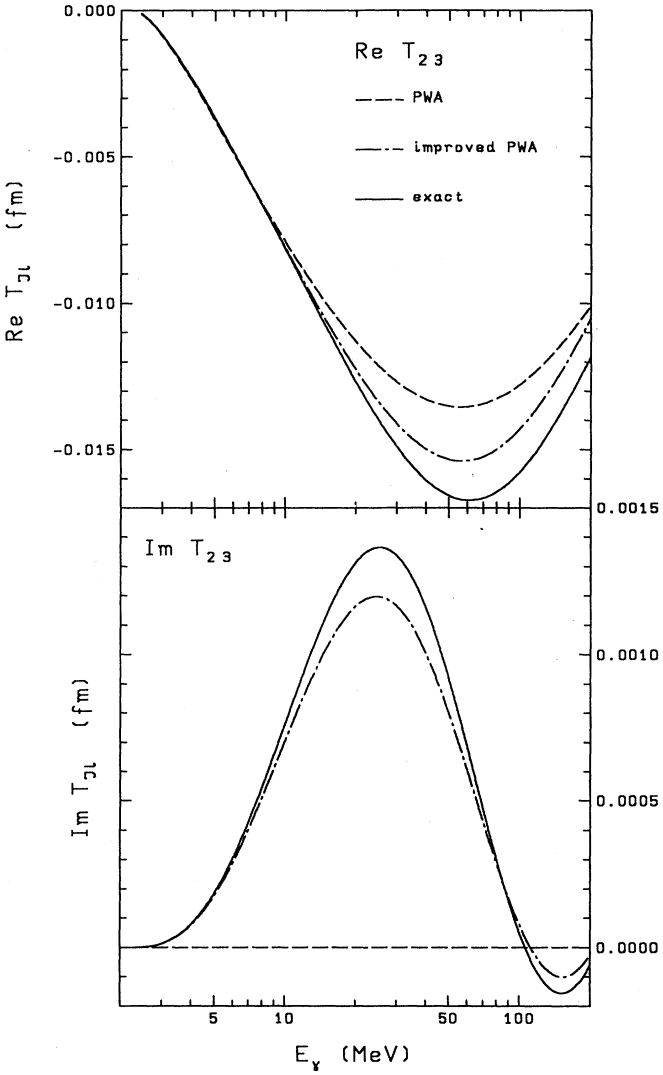

FIG. 4. Same as Fig. 1, for $\mathcal{T}_{23}^{E1}$.

and the Jost function (or, rather, Jost matrix for coupled channels) is given explicitly as

$$F_{ll'}^J(k^2+i0)=\delta_{ll'}-\int_0^\infty dq\, q^2\frac{q^l W_{ll'}^J(q,k;k^2)}{k^2+i0-q^2}\ . \tag{27}$$

The expression for the improved plane-wave approximation $M_{Bl}^J(k)$ discussed in subsection A follows upon replacing in Eq. (24) the full solutions χ_l^J of the integral equation (23) by their respective Born approximations B_l^J given in (18) and (19):

$$M_{Bl}^J(k)=B_l^J(k)+\sum_{l'} t_{ll'}^J(k)k^{-l'}\int_0^\infty dq\, q^2\frac{q^{l'}B_{l'}^J(q)}{k^2+i0-q^2}\ . \tag{28}$$

Equations (23)–(28) provide a complete summary of the detailed formulas for the calculation of deuteron photodi-

sintegration amplitudes within the framework of nonsingular integral equations. All observables can be expressed with the help of the amplitudes M_l^J; explicit expressions for some observables are given in the Appendix.

III. NUMERICAL RESULTS

We have applied the procedure described in the preceding section using the Reid soft-core potential.[12] The objectives of these test calculations were twofold; firstly, to verify the numerical usefulness of the nonsingular formulation developed above and secondly, to test the reliability of the improved PWA according to Eq. (28). As mentioned already, for simplicity we restrict ourselves to $E1$ transitions.

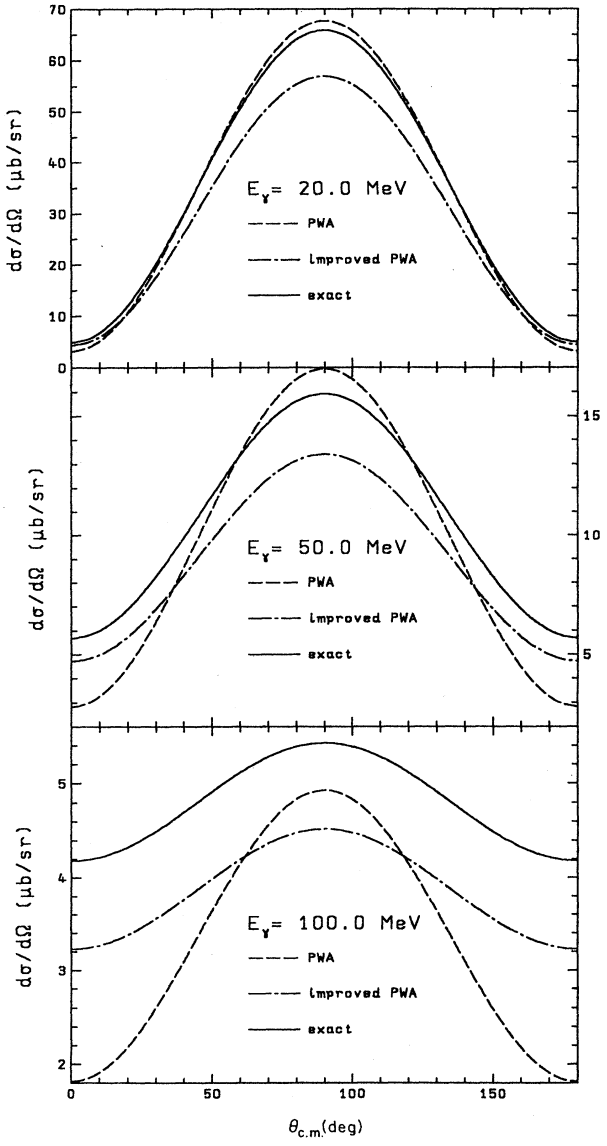

FIG. 5. Differential cross sections at photon lab energies of 20, 50, and 100 MeV [cf. Eq. (A5)]. Indicators as in Fig. 1.

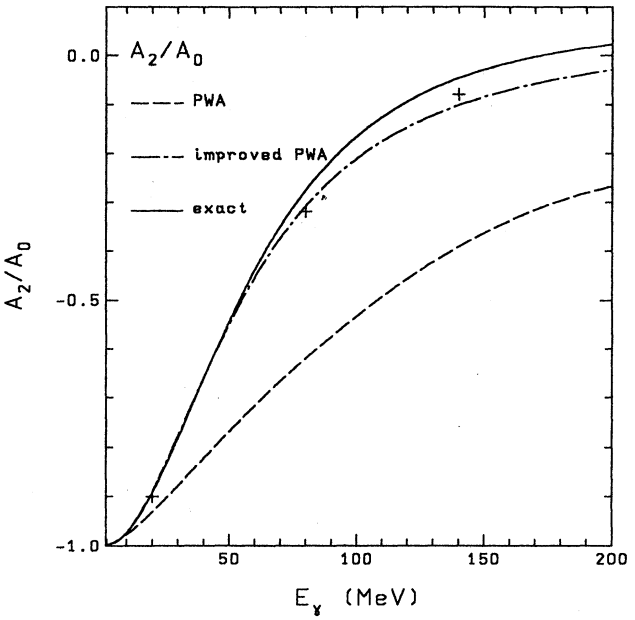

FIG. 6. Correlation coefficient A_2/A_0 (see the text); crosses correspond to values calculated from numbers given by Partovi (Ref. 1) for the Hamada-Johnston potential. Indicators as in Fig. 1.

In general, we find that the numerical solution of the nonsingular scattering equations (23) and (26), and the subsequent quadratures (24) and (27), present no practical problems whatsoever. In our opinion, the nonsingular formulation proposed here is definitely simpler in numerical applications than the usual approach in terms of Lippman-Schwinger-type integral equations.

In Figs. 1–4 we plot the real and imaginary parts of the four $E1$ amplitudes $\mathcal{T}_{Jl}^{E1}$ defined in Eq. (A3). Each figure contains the usual PWA according to Eqs. (18) and (19), our improved PWA defined in Eq. (28), and the corresponding exact result. As far as the real parts are concerned, we find that there is little difference between the usual PWA and the improved PWA. Although our approximation seems to work somewhat better at higher energies (in particular, for $\mathrm{Re}\mathcal{T}_{23}^{E1}$ in Fig. 4), both PWA's reproduce the corresponding exact results fairly well. The decisive difference between the two approximations appears in the imaginary parts: Whereas the imaginary parts are always identically zero for the usual PWA, our improved PWA is seen to follow the exact calculation to the same degree of reliability as found already for the real parts. From these findings it seems obvious, therefore, that our approximation can be expected to be at least of the same quality as the usual Born approximation for all observables in which the vanishing imaginary part plays no role, and to be quite superior in all other applications.

In assessing the quality of the approximations shown in Figs. 1–4, it should be noted that the overall normalization of the amplitudes $\mathcal{T}$ enters directly here; many observables, however, are calculated in the form of ratios (cf. the examples given in the appendix). One may hope

that in the latter cases differences between the approximate and the exact values due to normalizations might cancel to a certain degree. As a first example of this, we show in Fig. 5 the differential cross sections at photon energies of 20, 50, and 100 MeV, and in Fig. 6 the correlation coefficient A_2/A_0, where A_0 and A_2 are defined by expanding the differential cross section in Legendre polynomials, i.e.,

$$d\sigma/d\Omega = \Sigma_l A_l P_l(\cos\theta)$$

[cf. Eq. (A5)]. (This ratio can also be calculated from the numbers given by Partovi[1] for the Hamada-Johnston potential; the corresponding results are shown in Fig. 6 for comparison.) As can be seen from Fig. 5, for the differential cross sections our approximation is definitely better as far as the shape of the exact result is concerned, but it does not quite reproduce the overall normalization, which is given somewhat better by the usual PWA, at least at lower energies. However, for the ratio A_2/A_0 one sees from Fig. 6 that our improved PWA follows the exact result quite closely. It is definitely much better than its customary counterpart, which falls short by about 30 percent almost over the whole energy range.

Figure 7 provides a further observable obtained by forming a ratio; it shows the asymmetry for linearly polarized photons at lab energies of 20, 50, and 100 MeV. The corresponding formula is given in Eq. (A6) in the Appendix. We find here that the improved PWA definitely is much better than the usual PWA, being almost indistinguishable from the exact result at lower energies and fairly close to it even at an energy of 100 MeV.

In Figs. 8 and 9, we show the results for the proton polarization and for the target asymmetry [cf. Eqs. (A7) and (A8)]. These two examples, perhaps, demonstrate the superiority of the improved PWA most clearly: Being pure real, the usual Born approximation vanishes identically for these observables and thus cannot be applied at all here. The improved PWA, in contrast, not only does not vanish, but again is seen to reproduce the corresponding exact calculation very well, in the case of the proton polarization (Fig. 8) even extremely well.

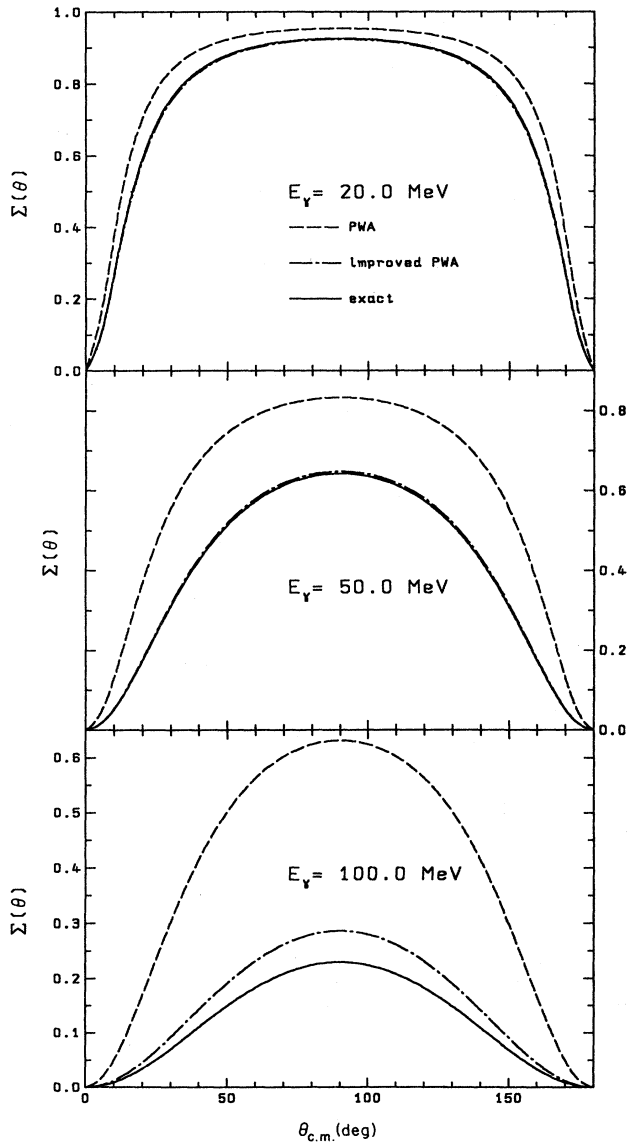

FIG. 7. Asymmetry for linearly polarized photons at lab energies of 20, 50, and 100 MeV [cf. Eq. (A6)]. Indicators as in Fig. 1.

IV. SUMMARY AND DISCUSSION

We have presented here a momentum-space formulation of the deuteron photodisintegration within the framework of nonsingular scattering integral equations. The decisive features of this approach are, firstly, numerical utility, secondly, the fact that the associated final-state amplitude is required only on shell and, thirdly, a lowest-order approximation which, given the usual PWA, can be calculated very easily and which provides a definite improvement over the usual PWA.

As mentioned already, the set of equations (23) and (24) is numerically much simpler to solve than the customary integral equation (17). This obviously is due to the fact that the auxiliary integral equation to be solved, viz., Eq. (23), does not contain any of the usual LS kernel-type singularities and is completely real. The customary singularities of the scattering problem enter here only through quadratures which can be dealt with very simply and efficiently numerically. In addition, we have shown that the lowest-order approximation suggested by the approach presented here yields results which are, for all observables considered, in good—sometimes even excellent—agreement with the corresponding exact calculations. These results were found to be at least of the same quality as the usual PWA, and in most cases definitely better. Particularly striking examples of the latter kind are all those observables for which the usual Born approximation vanishes altogether (cf. Figs. 8 and 9). We feel justified, therefore, to refer to this approximation as the improved PWA.

The formulation developed here allows experimental (i.e., on-shell) information about the final-state interaction to have a direct bearing on the final result. Apart

from the bound-state wave function in the Born term (which contains a certain amount of off-shell information, of course), the only place where the off-shell behavior of the nucleon-nucleon interaction enters is via the nonsingular kernel of the auxiliary integral equation (23). The overall quality of the improved Born approximation found here is a good indication, therefore, that this off-shell behavior does not seem to be very important and that the on-shell final-state amplitude, together with the off-shell information contained in the wave function, is quite sufficient to provide the main features of the corresponding exact result.

Let us add a few remarks concerning possible other applications. Although derived for the deuteron, the above approach can be quite easily taken over to the photodisintegration of more complex nuclei. This is particularly true for relatively small systems like ^{3}He, ^{3}He, or ^{4}He which involve only a few constituents and which can still be calculated exactly—at least in principle. Generalizing the two-body LS equation, the scattering problem for these three- and four-nucleon systems may be very conveniently formulated in terms of Faddeev-type integral equations. Recasting these in the form of *effective* two-body equations of an LS-type structure very similar to the genuine two-body scattering equations,[14] this then forms a starting point from which one derives an LS-type description of the corresponding photodisintegration processes exactly analogous to the one given in Eqs. (4) or (17) for the deuteron (for a treatment of three- and four-nucleon photodisintegration along these lines, see Refs. 5 and 6, respectively). Owing to the structural similarities between the deuteron case and these effective two-body formulations, it is obvious that one may now also take over the nonsingular approach presented above and ar-

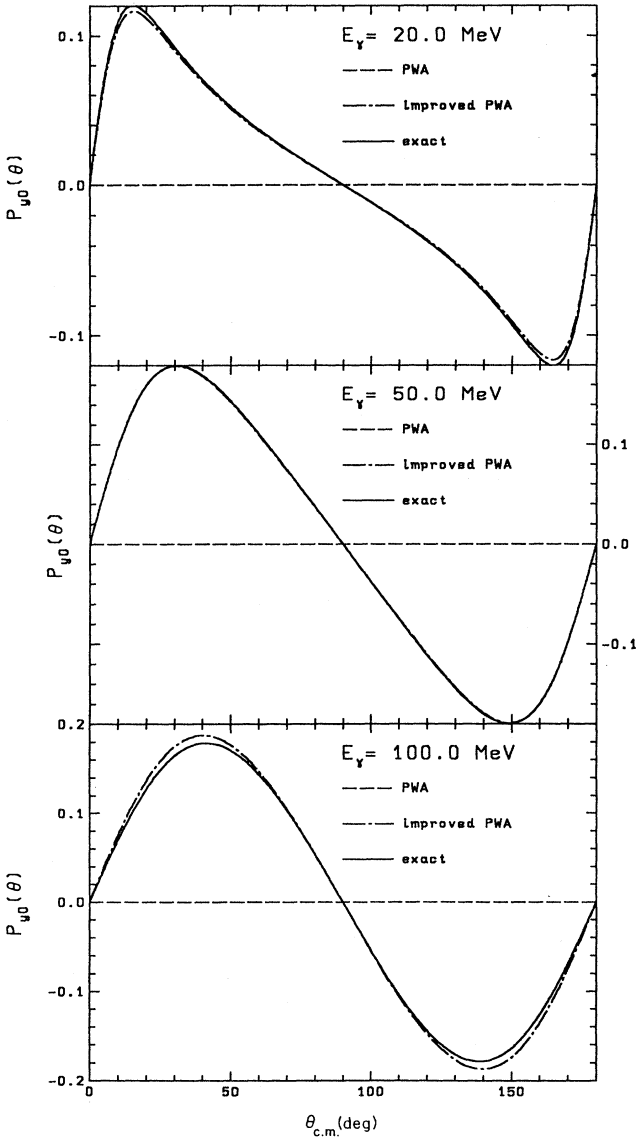

FIG. 8. Proton polarization at lab energies of 20, 50, and 100 MeV [cf. Eq. (A7)]. Indicators as in Fig. 1.

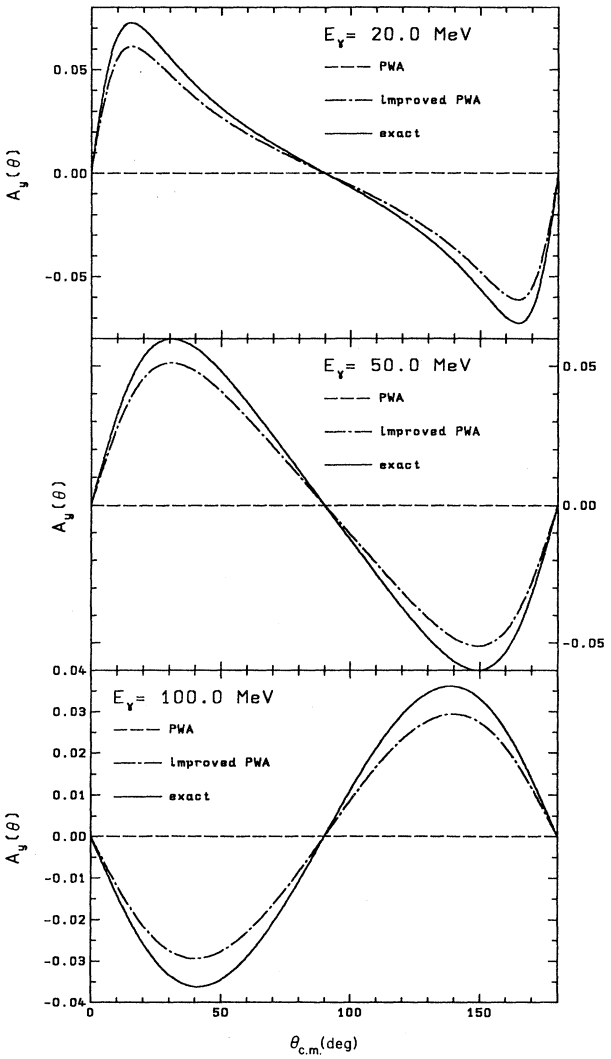

FIG. 9. Target asymmetry at lab energies of 20, 50, and 100 MeV. Indicators as in Fig. 1.

rive at a description of three- or four-nucleon photodisintegration processes in which the corresponding on-shell nuclear final-state amplitudes appear explicitly, just as in Eqs. (24) and (28). This then offers the opportunity to take this on-shell information *entirely from the experimental phase shifts.* If one restricts oneself to the analog of the improved plane-wave approximation (28), one may then altogether avoid the solution of an integral equation and is left only with a relatively simple integration over the corresponding Born amplitudes. If the quality of the improved PWA found for the deuteron carries over to these more complex systems, one then would have a relatively simple tool for the approximate calculation of few-nucleon photodisintegration amplitudes which would allow one to circumvent the nontrivial numerical complexities of the full scattering problem. It is in this respect that we consider the above nonsingular formulation—and, in particular, the improved PWA—a possibly important step towards a simplified yet reliable calculation of photodisintegration observables.

In summary, we believe that the formulation presented here is a viable alternative to the usual ways of calculating photodisintegration amplitudes and we are convinced that the modified plane-wave approximation which follows naturally from our approach constitutes a definite improvement over the customary PWA.

ACKNOWLEDGMENTS

The work of one of the authors (H.H.) was supported in part by the U.S. Department of Energy under Grant No. DE-FG05-86-ER40270. Moreover, H.H. would like to thank the University Committee on Research of The George Washington University for a travel grant through the University Facilitating Fund and the Physikalisches Institut der Universität Bonn for a Visiting Professorship during the time when this work was written up.

APPENDIX

The total cross section for $E1$ transitions is calculated according to

$$\sigma_{\rm tot}(\omega)=4\pi\sum_{J,l}\frac{(2J+1)}{6}|\mathcal{T}_{Jl}^{E1}|^2\ , \tag{A1}$$

where ω is the center-of-mass system (CMS) photon energy, related to the corresponding laboratory energy $\omega_{\rm lab}$ via

$$\left[\frac{\omega_{\rm lab}}{\omega}\right]^2=1+\frac{2\omega_{\rm lab}}{M_d}\ ; \tag{A2}$$

M_d is the deuteron mass. The quantity $\mathcal{T}_{Jl}^{E1}$ is determined with the aid of the amplitudes M_l^J of Eq. (24) as

$$\mathcal{T}_{Jl}^{E1}(k)=\left[1+\frac{\omega}{2M_d}\right]\left[\frac{\pi\omega k}{3}\right]^{1/2}\frac{e}{2}\frac{(-1)^J}{\sqrt{2J+1}}M_l^J(k)\ , \tag{A3}$$

with e being the elementary charge. The relation between the photon energy $\omega_{\rm lab}$ and the CMS kinetic energy k^2 of the two nucleons in the final state is

$$k^2=\omega_{\rm lab}+E_d+\tfrac{1}{2}\hat{E}_d(\omega_{\rm lab}+\tfrac{1}{2}E_d)\ . \tag{A4}$$

Here, E_d (<0) is the deuteron binding energy and $\hat{E}_d$ is a dimensionless quantity expressing the deuteron binding energy in units of the nucleon mass i.e.,

$$\hat{E}_d=-2.3694\times10^{-3}\ ;$$

the term containing this factor plays no role in practice for the energies considered here.

In the following, we give a list of all observables calculated in this work taking into account $E1$ transitions only. (We follow here the Madison convention; see also Ref. 13.) The functions $P_2(x)$ and $P_2^m(x)$ are the Legendre polynomials and associated Legendre polynomials of the first kind, respectively, for $l=2$. Differential cross section:

$$\frac{d\sigma}{d\Omega}(\theta)=A_0+A_2P_2(\cos\theta)\ ; \tag{A5}$$

asymmetry:

$$\Sigma(\theta)=\frac{B_2P_2^2(\cos\theta)}{A_0+A_2P_2(\cos\theta)}\ ; \tag{A6}$$

polarization:

$$P_{y0}(\theta)=\frac{C_2P_2^1(\cos\theta)}{A_0+A_2P_2(\cos\theta)}\ ; \tag{A7}$$

target asymmetry:

$$A_y(\theta)=\frac{T_2P_2^1(\cos\theta)}{A_0+A_2P_2(\cos\theta)}\ . \tag{A8}$$

Here, the coefficient A_0 is related to the total cross section of Eq. (A1) via $4\pi A_0=\sigma_{\rm tot}$. The definitions of the other coefficients A_2, B_2, and C_2 can be found in Ref. 13, and T_2 is given as

$$T_2=\sum_{JlJ'l'}b_{ll'}^{JJ'}\,{\rm Im}(\mathcal{T}_{Jl}^{E1}\mathcal{T}_{J'l'}^{E1*})\ , \tag{A9}$$

where the asterisk denotes complex conjugation and the eight nonvanishing recoupling coefficients $b_{ll'}^{JJ'}$ are[10]

$$b_{11}^{02}=-b_{11}^{20}=\tfrac{1}{12},\quad b_{13}^{02}=-b_{31}^{20}=-1/(4\sqrt{6})\ ,$$

$$b_{11}^{12}=-b_{11}^{21}=\tfrac{1}{8},\quad b_{13}^{12}=-b_{31}^{21}=1/(4\sqrt{6})\ .$$

More general formulas, going beyond $E1$ transitions, can be found in Ref. 13 and, in particular, for some cases not considered there, also in Ref. 10.

*Present address: Fachbereich Physik, Universität GH Essen, D-4300 Essen 1, Federal Republic of Germany.

[1]F. Partovi, Ann. Phys. (N.Y.) **27**, 79 (1964).

[2]H. Ahrenhövel, in *Electron and Photon Interactions at Intermediate Energies,* Vol. 234 of *Lecture Notes in Physics,* edited by D. Menze, W. Pfeil, and M. J. Schwille (Springer, New York, 1985), p. 276.

[3]For an overview, and for further references, see Proceedings of the Workshop on Radiative Processes in Few-Nucleon Systems, edited by N. E. Davison and D. A. Hutcheon [Can. J. Phys. **62**, 1019 (1984)].

[4]I. M. Barbour and A. C. Phillips, Phys. Rev. Lett. **19**, 1388 (1967); Phys. Rev. C **1**, 165 (1970).

[5]B. F. Gibson and D. R. Lehman, Phys. Rev. C **11**, 29 (1975).

[6]W. Böttger, A. Casel, and W. Sandhas, Phys. Lett. **B92**, 11 (1980); A. Casel and W. Sandhas, in Proceedings of the Ninth International Conference on Few Body Problems, Eugene, Oregon, 1980, edited by M. J. Moravcsik, Vol. I, p. 33; Czech. J. Phys. **B36**, 300 (1986).

[7]E. A. Bartnik, H. Haberzettl, and W. Sandhas, Phys. Rev. C **34**, 1520 (1986).

[8]K. L. Kowalski, Nucl. Phys. **A190**, 645 (1972).

[9]F. Coester, Phys. Rev. C **3**, 525 (1971).

[10]Chr. von Ferber, Universität Bonn Report No. BONN-IR-88-04, 1988.

[11]T. Sasakawa, Prog. Theor. Phys. Suppl. **27**, 1 (1963).

[12]R. V. Reid, Ann. Phys. (N.Y.) **50**, 411 (1968).

[13]A. Cambi, B. Mosconi, and P. Ricci, Phys. Rev. C **26**, 2358 (1982).

[14]See, for example, E. O. Alt, P. Grassberger, and W. Sandhas, Nucl. Phys. **B2**, 167 (1967); W. Sandhas, Acta Phys. Austriaca Suppl. IX, 57 (1972); H. Haberzettl and W. Sandhas, Phys. Rev. C **24**, 359 (1981).