Pion-nucleon interaction and neutral-pion photoproduction on the proton near threshold

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Neutral-pion photoproduction on the proton near threshold is treated in a Lippmann-Schwinger–type formalism. Rescattering effects are found to be important and depend strongly on the final-state πN S_{11} interaction.

Recently, absolute measurements of neutral-pion photoproduction on protons in the threshold region have been performed at Saclay[1] and Mainz[2] and values of (-0.5 ± 0.3) and $-0.35\times10^{-3}/m_{\pi^+}$ (these units will be used hereafter) are extracted for the dipole amplitude $E_{0^+}(p\pi^0)$, respectively. Both strongly disagree with the values of -2.47 predicted by the low-energy theorems[3] and the previously inferred experimental result[1] of (-1.8 ± 0.6). This large discrepancy between the low-energy-theorem prediction and the latest experimental results may indicate large rescattering effects[3] in the E_{0^+} amplitude.

In this Brief Report, we report an evaluation of the rescattering effects in E_{0^+} within a Lippmann-Schwinger-type formulation[4] of pion photoproduction from the nucleon. We begin by introducing a transition potential $v_{\gamma\pi}$ and express the transition matrix for $\gamma N\to\pi N$, to first order in e, as

$$t_{\gamma\pi}=t_{\pi N}G_0v_{\gamma\pi}+v_{\gamma\pi}\ , \tag{1}$$

where the πN t matrix $t_{\pi N}$ is generated by the πN potential $v_{\pi N}$ and $G_0=(E-H_0)^{-1}$ with H_0 as the free energy operator of the system. $v_{\gamma\pi}$ is constructed from an effective chiral Lagrangian[5] which includes the Born terms in pseudovector coupling and contributions from t-channel (ρ,ω) vector meson exchanges. Off-energy-shell matrix elements of the transition potential $v_{\gamma\pi}$ are needed in Eq. (1). The field-theoretic expression is not gauge invariant when the pion is off the energy shell, and we choose to retain only the dominant gauge-invariant part of it here in order to ensure the gauge invariance of $t_{\gamma\pi}$. Equation (1) implies the following simple and plausible physical picture. Namely, the pion is first photoproduced from a nucleon through some mechanisms which we take to be given by the field-theoretic diagrams with the restriction that the pion so produced has not had the chance to interact with the nucleon. The pion then scatters from the nucleon through a potential $v_{\pi N}$ before it escapes.

For each multipole channel α, multipole decomposition of Eq. (1), in the c.m. frame of πN, yields

$$t^{(\alpha)}_{\gamma\pi}(q_c,k_c;E)=v^{(\alpha)}_{\gamma\pi}(q_c,k_c)+\int_0^\infty dq_c' q_c'^2\frac{t^{(\alpha)}_{\pi N}(q_c,q_c';E)v^{(\alpha)}_{\gamma\pi}(q_c',k_c)}{E-E_{\pi N}(q_c')}\ , \tag{2}$$

where k_c and q_c are the momenta of the photon and pion, respectively, and

$$E_{\pi N}(q_c)=(m_N^2+q_c^2)^{1/2}+(m_\pi^2+q_c^2)^{1/2}\ .$$

For physical production processes, Eq. (2) then yields

$$t^{(\alpha)}_{\gamma\pi}(q_E,k_c;E+i\epsilon)=\exp(i\delta^{(\alpha)})\cos\delta^{(\alpha)}\left[v^{(\alpha)}_{\gamma\pi}(q_E,k_c)+\mathrm{P}\int_0^\infty dq_c'\frac{q_c'^2R^{(\alpha)}_{\pi N}(q_E,q_c';E)v^{(\alpha)}_{\gamma\pi}(q_c',k_c)}{E-E_{\pi N}(q_c')}\right]\ , \tag{3}$$

where $\delta^{(\alpha)}$ is the πN phase shift in channel α, $R^{(\alpha)}_{\pi N}$ is the πN reaction matrix, and q_E is the pion on-shell momentum, i.e.,

$$(m_\pi^2+q_E^2)^{1/2}+(m_N^2+q_E^2)^{1/2}=E=k_c+(m_N^2+k_c^2)^{1/2}\ .$$

The conventional multipole amplitudes are related to $t^{(\alpha)}_{\gamma\pi}(q_E,k_c;E+i\epsilon)$ by some kinematical proportional constant. The $\gamma\pi$ amplitude in Eq. (3) manifestly satisfies the Watson theorem and depends on the half-off-shell behaviors of $R^{(\alpha)}_{\pi N}$. This is because the final-state interactions have been included in Eq. (1) and the theory is inherently unitary. We mention that in actual calculations a dipole cutoff form factor

$$(\Lambda^2+q_E^2)^2/(\Lambda^2+q_c^2)^2$$

with $\Lambda=476.81$ MeV (Ref. 4) is included with the $v_{\gamma\pi}$ in order to suppress the high momentum contribution in the integral part of Eq. (3).

For $E_{0^+}(p\pi^0)$, the final πN state can either be in S_{11} or S_{13}. Three simple rank 1 separable S-wave πN interactions $\lambda v(q')v(q)$ are used to estimate the rescattering effects. The first is taken from Betz and Lee[6] (BL) which is of Gaussian form with single range. The other two are modifications of a potential constructed by Ernst and

Johnson (EJ) (Ref. 7) and of two ranges, i.e.,

$$v(q)=\frac{1}{\sqrt{\omega_q}}(e^{-q^2/\alpha_1^2}+Aq^2e^{-q^2/\alpha_2^2})\,, \tag{4}$$

where ω_q is the pion energy. For simplicity, the effects of inelasticity are not included here, as was done in Ref. 7. We thus readjust their parameters to fit the experimental phase shifts. Two sets of parameters are obtained which give the same quality of fit to the experiments. The first set is very close to that given in Ref. 7 and has $\lambda=-1.37\times10^{-2}/m_{\pi^+}$, $\alpha_1=1.612m_{\pi^+}$, $\alpha_2=16.12m_{\pi^+}$, and $A=0.04865/m_{\pi^+}^2$ for S_{11}, and $\lambda=8.082\times10^{-2}/m_{\pi^+}$, $\alpha_1=\alpha_2=3.247m_{\pi^+}$, and $A=0.4053/m_{\pi^+}^2$ for S_{13}. The second set has $\lambda=-0.803\times10^{-2}/m_{\pi^+}$, $\alpha_1=2.0175m_{\pi^+}$, $\alpha_2=20.984m_{\pi^+}$, and $A=0.042/m_{\pi^+}^2$ for S_{11} and same parameters as first set in S_{13}. These will be labeled as EJ1 and EJ2. The predicted phase shifts of these three potentials, along with the 1987 Arndt[8] phase shifts are shown in Figs. 1(a) and (b). The results obtained with potentials BL, EJ1, and EJ2 are represented by short-, medium- and long-dashed curves, respectively. This convention will be followed throughout the paper. The Arndt's phase shifts are denoted by solid curves. Potential BL describes the experimental results only at very low energy. Potentials EJ1 and EJ2 both give reasonable representations of the experiments up to 2 GeV, except the oscillation around 1.5 GeV.

Table I lists[9] the predictions for $E_{0^+}(p\pi^0)$ at threshold for different combinations of transition potentials and final-state interactions. One sees that the inclusion of (ρ,ω) vector meson exchanges does produce some differences, always making it more positive. However, the most striking difference comes from using different final-state πN interactions. If the potential of BL is used, a calculation with a full transition potential operator yields a value of -1.93, very close to the previously inferred experimental value. If a potential of EJ type is employed, calculations without (ρ,ω) contributions yield values of 0.25 and -0.11 for EJ1 and EJ2, respectively. Inclusion of t-channel (ρ,ω) exchanges, however, increases the discrepancy with experiment and changes the predictions to 0.52 and 0.14.

Our model predictions for multipoles $E_{0^+}(\frac{1}{2})$, $E_{0^+}(0)$, and $E_{0^+}(\frac{3}{2})$ from threshold to $E\sim1400$ MeV with BL and EJ potentials as the final-state interactions (FSI's), together with the experimental data from Refs. 10 and 11, are shown in Figs. 2(a)–(c). They are calculated with a transition potential which includes the Born terms and t-channel (ρ,ω) exchanges. Here the solid curves give the results with no FSI's included. In all three independent isospin channels, FSI's give non-negligible contributions. For $E_{0^+}(\frac{3}{2})$, where the final πN state is in S_{13}, a small change is produced with the use of different FSI's. For $E_{0^+}(\frac{1}{2})$ and $E_{0^+}(0)$, where both go through a S_{11} πN final state, large effects are seen with the use of different πN models. The sensitivity with respect to the FSI's is most conspicuous for $E_{0^+}(0)$ near threshold. There, even the differences in the predictions between EJ1 and EJ2 are sizable. This explains why $E_{0^+}(p\pi^0)$ is so sensitive with respect to FSI's since it is a linear combination of all three independent isospin amplitudes.

From Eq. (3), one sees that for two potentials with the same phase shift, the prediction would give rise to different multipoles only if they differ in their half-off-shell behaviors. In S_{11}, form factors of EJ potentials are

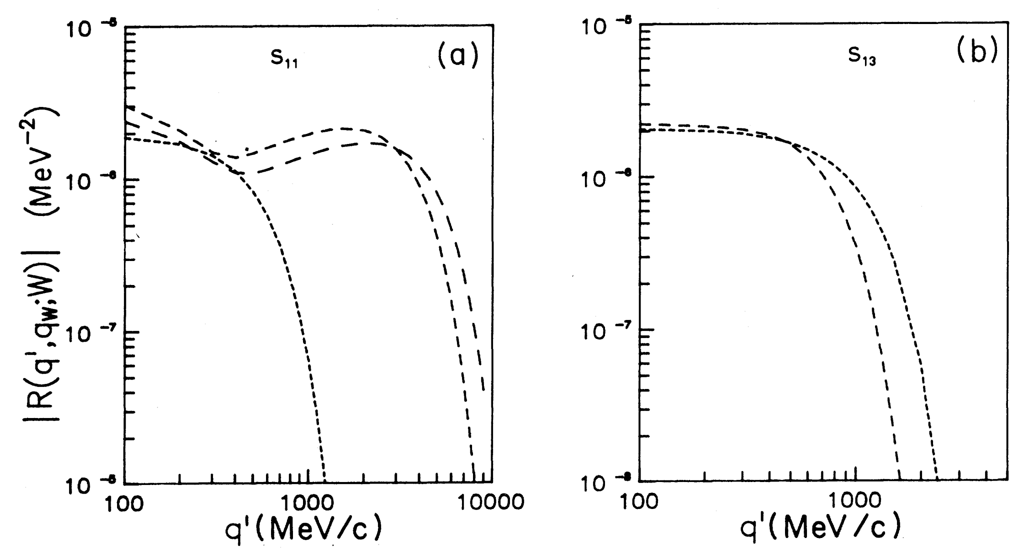

FIG. 1. (a) S_{11} and (b) S_{13} πN phase shifts given by three different separable potentials. Short-dashed curves are due to potentials of Betz and Lee (Ref. 6). Medium- and long-dashed curves are obtained with two different versions EJ1 and EJ2 of the potentials constructed by Ernst and John (Ref. 7). Solid curves represent the 1987 Arndt phase shift analysis results.

sums of two terms with long and short ranges, while in the BL potential they consist simply of one term of long range. The predictions of EJ potentials for S_{11} phase shifts are clearly preferred at higher energies, as seen in Fig. 1(a). Furthermore, they give rise to drastically different behaviors for the half-off-shell R-matrix elements at large off-shell momenta even at low energies, as

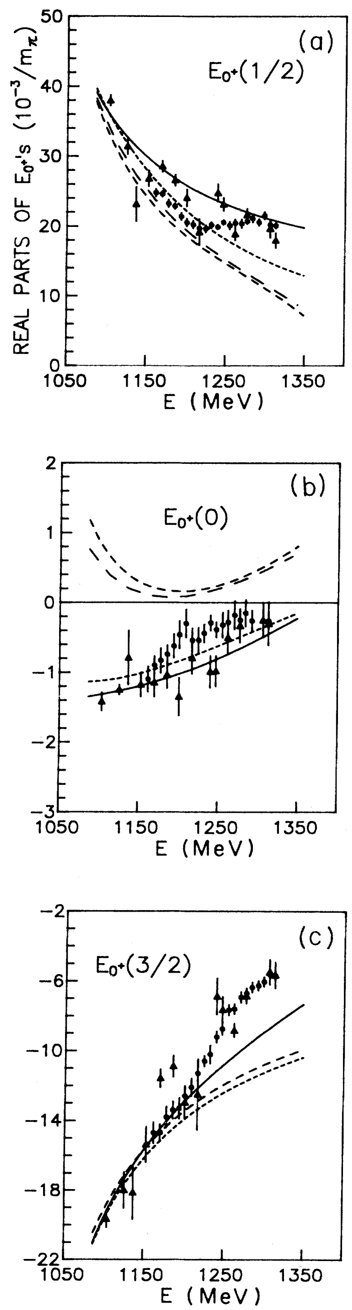

FIG. 2. Our model predictions for real parts of E_{0^+}'s. Results shown here use the full transition-potential, i.e., pseudovector coupling Born term plus (ρ,ω) exchanges but with different πN S-wave potentials. Notations are the same as in Fig. 1. Data denoted by ● and △ are from Refs. 10 and 11, respectively.

TABLE I. Results obtained for the threshold value of $E_{0^+}(p\pi^0)$ for various combinations of transition potentials with and without the inclusion of t-channel (ρ,ω) vector meson exchanges and S-wave πN interactions taken from Betz and Lee (Ref. 6) (BL) and Ernst and Johnson (Ref. 7) (EJ). Results obtained without including the final-state interaction (FSI) is also listed for comparison.

	No. FSI's	BL	EJ1	EJ2
Born	−2.51	−2.17	0.25	−0.11
Born+ρ	−2.40	−2.00	0.48	0.09
Born+ρ+ω	−2.37	−1.93	0.52	0.14

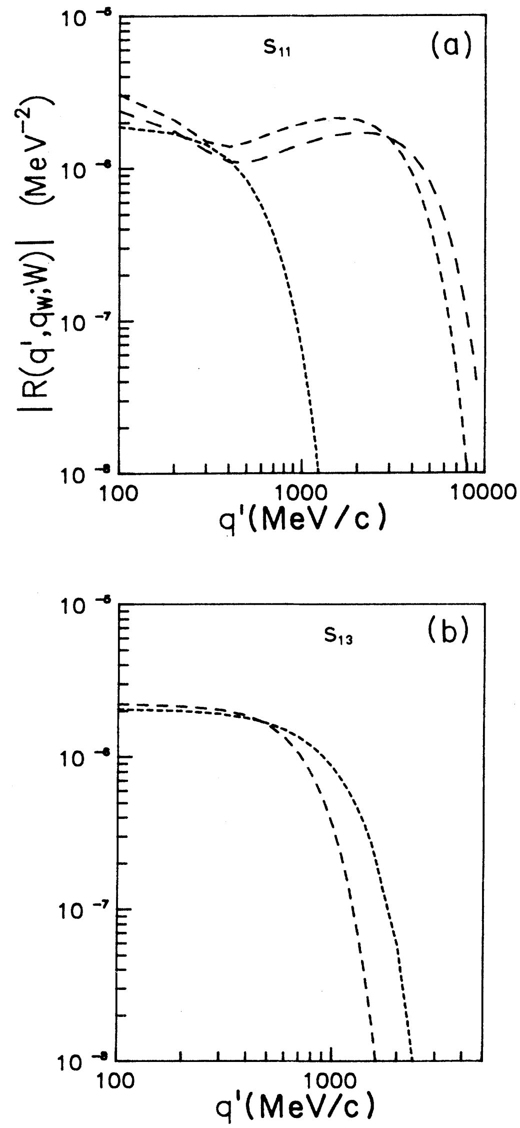

FIG. 3. Half-off-shell matrix elements $|R_{\pi N}(q',q_E;E)|$ of the πN reaction matrix for energy at 3 MeV above threshold given by different πN separable potentials. Notations are the same as in Fig. 1.

shown in Fig. 3(a). We see that at energy 3 MeV above threshold, the R-matrix element is a fast falling-off function of the off-shell momentum q' for the BL potential. In the case of EJ potentials, the R-matrix elements first decrease as q' increases, but start to increase around $q'=300$ MeV/c. They reach a secondary maximum around $q'=2.5$ GeV/c and then fall off rapidly thereafter. This secondary maximum gives a significant contribution to the second term in Eq. (3) which describes rescattering through off-shell interaction and leads to different predictions for $E_{0^+}(\frac{1}{2})$ and $E_{0^+}(0)$ from that predicted by BL potentials. In S_{13}, the form factors of the BL and EJ potentials are both of only one range. They give very similar phase shifts and have almost the same half-off-shell behavior as depicted in Fig. 3(b). We thus see little difference in their predictions for $E_{0^+}(\frac{3}{2})$ as depicted in Fig. 2(b). It is clear then that the difference in the predictions for $E_{0^+}(p\pi^0)$ by the three potentials considered here derives mostly from their different half-off-shell behaviors in S_{11}.

In conclusion, we find that within a Lippmann-Schwinger-type formulation of pion photoproduction from the nucleon, the rescatterings of the pion with the nucleon give a non-negligible contribution to the E_{0^+} multipoles. In channels where the final πN go through S_{11}, the rescattering effects depend sensitively on the half-off-shell behaviors of the final-state πN interaction. It would be of interest to explore this flexibility to see if the recent neutral pion production data near threshold could be explained.

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