Nuclear thermometers for heavy-ion collisions

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Measurement of the ratio of deuterons to excited deuterons is prescribed as a method for determining temperatures and breakup criteria in intermediate-energy heavy-ion collisions. The sudden coalescence model is contrasted with a thermal breakup picture where ratios are determined by Boltzmann factors.

During heavy-ion collisions where the center of mass energy is some significant fraction of the nuclear binding energy and the density is less than that of normal nuclear matter, the possibility of interesting phenomena such as phase separation, hydrodynamic instabilities,[1] or even critical phenomena[2] has been raised. Even if these are not possible due to the small size of the system, a change of reaction mechanisms should occur.[3] While at low energies liquid drops or slowly evaporating hot nuclei make an appropriate picture, more energetic collisions should enter a regime where a rapidly expanding gas provides a more reasonable scenario.

Within the last few years experiments have provided us with the first opportunities to test the various pictures. One pivotal measurement has been the inference of a temperature from the relative populations of excited states.[4] The ratio of two populations of the same fragment, for instance the number of excited lithium fragments to those in the ground state, should depend only on the temperature and the energy difference if the emitting source is thermal.

$$\frac{N_1}{N_2}=e^{-(E_1-E_2)/T} . \tag{1}$$

Since the energy difference E_1-E_2 is known, one can infer the temperature by measuring the ratio of the two states. In this paper we will discuss the validity of using Eq. (1). By studying the example of the ground state and quasiresonance of the deuteron, we will compare the ratio in Eq. (1), using two different formalisms. The first is sudden coalescence where two-body correlations are neglected until the breakup time, at which time the two-body potential suddenly appears, and the formation of the deuteron depends on the quantum overlap of the uncorrelated states with that of a free streaming deuteron. Secondly, a scattering formalism is developed and discussed where the last interaction of the two-body system with the remaining particles is considered to be a hard randomizing collision. This leads to an answer similar to Eq. (1), while the sudden coalescence model will be shown to predict a greater population of the excited state than of the ground state.

The temperature can also be chosen from fitting the cross sections as a function of kinetic energy. The temperatures from these two methods do not agree. Kinetic temperatures taken from the logarithmic slope of the energy-dependent cross section seem to reflect the beam energy, while the excited-state temperature from Eq. (1) seems to stay constant at around 4–5 MeV.[4] This occurs even at high beam energies, where kinetic temperatures are around 20 MeV.

The following simple explanation for this discrepancy was given by Barz, Bertsch, and Schulz.[5] The basis of the argument is that since there are only one or two collisions per nucleon at these energies, the cooling is not hydrodynamic in nature, but collisionless. A Gaussian-shaped thermal phase-space distribution will spread after a time t in the following manner:

$$f(p,x,t=0)=\exp\left[-\frac{p^2}{2mT_0}-\frac{x^2}{2R_0^2}\right] ,$$

$$f(p,x,t>0)=\exp\left[-\frac{p^2}{2mT_0}-\frac{(x-v_p t)^2}{2R_0^2}\right]$$

$$=\exp\left[-\frac{[p-p(x)]^2}{2mT_{\rm local}(t)}\right]F(x) , \tag{2}$$

$$\frac{1}{T_{\rm local}(t)}=\left[\frac{1}{T_0}+\frac{2t^2}{mR_0^2}\right] .$$

The momentum distributions always look thermal in the local rest frame of the matter, as F does not depend on momentum. Furthermore, the local temperature depends only on the time. One can even show that the entropy is conserved, as it should be when there are no collisions. Since the particles do not collide after some initially dense stage of the collision, the kinetic temperatures must reflect that initial temperature T_0. The formation of fragments cannot occur when the local temperature is high, and in fact, due to the screening, should not occur until some time late in the collision. The fragment yields should depend on the temperature corresponding to the time when nucleons undergo their final arrangement into various fragments.

Reference 5 uses an approximation of sudden coalescence[6] as the method under which the thermally equilibrated phase space rearranges itself into the bound and

excited states characteristic of the suddenly apparent two-body potentials. The quantum overlap of the thermal wave function and a bound state's wave function gives the probability of forming that state. For instance, the probability of forming a deuteron with momentum $\mathbf{K}$ would be the overlap of the wave function of a deuteron with that of the two-body wave function extracted from some dynamical code where there are assumed to be no two-body correlations until the breakup time t_b.

$$P(\mathbf{K},d)=|\langle \mathbf{K},d|\Psi_2(t_b)\rangle|^2 ,$$
$$\Psi_2=\Psi_{\text{com}}\Psi_{\text{rel}} . \tag{3}$$

Any dynamical model that yields the single-body phase-space distributions $f_{\text{rel}}(\mathbf{k},\mathbf{r},t_b)$ and $f_{\text{com}}(\mathbf{K},\mathbf{R},t_b)$ can yield the needed combinations products of wave functions. This can be seen from the quantum definition of the Wigner functions.

$$\Psi^*\left[\mathbf{K}+\frac{\mathbf{k}}{2},t_b\right]\Psi\left[\mathbf{K}-\frac{\mathbf{k}}{2},t_b\right]=\int d^3\mathbf{r}\, e^{i\mathbf{k}\cdot\mathbf{r}} f(\mathbf{K},\mathbf{r},t_b) . \tag{4}$$

Boltzmann codes, time-dependent Hartree-Fock calculations, or even hydrodynamic codes can all predict the phase-space-distribution function $f(\mathbf{K},\mathbf{r},t_b)$ and therefore can be used in Eq. (3). It is straightforward to extend this formalism to the n-body case.

The assumptions necessary for the sudden approximation to be valid are rather tenuous. For the potential to appear suddenly, the expansion time or the time it takes the screening to disappear must be less than the traversal time for a particle in the ground state. Otherwise an adiabatic approximation would be reasonable. Secondly, the expansion time should be less than the collision time. If a hard collision occurs after the screening has disappeared, a thermal result is expected. One should therefore look at the three time scales involved. The period τ_b for a particle to traverse its bound state is related to the size of the system if it is a harmonic oscillator state. The expansion time τ_{exp} and the collision time τ_{coll} depend on the beam energy and size of the participants, but in the neighborhood of $100A$ (MeV) they are approximately

$$\tau_{\text{exp}}=50 \text{ fm}/c ,$$
$$\tau_{\text{coll}}=50 \text{ fm}/c , \tag{5}$$
$$\tau_b=\pi m\langle r^2\rangle=50 \text{ fm}/c \text{ for deuterons} .$$

In the case of deuterons, the assumptions for the sudden approximation are uncertain because all three times are similar.

In the sudden approximation bound states are formed according to their overlap with the uncorrelated basically thermal states. For temperatures around 5 MeV, the thermal wavelength is on the order of 10 fm, which is larger than most bound states, and therefore the more extended states are populated rather than the least energetic ones. As an extreme example of this we consider the example of bound state formation in a very large box. Then the population of a state N_1 is given by

$$N_b=\int d^3k\, \phi^*(\mathbf{k})\phi(\mathbf{k})\exp\frac{-k^2}{2mT} . \tag{6}$$

For particles in a very cold large box, the population depends only on the square of the wave function in momentum space taken as the momentum k goes to zero.

$$\left[\frac{N_1}{N_2}\right]_{\lim_{T\to 0}}=\frac{|\phi_1(\mathbf{k}=0)|^2}{|\phi_2(\mathbf{k}=0)|^2} ,$$
$$\phi(\mathbf{k}=0)=\int d\mathbf{r}\,\phi(\mathbf{r}) \tag{7}$$
$$\approx\sqrt{L_b} \text{ for a symmetric state}$$
$$=0 \text{ for an antisymmetric state} .$$

The ratio in Eq. (7) is thus linearly proportional to the spatial extent of the wave function L_b if the two states are both symmetric. For the case of the p-n system, the d^*, which is an isospin triplet, spin singlet in an s state, is more extended than the ground state. Therefore, at low temperatures we expect more of the excited state than the ground state.

A hard scattering by definition allows the deuteron to sample all of the states regardless of their spatial shape. The result is then a more thermal spectra. In fact, even a hard scattering early in the collision can affect the thermalization at breakup. For a time T after a hard scattering, the energy is uncertain by π/T. Even when the time is 50 fm/c, the uncertainty in the energy is 12 MeV, quite a bit larger than the binding energy of the deuteron. Thus the high momentum components of the deuteron can still be sampled, leading to more Boltzmann-type ratios. Here we develop a formalism where we show that in an instantaneously disintegrating system, where the last interaction of the two nucleons with the other particles was a randomizing collision, the formation of deuterons and d^*'s will depend primarily on the temperature as in Eq. (1).

The proper normalization for the d^*'s will also be explained to take into account the fact that the d^* is not a complete resonance, and its number must be extracted from two-particle correlation measurements since it is particle unbound. We then end up with an effective ratio of d^*'s to deuterons which can be measured experimentally. For an instantaneously dissolving source at a given temperature, we make theoretical predictions of the ratio in both the hard scattering or thermal model and the coalescence model.

For the inelastic scattering of a particle from a potential, the T matrix element is very broad in momentum if the scattering was hard. The momentum distribution would be then determined more by the delta function at the end of the equation than the momentum dependence in the matrix element. For the emission of a single particle with momentum $\mathbf{p}$ from a collision where the final state is written as $|\mathbf{F},\mathbf{p}\rangle$, the probability can be written

$$P(\mathbf{p})=\frac{2\pi}{v_b}\sum_F |\langle\Psi|V|F\mathbf{p}\rangle|^2\delta(E_\psi-E_F-E_p) . \tag{8}$$

The exact eigenstate is represented by $|\Psi\rangle$ and is of course not calculable as it is a many-body system. The

potential V represents the interaction of the particle represented by $\mathbf{p}$ with the remainder of the system. The beam velocity is v_b. In order to see the momentum and space dependence in the matrix element we may rewrite this as

$$P(\mathbf{p})=\frac{2\pi}{v_b}\sum_F \int S_F(\mathbf{p},\mathbf{r})d\mathbf{r}\,\delta(E_\psi-E_F-E_p)\ , \tag{9}$$

where

$$S_F(\mathbf{p},\mathbf{r})\equiv\frac{1}{(2\pi)^3}\int d\delta\mathbf{p}\,e^{i\delta\mathbf{p}\cdot\mathbf{r}}\langle F\mathbf{p}+\delta\mathbf{p}/2|V|\Psi\rangle\langle\Psi|V|F\mathbf{p}-\delta\mathbf{p}/2\rangle\ .$$

The function $S_F(\mathbf{p},\mathbf{r})$ represents the matrix element for ejecting a particle with momentum $\mathbf{p}$ from the position $\mathbf{r}$ where the remainder of the particles go into a state F. Summing over the states F, one obtains

$$P(\mathbf{p})=\frac{2\pi}{v_b}\int S(\mathbf{p},\mathbf{r})d\mathbf{r}\,\rho_R(E_\psi-E_p)\ . \tag{10}$$

The function $\rho_R(E_\psi-E_p)$ is the density of available states for the remainder of the system and should behave with respect to E_p as $e^{(-E_p/T)}$. Thus if there is no momentum dependence in $S(\mathbf{p},\mathbf{r})$ the momentum distribution will be according to phase space, and thus be thermal. We define this as a hard collision. States of the system F are sampled in a way that does not depend on their energy or the momentum of the outgoing particle. This is similar to a hammer striking a bell, where all frequencies are initially excited before the characteristic ones dominate. If the matrix elements in Eq. (9) are transformed into coordinate space, the lack of momentum dependence requires that the matrix elements be so chaotic that there is no phase coherence at different spatial points. This is indeed a very strong and rather extreme assumption, but it allows us to see the effect of a hard collision in the population of final states.

In order to investigate the formation of a deuteron or d^* we must extend this formalism to the emission of two particles represented by the total momentum $\mathbf{K}$ and the relative momentum $\mathbf{k}$. If the final state is to be bound, the label $\mathbf{k}$ must be replaced by the discreet label "d". The potential V must now refer to the interaction of the two-particle subsystem with the remainder. Going through the same procedure as before, the result is

$$\begin{aligned}P(\mathbf{K},\mathbf{k})&=\frac{2\pi}{v_b}\int S(\mathbf{K}',\mathbf{k}',\mathbf{R},\mathbf{r})d\mathbf{R}\,d\mathbf{r}\,d\mathbf{K}'d\mathbf{k}'d\delta\mathbf{R}\,d\delta\mathbf{r}\Phi^*(\mathbf{K},\mathbf{k}|\mathbf{R}+\delta\mathbf{R}/2,\mathbf{r}+\delta\mathbf{r}/2)\Phi(\mathbf{K},\mathbf{k}|\mathbf{R}-\delta\mathbf{R}/2,\mathbf{r}-\delta\mathbf{r}/2)\\&\quad\times e^{-i\mathbf{K}'\cdot\delta\mathbf{R}-i\mathbf{k}'\cdot\delta\mathbf{r}}\rho_R(E_\psi-E_K-E_k)\\&=\frac{2\pi}{v_b}\int S(\mathbf{K},\mathbf{k}',\mathbf{R},\mathbf{r})d\mathbf{R}\,d\mathbf{r}\,d\mathbf{k}'d\delta\mathbf{r}\phi^*(\mathbf{k}|\mathbf{r}+\delta\mathbf{r}/2)\phi(\mathbf{k}|\mathbf{r}-\delta\mathbf{r}/2)e^{-i\mathbf{k}'\cdot\delta\mathbf{r}}\rho_R(E_\psi-E_K-E_k)\ .\end{aligned} \tag{11}$$

The upper case letters refer to center of mass coordinates and the lower case to relative coordinates. The relative wave function ϕ has all the information about the interaction of the two particles between themselves. The two-body matrix element $S(\mathbf{K},\mathbf{k}',\mathbf{R},\mathbf{r})$ can be factorized as the product of the single-body matrix elements. Making the hard scattering approximation of the preceding paragraph, this result can be made more tenable. Here we simply ignore the momentum dependence in S. This then assumes that both particles are scattered simultaneously. Otherwise the first wave packet would have spread correlating momentum to position.

$$\begin{aligned}P(\mathbf{K},\mathbf{k})=\frac{2\pi}{v_b}\int S(\mathbf{R},\mathbf{r})d\mathbf{R}\,d\mathbf{r}\,\phi^*(\mathbf{k}|\mathbf{r})\phi(\mathbf{k}|\mathbf{r})\\\times\rho_R(E_\psi-E_K-E_k)\ .\end{aligned} \tag{12}$$

The square of the wave function can be thought of as the corrected density of states due to the interaction. In the case of the deuteron, one replaces the label $\mathbf{k}$ with the discreet index d. In the case of the d^* one must use scattering states and integrate over the range of relative momenta, which would correspond to the d^*, and subtract the number of pairs produced in that range if there were no two-body interactions. Somewhat arbitrarily, we define that range to be from 0 to 18 MeV/c relative momenta. The number of deuterons and d^*'s in a system where the density of states is proportional to a Boltzmann factor is then

$$\begin{aligned}N_{\text{ther}}(\mathbf{K},d)&=\int S(\mathbf{R},\mathbf{r})d\mathbf{R}\,d\mathbf{r}\,\phi_d^*(\mathbf{r})\phi_d(\mathbf{r})\\&\quad\times e^{-2.2/T_e}e^{-K^2/2MT}\ ,\\N_{\text{ther}}(\mathbf{K},d^*)&=\int_0^{18\ \text{MeV}/c}\frac{d^3\mathbf{k}}{(2\pi)^3}S(\mathbf{R},\mathbf{r})d\mathbf{R}\,d\mathbf{r}\\&\quad\times[|\phi(\mathbf{k}|\mathbf{r})|^2-N]\\&\quad\times e^{-k^2/2\mu T_e}e^{-K^2/2MT}\ .\end{aligned} \tag{13}$$

The coefficient N is chosen so that the integrand goes to zero as k approaches 18 MeV/c. This is our method of subtracting out the background. The total and reduced mass of the two-nuclear system are M and μ, respectively. Using the coalescence model described earlier, the populations can be written

$$N_{\rm coal}(\mathbf{K},d)=\int F(\mathbf{R},\mathbf{r})d\mathbf{R}\,d\mathbf{r}\,e^{-K^2/2MT}e^{-q^2/2\mu T}d\mathbf{q}\,e^{i\mathbf{q}\cdot\delta\mathbf{r}}\phi_d^*(\mathbf{r}+\delta\mathbf{r}/2)\phi_d(\mathbf{r}-\delta\mathbf{r}/2)e^{-2.2/T}\,,$$
$$N_{\rm coal}(\mathbf{K},d^*)=\int_0^{18\ {\rm MeV}/c}\frac{d^3\mathbf{k}}{(2\pi)^3}F(\mathbf{R},\mathbf{r})d\mathbf{R}\,d\mathbf{r}\,e^{-K^2/2MT}e^{-q^2/2\mu T}d\mathbf{q}\,e^{i\mathbf{q}\cdot\delta\mathbf{r}}[\phi^*(\mathbf{k}|\mathbf{r}+\delta\mathbf{r}/2)\phi(\mathbf{k}|\mathbf{r}-\delta\mathbf{r}/2)-N]\,. \tag{14}$$

The integration over q in Eq. (14) can be performed explicitly, resulting in a Gaussian weight to the integration over $\delta\mathbf{r}$. This width goes to zero as the temperature approaches infinity and two formulas, Eqs. (13) and (14), become identical. Experimentally, the two numbers are easily defined. To count the d^*'s one must count all pairs with relative momentum less than 18 MeV/c, then subtract out the background or uncorrelated pairs normalized such that the number of d^*'s at exactly 18 MeV/c relative momentum is zero.

The ratio of d^*'s to deuterons is Boltzmann in the thermal case as long as the temperature is larger than (18 MeV/c)^2/940 MeV=0.3 MeV.

$$\frac{N_{\rm ther}(d^*)}{N_{\rm ther}(d)}(T)\approx\exp\left[-\frac{2.2}{T}\right]\frac{N(d^*)}{N(d)}(T\to\infty)\,. \tag{15}$$

The ratio of populations for the coalescence case is quite different, and in fact does not even rise with increasing temperature. At $T\to\infty$ the coalescence ratio is the same as the thermal ratio of Eq. (14), about one tenth, which is a measure of the fraction of a true resonance to which the increase in the density of states corresponds. These two ratios defined below are plotted in Fig. 1 for the thermal and coalescence case where the infinite temperature ratio is factored out so that the normalized ratios approach unity at infinite temperature. This infinite temperature normalization does depend on the size of the source but not strongly. Thus one must also estimate the source size which can be done by looking at two-particle correlation measurements.[7] We use a Gaussian source size of 7 fm for $F(r)$ when calculating Fig. 1. Wave functions were calculated by using a Coulomb plane wave modified by the corrections in the two s-wave channels. The Reid soft core potential[8] was used for the isospin triplet, spin singlet channel, and somewhat modified version of the Reid potential was devised for the isospin singlet, spin triplet channel. Modifications were necessary since the mixing with the d channel was ignored, and we wanted to preserve the binding energy of 2.2 MeV and the effective range expansion for the phase shift.

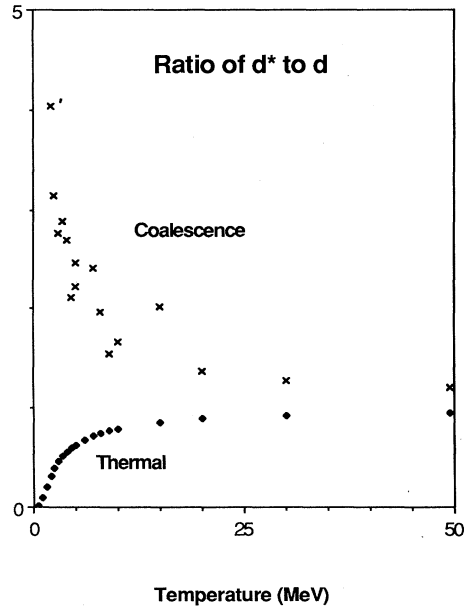

FIG. 1. The ratio of excited deuterons to deuterons is normalized so that it approaches unity at high temperatures. The coalescence formalism yields the peculiar result that the ratio rises as the temperature falls. The scatter in the points is due to the Monte Carlo nature of the integration.

$$R_{\rm ther}(T)\equiv\frac{[N_{\rm ther}(d^*)/N_{\rm ther}(d)](T)}{[N(d^*)/N(d)](T\to\infty)}\,,$$
$$R_{\rm coal}(T)\equiv\frac{[N_{\rm coal}(d^*)/N_{\rm coal}(d)](T)}{[N(d^*)/N(d)](T\to\infty)}\,. \tag{16}$$

By looking at Fig. 1, one can see that if local temperatures are 5 MeV, one could distinguish between the coalescence and the thermal pictures by simply seeing whether this ratio is greater or less than unity. Another key ingredient is seeing whether the ratio would rise or fall with the beam energy, although the experimental answer might easily be between these two values.

The most important neglect of this analysis is disregarding the effects of the Coulomb field which could affect the existence of the d^* near the residual system. This is discussed in the context of the compound nucleus by Bernstein.[9] For the case of instantaneous disintegration of a low density system the Coulomb effects should not be as large as the compound nucleus example of Ref. 9. The thermal formalism which is meant only as an extreme example should be modified for the case where the nucleons are emitted not from a single instantaneous matrix element but separately, at different times. By the time of the emission of the final particle, the first particle will have become a wave packet, correlating its position to its momentum, and narrowings its uncertainty in energy by $1/(t_1-t_2)$. Nonetheless, Eq. (13) gives insight into the coupling of the wave function and the density distribution of a violently disintegrating system.

If the temperatures extracted from a deuteron to d^* experiment would correspond to the 4–5 MeV found from ratios of other states, the thermal picture would be strengthened. This ratio is also quite important since the emission of protons, deuterons, and neutrons does not have the temperature-window effect of the compound nucleus.[10] Heavier ions can only escape at temperatures high enough to raise the several charges above the Coulomb barrier, and low enough that the proton and neutron cooling is not so rapid that the temperature falls in too short a time to emit a heavier particle. We con-

clude by stating that a good phenomenology of the break-up mechanism is necessary before a good phenomenology can be tested for previous stages.

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