Instability of hot nuclei

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(Received 3 May 1989)

A hot nucleus is unstable if its temperature exceeds a certain limiting value due to the Coulomb repulsion between its protons. This instability is investigated using a finite-temperature generalization of the liquid-drop model. For this purpose an equation of state for asymmteric nuclear matter derived in a previous work with the use of a Skyrme-type effective interaction is generalized to the case of a density-dependent effective mass. Standard parametrizations of the Skyrme interaction are used to study the properties of a hot ^{208}Pb nucleus immersed in a uniform vapor. Good agreement is achieved between the present model and the results of finite-temperature Hartree-Fock calculations only if the Coulomb interaction between the vapor and the drop is properly taken into account. The results are seen to yield valuable information on the temperature dependence of the surface tension of hot nuclei.

I. INTRODUCTION

In an earlier work,[1] hereafter referred to as I, an equation of state for asymmetric nuclear matter was derived and used to investigate the instability of hot nuclei caused by the Coulomb force. The investigation was carried out with the use of a model suggested by Levit and Bonche[2] which treats the hot nucleus as a spherical liquid drop with uniform density, a sharp edge, and a surface tension immersed in a uniform vapor. This is an extension of the liquid-drop model which is successful at very low temperatures. The equation of state used in I to describe the liquid and vapor phases was derived with the use of a Skyrme-type effective nucleon-nucleon interaction that did not include the so-called finite-range terms and hence did not include the important effects of the density dependence of the effective mass $m^*(\rho)$.

This inadequacy is remedied in the present work where a more general Skyrme-type interaction is used that not only allows for an examination of the role of the effective mass but also makes possible in the present calculations the use of any of the various parametrizations of the Skyrme interaction currently found in the literature since these parametrizations always yield an effective mass $m^*(\rho)\neq m$. The advantage of using such parametrizations is that it allows a comparison between the current results and those of finite-temperature Hartree-Fock calculations such as those carried out by Bonche *et al.*[3]

Another inadequancy of the work reported in I is that it assumed the vapor surrounding the nuclear drop to be electrically screened. Here this assumption is found to appreciably alter the properties of the drop-vapor system and therefore a proper treatment of the charge of the vapor is carried out so that the model becomes capable of yielding reliable results.

In Sec. II the role of the effective mass is investigated for the case where protons and neutrons are treated equally and the equation of state of symmetric nuclear matter is used to study the properties of hot nuclei. The equation of state is then generalized in Sec. III to include the effect of the neutron-proton asymmetry and used to investigate the instability of hot nuclei. The effects of the charge of the vapor are taken into consideration and found to be important. Section IV contains a discussion and concluding remarks about the present work.

II. EQUATION OF STATE OF SYMMETRIC NUCLEAR MATTER AND THE COULOMB INSTABILITY OF HOT NUCLEI

The equation of state of nuclear matter depends on the assumed form for the effective nucleon-nucleon interaction. In this work the following general form for a Skyrme-type interaction is used:

$$\begin{aligned} v_{12} = & -t_0(1+x_0P_\sigma)\delta(\mathbf{r}_1-\mathbf{r}_2) \\ & +\tfrac{1}{2}t_1[\overleftarrow{P}'^2\delta(\mathbf{r}_1-\mathbf{r}_2)+\delta(\mathbf{r}_1-\mathbf{r}_2)\mathbf{P}^2] \\ & +t_2\overleftarrow{P}'\cdot\delta(\mathbf{r}_1-\mathbf{r}_2)\mathbf{P} \\ & +\tfrac{1}{6}t_3(1+x_3P_\sigma)\rho^\sigma\left[\frac{\mathbf{r}_1+\mathbf{r}_2}{2}\right]\delta(\mathbf{r}_1-\mathbf{r}_2)\,, \end{aligned} \tag{2.1}$$

where P_σ is the spin-exchange operator and $\mathbf{P}$ and $\overleftarrow{\mathbf{P}}'$ are $(\nabla_1-\nabla_2)/2i$ and $-[(\overleftarrow{\nabla}_1-\overleftarrow{\nabla}_2)/2i]$, respectively. The parameters t_0, x_0, t_1, t_2, t_3, and x_3 are determined phenomenologically by fitting the ground-state properties of nuclei and/or nuclear matter. The nuclear density is denoted by ρ and the exponent σ of the density-dependent term is taken as a parameter that controls the compressibility of nuclear matter.[4] The interaction (2.1) differs from the one employed in I by the inclusion of the finite-range terms t_1 and t_2 and also by allowing for the possibility of varying the value of x_3 which was fixed at $x_3=1$ in I.

For symmetric nuclear matter the equation of state is independent of x_0 and x_3 and is given in terms of an ex-

pansion in the degree of degeneracy of the system[4]

$$\tilde{P}(T,\rho)=-\tfrac{3}{8}t_0\rho^2+\frac{t_3}{16}(1+\sigma)\rho^{2+\sigma}+\left[1-\frac{3}{2}\frac{\rho}{m^*}\frac{dm^*}{d\rho}\right]T\rho\sum_{n=0}^{\infty}b_n\left[\frac{\lambda_T^3\rho}{g}\right]^n, \tag{2.2a}$$

or

$$\tilde{\mu}(T,\rho)=-\tfrac{3}{4}t_0\rho+\frac{t_3}{16}(2+\sigma)\rho^{1+\sigma}-\tfrac{3}{2}T\frac{\rho}{m^*}\frac{dm^*}{d\rho}\sum_{n=0}^{\infty}b_n\left[\frac{\lambda_T^3\rho}{g}\right]^n+T\left[\ln\left[\frac{\lambda_T^3\rho}{g}\right]+\sum_{n=1}^{\infty}\frac{n+1}{n}b_n\left[\frac{\lambda_T^3\rho}{g}\right]^n\right], \tag{2.2b}$$

where $\tilde{P}$ is the pressure, $\tilde{\mu}$ is the chemical potential, T is the temperature in energy units, g is the spin-isospin degeneracy ($g=4$ for symmetric nuclear matter), and λ_T is the thermal wavelength of the nucleon

$$\lambda_T=\left[\frac{2\pi\hbar^2}{m^*T}\right]^{1/2}. \tag{2.2c}$$

The effective mass m^* is given by

$$m^*=m\left[1+\frac{m\rho}{8\hbar^2}(3t_1+5t_2)\right]^{-1}. \tag{2.2d}$$

Note that $m^*=m$ if $t_1=t_2=0$ as was used in I. The b_n's are the coefficients of the virial series for an ideal Fermi gas with $b_0=1$ and the remaining coefficients, up to $n=5$, are listed in I. Only terms up to fifth order in degeneracy ($n<5$) are included in all calculations in the present work as well as in I.

With the equation of state (2.2), the coexistence equations between the liquid drop, assumed to be a uniform sphere with radius R_L, and the surrounding uniform vapor are determined by the standard requirements of the equality of the liquid (L) and vapor (V) temperatures, pressures, and chemical potentials:[2]

$$\tilde{P}(T,\rho_L)+P_{\text{Coul}}(\rho_L)+P_{\text{surf}}(T,\rho_L)=\tilde{P}(T,\rho_V), \tag{2.3a}$$

$$\tilde{\mu}(T,\rho_L)+\frac{Z}{A}\mu_{\text{Coul}}(\rho_L)=\tilde{\mu}(T,\rho_V), \tag{2.3b}$$

where P_{Coul} and $(Z/A)\mu_{\text{Coul}}$ are the Coulomb contributions to the pressure and chemical potential of the drop which contains Z protons and $N=A-Z$ neutrons

$$P_{\text{Coul}}=\left[\frac{4\pi\rho_L}{3A}\right]^{1/2}\frac{Z^2e^2}{5A}\rho_L, \tag{2.4a}$$

$$\mu_{\text{Coul}}=\tfrac{6}{5}Ze^2\left[\frac{4\pi\rho_L}{3A}\right]^{1/3}. \tag{2.4b}$$

P_{surf} is the contribution of the surface tension $\gamma(T)$ of the liquid drop

$$P_{\text{surf}}=-\frac{2\gamma(T)}{R_L}=-2\gamma(T)\left[\frac{4\pi\rho_L}{3A}\right]^{1/3} \tag{2.5a}$$

with

$$\gamma(T)=1.14\ \text{MeV fm}^{-2}\left[1+\frac{3}{2}\frac{T}{T_c}\right]\left[1-\frac{T}{T_c}\right]^{3/2} \tag{2.5b}$$

as used in I.

As found in Ref. 2 and also in I, the coexistence equations (2.3) have a real solution only up to a limiting temperatures T_{lim} indicating that the liquid drop is unstable above this temperature. This instability is due to the Coulomb force since it disappears if the protons are assumed chargeless[2] and the nucleus can then survive up to the critical temperature T_c which is the temperature above which there is a single fluid phase. The values of T_{lim} calculated for two values of the parameter σ of the density-dependent term and for various values of $m^*(\rho_0)$, the effective mass as evaluated at nuclear saturation density ρ_0, are given in Table I for the case of a ^{208}Pb nucleus together with the corresponding values of the critical temperature of symmetric nuclear matter. In each case, for every pair of values of $m^*(\rho)$ and σ, the parameters of the Skyrme interaction are determined in terms of the binding energy per particle $E_B(=16$ MeV) and the kinetic energy per particle $E_K(=24$ MeV) in the ground state of nuclear matter:[4]

$$\tfrac{3}{8}t_0\rho_0\sigma=(1+\sigma)E_B+E_K+(\sigma-\tfrac{2}{3})\frac{m}{m^*(\rho_0)}E_K, \tag{2.6a}$$

$$\tfrac{1}{16}t_3\rho_0^{1+\sigma}\sigma=E_B+E_K-\frac{2}{3}\frac{m}{m^*(\rho_0)}E_K. \tag{2.6b}$$

By examining Table I it is obvious that the limiting temperature varies with the value of the effective mass and correspondingly, through Eq. (2.2d), with the values of the finite-range terms of the Skyrme interaction. This variation correlates to a large extent with the correspond-

TABLE I. Values of the limiting temperature of a ^{208}Pb nucleus for various values of the effective mass at nuclear saturation density (ρ_0) and for two values of σ. Also listed are the corresponding values for the critical temperature of symmetric nuclear matter. Note that for $m^*(\rho_0)=0.40$ m the results are independent of σ since $t_3=0$ in this case.

	$\sigma=1$		$\sigma=\frac{1}{4}$	
$\frac{m^*(\rho_0)}{m}$	T_{lim} (MeV)	T_c (MeV)	T_{lim} (MeV)	T_c (MeV)
1.0	8.51	22.98	6.27	17.34
0.9	8.30	21.47	6.34	16.76
0.8	8.17	20.27	6.46	16.36
0.7	8.10	19.37	6.66	16.19
0.6	8.10	18.83	6.99	16.43
0.5	8.25	18.81	7.61	17.37
0.45	8.48	19.11	8.26	18.30
0.4	9.23	19.73	9.23	19.73

TABLE II. Parameters of the SIII and SKM interactions used in the present calculations. Also listed are the values of the critical temperature of symmetric nuclear matter and the effective mass for $\rho=\rho_0$ calculated with these interactions. Note that Ref. 3 gives a value $x_0=0.34$ for the SIII interaction whereas a value $x_0=0.45$ appears in the original work of Beiner *et al.* (Ref. 5). The value 0.45 is used in the present calculations as it is assumed that the 0.34 value is a result of a typing error. In any case, it has been checked that changing x_0 from 0.45 to 0.34 affects the limiting temperature of ^{208}Pb by $\lesssim 0.2$ MeV.

	t_0 (MeV fm^3)	x_0	t_1 (MeV fm^5)	t_2 (MeV fm^5)	t_3 (MeV fm$^{3\sigma+3}$)	x_3	σ	$\frac{m^*(\rho_0)}{m}$	T_c (MeV)
SIII	1128.75	0.45	395.0	−95.0	14 000	1.0	1	0.760	17.85
SKM	2645.0	0.09	385.0	−120.0	15 595.0	0.0	$\frac{1}{6}$	0.789	14.58

ing variation in the critical temperature. The value of $T_{\rm lim}$ becomes increasingly sensitive to the value of $m^*(\rho_0)$ as the latter approaches its lowest possible value of $m^*(\rho_0)=0.4$ m that corresponds to $t_3=0$ in Eq. (2.6b), in which case the saturation of nuclear matter is determined solely by the finite-range terms. Lower values of $m^*(\rho_0)$ are not physically acceptable since they would imply that the density-dependent term is attractive. The results in Table I do not include the isospin effects associated with the neutron-proton asymmetry. Nevertheless, they serve to illustrate the importance of the inclusion of the proper value of the effective mass especially for the case with $\sigma=\frac{1}{4}$ where $T_{\rm lim}$ changes by about 3 MeV in going from $m^*(\rho_0)=m$ to $m^*(\rho_0)=0.4$ m.

Finally it would be interesting to calculate $T_{\rm lim}$ with some of the various parametrizations of the Skyrme interaction currently found in the literature. Because of their relevance to the remainder of the present work two such sets of parameters have been chosen, one corresponding to the standard SIII interaction of Ref. 5 while the other is obtained from Ref. 6 where it is denoted the SKM interaction. The parameters of both interactions are shown in Table II. Without including any corrections connected with the proton-neutron asymmetry the SIII interaction yields a value $T_{\rm lim}=6.73$ MeV while the SKM interaction gives $T_{\rm lim}=5.25$ MeV. The values of the corresponding critical temperatures are listed in Table II.

III. EQUATION OF STATE OF ASYMMETRIC NUCLEAR MATTER AND THE COULOMB INSTABILITY OF HOT NUCLEI

In a proper treatment one must distinguish between the neutrons (n) and the protons (p) and assign different densities, effective masses, and chemical potentials to each species of nucleons. The chemical potential for species q (where $q=p$ or n) is then given by[7]

$$\mu_q=\varepsilon_q+T\left[\ln\left[\frac{\lambda_T^3\rho_q}{g_s}\right]+\sum_{n=1}^{\infty}\frac{n+1}{n}b_n\left[\frac{\lambda_T^3\rho_q}{g_s}\right]^n\right], \tag{3.1}$$

where $g_s=2$ is the spin degeneracy of each species and ε_q is the single-particle potential energy for species q (Ref. 8)

$$\varepsilon_q=-t_0\left[\left[1+\frac{x_0}{2}\right]\rho-(x_0+\tfrac{1}{2})\rho_q\right]+\tfrac{1}{4}(t_1+t_2)\tau+\tfrac{1}{8}(t_2-t_1)\tau_q+\frac{t_3}{12}\left\{\left[2+\frac{\sigma}{2}+x_3\left[1-\frac{\sigma}{2}\right]\right]\rho^{1+\sigma}-2(x_3+\tfrac{1}{2})\sigma\rho^{\sigma-1}\rho_q^2+2(x_3+\tfrac{1}{2})(\sigma-1)\rho^{\sigma}\rho_q\right\}. \tag{3.2}$$

In (3.2) τ_q is the kinetic energy density of species q and $\tau=\tau_n+\tau_p$,

$$\begin{aligned}\tau_q&=\sum_i n_i|\nabla\phi_i(\mathbf{r},q)|^2\\&\to\left[\frac{L}{2\pi}\right]^3 g_s\int d^3k|\nabla\phi_{\mathbf{k}}(\mathbf{r},q)|^2 n_{\mathbf{k}}\\&=\frac{1}{(2\pi)^3}g_s\int d^3k(k^2)n_{\mathbf{k}}\\&=\tfrac{3}{2}T\frac{2m_q^*}{\hbar^2}\rho_q\sum_{n=0}^{\infty}b_n\left[\frac{\lambda_T^3\rho_q}{g_s}\right]^n,\end{aligned} \tag{3.3}$$

where the summation has been changed into an integration in the standard manner and the last step follows from the definition of the b_n's.[1,4] In (3.3) n_i is the occupation probability of the state i at temperature T and the effective mass for each species is given by

$$m_q^*=m\left[1+(t_1+t_2)\frac{m\rho}{2\hbar^2}+(t_2-t_1)\frac{m\rho_q}{4\hbar^2}\right]^{-1}. \tag{3.4}$$

The contribution of the Coulomb force does not appear in Eq. (3.2) since it will be included separately in the coexistence equations.

The final expressions for the chemical potentials to second order in the asymmetry parameter

$$\alpha=\frac{\rho_n-\rho_p}{\rho} \tag{3.5}$$

are then given by

$$\mu_q(T,\rho,\alpha)=\tilde{\mu}(T,\rho)\pm\mu_1(T,\rho)\alpha+\mu_2(T,\rho)\alpha^2, \tag{3.6a}$$

where the + (−) sign applies for neutrons (protons) and $\tilde{\mu}(T,\rho)$ is the chemical potential for symmetric nuclear matter given by Eq. (2.2b). In addition,

$$\mu_1(t,\rho)=\frac{t_0}{2}(x_0+\tfrac{1}{2})\rho-\frac{t_3}{12}(x_3+\tfrac{1}{2})\rho^{\sigma+1}+T\sum_{n=0}^{\infty}[n+1+\tfrac{3}{2}\beta(n+1-\beta)]b_n\left[\frac{\lambda_T^3\rho}{g}\right]^n \tag{3.6b}$$

and

$$\mu_2(T,\rho)=-\frac{t_3}{24}(x_3+\tfrac{1}{2})\sigma\rho^{\sigma+1}+T\sum_{n=0}^{\infty}\left\{\frac{n^2-1}{2}-\frac{3}{2}\frac{\rho}{m^*}\frac{dm^*}{d\rho}\left[\beta^2-\beta n-\beta+\frac{n(n+1)}{2}\right]\right\}b_n\left[\frac{\lambda_T^3\rho}{g}\right]^n \tag{3.6c}$$

with

$$\beta=\frac{m^*}{8\hbar^2}(t_2-t_1)\rho=\frac{t_2-t_1}{3t_1+5t_2}\left[1-\frac{m^*}{m}\right]\equiv\delta\left[1-\frac{m^*}{m}\right]. \tag{3.6d}$$

The pressure of the system is then determined through the Gibbs-Duhem relation

$$\frac{\partial P}{\partial\rho}=\rho\frac{\partial}{\partial\rho}\left[\frac{Z}{A}\mu_p+\frac{N}{A}\mu_n\right], \tag{3.7}$$

which leads, up to order α^2, to the expression

$$P(T,\rho,\alpha)=\tilde{P}(T,\rho)+P_{\rm sym}(T,\rho)\alpha^2, \tag{3.8a}$$

where $\tilde{P}$ is the pressure for symmetric nuclear matter given by Eq. (2.2a) and

$$P_{\rm sym}(T,\rho)=\frac{t_0}{4}(x_0+\tfrac{1}{2})\rho^2-\frac{t_3}{24}(x_3+\tfrac{1}{2})(1+\sigma)\rho^{2+\sigma}+T\rho\sum_{n=1}^{\infty}\frac{n(n+1)}{2}b_n\left[\frac{\lambda_T^3\rho}{g}\right]^n+\rho\mu_3-\int\mu_3 d\rho, \tag{3.8b}$$

where

$$\mu_3=\tfrac{3}{2}T\frac{m(3t_1+5t_2)}{8\hbar^2}\rho\sum_{n=0}^{\infty}b_n\left[\frac{\lambda_T^3\rho}{g}\right]^n\left[\delta^2\left[\frac{m^*}{m}\right]^3+(\delta+n\delta-\delta^2)\left[\frac{m^*}{m}\right]^2+\frac{n(n+1)}{2}\left[\frac{m^*}{m}\right]\right] \tag{3.8c}$$

with δ as defined in Eq. (3.6d). The terms that appear in μ_3 cannot be integrated exactly. From the dependence of m^* on ρ given by Eq. (2.2d) it is, however, possible to successively integrate each term by parts and express it as a convergent series

$$\int\left[\frac{m^*}{m}\right]^s(\lambda_T^3)^{\nu-1}\rho^\nu d\rho\sim\int\frac{\rho^\nu d\rho}{(1+\omega\rho)^\varepsilon}=\frac{\rho^{\nu+1}}{(\nu+1)(1+\omega\rho)^\varepsilon}\left[1+\frac{\varepsilon\omega\rho}{(\nu+2)(1+\omega\rho)}+\frac{\varepsilon(\varepsilon+1)\omega^2\rho^2}{(\nu+2)(\nu+3)(1+\omega\rho)^2}+\cdots\right], \tag{3.9}$$

where

$$\varepsilon=\tfrac{3}{2}-\frac{3\nu}{2}+s$$

and

$$\omega=\frac{m(3t_1+5t_2)}{8\hbar^2}.$$

In practice it was found that the first three terms in the series (3.9) are sufficient for the accuracy required in the present calculations.

With the neutron and proton chemical potentials given by Eq. (3.6a) and the pressure given by Eq. (3.8a) it is straightforward to write down the coexistence equations which express the thermal, chemical, and mechanical equilibrium between the drop and the surrounding vapor

$$P(T,\rho_L,\alpha_L)+P_{\rm Coul}(\rho_L)+P_{\rm surf}(T,\rho_L)=P(T,\rho_V,\alpha_V), \tag{3.10a}$$

$$\mu_n(T,\rho_L,\alpha_L)=\mu_n(T,\rho_V,\alpha_V), \tag{3.10b}$$

$$\mu_p(T,\rho_L,\alpha_L)+\mu_{\rm Coul}(\rho_L)=\mu_p(T,\rho_V,\alpha_V). \tag{3.10c}$$

These equations are a generalization of Eqs. (2.3). They were solved by the method described in I for the case of a drop corresponding to a ^{208}Pb nucleus with the use of the SIII and SKM interactions whose parameters are shown in Table I. These two interactions were used in Ref. 3 in finite-temperature Hartree-Fock (HF) calculations for the same nucleus which permits a direct comparison between the present results and those of Ref. 3 in order to examine the validity of the present model.

With the SIII interaction the limiting temperature of a ^{208}Pb nucleus was determined to be 5.8 MeV (as opposed

to 6.7 MeV obtained in the previous section where symmetry corrections were neglected). This is to be compared with the result of the HF calculation,[3] where it was found, with the use of the same interaction, that the ^{208}Pb nucleus can survive up to a temperature of about 10 MeV. This serious discrepancy indicates the need to examine the possibility of introducing various corrections and modifications into the present model in order to see if the discrepancy can be removed.

One such correction is a better treatment of the Coulomb energy to include both exchange and diffuseness contributions

$$E_{\mathrm{Coul}}=\frac{3}{5}\frac{Z^2e^2}{R_L}\left[1-\frac{5}{2}\frac{b^2}{R_L^2}\right]-\frac{3}{4}\left[\frac{3}{2\pi}\right]^{2/3}\frac{e^2Z^{4/3}}{R_L}\ , \tag{3.11}$$

where b is the surface thickness of the nucleus as defined by Myers.[9] The last term in (3.11) is the Slater approximation for the exchange part of the Coulomb energy[10] while the term depending on b gives the correction due to the fact that the nucleus has a diffuse rather than a sharp edge.[11] The corresponding contributions to the pressure and chemical potential inside the nucleus are then determined by the relations

$$P_{\mathrm{Coul}}=-\left[\frac{\partial E_{\mathrm{Coul}}}{\partial V_L}\right]_Z$$

and

$$\mu_{\mathrm{Coul}}=\left[\frac{\partial E_{\mathrm{Coul}}}{\partial Z}\right]_{V_L}\ ,$$

where

$$V_L=\frac{4\pi}{3}R_L^3$$

is the volume of the liquid drop. The value of b used in the present calculations is taken from Ref. 11 where it was evaluated by a thermal Hartree-Fock calculation

$$b\approx 0.72(1+9\times 10^{-3}T^2)\ \mathrm{fm}\ . \tag{3.12}$$

This value somewhat underestimates the diffuseness of the nucleus which is found experimentally to correspond to $b\approx 1$ fm at $T=0$. However, (3.12) is used here because it is obtained by a HF calculation with a Skyrme interaction very close to SIII which makes it possible to continue the comparison between the present results and those of Ref. 3. With these corrections to the Coulomb terms it is found that T_{lim} for ^{208}Pb is raised slightly from 5.8 to 6.1 MeV.

Another ingredient that can be investigated in order to see its effect on T_{lim} is the temperature dependence of the surface tension. In principle, the surface tension is related to the equation of state since they are both determined by the same effective interaction. This relationship is, however, not available, and the temperature dependence of the surface tension is treated independently of the equation of state except for the fact that the surface tension vanishes as $T\to T_c$ where the critical temperature T_c is determined by the equation of state. The surface tension given by Eq. (2.5b) has the correct value and quadratic dependence on T for small temperatures. It also vanishes at the critical temperature but it is not clear whether it has the correct magnitude at intermediate temperatures. The calculation of T_{lim} is therefore repeated but with the temperature dependence of the surface tension given by

$$\gamma(T)=1.14\ \mathrm{MeV\,fm^{-2}}\left[1-\frac{T}{T_c}\right]^2 \tag{3.13}$$

rather than by Eq. (2.5b). This expression also vanishes as $T\to T_c$ but it yields much lower values of the surface tension at intermediate temperatures than does Eq. (2.5b). This change in the temperature dependence of the surface tension raises the limiting temperature from 6.1 to 7.1 MeV for the SIII interaction. This change in T_{lim} is in the right direction but it is not enough to resolve the discrepancy with the HF calculations. It may be argued that at intermediate temperatures the surface tension should be lower than that given by Eq. (3.13). This, however, will not be sufficient to raise the limiting temperature appreciably for even if the surface tension is assumed to vanish T_{lim} is found to reach 7.9 MeV only for the SIII interaction.

Another contribution that has been neglected so far is the effect of the charge of the vapor. Treating the vapor as completely screened is an oversimplification that must be examined carefully. If the vapor is assumed to extend from R_L up to a radius R_V with a total proton number Z_V surrounding the nucleus which has Z protons, then the total Coulomb energy of the system is given by

$$E_C=E_C(\mathrm{dd})+E_C(\mathrm{dv})+E_C(\mathrm{vv})\ , \tag{3.14}$$

where $E_C(\mathrm{dd})$ represents the Coulomb interaction of the drop charges with each other and is taken to be given by Eq. (3.11), while $E_C(\mathrm{dv})$ and $E_C(\mathrm{vv})$ represent, respectively, the Coulomb interaction between the drop and vapor charges and between the vapor charges themselves. These are given (for uniform liquid and vapor densities) by the expression[12]

$$E_C(\mathrm{dv})=\tfrac{3}{2}e^2ZZ_V\frac{R_V^2-R_L^2}{R_V^3-R_L^3}\ , \tag{3.15a}$$

$$E_C(\mathrm{vv})=\tfrac{3}{5}e^2Z_V^2\frac{[(R_V^5-R_L^5)-\frac{5}{2}R_L^3(R_V^2-R_L^2)]}{(R_V^3-R_L^3)^2}\ . \tag{3.15b}$$

With the expression (3.14) for the Coulomb energy the Coulomb contribution to the pressure difference across the liquid-vapor interface is given by $-(\partial E_C/\partial V_L)_{Z,Z_V}$, while its contribution to the chemical potential of the protons in the drop and in the vapor is given by $(\partial E_C/\partial Z)_{Z_V,V_L}$ and $(\partial E_C/\partial Z_V)_{Z,V_L}$, respectively. It is worth mentioning that the HF calculations of Ref. 3 are carried out inside a spherical box whose radius corresponds to R_V in the present work. This makes the comparison between the present results and those of Ref. 3 even more meaningful. Using a value of $R_V=16$ fm (the

same as in Ref. 3) and with the surface tension given by Eq. (2.5b), the SIII interaction yields, for the limiting temperature of ^{208}Pb, the value of 10.0 MeV which is in excellent agreement with the results of the HF calculation.[3] This value is to be compared with $T_{\rm lim}=6.1$ MeV obtained with the same interaction and surface tension but with the vapor assumed uncharged. This large change in the value of $T_{\rm lim}$ indicates the importance of the inclusion of the effects of the electric charge of the vapor.

A question naturally arises as to the effect of the value of R_V used on the results of the calculation. In principle, these results should be independent of R_V and this was the case with the HF calculations for $R_V \geq 14$ fm.[3] In the present case changing R_V affects the results of the calculation; for example, with the SIII interaction, changing R_V from 16 to 18 fm changes $T_{\rm lim}$ from 10.0 to 9.4 MeV while decreasing R_V to 14 fm raises $T_{\rm lim}$ to 10.7 MeV. This dependence on R_V reflects the effects of the increasing size of the vapor charge and is due to the assumption of uniform vapor charge density used in calculating the Coulomb energy of Eqs. (3.14) and (3.15). In contrast the self-consistent HF equations lead to the polarization of the vapor with the vapor proton density increasing with the distance away from the surface of the nucleus. This polarization reduces the effect of the charge of the vapor.

Using the SKM interaction with the surface tension (2.5b) and $R_V=16$ fm, the limiting temperature for ^{208}Pb is found to be 7.7 MeV. This is also in good agreement with the HF calculations[3] which yield $T_{\rm lim}\sim 8$ MeV for the SKM interaction. The inclusion of the effect of the charge of the vapor is seen again to produce satisfactory results. Moreover, the expression (2.5b) is found to adequately describe the temperature dependence of the surface tension at intermediate temperatures when used with the SKM as well as SIII interactions. In contrast, expression (3.13) is found unacceptable as it yields values of $T_{\rm lim}$ for ^{208}Pb that are higher than the HF values by about 1.3 MeV (it yields $T_{\rm lim}=11.3$ MeV with the SIII interaction and $T_{\rm lim}=8.95$ MeV with the SKM interaction). This illustrates how information on $T_{\rm lim}$ can be used to study the surface properties of hot nuclei.

The effect of polarization can be investigated with a simple model using a linearly increasing vapor proton density of the form

$$\rho_{pV}=\bar{\rho}_{pV}[1+\varepsilon(r-\bar{R})], \quad R_L<r<R_V , \tag{3.16}$$

where $\bar{R}$ is chosen so that the total number of protons in the vapor (Z_V) is independent of ε, i.e., Z_V depends only on R_V and $\bar{\rho}_{pV}$. This is achieved by setting

$$\bar{R}=\frac{3}{4}\frac{R_V^4-R_L^4}{R_V^3-R_L^3} . \tag{3.17}$$

This choice makes it possible to investigate the effect of changing the polarization of the vapor without changing Z_V. It is, of course, understood that the vapor neutron density distribution remains uniform.

With the density distribution (3.16), the total Coulomb energy for the drop plus vapor system is then found to be

$$E_C(\varepsilon)=E_C(0)-F\varepsilon+G\varepsilon^2 , \tag{3.18a}$$

where $E_C(0)$ is the Coulomb energy for the case of a uniform vapor density and is given by Eqs. (3.14) and (3.15), while F and G are given by

$$F=\frac{(R_V-R_L)^4}{(R_V^3-R_L^3)}\left[\frac{\pi\bar{\rho}_{pV}Ze^2}{6}(R_V^2+4R_VR_L+R_L^2)+\frac{2\pi^2\bar{\rho}_{pV}^2e^2}{45}\times(R_V-R_L)(R_V^4+5R_V^3R_L+15R_V^2R_L^2+20R_VR_L^3+4R_L^4)\right] \tag{3.18b}$$

and

$$G=\frac{\pi^2\bar{\rho}_{pV}^2e^2}{210}\frac{(R_V-R_L)^7}{(R_V^3-R_L^3)^2}\times(R_V^6+7R_V^5R_L+28R_V^4R_L^2+54R_V^3R_L^3+28R_V^2R_L^4+7R_VR_L^5+R_L^6) . \tag{3.18c}$$

Assuming that for a given R_V and Z_V the distribution of the vapor charge is such that it minimizes the Coulomb repulsion energy, one is led to minimizing $E_C(\varepsilon)$. This has a minimum at

$$\varepsilon=\bar{\varepsilon}=\frac{F}{2G} \tag{3.19a}$$

so that

$$E_C(\bar{\varepsilon})=E_C(0)-\frac{F^2}{4G} . \tag{3.19b}$$

It is seen that polarization reduces the value of the total Coulomb energy for the vapor plus drop system by an amount equal to $F^2/4G$ as compared with the case in which the vapor is assumed to have a uniform charge density. Moreover, the change of $\bar{\varepsilon}$ with R_V is found to partially offset the effect of the corresponding change in the total charge of the vapor so that the total Coulomb energy given by Eq. (3.19b) increases more slowly with R_V than does the corresponding expression for a uniform vapor charge density. As an example, for a ^{208}Pb nucleus it is found that $\bar{\varepsilon}$ increases from 1.64 to 2.92 fm^{-1} as R_V decreases from 16 to 14 fm. The corresponding decrease in the total Coulomb energy is

$$E_C\ (R_V=16\text{ fm})-E_C(R_V=14\text{ fm})=405.8\text{ MeV}$$

as compared to 531.5 MeV for the case where no polarization is assumed. Similarly, when R_V is increased from 16 to 18 fm, $\bar{\varepsilon}$ decreases to 1.05 fm^{-1} and E_C increases by only 601.6 MeV as opposed to an increase of 752.3 MeV when a uniform density is assumed. These calculations are carried out using $R_L=7.349$ and $\bar{\rho}_{pV}=0.006\,14$ which are obtained in the calculation reported above for $T_{\rm lim}=10.0$ MeV with $R_V=16$ fm.

Expression (3.19b) does not, however, completely remove the dependence on R_V although it diverges more slowly with R_V than do Eqs. (3.15). This is because the

variation carried out has been a restricted one with the vapor charge density allowed to increase only linearly with r. In contrast, the thermal HF calculations[3] lead to a vapor charge density that increases exponentially with r. The present model given by Eq. (3.16) is used here because of its simplicity and its ability to yield analytical results. Moreover, no attempt is made here to incorporate the effect of polarization into the coexistence equations as it is felt that is goes beyond the scope of the drop-vapor coexistence model used in this work with its associated uniform densities. Problems would arise in finding the equation of state for such a variable-density system and in calculating the neutron-proton symmetry energy.

IV. DISCUSSION AND CONCLUSION

In the present work hot nuclei are treated as liquid drops immersed in a vapor of nucleons. Both the liquid and vapor states are described by an equation of state for nuclear matter that includes both degeneracy corrections as well as corrections due to the neutron-proton asymmetry. This equation generalizes the equation of state of asymmetric nuclear matter derived in I for the case $m^*=m$ to the case of a density-dependent effective mass. This generalization allows for an investigation of the effect of the density dependence of the nucleonic effective mass on the stability of hot nuclei and also makes possible the use of standard parametrizations of the Skyrme interaction. The other modification introduced in the present work is to include the effects of the electric charge of the vapor but with the simplifying assumption that the drop and vapor have uniform charge density.

These improvements remove two of the simplifying features of Refs. 1 and 2 and, together with the corrections introduced in I, are found to lead to very good agreement between the results of the present model and those of finite-temperature HF calculations for both the SIII and SKM interactions. This agreement is seen to be useful in gaining information on the temperature dependence of the surface $\gamma(T)$. In particular, expression (2.5b) for $\gamma(T)$, which satisfies many of the properties expected on physical grounds,[13] is found to yield values of $T_{\rm lim}$ for ^{208}Pb close to those predicted by HF calculations for both interactions while expression (3.13) yields values of $T_{\rm lim}$ that are too high by about 1.3 MeV. Another correction that has been included is the effect of the diffuseness of the nuclear surface on the Coulomb energy. This is found to be rather small, leading to a change in the value of $T_{\rm lim}$ by $\sim$0.3 MeV. In contrast, the effect of including the charge of the vapor leads to an increase in $T_{\rm lim}$ of about 4 MeV.

Despite these improvements the present model still has some simplifying features that are carried over from the previous work of Refs. 1 and 2, namely, the neglect of the temperature dependence of the Coulomb contribution and the assumption of a uniform density distribution both inside the nucleus and for the surrounding vapor. The last feature results in the absence of any polarization of the vapor in the present work which is reflected in the fact that the results depend on the radius R_V of the spherical box in which the vapor is enclosed. A simple model involving a linearly increasing vapor charge density has been used to illustrate the effect of polarization in reducing the dependence of the Coulomb repulsion energy on the radius R_V. It was not, however, attempted to include the effect of polarization into the coexistence equations.

The exchange part of the Coulomb interaction has been included in Eq. (3.11) for the interaction between the protons in the liquid drop but not for the interactions between the vapor protons or between them and the liquid drop. This is not, however, expected to have an appreciable effect. Curvature effects have also not been included. Empirical evidence suggests that the curvature energy of a nucleus is consistent with zero although a semiclassical study of nuclear properties with Skyrme-type effective interactions yields a nonzero value.[14] The curvature contribution is expected to be important near the critical temperature where the surface tension goes to zero. However, since $T_{\rm lim}$ is much lower than T_c, the curvature term is not expected to have a significant effect on the results of the present work. Corrections to the surface tension due to the neutron-proton asymmetry are also neglected. At zero temperature these corrections for a ^{208}Pb nucleus are approximately 8 and 15 % for the SIII and SKM forces, respectively.[14]

The results of the present calculations indicate that the liquid-drop model, which has been very successful in studying nuclear properties at very low temperatures, is equally successful in studying hot nuclei provided that care is taken to include the important corrections discussed here and in I. With these corrections properly taken care of, the largest uncertainty in the values calculated for $T_{\rm lim}$ is that due to the equation of state used (or the corresponding effective interaction). Measurements of $T_{\rm lim}$ can therefore be used to study the equation of state of nuclear matter.

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