Temporal development of the plasma phase transition

G. Bertsch
Department of Physics and Cyclotron Laboratory, Michigan State University, East Lansing, Michigan 48824

G. E. Brown
Department of Physics, State University of New York, Stony Brook, New York 11794
(Received 17 October 1988)

We present a simple argument to show why a high entropy phase must exhibit an extended pion source lifetime. The plasmas that would be formed under the conditions of the recent heavy ion experiments would emit pions over a time of the order of 10 fm/c or longer, if the plasma has an entropy density larger than that of a pion gas by an order of magnitude.

A promising diagnostic for the formation of quark-gluon plasma in heavy-ion collisions is the space-time distribution of the hadronic final state as measured in pion interferometry.[1–3] In particular, the time distribution of the pion emission is considerably lengthened by a phase transition. That the system should persist in a mixed phase for a substantial time interval is expected on the basis of qualitative arguments about rarefaction wave propagation,[4] one-dimensional hydrodynamics,[5] and numerical hydrodynamics studies of more realistic geometries.[6–9] However, the duration of the pion emission, as measured by the interferometry technique, depends on additional assumptions about the decoupling of the pions.[10] In fact, the pions decouple rather quickly once they are outside the formation zone, according to numerical studies in the cascade mode.[11] A rather simple argument can then be made to estimate the duration of the pion emission source. Our object in this work is to present this argument, which shows better than the detailed numerical studies how the predicted long-duration source depends on the physical assumptions.

Our two main assumptions are (1) a high entropy phase with entropy density s_{qg} large enough for a phase transition is formed in the collision; and (2) the system comes into local equilibrium at a short time τ_0 after the collision, say $\tau_0 \sim 1$ fm/c. At that time the system has the shape of a squat cylinder, which then expands rapidly along its axis.

Because of the Lorentz contraction, the pressure gradient in the longitudinal direction is much greater than in the transverse, so the expansion will be chiefly longitudinal. We examine the matter coming through the sides of this cylinder. In the hydrodynamic modeling of the plasma,[6–9] the matter can only be a hadronic gas—the plasma phase shrinks as it is converted to hadrons. That is, the radius of the cylinder decreases slowly, rather than undergoing a transverse expansion in the plasma phase. We shall show how this happens in our simple model, which reproduces the same general features as the detailed modeling with kinetics of plasma droplets and pions.[11]

The modeling also shows that the particles passing through the surface substantially retain their transverse momentum into the final state. Collisions occur outside the formation zone, but they are ineffective in reducing the average transverse momentum.

With these ingredients we can apply to pion emission the same kinds of entropy arguments used elsewhere, for example, to discuss the time duration of neutrino emission from neutron stars.[12] The maximum current of particles from a thermal source is given by the black-body emissivity

$$j = nc/4 \; , \tag{1}$$

where

$$n = \frac{3\zeta(3)T^3}{\pi^2} \sim 0.37T^3 \tag{2}$$

is the number density of the particles and T is the temperature. In applying this to the pion gas, we have neglected the mass of the pions. In our argument we will not need the detailed form (2).

We apply Eq. (1) to estimate the minimum time duration of the mixed phase, requiring that the emitted pions carry off the entropy. Since the pions are treated by black-body emission, the pion entropy current can be calculated the same way. Comparing the initial entropy and the entropy flux through a slice of the cylindrical surface yields the equation

$$\pi R^2 \tau_0 s_{qg} \Delta y = \int_0^t 2\pi R c t \Delta y \frac{c s_\pi}{4} dt \; . \tag{3}$$

Here s_{qg} is the entropy density in the quark-gluon plasma, and Δy is the rapidity width of the slice, so that the left-hand side represents the initial entropy in that interval of rapidity. This remains unchanged in the following longitudinal expansion, according to Bjorken's boost-invariant expansion model.[5] Although this is not exactly satisfied, it is a reasonable approximation since the high entropy density extends over a couple of units of rapidity. On the right-hand side of Eq. (3), $2\pi R c t \Delta y$ is the surface area of the cylinder; s_π is the pion entropy density just outside the cylinder (at temperature T_c). The factors

$cs_\pi/4$ represent the entropy current carried away by the pions. One easily obtains

$$t^2 = \frac{4R}{c}\tau_0\frac{s_{qg}}{s_\pi}, \tag{4}$$

showing that t is proportional to $\tau_0^{1/2}$. With $\tau_0 = 1$ fm/c, $s_{qg}/s_\pi = 12$, and [see Eq. (2)] $R = 3$ fm, we find

$$t = 12 \text{ fm}/c . \tag{5}$$

Published results of the NA35 collaboration[13] for 200 Gev/nucleon ^{16}O+Au collisions give transverse dimension $R_T = 8.1 \pm 1.6$ fm for the more central rapidity region, $2 < y < 3$, in which one might hope to form a high entropy phase. This has been interpreted as indicating a large transverse expansion of the quark-gluon plasma, but our model suggests a different interpretation. Pions will emerge from the cylindrical formation zone as shown in Fig. 1. Pions emitted at the rate (1) do not rescatter much in the final state. Thus the sideward dimension does not increase much. On the other hand, the outward dimension is now long because of the time duration of the emission process. A relatively long time such as the estimate (5) would be manifested in the outward correlation.

Recent results from the NA35 collaboration yield[14]

$$R_T|_{\text{sideward}} = 6.6 \pm 1.8 \text{ fm} ,$$
$$R_T|_{\text{outward}} = 11.2 \pm 2.3 \text{ fm} \tag{6}$$

for $2 < y < 3$, where the longitudinal correlation is $R_L = (5.6+1.2)/-0.8$ fm. Although the statistics are not good, these results support our picture and definitely show that the averaged $R_T = 8.1 \pm 1.6$ fm does not result from transverse expansion of the high entropy phase.

We now return to the discussion of the radial motion in the high entropy phase. The main point is that the acceleration of matter in the high entropy phase will be small if it exists in the form of droplets. The mixed phase is very likely to take the form of massive droplets separated by the hadronic gas, to minimize the surface energy of the interface. The pressure in the mixed phase is the same as in the pion gas, but the energy density within the droplets is very high. Furthermore, the pions streaming out through the surface of the cylinder carry momentum which partially balances the pressure on the inside.

In the hydrodynamic model the radius of the cylinder defining the boundary between mixed phase and hadronic phase does not remain fixed, but shrinks to zero, and the rarefaction wave moves inward. Our schematic model shows that the rate at which the hadronization wave moves inward is controlled by the rate at which j_π can carry off entropy. Since this latter rate is rather slow, due to the few degrees of freedom represented by the pions, the lifetime of the plasma is ~ 10 fm/c rather than ~ 1 fm/c.

It is useful to compare our schematic development with the detailed numerical study of the droplet cascade model.[11] We have examined the flux of pions and droplet matter through the 3 fm cylindrical surface in that model to make the comparison. One difference we found is that the cascade does not predict a shrinking radius for the dense phase matter. The droplets drift somewhat, and in fact about $\frac{1}{4}$ of the pions are emitted from droplets that have expanded beyond the cylinder boundary. Since the pion flux through the surface is lower, this would decrease the minimum time duration. However, there are other effects that more than compensate in the other direction. The emissivity through the surface is less than black body, because during the longitudinal expansion the optical thickness of the matter inside the cylinder becomes less than its transverse dimension. This lowered emissivity increases the effective time scale for the pion emission. This is seen in Fig. 2, which shows the flux of

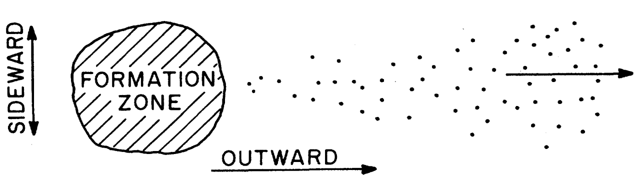

FIG. 1. A transverse cross section of the reaction cylinder. The direction of one of the pions, measured in a frame such that its longitudinal momentum is zero, defines the outward direction. Correlation of the second pion is measured either sidewards or outwards. The long tail is due to the time it takes to carry off entropy.

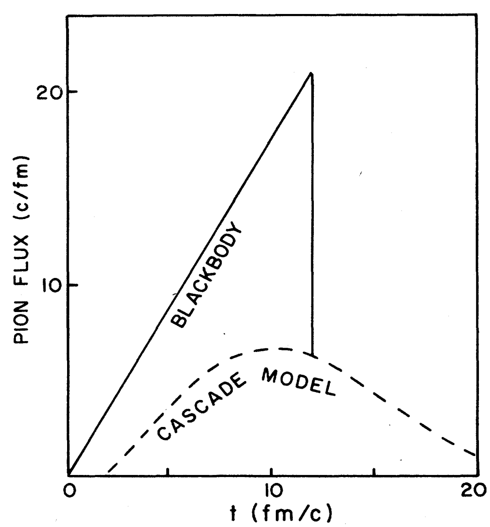

FIG. 2. The flux of pions through the cylinder surface is graphed for our black-body model, and compared with the calculated flux from the cascade model of Ref. 11. It may be seen that the emissivity in the cascade does not approach the full black-body value. In addition, about $\frac{1}{4}$ of the final-state entropy is transported through the surface by outward-moving droplets.

pions through the surface in the two models.

An alternative suggestion for a long-time scenario has been given by Gyulassy and Padula,[15] who suggest that correlated pions come from the relatively long-lived ω meson. We expect these effects to be small if the ω mesons are in chemical equilibrium, but a measurement of the meson abundances is necessary to distinguish between the scenarios.

With present experiments it is unlikely that initial entropy densities much in excess of the s_{qg} for the high entropy phase are produced. Indeed, the initial entropy density will be halved by time $2\tau_0$, so relatively few pions would be emitted from the pure high-density phase. Most of the pions come off during the mixed phase (or the phase in which droplets of plasma are present).

We conclude with some remarks about the final cooling of the matter in the cylinder. If the baryon density is low, we do not expect any differences between the spectrum of the late pions compared to ones that are emitted earlier. The temperature remains the same until the dense phase is exhausted, and after this point the pions are essentially decoupled. On the other hand, if the baryon density is high enough, we expect the very latest pions to arise from the strongly interacting and cooling baryonic matter. A simple estimate of the possible baryon density in the final state is useful for an orientation. In the O+Au collisions, the number of nucleons in the cylinder is about 70, taking the cylinder radius as 3 fm. If the initial equilibration is strong enough to distribute the baryons in proportion to the energy density in rapidity, we estimate that there would be about 20 baryons per unit rapidity in the central region (the distribution is approximately Gaussian with a variance $y=1.5$). At the time $t=12$ fm, when the dense phase has just boiled away, the baryon density would be

$$\rho_B \sim \frac{dN_b/dy}{\pi R^2 t} \sim \frac{20}{\pi(3\text{ fm})^2(12\text{ fm})} \sim 0.06/\text{fm}^3 .$$

This is about $\frac{1}{3}$ nuclear matter density. Although the density is low, the coupling to pions remains strong because of the resonant scattering between pions and nucleons in the Δ. Taking the cross section to be 100 mb, the mean free path of the pions is $\lambda = 1/\sigma\rho \sim 1.7$ fm, smaller than the radius of the cylinder. The cooling baryonic matter would emit lower momentum pions that would be evident in the final state.

Experimentally, the pion spectrum shows a two-component structure[16,17] with a low momentum component comprising about $\frac{1}{4}$ of the pions; this has been ascribed to cooling by Shuryak.[18] The cool component contains about 30 pions out of the 120 per unit rapidity in the final state. So, many pions could possibly come from a baryon gas, but the number of baryons we estimate, 20 per unit rapidity, seems somewhat small to accomplish this. More detailed estimates are required, and we especially need better experimental information about the baryon distribution in the central rapidity region. The two-component structure has also been attributed to collective flow.[17,19] However, this does not seem to be supported by more detailed hydrodynamic studies.[20,21]

We would like to thank Miklos Gyulassy and Madappa Prakash for helpful discussions. This work was supported in part by the National Science Foundation under Grant No. 87-14432 and in part by the U.S. Department of Energy (USDOE) under Grant No. DE-FG02-88ER40388.

[1] S. Pratt, Phys. Rev. D **33**, 1314 (1986).
[2] G. Bertsch, M. Gong, and M. Tohyama, Phys. Rev. C **37**, 1896 (1988).
[3] G. Bertsch, Proceedings of Quark Matter '88, Lenox [Nucl. Phys. **A498**, 173c (1989)].
[4] L. Van Hove, Z. Phys. C **27**, 135 (1985).
[5] J. D. Bjorken, Phys. Rev. D **27**, 140 (1983).
[6] M. Kataya, P. Ruuskanen, L. McLerran, and H. von Gersdorff, Phys. Rev. D **34**, 2755 (1986).
[7] G. Baym, B. Friman, J. Blaizot, M. Soyeur, and W. Czyz, Nucl. Phys. **A407**, 541 (1983).
[8] J. P. Blaizot and J. Y. Ollitrault, Nucl. Phys. **A458**, 745 (1986).
[9] P. Danielewicz and P. V. Ruuskanen, Phys. Rev. D **35**, 344 (1987).
[10] In particular, a common assumption, that freezeout occurs at a fixed proper time leads to a short-duration source (see Refs. 1 and 4).
[11] G. Bertsch, M. Gong, L. McLerran, V. Ruuskanen, and E. Sarkkinen, Phys. Rev. D **37**, 1202 (1988).
[12] S. H. Kahana, J. Cooperstein, and E. Baron, Phys. Lett. B **196**, 259 (1987).
[13] A. Bamberger *et al.*, Phys. Lett. B **203**, 320 (1988).
[14] John Harris, Proceedings of Quark Matter '88, Lenox [Nucl. Phys. **A498**, 133c (1989)].
[15] M. Gyulassy and S. Padula, Proceedings of Quark Matter '88, Lenox [Nucl. Phys. **A498**, 555 (1989)].
[16] A. Bamberger *et al.*, Z. Phys. C **38**, 88 (1988).
[17] T. W. Atwater, P. S. Freier, and J. Kapusta, Phys. Lett. B **199**, 30 (1987).
[18] E. V. Shuryak, Phys. Lett. B **207**, 345 (1988).
[19] K. S. Lee and U. Heinz, Z. Phys. (to be published).
[20] V. Ruuskanen (unpublished).
[21] D. Kusnezov and G. Bertsch (submitted to Phys. Rev. C).