Gamow-Teller and $M1$ strength sums for sd shell nuclei by spectral distribution methods

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The spectral distribution method is applied to evaluate the strength sums for Gamow-Teller and isovector $M1$ excitations and compared with shell-model predictions for (sd) shell nuclei.

Extensive work on the Gamow-Teller (GT) giant resonances by means of the charge-exchange (p,n) and (n,p) reactions has renewed interest in the theoretical evaluation of the GT excitation strength and strength sums. Whereas careful and detailed study of the excitation strength[1] can answer questions relating to the origin of the quenching of strength observed experimentally, reliable estimates of the GT strength sum itself is required to predict the amount of quenching. For blocked proton shells, the Ikeda sum rule[2] is an exact result for the total β^- strength. For open neutron and proton shells, the spectral distribution theory, a statistical theory which evaluates average strength and sum rules, was recently used and examples were worked out in the (fp) shell.[3] For (sd) shell nuclei, though full shell-model predictions of Wildenthal[4] are available nowadays, it is still worthwhile to explore simple average forms for the strength sums for a global understanding of the structural effects. Assuming the protons and neutrons both to have zero total spin, separate forms for S_{β^-} and S_{β^+}, the total β^- and β^+ strengths, were formulated[5] and recently Hino, Muto, and Oda explored[6] the validity and ways to improve them for the ground states of doubly open (sd) shell nuclei.

On the other hand, there is also considerable interest in the problem of $M1$ excitation strength through (p,p') and (e,e') reactions, and shell-model and other theoretical estimates of $M1$ strength sums for (sd) shell nuclei are available.[7] In this Brief Report we apply the spectral distribution theory to evaluate the GT and isovector $M1$ strength sums for (sd) shell nuclei and compare them with other estimates, primarily shell-model ones. We note here that earlier attempts to work out the isovector $M1$ strength sums using the spectral distribution methods in the (sd) shell were not successful[8] because there the truncation of the polynomial expansion of the sum-rule operator was done keeping only the first two terms. We go beyond that and use spectral distributions in two distinct approaches and make comparison of their relative merits. We also observe that this formalism can evaluate the strength sums for excited states as well and, as a result, is very useful for astrophysical problems like supernova triggered by electron capture.[9]

In a shell-model space of dimensionality $d(m)$ and density of energy eigenvalues $\rho(E)$, the expectation value of an operator K is defined as

$$K(E)\equiv\left[\sum_{\alpha\epsilon E}\langle E\alpha|K|E\alpha\rangle\right]/d(m)\rho(E)=S_K(E)/\rho(E)\ , \qquad (1)$$

where

$$S_K(E)=K(E)\rho(E)=\langle K\delta(H-E)\rangle^m\ . \qquad (2)$$

$S_K(E)$ is called the expectation value density of operator K. To evaluate the non-energy-weighted (NEW) strength sum at energy E, $M_0(E)$, for GT (isovector $M1$) excitation one uses the sum-rule operator $K=0^+0$ where

$$0=\sum_{i=1}^{A}\sigma(i)t_{\binom{-}{+}}(i)\quad\text{for } \beta^{(\mp)}\text{GT}$$

$$\left[0=\tfrac{1}{2}\mu_0\sum_i[\mathbf{j}(i)+(g_p+g_n-1)\mathbf{s}(i)]\quad\text{for isovector } M1\right]$$

transition. It is seen that the asymptotic form of $S_K(E)$ under the action of central limit theorem (CLT) acquires a Gaussian form. This follows from a bivariate Gaussian form for the actual strength density distribution.[10] For a discussion of this point, we refer to Kota and Kar.[11] Thus the NEW strength sum takes the form

$$M_0(E)=\langle 0^+0\rangle\frac{\sigma_c}{\sigma_s}\exp\left[-\frac{1}{2}\left[\frac{E-\epsilon_s}{\sigma_s}\right]^2+\frac{1}{2}\left[\frac{E-\epsilon_c}{\sigma_c}\right]^2\right]\ . \qquad (3)$$

In Eq. (3) (ϵ_c,σ_c) and (ϵ_s,σ_s) stand for the centroid and width of $\rho(E)$ and $S_K(E)$, respectively. For ϵ_s and σ_s we use

$$\epsilon_s=\langle 0^+0H\rangle^m/\langle 0^+0\rangle^m$$

and

$$\sigma_s^2=\langle 0^+0H^2\rangle^m/\langle 0^+0\rangle^m-\epsilon_s^2\ .$$

We note that these arguments can be extended to spaces with fixed particle and isospin (mT) and for actual applications we calculate the (mT) traces instead of the traces in the scalar (m) space.

The other well-studied spectral distribution method for expectation values $K(E)$ uses the orthonormal polynomials $P_\mu(E)$ [defined with $\rho(E)$ as the weight function], and writes the expansion

TABLE I. Correlation coefficient η_{H-K} with $K=0^{+}0$ where 0 is the excitation operator for both GT and $M1$ excitations for five self-conjugate nuclei in (sd) shell.

Nuclear	Correlation coefficient (η_{H-K}) Gamow-Teller	Isovector $M1$
^{20}Ne	0.554	0.342
^{24}Mg	0.600	0.485
^{28}Si	0.592	0.534
^{32}S	0.549	0.556
^{36}Ar	0.442	0.562

$$K(E)=\sum_{\mu}\langle KP_\mu(H)\rangle^m P_\mu(E)$$
$$=\langle K\rangle^m+\left\langle\frac{K(H-\epsilon_c)}{\sigma_c}\right\rangle^m\frac{E-\epsilon_c}{\sigma_c}$$
$$+\sum_{\mu\geq 2}\langle KP_\mu(H)\rangle^m P_\mu(E)\ . \tag{4}$$

Equation (4) uses $P_0(E)=1$ and $P_1(E)=(E-\epsilon_c)/\sigma_c$. Assuming that the density $\rho(E)$ does not change its shape when $H\rightarrow H+\alpha K$ (for small α) it is shown that terms beyond the first two in Eq. (4) are inhibited and this is called the CLT result. This is clearly the case when the eigenvalue densities of H and $H+\alpha K$ are both Gaussian. Otherwise one should take into account the higher terms, and convergence properties of the expansion need to be probed. The CLT form for $K(E)$ is simple to understand in terms of the geometry of the operator space. Defining the correlation coefficient

$$\eta_{H-K}=\langle(K-\langle K\rangle)(H-\langle H\rangle)/\sigma_H\sigma_K\rangle^m$$

with $\langle K\rangle$ and σ_K being the centroid and width of the operator K in the relevant space, one sees that the CLT result for the strength sum is small, large, or just $\langle K\rangle^m$ for η_{H-K} taking values large positive, large negative, or zero, respectively. η_{H-K}, of course, is bounded by $-1\leq\eta_{H-K}\leq 1$. In the (fp) shell η_{H-K} was found to be very small and could be approximated as zero. As a result the GT strength sum there becomes easy to evaluate.[3] In the sd shell the situation is quite different. Table I gives the correlation coefficient of the Wildenthal interaction with the sum-rule operator for GT and isovector $M1$ excitations. We see that η_{H-K} is quite large for both the operators throughout the shell, and as a consequence the CLT result for the ground-state NEW strength sum gives very small and for some cases even negative values. We note here that earlier Halemane and French,[8] in calculating isovector $M1$ strength sums, were confronted with this difficulty and as a result could not give proper estimates. We realize that in this case the truncation of the expansion with only the CLT terms is inadequate and one needs to go beyond. The first term beyond the CLT limit in Eq. (4) is given by

$$K_2(E)=\langle KP_2(H)\rangle^m P_2(E)\ .$$

Using

$$P_2(\hat{x})=(\hat{x}^{\,2}-\gamma_1\hat{x}-1)/(\gamma_2+2-\gamma_1^2)^{1/2}\ ,$$

where $\hat{x}$ is the standardized variable $\hat{x}=(x-\epsilon)/\sigma_x$ with ϵ_x and σ_x being the centroid and width of x in the m-particle space and (γ_1,γ_2) being the skewness and excess of the density $\rho(E)$. Thus including this term, for $\gamma_1=\gamma_2=0$ one gets, for the sum-rule expansion,

$$K(E)=K_{\rm CLT}(E)+K_2(E)$$
$$=\langle K\rangle^m+\langle K\hat{H}\rangle^m\hat{E}+\tfrac{1}{2}\langle K(\hat{H}^{\,2}-1)\rangle^m(\hat{E}^{\,2}-1)\ . \tag{5}$$

Extension of this expression for nonzero (γ_1,γ_2) is straightforward.

In Table II we give the strength sums for GT and isovector $M1$ excitations for five self-conjugate nuclei in (sd) shell evaluated using (mT) traces. The polynomial expansion form (called R strength) is given including contribution from $K_2(E)$ as in Eq. (5) along with $\langle K\rangle^{mT}$ for the purpose of comparison. We also use the S-strength

TABLE II. The NEW strength sum by the R and S strength for GT and isovector $M1$ excitation by spectral distribution method compared with the shell-model values for five self-conjugate nuclei in (sd) shell. We note that for these $N=Z$ nuclei $S_{\beta^-}=S_{\beta^+}$ and so the table gives both of them.

Excitation operator	Nucleus	$\langle K\rangle^{mT}$	$M_0(E)$ by R strength $\sum_{\nu=0}^{2}\langle KP_\nu(H)\rangle^{mT}P_\nu(E)$ with $(\gamma_1,\gamma_2)\neq 0$	with $(\gamma_1,\gamma_2)=0$	$M_0(E)$ by S strength	$M_0(E)$ by shell model
	^{20}Ne	5.03	2.63	2.36	2.04	0.55
	^{24}Mg	8.05	4.29	3.94	2.68	2.33
	^{28}Si	9.06	5.74	4.96	3.00	3.89
GT	^{32}S	8.05	5.16	3.88	2.82	4.00
	^{36}Ar	5.03	3.50	2.63	2.50	2.10
	^{20}Ne	14.31	5.72	5.86	6.54	2.56
	^{24}Mg	22.90	10.43	10.00	8.66	6.58
Isovector	^{28}Si	25.77	13.38	13.28	9.15	9.76
$M1$	^{32}S	22.90	14.57	10.79	7.65	9.70
	^{36}Ar	14.31	9.55	6.12	5.41	5.20

form of Eq. (3) and give the shell-model strength[6,7] too. The Hamiltonian used in our calculation as well as in the shell model is the Wildenthal's Universal-*sd* interaction.[4] The strength sums with $(\gamma_1,\gamma_2)\neq 0$ use the values of γ_1 and γ_2 evaluated in m spaces[12] due to nonavailability of the (mT) space values.

We see that in most cases the predictions using the S strength agree better with the shell model than that of the R strength. The S-strength form through its very construction is always positive definite, whereas truncated R strength in some extreme cases does not have this very desirable feature.

We have calculated the GT and $M1$ strength sums in configuration isospin $(\tilde{m}T)$ spaces for terms up to CLT but report here only the (mT) values to be consistent with the terms beyond CLT. Also, it is important to note here that Hino, Muto, and Oda[6] in their "occupation number approximation" get much larger values for the strength sums compared to the full shell-model values, and these values are nothing but the first term of our expansion of Eq. (4) evaluated and averaged over the configurations.

To probe the convergence of the R-strength expansion, we also estimate the term beyond the first three of Eq. (4) given by

$$\langle KP_3(H)\rangle^m P_3(E) \ .$$

Because of present limitations in the evaluation of traces of the product of four two-body operators in (mT) spaces, we evaluate this term using traces in m spaces. Assuming that this does not change matter much, we see that the addition of this term in the strength sum for GT changes the values 2.63, 4.29, 5.74, 5.16, and 3.50 for ^{20}Ne, ^{24}Mg, ^{28}Sr, ^{32}S, and ^{36}Ar to the values 0.41, 3.83, 6.80, 4.07, and 1.68, respectively. We see that except for ^{20}Ne/^{36}Ar (the 4-particle/hole cases) this fourth term of the expansion is less than about 25 percent of the third. The density of energy eigenvalues for four particles or holes in (sd) shell deviate somewhat from the Gaussian,

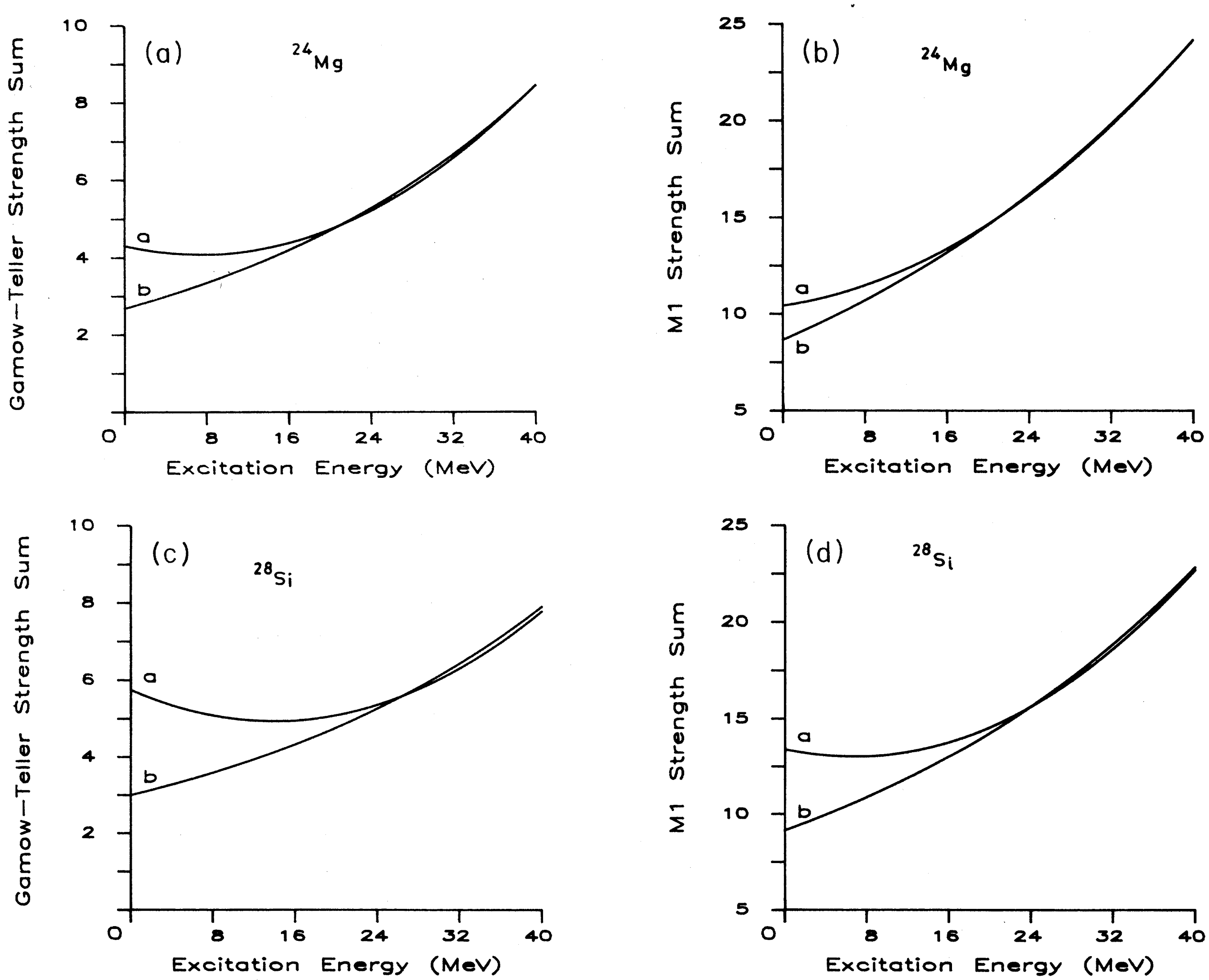

FIG. 1. Strength sum as a function of excitation energy of the nuclei for both GT and $M1$ excitations with the $T=0$ states of ^{24}Mg and ^{28}Si. Curve a is by the R-strength and b is by the S-strength method.

and this may be the reason for larger values for terms beyond CLT in the strength sum for these two cases. Similar features are seen for $M1$ strength sums also and this indicates a convergence but not a very rapid one.

This method gives the strength sum as a function of the energy of the nucleus and can be used to evaluate it for any excited state. In Fig. 1 we show the strength sum for GT as well as $M1$ excitation as a function of the excitation energy of the initial state for the 8- and 12-particle cases by the S as well as the R strength. The significant departure of one from the other in the low-energy region can be ascribed to using two distinct expansions having very different convergence properties. But this difference and in fact the contributions from all the higher polynomials go down as one moves up in the excitation energy from the ground-state region. This evaluation for excited states can be very useful in the astrophysical problem of evolution of stars in the mass range of $8-12\ M_{\odot}$, where electron-capture rates of excited nuclei like ^{20}Ne, ^{24}Mg, etc., are needed at the presupernova range.

In conclusion, we remark that in this work we, for the first time, apply the formalism of the expectation value density of the sum-rule operator for GT and $M1$ strength sums in the (sd) shell, and also show that the polynomial expansion of the strength sum in spectral distributions gives meaningful results when taken beyond the CLT limit for these excitations in the (sd) shell.

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