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Pions in the deuteron

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A potential containing a one-pion-exchange tail and regularized at the origin by means of three parameters is used to construct several families of deuteron wave functions, which are employed in the assessment of the influence of the inner parts of the potential over observables such as r_m, η, and Q. The off-energy shell extrapolation of the results is considered, so as to provide guidelines for the treatment of other systems such as the triton. We also show that, provided the central potential contains a one-pion-exchange tail, the value of η is determined with great precision by just the inner part of the tensor component of the interaction.

I. INTRODUCTION

More than 25 years ago Glendenning and Kramer[1] established a close relationship between several deuteron properties and the pionic tail of the nucleon-nucleon potential. Since then, this relationship has been both extended and refined, and nowadays it is well accepted that pion dynamics dominates observables such as the quadrupole moment Q and the asymptotic D/S ratio η. The latter, in particular, deserved considerable attention in recent times, for it was realized that its value is a direct consequence of the NN interaction, whereas Q is influenced by meson-exchange currents, and hence is subject to additional theoretical uncertainties.[2,3]

The theoretical association between η and the one-pion exchange potential (OPEP) proved to be very fruitful. For instance, it paved the way for the derivation of rather narrow bounds for allowed values of this asymptotic ratio[4] and also motivated an effort to disentangle effects due to single-pion exchange from those arising from multiple pion scattering, form factors, and quark bags.[5,6] Indeed, the domination of single-pion dynamics was found to be so strong as to allow η to be taken as a sort of evidence for the very existence of pions in nuclei.[7] And it is worth noting that this is a substantive question, at a time when both nucleons and pions are understood as objects made of bound quarks.

In the framework of nonrelativistic potential theory, the value of any observable can be obtained from the solution of the Schrödinger equation for a given NN interaction and hence, in principle, it could depend on all the parameters used to describe that interaction. In the case of η, for instance, this possibility may be summarized by the expression $\eta=\eta[g,\mu;(\text{o.p.})]$, where g is the πN coupling constant, μ is the pion mass, and (o.p.) indicates collectively all the other parameters on which the potential depends. On the other hand, when an observable is dominated by pion dynamics, one expects the influence of the other parameters to be an indirect one, occurring through the values of either the binding energy ($E>0$) or $\alpha(\alpha\equiv\sqrt{mE}\,)$. Again in the case of η, this idea could be formalized by writing $\eta=\eta(g,\mu;\alpha)$, where α depends on all the parameters of the potential. This amounts to stating that η is an external quantity, depending only on the tail of the wave function, which is determined by the potential at large distances.

In recent papers, Ericson and Rosa-Clot studied in great detail the dynamical content of η, employing a method based on a Green's function for the deuteron differential equation.[5,6,8] Their analysis showed that 95% of the value of this ratio can be ascribed to the exchange of a single pion, whereas the leading correction would come from the two-pion exchange process, whose contribution would be about 4%. The formalism developed by Ericson and Rosa-Clot seems also to be suited to the study of external observables of other few-body systems such as, for instance, the asymptotic D/S ratio η_t of the triton-deuteron-neutron (tdn) vertex. This quantity deserved some attention lately, both by experimentalists[9] and theoreticians[10,11] and it may, in principle, provide information about the dynamical content of this vertex, especially concerning the role of three-nucleon forces. The longest-range three-nucleon interaction is the two-pion exchange three-nucleon potential ($\pi\pi$E-3NP),[12] and recent calculations leave no doubt that it does influence the triton binding energy.[13,14] The present stage of the problem consists in quantifying precisely this statement, since numerical results depend quite strongly on the πN form factors included in the $\pi\pi$E-3NP. Nevertheless, the conclusion is possible that this force can influence the tdn asymptotic D/S ratio at least indirectly, through the value of the separation energy. However, in the tdn vertex, the $\pi\pi$E-3NP has components whose ranges are comparable to either the OPEP or to the two-pion exchange component of the NN force. The former case occurs when one of the pions of the 3NP is hidden within the deuteron, whereas the latter happens when both pions are outside it. These dynamical features, when combined with the fact that the tdn separation energy is about three times larger than the deuteron binding energy, allow the hope of a non-negligible direct influence of the $\pi\pi$E-3NP over η_t.

In the study of the deuteron performed by Ericson and Rosa-Clot,[5,6] the binding energy was always kept fixed, while the sensitivity of η to changes of the various parameters of both the interaction and the S wave function was estimated. The assessment of the implications of this crucial element of their analysis is one of the purposes of the present work. The motivation for this assessment is twofold. First, we note that the study of the deuteron was made possible due to the availability of several wave functions, constructed by means of realistic NN potentials and having the correct binding energy. The case of the triton, on the other hand, is different. Theoretical calculations based exclusively on pair interactions yield binding energies which are both dependent on the realistic NN potential adopted and below the experimental value.[13,14] The inclusion of three-nucleon potentials is able to increase the binding energy, but uncertainties associated with the πN form factor do not allow a unique value to be obtained.[13,14] Therefore, we take the deuteron as a test ground for studying how changes in the binding energy of a system affect its external observables. Our second motivation concerns the theoretical method developed by Ericson and Rosa-Clot.[5,6] In their study of the deuteron, they found a quite important dependence of the asymptotic D/S ratio on πN form factors. This finding came as a surprise, since changes in form factors are supposed to be confined to small internucleon distances, and hence might be expected to be unable to influence external observables. In the case of the tdn vertex the situation may become worse, since its bigger separation energy tends to expose more the nonpionic content of the interaction. Thus it is important to understand the implications of their method before considering applications to other systems. In this work we also estimate the importance of nonpionic effects in several external deuteron observables and check the role of the pion in correlating them.

II. DYNAMICS

In this work we are concerned with the structural dependence of the deuteron external observables on g, μ, and α rather than with precise phenomenology, and hence we study the solutions of the standard Schrödinger equation using a simplified NN potential which contains the OPEP tail and is regular at the origin. This regularization is achieved by means of effective monopole form factors, which are formally introduced by multiplying each πN coupling constant g by a factor

$$[(\Lambda^2-\mu^2)/(\Lambda^2+\mathbf{k}^2)]\ ,$$

where Λ is a parameter that accounts for the non-OPEP nucleon-nucleon dynamics. The pure OPEP case is recovered in the limit $\Lambda\to\infty$.

In order to prevent misunderstandings, we would like to stress that our Λ does not represent the true πN form factor, which is associated with short-range exchanges that occur when a pion interacts with a nucleon. For instance, these vertex effects correspond, at the hadron level, to the meson cloud that dresses a pointlike nucleon, whereas in a bag model they are related to the size of the bag. True form factors are those considered in corrections to the Goldberger-Treiman relation[15] and in the study of Ericson and Rosa-Clot.[5,6] In our case Λ represents both the true πN form factor and other dynamical effects such as multimeson nucleon-nucleon exchanges. In particular, our Λ includes the central and tensor contributions of the two-pion exchange. Therefore our Λ describes, at best, an effective form factor.

With the purpose of allowing for more flexibility in our potential, we let its central and tensor components be regularized by different parameters, denoted, respectively, by Λ_C and Λ_T. Its explicit components are written as[16]

$$V_C(r)=-g[U_C(r)-\delta G(r)]\ , \tag{1}$$

$$V_T(r)=-g[U_T(r)]\ , \tag{2}$$

where g is the πN coupling constant and

$$U_C(r)=\frac{e^{-\mu r}}{\mu r}-\frac{\Lambda_C}{\mu}\frac{e^{-\Lambda_C r}}{\Lambda_C r}-\frac{1}{2}\frac{\mu}{\Lambda_C}\left[\frac{\Lambda_C^2}{\mu^2}-1\right]e^{-\Lambda_C r}\ , \tag{3}$$

$$G(r)=\frac{1}{2}\frac{\mu}{\Lambda_C}\left[\frac{\Lambda_C^2}{\mu^2}-1\right]^2 e^{-\Lambda_C r}\ , \tag{4}$$

$$U_T(r)=\left[1+\frac{3}{\mu r}+\frac{3}{\mu^2r^2}\right]\frac{e^{-\mu r}}{\mu r}-\frac{\Lambda_T^3}{\mu^3}\left[1+\frac{3}{\Lambda_T r}+\frac{3}{\Lambda_T^2r^2}\right]\frac{e^{-\Lambda_T r}}{\Lambda_T r}-\frac{1}{2}\frac{\Lambda_T}{\mu}\left[\frac{\Lambda_T^2}{\mu^2}-1\right](1+\Lambda_T r)\frac{e^{-\Lambda_T r}}{\Lambda_T r}\ . \tag{5}$$

The function $G(r)$ is such that in the case of pointlike nucleons we have

$$\lim_{\Lambda\to\infty} G(r)=\frac{4\pi}{\mu^3}\delta^3(r)\ . \tag{6}$$

So, in this limit, $G(r)$ represents the well-known contact interaction present in the pure OPEP. On the other hand, when Λ is finite, the nucleon is surrounded by a meson cloud and $G(r)$ may be interpreted as a contact interaction between extended objects, in the sense discussed in Ref. (16). The parameter δ in V_C allows this interaction to be switched on ($\delta=1$) or off ($\delta=0$). For the sake of the flexibility of the model, other values of δ may also be used. When we set $\Lambda_C=\Lambda_T=\Lambda$ and $\delta=1$, the monopole interaction employed recently by Friar, Gibson, and Payne[17] in their OPEP study of the deuteron is recovered. On the other hand, the OPEP potential used by Ericson and Rosa-Clot[6] corresponds to $\Lambda_C = \Lambda_T = \Lambda$ and $\delta = 0$, whereas that adopted by Righi and Rosa-Clot[18] is given by $\Lambda_T = \Lambda, \Lambda_C \to \infty$ and $\delta = 0$. In order to produce a feeling for the role of the parameters Λ_C, Λ_T and δ, we plot in Fig. 1 the potentials V_C and V_T for the cases $\delta = 0$ and $\delta = 1$, when the nucleons are either pointlike ($\Lambda_C = \Lambda_T \to \infty$) or corrected by form factors, with the realistic choice $\Lambda_C = \Lambda_T = 900$ MeV. Inspecting this figure we note that the main effect of the contact term is to produce a repulsive core for the central potential, that would otherwise be entirely attractive. The amount of repulsion can be controlled through the parameters Λ_C and δ. An increase in Λ_C would move the point P upwards and the point Q to the left, in such a way that, for $\Lambda_C = \infty$, the curve c_1 becomes identical to the curve C plus a repulsive δ function at the origin. Similarly, an increase in Λ_C and Λ_T would move the curves c_0 and t towards the curves C and T. In the opposite limit, the value $\Lambda_C = \mu$ allows the entire central potential to be turned off.

The deuteron wave function is written as

$$\psi = \frac{1}{\sqrt{4\pi} r} \left[u(r) + \frac{1}{\sqrt{8}} w(r) S_{12} \right] \chi_1^{\xi} , \tag{7}$$

where S_{12} is the tensor operator and χ_1^{ξ} is a triplet spinor. The Schrödinger equation for our OPEP is

$$u'' - (\alpha^2 + m V_C) u = \sqrt{8} m V_T w , \tag{8}$$

$$w'' - \left[\alpha^2 + \frac{6}{r^2} + m(V_C - 2V_T) \right] w = \sqrt{8} m V_T u . \tag{9}$$

The solutions u and w of these coupled equations allow the asymptotic normalizations to be obtained as

$$A_S \equiv \lim_{r \to \infty} u(r) / e^{-\alpha r} , \tag{10}$$

$$A_D \equiv \lim_{r \to \infty} w(r) \Big/ \left[\left[1 + \frac{3}{\alpha r} + \frac{3}{\alpha^2 r^2} \right] e^{-\alpha r} \right] . \tag{11}$$

Following the common usage, $\eta \equiv A_D / A_S$.

In order to test the numerical stability of our results, we have also evaluated A_S and A_D with the help of the free Green's functions for Eqs. (8) and (9), through the equations

$$A_S = m \int_0^\infty dr \, r j_0(i\alpha r) [V_C(r) u(r) + \sqrt{8} V_T(r) w(r)] , \tag{12}$$

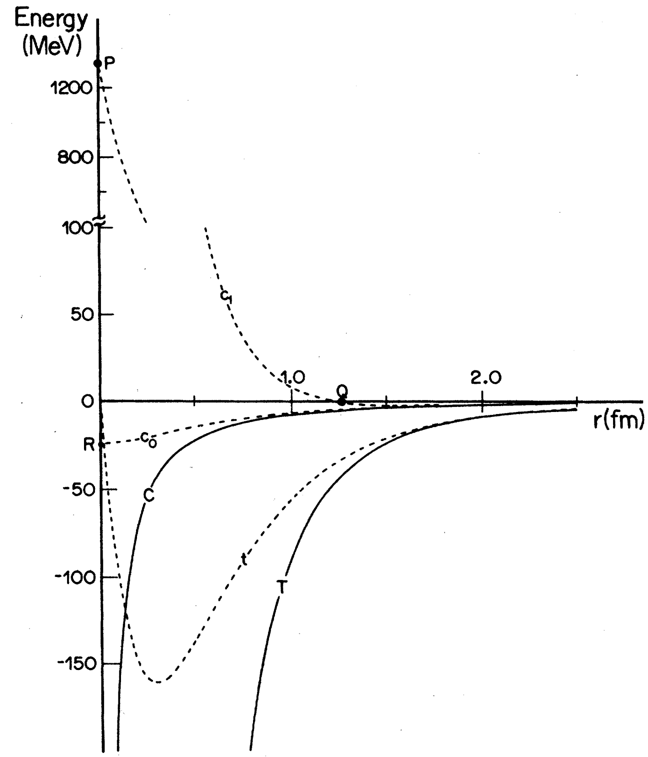

FIG. 1. The components of the potential for different values of the parameters. The central part V_C is represented by the curves $C(\Lambda_C \to \infty, \ \delta = 0)$, c_0 ($\Lambda_C = 900$ MeV, $\delta = 0$), and c_1 ($\Lambda_C = 900$ MeV, $\delta = 1$). The tensor component is given by the curves T ($\Lambda_T \to \infty$) and t ($\Lambda_T = 900$ MeV).

$$A_D = m \int_0^\infty dr \, r j_2(i\alpha r) \{ \sqrt{8} V_T(r) u(r) + [V_C - 2V_T(r)] w(r) \} \tag{13}$$

where j_0 and j_2 are spherical Bessel functions. This last expression is totally equivalent to that derived by Ericson and Rosa-Clot[5,6] in terms of $\mathcal{J}_2$, the exact Green's function for the right-hand side of Eq. (9), and which can be written as

$$A_D = m \int_0^\infty dr \, \mathcal{J}_2(i\alpha r) \sqrt{8} V_T(r) u(r) . \tag{14}$$

For future purposes, we also quote their WKB approximation for $\mathcal{J}_2$, suitably adapted for our case

$$\mathcal{J}_2(i\alpha r) = \exp \left\{ -\frac{m}{5} \int_\infty^r dt \, t [V_C(r) - 2V_T(r)] \right\} \left\{ 1 + \frac{4m}{25} r^2 [V_C(r) - 2V_T(r)] \right\}^{-1/4} r j_2(i\alpha r) . \tag{15}$$

For the sake of completeness, we give the expressions for the single nucleon contribution to the quadrupole moment Q_P and the root mean square r_m

$$Q_P = \frac{1}{\sqrt{50}} \int_0^\infty dr \, r^2 \left[u(r) w(r) - \frac{1}{\sqrt{8}} w(r)^2 \right] , \tag{16}$$

$$r_m = \tfrac{1}{4} \int_0^\infty dr \, r^2 [u(r)^2 + w(r)^2] . \tag{17}$$

III. RESULTS AND DISCUSSION

A. The pion in the deuteron

In order to study the stability of various deuteron observables under variations of the inner parts of the potential, we fixed the πN coupling constant as $g = 10.904\,370$ MeV, corresponding to $f^2 = 0.079$, and searched out the values of Λ_C and Λ_T that, for a given δ, would yield the experimental binding energy $E = 2.2246$ MeV. The numerical integration of Eqs. (8) and (9) was performed up to 26 fm, using a variable step and employing 890 points. The value of the S wave constant A_S was extracted using both the direct ratio [Eq. (10)] and the integral method [Eq. (12)], and results agree within 0.03%. The D wave constant A_D was evaluated by four different methods, namely the direct ratio [Eq. (11)] at both 16 and 26 fm, the complete integral method [Eq. (13)], and its WKB approximation [Eqs. (14) and (15)]. The agreement between the results from both integral methods and that extracted at 16 fm is better than 0.3%, whereas the value extracted at 26 fm diverges by less than 4% and was not considered.

The values of Λ_C and Λ_T obtained for the choices $\delta = 0$, 1.0, 1.5, and 2.0 allowed the construction of the curves shown in Fig. 2, where the triangle indicates the dipole case considered by Friar *et al.*[17] A remarkable feature of this plot is that the range of variation of Λ_C is much greater than that of Λ_T. The former can have any value between μ and infinity, whereas the latter is confined to the interval 750 MeV $< \Lambda_T <$ 1200 MeV. This result reflects the well-known dominance of the tensor force in the binding of the deuteron. The values of Λ_T for the cases including contact interactions ($\delta \neq 0$) lie generally above that corresponding to $\delta = 0$. This is easily understood, since contact interactions are repulsive and hence, for a given Λ_C, the binding of the deuteron demands a deeper tensor potential. The potentials corresponding to $\delta \neq 0$ are much closer to reality, due to the presence of the central repulsive core. In order to test the extension of the influence of the contact term over the results, we have analyzed the behavior of the curves for large values of Λ_C, when its range tends to zero, and found that they approach each other at a slow pace; for instance, for Λ_C around 20 GeV, there is still a 3% difference between the corresponding values of Λ_T. In the case $\delta = 0$, the value of Λ_T remains almost constant after $\Lambda_C = 800$ MeV, meaning a saturation of the influence of Λ_C over the potential. It is interesting to note that all the curves converge to the point $\Lambda_C = \mu$, as this means that the central potential vanishes and the binding is achieved by the tensor force alone. Therefore, when Λ_C is close to μ, the central OPEP is suppressed by a term having comparable range and only for values of Λ_C larger than 800 MeV does the central potential recover its characteristic pionic tail.

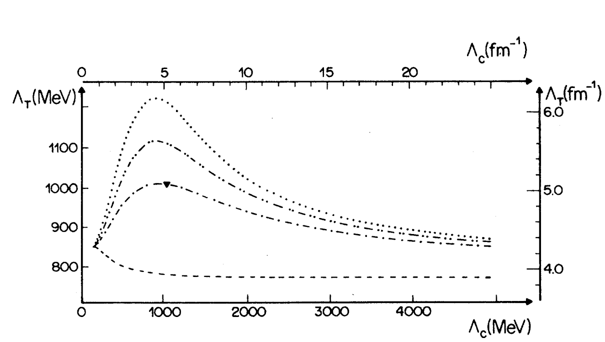

FIG. 2. Curves relating the values of Λ_C and Λ_T that yield the experimental deuteron binding energy for central potentials with $\delta = 0$ (— — —), $\delta = 1.0$ (—·—·—·), $\delta = 1.5$ (—··—··), and $\delta = 2.0$ (· · · ·). The triangle corresponds to the potential of Ref. 17.

In Figs. 3(a)–3(d) we display our results for A_S, r_m/A_S, η, and Q_P/A_S^2 as functions of Λ_C. In all curves one finds a rapid variation for small values of Λ_C, whereas they become much more stable for Λ_C bigger than 1000 MeV. All the curves show an influence of the repulsive core, represented by the parameter δ, and those corresponding to $\delta = 0$ are flatter than the others, due to the saturation of the central potential for large values of Λ_C. All the curves with $\delta \neq 0$ tend to that with $\delta = 0$ for very large values of Λ_C since, as mentioned before, in this case the corresponding central potentials differ only by a δ function located at the origin.

One of the purposes of this work consists in assessing the extent of single-pion dominance over deuteron observables. In our study the binding energy was kept fixed and hence, for given values of g and μ, the parameters δ, Λ_C, and Λ_T are constrained by the condition $\alpha(g,\mu;\delta,\Lambda_C,\Lambda_T) =$ constant. This means that a result which does not depend on δ and Λ_C is also independent of Λ_T and can, in principle, be ascribed to the pion. In the plots of Fig. 3, independence of short-range dynamics manifests itself through the convergence of all the curves to a single horizontal line.

The ratio r_m/A_S, given in Fig. 3(b), is remarkably stable under changes in the inner parts of the potential. This means that our model potential yields the same linear correlation between r_m and A_S as found empirically by Ericson[19] and Klarsfeld *et al.*[20] As pointed out by the former, this happens because the bulk of the contribution to r_m in Eq. (17) is due to the asymptotic part of the S wave function. In order to check the extent that the ratio r_m/A_S is influenced by pionic parameters, in Fig. 4 we display the effects of changes in the πN coupling constant and pion mass, together with values corresponding to various realistic NN potentials. Once again, one finds a great stability.

The behavior of η is shown in Fig. 3(c), and one notes a dependence on the inner components of the potential, represented by δ and Λ_C. Nevertheless, in spite of the very wide variation of parameters considered here, the value of η does not change more than 5%. If Λ_C were confined to a more conservative interval such as, for instance, 700 MeV $< \Lambda_C <$ 1500 MeV, the variation induced in η would be considerably reduced. Thus our results agree qualitatively with those of Friar, Gibson, and Payne,[17] as well as with the refined analysis of Ericson and Rosa-Clot.[5,6]

For a given δ, the curves for η and Q_P/A_S^2 resemble each other for values of Λ_C larger than 800 MeV. In order to explore this similarity, we display the correlation between Q_P/A_S^2 and η in Fig. 5, where it is possible to note that, for large values of Λ_C, all the curves merge into a single straight line. This line is universal in the sense that it is independent of δ, Λ_C, and, due to the constraint imposed by the fixing of the binding energy, also of Λ_T. This linear relation between Q and η has a long history, going back to Blatt and Biedenharn,[27] and a recent discussion of the subject can be found in Butler and Sprung.[28] On the other hand, the relationship between Q_P/A_S^2 and η is dependent on the inner parts of the interaction for small values of Λ_C, when the pionic tail of the central potential is very disturbed by the form factor. This result sheds light on the fact that the universal line derives from the joint action of the tails of both central and tensor potentials.

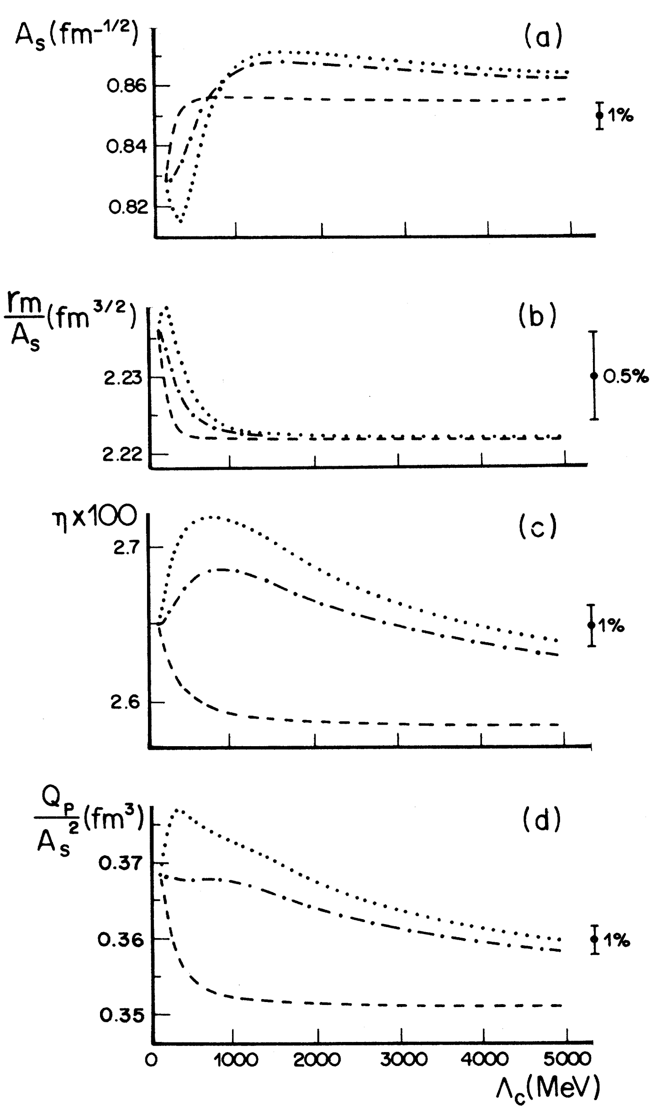

FIG. 3. The plots express the values of A_S (a), r_m/A_S (b), η (c), and Q_P/A_S^2 (d) as functions of Λ_C for the values of $\delta=0$ (— — —), $\delta=1.0$ (—·—·—·), and $\delta=2.0$ (· · · ·). Please note that a scale in terms of percentage is supplied on the right of the figures.

B. The role of the binding energy

We now study the sensitivity of deuteron observables to changes in the binding energy, with two motivations. First, we are interested in using the deuteron as a test ground for other systems, such as the triton, where calculated wave functions fail to reproduce the measured binding energy. The second one is to check the geometrical character of some results. A quantity can be considered as geometrical when its value is determined by the energy scale of the system. If this is the case, the multiplication by a suitable power of α yields a dimensionless quantity that is independent of the energy.

The ratio r_m/A_S, for instance, is quite insensitive to both internal and pion dynamics. Its asymptotic value, obtained by using Eq. (10) into Eq. (17), is[19] $\alpha^{-3/2}/4$. Therefore in Fig. 6 we plot the dimensionless form $(4r_m\alpha^{3/2}/A_S)$ against the deuteron binding energy, which was allowed to vary between 2 and 10 MeV. In spite of the various choices of parameters employed, all the points fall within the narrow shaded band. Over the whole energy interval considered, the dimensionless ratio does not vary more than 4%, and it is instructive to compare this value with those corresponding to $(r_m\alpha)$ and $(A_S/\sqrt{\alpha})$, which are 20% and 24%, respectively. This means that, to a few percent, the ratio r_m/A_S behaves as a geometric quantity.

The behavior of η as a function of α, in the interval 2 MeV $< E <$ 10 MeV, is given by the shaded band in Fig. 7, for a wide variety of inner parameters of the potential. Its width indicates an almost uniform 4% spread in the values of η for a given energy, and can be ascribed to the

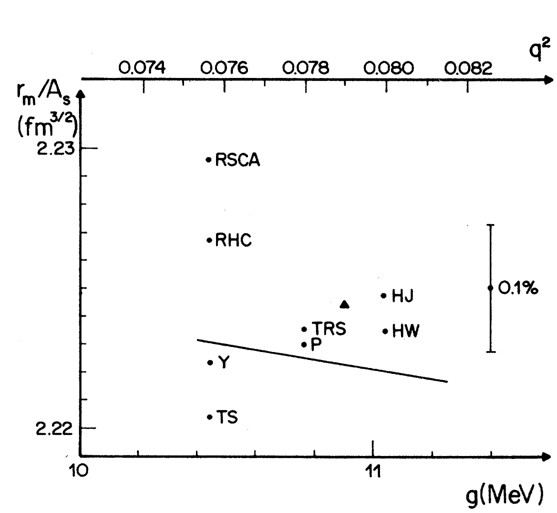

FIG. 4. Influence of pion parameters over the ratio r_m/A_S. The continuous line indicates the sensitivity to changes in the πN coupling constant, whereas the triangle indicates the effect of an increase of 3% in the pion mass. The dots correspond to the selected values quoted in Ref. 4, for the following NN potentials: (HJ,HW), corrected versions of Hamada and Johnston (Ref. 21); (Y), Yale (Ref. 22); (RHC,RSCA), Reid (Ref. 23); TS (super soft core C), (Ref. 24); (TRS), de Tourreil *et al.* (Ref. 25), and (P), Paris (Ref. 26).

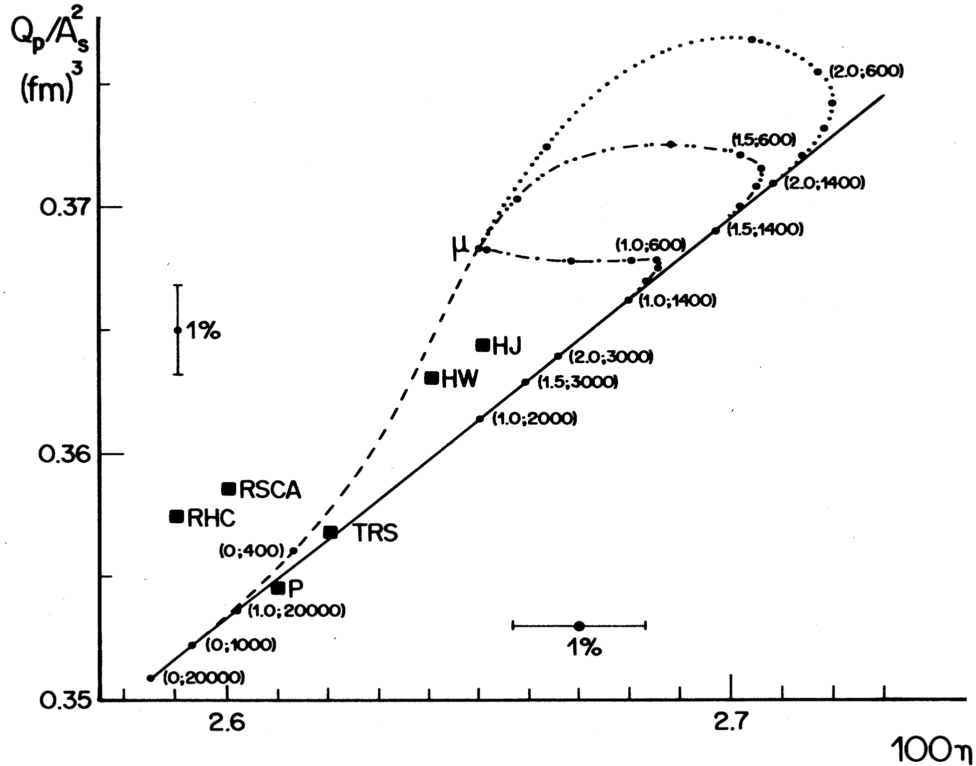

FIG. 5. The universal correlation between Q/A_S^2 and η is represented by the continuous line, and it is given by the merging of the various broken lines associated with the potentials corresponding to $\delta=0$ (— — —), $\delta=1.0$ (—·—·—·), $\delta=1.5$ (—··—··), and $\delta=2.0$ (· · · ·). Some points indicated by [δ; Λ_C (in MeV)] illustrate the universality of the correlation, whereas the dark squares correspond to realistic *NN* potentials (conventions as in Fig. 4).

short distance behavior of the interaction. The roughly linear relationship between η and α, on the other hand, reflects indirectly the influence of pionic parameters. This can be understood by noting that η is a dimensionless quantity, and hence the dependence on these parameters occurs through the ratios g/α and μ/α. Thus an in-

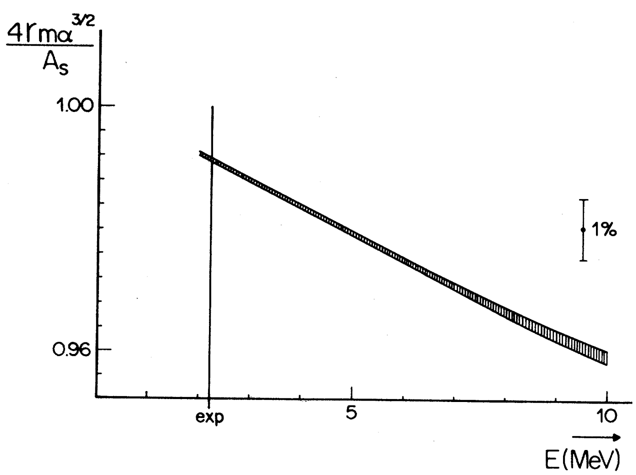

FIG. 6. The ratio $(4r_m\alpha^{3/2}/A_S)$ as a function of the energy, showing its geometrical character. The shaded band was obtained by fixing δ and Λ_C as (0, 1000 MeV), (1, 800 MeV), (1, 1000 MeV), (1, 1600 MeV), and (1.5, 1000 MeV) and looking for the values of Λ_T that produce the desired binding energy.

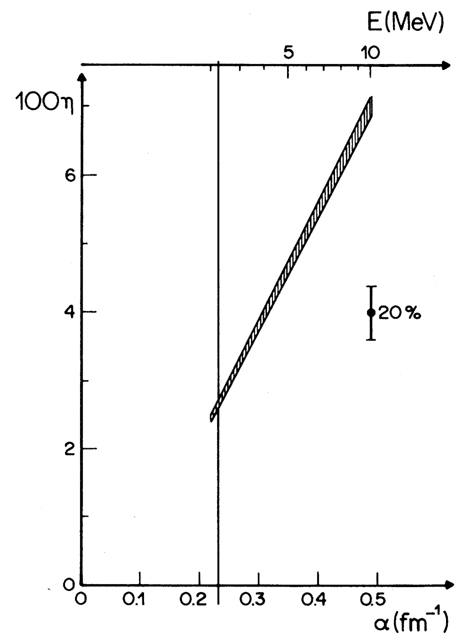

FIG. 7. The dependence of the asymptotic D/S ratio on the deuteron binding energy. The shaded band was obtained as in Fig. 6.

crease in α is equivalent to a simultaneous decrease of both g and μ. In cases such as the triton-deuteron-nucleon vertex, theoretical calculations yield the function $\eta(\alpha)$, where α is in general different from the observed value. However, one is interested in finding out the "true" value of η, which is associated with the observed binding energy. The study of the deuteron allows us to expect a width in the theoretical correlation between the separation energy and the asymptotic ratio, rendering very difficult the solution of this problem. It is worth noting that a spread in the values of the tdn asymptotic ratio is indeed present in a recent study by Friar, Gibson, Lehman, and Payne.[11]

C. The method of Ericson and Rosa-Clot

The third problem we are interested in concerns the relationship between η and Λ_T, that is displayed in Fig. 8. There one finds a behavior analogous to that encountered in the $\eta-Q/A_S^2$ correlation, namely the existence of a line which is independent of the central potential when Λ_C is not too small and it already possesses the OPEP tail. In other words, this line indicates that the value of the asymptotic ratio depends on the existence of the OPEP tail of the central potential, but is totally independent of its internal structure. On the other hand, there is a clear dependence on the non-OPEP structure of the tensor potential, which allows it to be constrained by the experimental value of η. This conclusion is, of course, quite similar to that reached by Ericson and Rosa-Clot in their study of the deuteron.[5,6] There are, however, important conceptual differences between their result and ours. One of them concerns the meaning attached to the parameter Λ, which in their case represents the πN form factor, whereas in ours it is associated with the whole non-OPEP content of the tensor interaction. As discussed in the introduction, the latter contains an important contribution from the two-pion exchange NN interaction.

There is also an important difference concerning the methods employed in both studies. The work of Ericson and Rosa-Clot is based on Eq. (14), which is an exact result only when the function $u(r)$ is consistent with the potential used in it. This means that Eq. (14) is a mathematical identity only when the $u(r)$ used in it is the solution of the Schrödinger equation for that potential. In this case, and in this case only, Eqs. (13) and (14) are fully equivalent. In their work, the sensitivity of η to the πN form factor was investigated by keeping $u(r)$ fixed while the parameter Λ in the potential was varied. This procedure may induce changes in the binding energy of the system, turning Eq. (14) into a method for extrapolating A_D to values of Λ other than that corresponding to consistency between $u(r)$ and the potential. In our work, on the other hand, we always varied both the potential and the wave function simultaneously, while α was kept fixed.

In the context of our model, Eq. (14) may be rewritten as

$$A_D(\delta,\Lambda_C,\Lambda_T;\delta',\Lambda_C',\Lambda_T')=m\int_0^\infty dr\,\mathcal{J}_2(i\alpha r;\delta,\Lambda_C,\Lambda_T)\sqrt{8}V_T(r;\Lambda_T)U(r;\delta',\Lambda_C',\Lambda_T') \,. \tag{18}$$

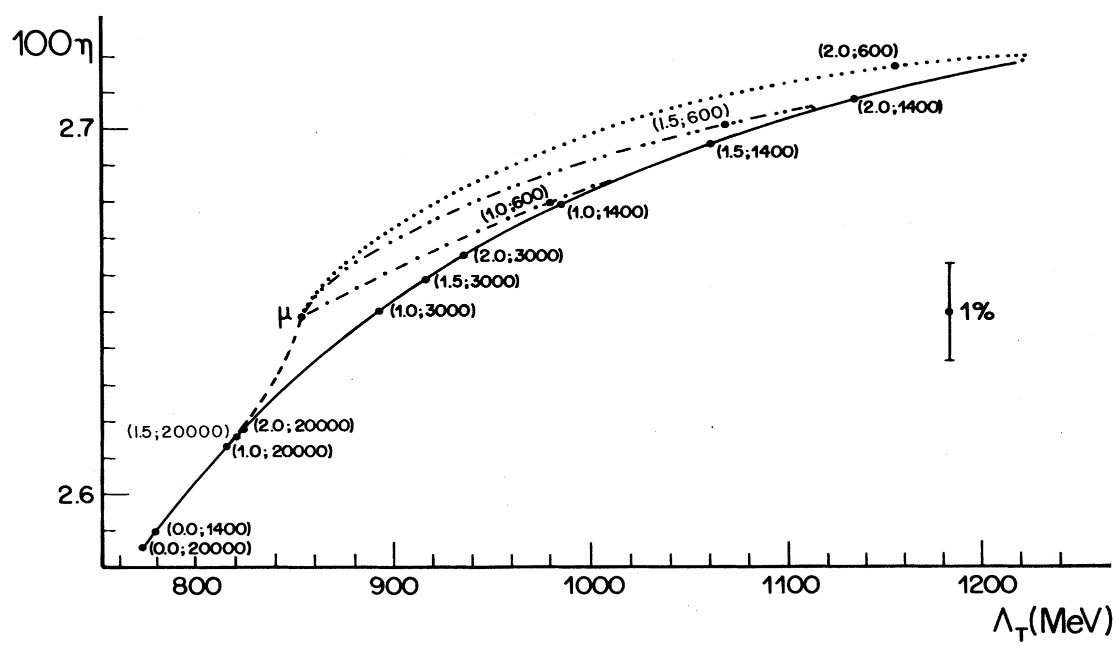

FIG. 8. The correlation between η and Λ_T; conventions are the same as in Fig. 5.

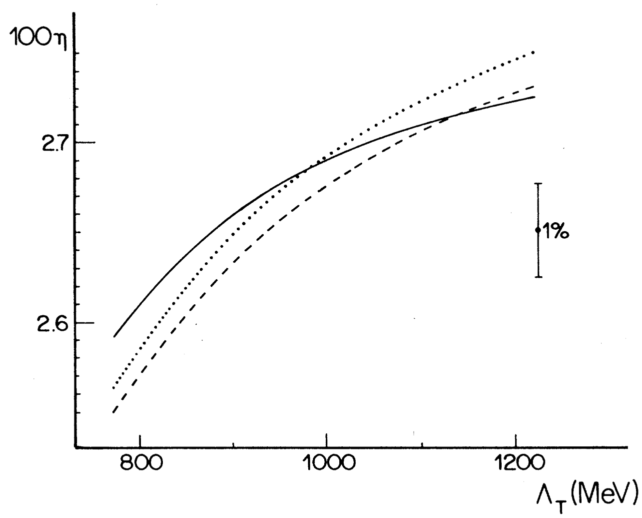

FIG. 9. The correlation between η and Λ_T. The continuous line is the same as in Fig. 8; the dashed and dotted curves were obtained from Eq. (18) with the following choices: $\delta=\delta'=2$, $\Lambda_C=\Lambda'_C=1381.3$ MeV, $\Lambda'_T=1135.5$ MeV, and $\delta=\delta'=2$, $\Lambda_C=\Lambda'_C=2367.9$ MeV, $\Lambda'_T=978.8$ MeV.

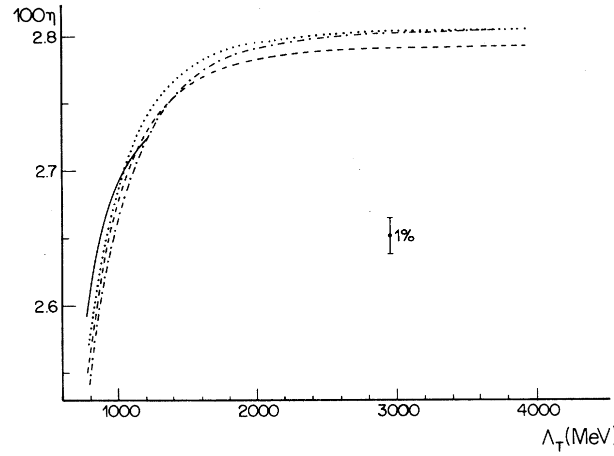

FIG. 10. The influence of the central potential over the $\eta-\Lambda_T$ correlation produced by Eq. (18). In all the curves the parameters of $u(r)$ were fixed at $\delta'=2$, $\Lambda'_C=1381.3$ MeV, $\Lambda'_T=1135.5$ MeV, whereas the dotted, dot-dashed, and dashed lines correspond, respectively, to the choices $\delta=0$, $\Lambda_C=\Lambda_T$; $\delta=2$, $\Lambda_C=\Lambda_T$; and $\delta=2$, $\Lambda_C=\Lambda'_C$. The continuous line is the same as in Figs. 8 and 9 and was included for the sake of comparison.

The consistent situation is attained when the primed and unprimed parameters are identical.

With the purpose of estimating the significance of the use of a potential which is not consistent with the wave function, we have fixed all the parameters but Λ_T in Eq. (18), which was then used in order to generate a relationship between η and Λ_T. In Fig. 9 we display the results of this procedure for two sets of fixed parameters, together with our $\eta-\Lambda_T$ correlation, and it is possible to note that our method and that based on Eq. (18) do not produce identical results. Nevertheless, the main trends of all the curves are the same and the numerical differences are less than 1% for the range of Λ_T considered. For small values of Λ_T these differences tend to be more pronounced, but we should recall that we have learned from Fig. 2 that this parameter must not be arbitrarily decreased. For instance, even for the extreme case of a potential whose central part does not have a repulsive core, Λ_T should not be made smaller than 750 MeV.

Another interesting feature of Fig. 9 is that there are two curves generated by Eq. (18), and the difference between them is due to the value of Λ_C adopted in each case. This means that the results of Eq. (18) can be influenced by the central interaction. In order to explore this dependence, we plot in Fig. 10 the relationship between η and Λ_T for various forms of the central potential in the interval 750 MeV $<\Lambda_T<$ 4000 MeV. The curves indicate the extent of the influence of both δ and Λ_C over the results. The dotted line corresponds to the case $\Lambda_C=\Lambda_T$ and $\delta=0$, considered by Ericson and Rosa-Clot.[5,6] Inspecting this figure one learns that a change in the value of δ from zero to 2 modifies the curve in the region of small Λ_T. On the other hand, the fixing of Λ_C while Λ_T is varied influences the results over the whole interval considered. The main conclusion to be drawn from Fig. 10 is that the influence of the central potential over the $\eta-\Lambda_T$ relationship produced by Eq. (18) is small, but by no means negligible. This is in qualitative disagreement with our Fig. 9, which shows that, provided the central potential has an OPEP tail, the value of η is very precisely determined by Λ_T only.

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