

Important configurations in six-quark N - N states. II. Current quark model

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Quark basis states constructed from molecular-type orbitals were shown previously to be more convenient to use than cluster model states for N - N processes. The usual cluster model representation omits configurations which emerge naturally in a molecular basis which contains the same number of spatial functions. The importance of the omitted states was demonstrated for a constituent quark model. The present work extends the study to the prototypical current quark model, namely the MIT bag. In order to test the expansion for short-range N - N interactions, the eigenstates and eigenenergies of six quarks in a spherical bag, including one-gluon exchange, are calculated. The lowest eigenenergies are lowered significantly with respect to the usual cluster model. This reaffirms the importance of dynamics for obtaining the needed short-range repulsion.

I. INTRODUCTION

In previous papers^{1,2} we discussed the advantages of molecular orbital wave functions to describe processes involving six quarks, especially N - N scattering. In contrast to cluster model functions (see, e.g., Ref. 3), single-particle molecular wave functions are orthogonal and possess definite parity with respect to the center of coordinates of the system. Molecular-type states, which have simple structure and yet preserve these characteristics, can be constructed from elementary functions, such as linear combinations of the displaced oscillator functions that are used in cluster model calculations. Single-particle orthogonality and parity are preserved during collision within the independent particle model (IPM) approximation; however, one needs to go beyond the IPM, including two-body interactions and correlations when calculating N - N processes.

The required algebra is considerably simplified by the use of molecular functions. Of course, both cluster and molecular bases are complete, but the natural truncations of the sets can lead to quite different results. Our objective has been to test various approximation schemes, and to offer to the practitioners the algebra and a selection of the important configurations for calculations based upon various models.

The structure of six-quark configurations was discussed in Ref. 1. In Ref. 2, we compared the use of displaced Gaussian cluster model functions with the corresponding molecular basis for the case of a constituent quark model of the kind used by Harvey and Le Tourneur.⁴ We diagonalized the Hamiltonian for united-cluster geometry; this was deemed relevant to short and intermediate ranges in the collision process. In both cases, we used configurations based on pseudo-left and -right Gaussians. In contrast with the usual cluster model configurations $R^3 L^3$, we found that the lowest eigenenergies were dramatically reduced by using the same number of ap-

propriately chosen molecular functions, both in the $T=0$, $S=1$ and $T=0$, $S=0$ sectors.

II. THE MIT BAG

In the present work, we consider a current quark model, namely the prototypical MIT model. The Hamiltonian contains a volume energy, the (somewhat dubious) Casimir zero-point energy, a boundary condition for the quarks, and one-gluon exchange, where the gluons are confined by the perfect dielectric properties of the vacuum. In analogy with what we did in the constituent quark model case, the energies of two separated nucleon bags, each containing three quarks in s states with respect to their respective bag centers, is compared with the energy obtained for a united spherical bag of six quarks containing only $s_{1/2}$ and $p_{3/2}$ ($|m| = \frac{1}{2}$) states. These are the spherical limits of the lowest molecular states and are also the only ones which (usually) survive in cluster-type calculations.

The analytic form of the Hamiltonian is

$$H = E_B + E_Q + E_{GE} . \quad (1)$$

The energy E_B associated with a bag of radius R is

$$E_B = \frac{4}{3}\pi BR^3 - Z_0/R , \quad (2)$$

where B , the pressure of the vacuum on the bag, and Z_0 , the zero-point energy constant, are phenomenological parameters, taken as in Ref. 5.

$$B = (145 \text{ MeV})^4 = 57 \text{ MeV}/\text{fm}^3, \quad Z_0 = 1.84 . \quad (3)$$

The total energy E_Q of the quarks moving freely in the bag is

$$E_Q = \sum_{i=1}^6 N_i \frac{x_i}{R} , \quad (4)$$

TABLE I. The two-quark interaction matrix elements obtained from Table I of Ref. 6. First column: the type of transition El or Ml and the type of single-particle states $s \equiv s_{1/2}$ and $p \equiv p_{3/2}$ exchanging the gluon. Second column: the coefficient C_{ij} of Eq. (6). Third column: the matrix element of the spin operator between two-body states, defined in Eq. (9).

Matrix element	C_{ij}	$\langle SM S_{ij} SM \rangle$	$S;M$
$\langle ss M1 ss \rangle$	-0.177	-3	0;0
		1	1;0,±1
$\langle pp M1 pp \rangle$	-0.134	$-\frac{9}{4}$	0;0
		$\frac{7}{4}$	1;0
		$\frac{1}{4}$	1;±1
$\langle pp E2 pp \rangle$	0.036	$\frac{3}{2}$	0;0
		$\frac{3}{2}$	1;0,±1
$\langle pp M3 pp \rangle$	-0.0296	$-\frac{99}{25}$	0;0
		$\frac{153}{100}$	1;0
		$\frac{81}{40}$	1;±1
$\langle sp M1 sp \rangle$	-0.153	$-\frac{5}{2}$	0;0
		$\frac{3}{2}$	1;0
		$\frac{1}{2}$	1;±1
$\langle sp E1+M1 ps \rangle$	0.636	$\frac{1}{3}$	0;0
		1	1;0
		$\frac{2}{3}$	1;±1
$\langle sp E2+M2 ps \rangle$	-0.093	$-\frac{5}{2}$	0;0
		$\frac{1}{2}$	1;0
		1	1;±1

where N_i is the number of quarks in an i state and $\omega_i = x_i/R$ its eigenenergy. At zero-quark mass the values of x_i needed are⁵

$$x_{s_{1/2}} = 2.043, \quad x_{p_{3/2}} = 3.204. \quad (5)$$

The gluon exchange E_{GE} is the two-body part of the Hamiltonian, and in the present case represents the one-gluon exchange contribution of the diagrams in Fig. 1. As indicated in Ref. 6, this contribution is equivalent to

TABLE II. Eigenvalues of the Hamiltonian in selected six-quark bases for $TS=(01)$. First column: the state; second column: diagonal matrix elements in the cluster model basis R,L ; third column: eigenvalues for the cluster model; fourth column: diagonal matrix elements in the molecular-type basis, r,l ; fifth to ninth columns: eigenvalues for bases for various sizes; last column: the components of the lowest state.

Channel	R,L (cluster)		r,l (molecular)							Amplitude
	1×1	3×3	1×1	3×3	4×4	5×5	6×6	7×7		
NN	71	10	21	19	1	-18	-48	-51	-0.933 75	
$\Delta\Delta$	354	237	434	426	352	298	295	294	0.018 11	
CC	543	721	1021	1032	654	633	497	497	-0.049 15	
$(42^+[6]\{33\})$			768		1238	943	703	689	-0.228 47	
$(42^+[42]\{33\})$			990			1379	957	894	-0.171 06	
$(42^+[42]\{51\})$			550				1380	964	0.201 65	
$(51^+[6]\{33\})$			737					1384	0.056 95	

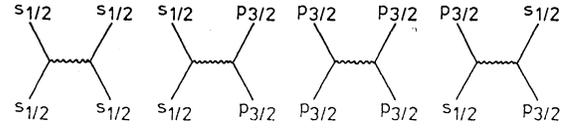


FIG. 1. One-gluon exchange diagrams for the six-quark states considered.

calculating the color electric and magnetic energies from currents of quarks by solving Maxwell's equations. In the following we shall use the results of Ref. 6, obtained in the static cavity approximation of the MIT bag model. Accordingly, E_{GE} can be written as

$$E_{GE} = \frac{\alpha_s}{R} \sum_{i<j} \frac{\hat{\lambda}_i}{2} \cdot \frac{\hat{\lambda}_j}{2} C_{ij} S_{ij}, \quad (6)$$

where $\hat{\lambda}_i/2$ are the generators of the SU(3) color group and⁵

$$\alpha_s = 2.2. \quad (7)$$

The coefficients C_{ij} and the matrix elements of the spin operators S_{ij} are given explicitly in Table I for all two-body matrix elements necessary in these calculations. Both C_{ij} and S_{ij} have been extracted from Table I of Ref. 6. In each of the matrix elements of Table I we found it useful to indicate the multipole producing the transition from the initial to final state. The spin operators require special discussion due to our truncation of the p -state space to $p_{3/2}$ states only. First one must project into this space as indicated in Ref. 6. Second one has to define two-body "pseudospin" states in order to apply techniques conveniently used previously for displaced oscillator basis functions in nonrelativistic models.²⁻⁴ In fact, the truncation of the p space helps to make a better comparison with these models.

If we denote the spin states by

$$\chi_{(3/2),(1/2)} = \uparrow, \quad \chi_{(3/2),-(1/2)} = \downarrow, \quad (8)$$

the pseudospin states $|S,M \rangle$ are

$$\begin{aligned} |0,0 \rangle &= \frac{1}{\sqrt{2}} (\uparrow\downarrow - \downarrow\uparrow), \\ |1,1 \rangle &= \uparrow\uparrow, \\ |1,0 \rangle &= \frac{1}{\sqrt{2}} (\uparrow\downarrow + \downarrow\uparrow), \\ |1,-1 \rangle &= \downarrow\downarrow. \end{aligned} \quad (9)$$

TABLE III. Eigenvalues of the Hamiltonian in six-quark bases for $TS=(00)$ for up to seven molecular basis states.

Channel	R,L (cluster)		r,l (molecular)						Amplitude
	1×1	3×3	1×1	3×3	4×4	5×5	6×6	7×7	
NN	107	-60	118	-69	-77	-100	-104	-116	-0.839 49
$\Delta\Delta$	393	160	324	286	128	109	101	80	0.307 48
CC	578	978	1015	1239	555	508	504	409	-0.329 26
$(42^- 51 \{42\})$			427		1279	755	754	562	0.203 21
$(51^- 51 \{42\})$			668			1280	943	763	0.163 58
$(33 33 \{222\})$			1002				1355	1335	0.107 27
$(33 33 \{3111\})$			1217					1738	0.112 54

Then the isospin-spin part of the wave function can be classified according to $SU(4)$.

The six-quark basis states are given in Ref. 1. In order to calculate matrix elements of Eq. (1) it is necessary to write the six-quark states as linear combinations of products of four- and two-quark states. These are obtained with the help of fractional parentage coefficients, alternatively called \bar{K} matrices.³ All necessary \bar{K} matrices are given in Refs. 1 and 7. They have been obtained with a consistent phase correction. Then the six-body matrix elements reduce to the two-body matrix elements of Table I. One can notice that the spin operator S_{ij} behaves generally as a tensor-type operator, i.e., its matrix elements depend on M for given S except for $\langle ss|M1|ss\rangle$ and $\langle pp|E2|pp\rangle$, where the behavior is of spin-spin type and central type (independent of S), respectively. The nonrelativistic analog of S_{ij} are the tensor and the spin-spin terms of the hyperfine interaction.

III. NUMERICAL RESULTS

In Ref. 1 we proposed a classification scheme with 16 channels in the $TS=(01)$ or (10) sector and seven channels in the $TS=(00)$ sector. Among these, there are two asymptotically physical channels, NN and $\Delta\Delta$. The rest of these channels couple to the "physical" states at short separations, but the expectation values of their energies become infinite at infinite separation. The new configurations appearing in our scheme as a result of using molecular orbitals are 42^\pm states with four quarks in one bag and two in the other, or 51^\pm states with five quarks in one bag and one in the other. These kinds of configurations do not appear in the usual cluster model. In Ref. 2 we found that only a restricted number of channels couple strongly to NN and have to be retained in the description of the lowest six-quark state at zero separation ($Z=0$). The significant 42^\pm or 51^\pm states have orbital symmetry [6] for $TS=(01)$ and [51] for $TS=(00)$. Their isospin-spin symmetry is {33} and {42}, respectively, i.e., common to that of the NN , $\Delta\Delta$ or CC channels.³ Six-quark MIT bag eigenvalues are given in MeV in Tables II and III for $TS=(01)$ and $TS=(00)$, respectively. They correspond to a bag radius of

$$R = 6.5 \text{ GeV}^{-1} \simeq 1.28 \text{ fm}, \quad (10)$$

i.e., close to the equilibrium value, obtained through the minimization of the lowest eigenvalue. The value in Eq. (10) is near the equilibrium radius of a six-quark system, all in s state, found to be 6.7 GeV^{-1} in Ref. 8.

The MIT model calculations presented here have some

similarities with and differences from the constituent model calculations reported earlier. In both cases we find that configurations of the type $l^4 r^2 \pm l^2 r^4$ are important and lead to a significant lowering of the energies.

In the case $TS=(01)$ (Table II), we find that four of the configurations listed are dominant and give a significant lowering of the lowest eigenvalue with respect to the cluster model. These four states account for 99.40% of the wave-function probability. The importance of some other members of the complete set of 16 basis states was tested by diagonalizing matrices containing them along with the first four states. Their importance was found to be negligible.

In the (00) case, Table III, all seven basis states were included and all seven were found to be significant. This is in contrast to the constituent model calculations where "nonasterisked" states were not important. In all, the energy lowering is 56 MeV below the 3×3 cluster model.

IV. CONCLUSIONS

As in the nonrelativistic quark model studied earlier, we find that for a spherical bag with six quarks coupled to NN quantum numbers, calculations using configurations based on molecular orbitals lead to a very significant lowering of the energy compared with configurations based on the cluster model. In the $TS=(01)$ case, we find that only four states contribute significantly, while in the (00) case all seven states are important. Although we consider more configurations than cluster model calculations, the number of *spatial* functions is the same, being only the lowest s and p states. The results clearly indicate the importance of molecular-type orbitals at short (and presumably intermediate) bag separations, and bring into serious question conclusions based on *limited* cluster model basis states.

As in our previous work, we wish to emphasize here that the collision process involves dynamics as well as statics, and the present study addresses only the issue of statics. The importance of dynamics in obtaining short-range repulsion has been demonstrated by Schuh, Pirner, and Wilets.⁹ The present work is tendered as a guide to practitioners in the field of quark models of N - N interactions.

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- ¹Fl. Stancu and L. Wilets, Phys. Rev. C **36**, 726 (1987).
²Fl. Stancu and L. Wilets, Phys. Rev. C **38**, 1145 (1988).
³M. Harvey, Nucl. Phys. **A352**, 301 (1981).
⁴M. Harvey and J. Le Tourneux, Nucl. Phys. **A424**, 419 (1984).
⁵T. De Grand, R. L. Jaffe, K. Johnson, and J. Kiskis, Phys. Rev. D **12**, 2060 (1975).
⁶J. Wroldsen and F. Myhrer, Z. Phys. C **25**, 59 (1984).
⁷Fl. Stancu, Phys. Rev. C **39**, 2030 (1989).
⁸Th. M. Aerts, P. J. G. Mulders, and J. J. De Swart, Phys. Rev. D **17**, 1260 (1978).
⁹A. Schuh, H. J. Pirner, and L. Wilets, Phys. Lett. B **174**, 10 (1986).