Electromagnetic interactions of extended nucleons

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An electromagnetic current operator is deduced from the most general form of the extended pion-nucleon vertex function using the minimal substitution prescription. It is proved that the sum of the obtained current operator and the isolated-pole contribution satisfies the Ward-Takahashi identity derived for the pion photoproduction. The minimal-coupling interaction is applied to the calculation of the one-pion exchange current regularized by the pion-nucleon form factors. It is found that the one-pion exchange current operator including hadronic and electromagnetic form factors satisfies the Ward-Takahashi equation for the nucleon-nucleon interaction.

I. INTRODUCTION

The nuclear constituents, nucleon, pion, Δ, and so forth, are particles which possess finite space-time extension. The strong, electromagnetic, and weak interactions of these particles are characterized by the form factors which are the consequences of all detailed contributions from the processes that take place inside these particles. If a form factor is inserted, for example, in the pion-nucleon vertex, one must inevitably presume that some current flows within the pion-nucleon interaction range delimited by the form factor. The existence of such a current, which is called the interaction current, has been recognized for a long time,[1] but the general prescription of evaluating the interaction current is yet to be established.[2] Recently, the present author[3] has proposed a method of constructing an electromagnetic current operator for a static, extended meson-nucleon vertex function. The method is extended to the nucleon-nucleon interaction, and it is found that when applied to the one-pion-exchange potential corrected by the pion-nucleon form factors, our method reproduces the static limit of the result of Gross and Riska.[4] Since they derived exchange-current operators for a more general, relativistic interaction with electromagnetic form factors of nucleon and pion, the coincidence of the two approaches in the static limit makes it intriguing to expand our method developed in the nonrelativistic theory into the relativistic one. The primary purpose of this paper is to derive an electromagnetic current operator from the pion-nucleon vertex function using a fully relativistic formalism.

The Ward-Takahashi (WT) identity[5] is the key to the development made by Gross and Riska.[4] In investigating the electromagnetic interactions of extended particles, the WT equation plays an essential role. As is emphasized by Nishijima,[6,7] the WT equations are valid for composite particles as well as for elementary particles.[8] The presence of hadronic and electromagnetic form factors of interacting hadrons should not violate the WT equation that is a direct manifestation of gauge invariance. The electromagnetic current operator associated with the hadronic form factors should arise in such a way that the WT equation is satisfied.

In Sec. II we develop a method of extracting the electromagnetic current operator from the most general form of the pion-nucleon vertex function on the basis of the minimal-substitution prescription. In Sec. III we derive the WT equation for the pion photoproduction process, and show that the sum of the isolated-pole term and the minimal-coupling interaction satisfies the WT equation. In Sec. IV we derive the WT equation for nucleon-nucleon interaction and prove that our current operator is essentially required for the WT equation to be valid. We apply our method to the calculation of the one-pion exchange current and compare the result with that of Gross and Riska.[4] We also present the most general form of the one-pion exchange current and show that it satisfies the WT equation rigorously. Finally in Sec. V we give a brief summary.

II. GAUGE INVARIANCE IN EXTENDED VERTICES

A. Pion-nucleon vertex function

The vertex function for the interaction between nucleon and pion is defined by the vacuum expectation value

$$\langle T[\psi(x')\bar{\psi}(x)\phi_i(y)]\rangle = -\int d^4\xi' d^4\xi d^4\eta S_F'(x'-\xi')\tau_i\Gamma(\xi'\xi:\eta)S_F'(\xi-x)\Delta_F'(y-\eta) \ , \tag{2.1}$$

where ψ, $\bar{\psi}$, and ϕ_i refer to the nucleon and pion field operators, the subscript i denotes the pion isospin index, and S_F' and Δ_F' are the Feynman propagation functions for the nucleon and the pion, respectively. The vertex function in momentum space is defined by

$$\Gamma(x'x:y) = \frac{-i}{(2\pi)^8}\int d^4p' d^4p \, e^{ip'\cdot(x'-y)+ip\cdot(y-x)}\Gamma(p',p) \ . \tag{2.2}$$

In Eq. (2.2), we replace p' and p by $-i\partial/\partial x'$ and $i\partial/\partial x$ acting on the exponential, and take Γ out of the integrand to obtain

$$\Gamma(x'x:y)=-i\Gamma(-i\partial/\partial x',i\partial/\partial x)\delta(x'-y)\delta(y-x)\ . \tag{2.3}$$

The gradient operators now act on the δ functions. The Lagrangian density for the pion-nucleon interaction can be expressed as

$$\begin{aligned}\mathcal{L}(x'x:y)&=\bar\psi(x')\tau_i\Gamma(x'x;y)\psi(x)\phi_i(y)\\ &=-i\bar\psi(x')\tau_i[\Gamma(-i\partial/\partial x',i\partial/\partial x)\\ &\qquad\times\delta(x'-y)\delta(y-x)]\psi(x)\phi_i(y)\ .\end{aligned}\tag{2.4}$$

Integrating by parts, we can make the gradient operators act on the nucleon field operators,

$$p_\mu\psi(x)=-i(\partial/\partial x_\mu)\psi(x)\ , \tag{2.5}$$

$$\bar\psi(x)p'_\mu=i(\partial/\partial x_\mu)\bar\psi(x) \tag{2.6}$$

(we use the same notations p' and p both for c numbers in momentum space and for gradient operators in position space), and write the Lagrangian in a local form

$$\mathcal{L}(x'x:y)=-i\delta(x'-x)\mathcal{L}(x:y)\ . \tag{2.7}$$

Here

$$\mathcal{L}(x:y)=\bar\psi(x)\tau_i\Gamma(x:y)\psi(x)\phi_i(y) \tag{2.8}$$

and

$$\Gamma(x:y)=\Gamma(p',p)\delta(y-x)\ , \tag{2.9}$$

in which p' and p differentiate the nucleon fields only. The nonlocal vertex function thus takes the form

$$\Gamma(x'x:y)=-i\delta(x'-x)\Gamma(x:y)\ . \tag{2.10}$$

This examples illustrates that an arbitrary nonlocal operator (nonlocal with respect to the nucleon position) can be expressed in the form of a momentum-dependent local operator,

$$\mathcal{O}(x'x:yz\cdots)=-i\delta(x'-x)\mathcal{O}(x:yz\cdots)\ , \tag{2.11}$$

where $y,z,\cdots$ are positions of fields other than the nucleon field.

The most general form of the pion-nucleon vertex function is given by[9]

$$\begin{aligned}\Gamma(p',p)&=i\gamma_5 g_1+i\gamma_5(i\gamma\cdot p+m)g_2\\ &\quad+(i\gamma\cdot p'+m)i\gamma_5 g_3\\ &\quad+(i\gamma\cdot p'+m)i\gamma_5(i\gamma\cdot p+m)g_4\ ,\end{aligned}\tag{2.12}$$

with m being the nucleon mass. The form factors g_1, g_2, g_3, and g_4 are functions of p^2, p'^2, and $(p-p')^2$. Integrating by parts, we can prove that

$$\begin{aligned}&\bar\psi(x)(p'-p)_\mu\psi(x)\delta(y-x)\\ &\quad=-i\bar\psi(x)\psi(x)(\partial/\partial x_\mu)\delta(y-x)\ .\end{aligned}\tag{2.13}$$

As a consequence, the $(p-p')^2$ in the form factors can be converted to $-\partial_x^2$. To make this fact explicit, we write the local vertex function as

$$\Gamma(x:y)=\Gamma(i\partial_x,p',p)\delta(y-x)\ , \tag{2.14}$$

in which $i\partial_x$ acts on the δ function, while p and p' act on the nucleon field operators. $\Gamma(p',p)$ in Eq. (2.9) and $\Gamma(i\partial_x,p',p)$ in Eq. (2.14) are the identical quantities in different representations. Utilizing

$$\Gamma(i\partial_x,p',p)\delta(y-x)=\frac{1}{(2\pi)^4}\int d^4q\, e^{iq\cdot(y-x)}\Gamma(q,p',p)\ , \tag{2.15}$$

and retracing our steps to restore p' and p to c numbers in momentum space, we can regard Eq. (2.12) as a function of q, p', and p.

Electrodynamics is made invariant by introducing the photon field $A_\mu(x)$ such that a gradient of the nucleon field is allowed to appear only in conjunction with the photon field in the combination

$$p_\mu\psi(x)\to[p_\mu-e_N A_\mu(x)]\psi(x)\ , \tag{2.16}$$

$$\bar\psi(x)p'_\mu\to\bar\psi(x)[p'_\mu-e_N A_\mu(x)]\ , \tag{2.17}$$

where e_N is the nucleon charge operator

$$e_N=\tfrac{1}{2}e(1+\tau_3)\ . \tag{2.18}$$

Since the pion-nucleon vertex contains τ_i, we have to be careful about the ordering of the isospin matrices. In Eq. (2.16) e_N should be placed to the right-hand side of τ_i while in Eq. (2.17) e_N should be placed to its left-hand side. We distinguish e_N as e_R or e_L depending on where it appears. The minimal substitution is expressed as

$$p_\mu\to p_\mu\to e_R A_\mu(x)\ , \tag{2.19}$$

$$p'_\mu\to p'_\mu-e_L A_\mu(x)\ . \tag{2.20}$$

The ∂_x, on the other hand, is subject to the minimal substitution

$$\partial/\partial x_\mu\to\partial/\partial x_\mu-i(e_L-e_R)A_\mu(x)\ . \tag{2.21}$$

In the presence of an external electromagnetic field A^e, the vertex function Γ undergoes a modification and it becomes a functional of A^e. We denote this with a tilde, $\tilde\Gamma$. The electromagnetic interaction is given by the functional derivative

$$\Delta M^i_\mu(x:yz)=\tau_i\left[\frac{\delta}{\delta A^e_\mu(z)}\tilde\Gamma(x:y)\right]_{A^e\to 0}\ . \tag{2.22}$$

The purpose of this section is to derive this interaction explicitly.

B. Minimal coupling in extended vertices

To make the argument transparent, let us start with the first term in Eq. (2.12),

$$\Gamma_1(q,p',p)=i\gamma_5 g_1(q^2,p'^2,p^2)\ . \tag{2.23}$$

Equation (2.23) as it stands is not convenient for minimal substitution. We make use of a formal Taylor expansion

$$\Gamma_1(q,p',p)=i\gamma_5\sum_{lmn} C_{lmn}q^{2l}p'^{2m}p^{2n} . \tag{2.24}$$

In the position space,

$$\Gamma_1(x:y)=i\gamma_5 g_1(-\partial_x^2,p'^2,p^2)\delta(y-x)$$
$$=i\gamma_5\sum_{lmn} C_{lmn}(-1)^l\partial_x^{2l}p'^{2m}p^{2n}\delta(y-x) . \tag{2.25}$$

The minimal substitution leads to the modified vertex function

$$\tilde{\Gamma}_1(x:y)=i\gamma_5\sum_{lmn} C_{lmn}(-1)^l[\partial_x-i(e_L-e_R)A^e]^{2l}$$
$$\times(p'-e_L A^e)^{2m}(p-e_R A^e)^{2n}\delta(y-x) . \tag{2.26}$$

The interaction (2.22) can be obtained by expanding the modified vertex function in terms of A^e. Consider the factor

$$[\partial_x-i(e_L-e_R)A^e]^{2l}=\partial_x^{2l}-i(e_L-e_R)[\partial_x^{2(l-1)}(\partial_x\cdot A^e+A^e\cdot\partial_x)+\partial_x^{2(l-2)}(\partial_x\cdot A^e+A^e\cdot\partial_x)\partial_x^2+\cdots$$
$$+(\partial_x\cdot A^e+A^e\cdot\partial_x)\partial_x^{2(l-1)}]+\cdots . \tag{2.27}$$

Since the ∂_x's lying on the right of A^e can be replaced with $-\partial_y$'s, the terms of order e become

$$-i(e_L-e_R)(\partial_x-\partial_y)_\mu(\partial_x^{2(l-1)}+\partial_x^{2(l-2)}\partial_y^2+\cdots+\partial_y^{2(l-1)})A_\mu^e(x) . \tag{2.28}$$

The second factor in (2.26) is expanded in the form

$$(p'-e_L A^e)^{2m}=p'^{2m}-e_L[p'^{2(m-1)}(p'\cdot A^e+A^e\cdot p')+p'^{2(m-2)}(p'\cdot A^e+A^e\cdot p')p'^2+\cdots$$
$$+(p'\cdot A^e+A^e\cdot p')p'^{2(m-1)}]+\cdots . \tag{2.29}$$

Here it should be noted that the p' appearing on the left of A^e acts on A^e as well as on $\bar{\psi}(x)$, i.e.,

$$\bar{\psi}A^e\cdot p'=i\partial_x\cdot(\bar{\psi}A^e)=\bar{\psi}p'\cdot A^e+i\bar{\psi}\partial_x\cdot A^e , \tag{2.30}$$

where the last differentiation does not act on the δ function but acts only on A^e. In order to distinguish the differentiation of the δ function and that of A^e, we introduce the integration

$$A_\mu^e(x)=\int d^4z\, A_\mu^e(z)\delta(z-x) , \tag{2.31}$$

and replace the differentiation of A^e with that of $\delta(z-x)$. The repeated use of this procedure converts all the p' on the right of A^e into $p'-i\partial_z$ and leads to

$$-e_L(2p'-i\partial_z)_\mu[p'^{2(m-1)}+p'^{2(m-2)}(p'-i\partial_z)^2$$
$$+\cdots+(p'-i\partial_z)^{2(m-1)}]\delta(z-x) . \tag{2.32}$$

In the third factor in (2.26), $(p-e_R A^e)^{2n}$, on the other hand, the p appearing on the left of A^e becomes $p+i\partial_z$ and we obtain

$$-e_R(2p+i\partial_z)_\mu[(p+i\partial_z)^{2(n-1)}+(p+i\partial_z)^{2(n-2)}p^2$$
$$+\cdots+p^{2(n-1)}]\delta(z-x) . \tag{2.33}$$

The pion-nucleon vertex function modified by the minimal substitution is now of the form of a series expansion,

$$\tilde{\Gamma}_1(x:y)=\Gamma_1(x:y)+\int d^4z\,\Delta\Gamma_\mu(x:yz)A_\mu^e(z)+\cdots , \tag{2.34}$$

and all terms linear in e are collected in

$$\Delta\Gamma_\mu(x:yz)=\left[\frac{\delta}{\delta A_\mu^e(z)}\tilde{\Gamma}_1(x:y)\right]_{A^e\to 0} . \tag{2.35}$$

Introducing the function of two arbitrary operators a and b,

$$\Phi_l(a,b)=a^{l-1}+a^{l-2}b+\cdots+a^{l-1} , \tag{2.36}$$

we can write the electromagnetic interaction in the form

$$\Delta\Gamma_\mu(x:yz)=i\gamma_5\Big[-i(e_L-e_R)(\partial_x-\partial_y)_\mu\sum_{lmn} C_{lmn}(-1)^l\Phi_l(\partial_x^2,\partial_y^2)p'^{2m}p^{2n}$$
$$-e_L(2p'-i\partial_z)_\mu\sum_{lmn} C_{lmn}(-1)^l\partial_y^{2l}\Phi_m(p'^2,(p'-i\partial_z)^2)p^{2n}$$
$$-e_R(2p+i\partial_z)\sum_{lmn} C_{lmn}(-1)^l\partial_y^{2l}p'^{2m}\Phi_n((p+i\partial_z)^2,p^2)\Big]\delta(y-x)\delta(z-x) . \tag{2.37}$$

In the last two terms in the brackets, we have replaced ∂_x^{2l} with ∂_y^{2l} because it does not operate on the external field A^e.

In the same way as in Eq. (2.15), we introduce the representation

$$\Delta M_\mu^i(x:yz)=\tau_i\Delta\Gamma_\mu(x:yz)=\frac{1}{(2\pi)^8}\int d^4q\, d^4k e^{iq\cdot(y-x)+ik\cdot(x-z)}\Delta M_\mu^i , \tag{2.38}$$

thereby ∂_x, ∂_y, and ∂_z becoming $-i(q-k)$, iq, and $-ik$, respectively, and restore p' and p to c numbers to find the electromagnetic interaction in momentum space (momentum variables q, k, p', and p are suppressed). The result is

$$\Delta M^i_\mu = i\gamma_5 \Big[ie\epsilon^{ij3}\tau_j (2q-k)_\mu \sum_{lmn} C_{lmn}\Phi_l(q^2,(q-k)^2)p'^{2m}p^{2n} - e_N\tau_i(2p'-k)_\mu \sum_{lmn} C_{lmn} q^{2l}\Phi_m(p'^2,(p'-k)^2)p^{2n} - \tau_i e_N (2p+k)_\mu \sum_{lmn} C_{lmn} q^{2l} p'^{2m}\Phi_n((p+k)^2,p^2) \Big] . \tag{2.39}$$

We have used the definitions of e_L and e_R,

$$\begin{aligned} e_L\tau_i &= e_N\tau_i , \\ e_R\tau_i &= \tau_i e_N , \\ (e_L-e_R)\tau_i &= [e_N,\tau_i] = ie\epsilon^{ij3}\tau_j . \end{aligned} \tag{2.40}$$

Since two arguments of the functions Φ_l, Φ_m, and Φ_n become c numbers, we can simplify them using

$$\Phi_l(a,b) = \frac{a^l - b^l}{a-b} . \tag{2.41}$$

With this form, the summation over l, m, and n can be performed with the result

$$\begin{aligned} \Delta M^i_\mu = i\gamma_5 \Bigg\{ & ie\epsilon^{ij3}\tau_j \frac{(2q-k)_\mu}{q^2-(q-k)^2}[g_1(q^2,p'^2,p^2) - g_1((q-k)^2,p'^2,p^2)] \\ & - e_N\tau_i \frac{(2p'-k)_\mu}{p'^2-(p'-k)^2}[g_1(q^2,p'^2,p^2) - g_1(q^2,(p'-k)^2,p^2)] \\ & - \tau_i e_N \frac{(2p+k)_\mu}{(p+k)^2-p^2}[g_1(q^2,p'^2,(p+k)^2) - g_1(q^2,p'^2,p^2)] \Bigg\} . \end{aligned} \tag{2.42}$$

Next consider the second term in Eq. (2.12),

$$\Gamma_2(q,p',p) = i\gamma_5(i\gamma\cdot p + m)g_2(q^2,p'^2,p^2) , \tag{2.43}$$

which defines the operator in position space,

$$\Gamma_2(x:y) = i\gamma_5(i\gamma\cdot p + m)g_2(-\partial_x^2,p'^2,p^2)\delta(y-x) . \tag{2.44}$$

The only difference from the g_1 term is the presence of the factor $i\gamma\cdot p + m$, which produces the minimal coupling $-ie_R\gamma\cdot A^e$. Following exactly the same procedure as above, we find the interaction

$$\begin{aligned} \Delta M^i_\mu = i\gamma_5(i\gamma\cdot p + m) \Bigg\{ & ie\epsilon^{ij3}\tau_j \frac{(2q-k)_\mu}{q^2-(q-k)^2}[g_2(q^2,p'^2,p^2) - g_2((q-k)^2,p'^2,p^2)] \\ & - e_N\tau_i \frac{(2p'-k)_\mu}{p'^2-(p'-k)^2}[g_2(q^2,p'^2,p^2) - g_2(q^2,(p'-k)^2,p^2)] \\ & - \tau_i e_N \frac{(2p+k)_\mu}{(p+k)^2-p^2}[g_2(q^2,p'^2,(p+k)^2) - g_2(q^2,p'^2,p^2)] \Bigg\} + \tau_i e_N g_2(q^2,p'^2,(p+k)^2)\gamma_5\gamma_\mu . \end{aligned} \tag{2.45}$$

We can rearrange terms to get

$$\begin{aligned} \Delta M^i_\mu = {} & ie\epsilon^{ij3}\tau_j \frac{(2q-k)_\mu}{q^2-(q-k)^2}[\Gamma_2(q,p',p) - \Gamma_2(q-k,p',p)] - e_N\tau_i \frac{(2p'-k)_\mu}{p'^2-(p'-k)^2}[\Gamma_2(q,p',p) - \Gamma_2(q,p'-k,p)] \\ & - \tau_i e_N \frac{(2p+k)_\mu}{(p+k)^2-p^2}[\Gamma_2(q,p',p+k) - \Gamma_2(q,p',p)] + \tau_i e_N g_2(q^2,p'^2,(p+k)^2)\gamma_5 \left[\gamma_\mu - \frac{(2p+k)_\mu}{(p+k)^2-p^2}\gamma\cdot k \right] . \end{aligned} \tag{2.46}$$

We are now in the position to present the result of the minimal replacement applied to the most general form of Γ as given by Eq. (2.12). It is straightforward to get

$$\Delta M^i_\mu = ie\epsilon^{ij3}\frac{(2q-k)_\mu}{q^2-(q-k)^2}[\Gamma(q,p',p)-\Gamma(q-k,p',p)] - e_N\tau_i\frac{(2p'-k)_\mu}{p'^2-(p'-k)^2}[\Gamma(q,p',p)-\Gamma(q,p'-k,p)]$$
$$-\tau_i e_N\frac{(2p+k)_\mu}{(p+k)^2-p^2}[\Gamma(q,p',p+k)-\Gamma(q,p',p)]$$
$$+e_N\tau_i\left[\gamma_\mu-\frac{(2p'-k)_\mu}{p'^2-(p'-k)^2}\gamma\cdot k\right]\gamma_5[g_3(q^2,(p'-k)^2,p^2)+(i\gamma\cdot p+m)g_4(q^2,(p'-k)^2,p^2)]$$
$$+\tau_i e_N[g_2(q^2,p'^2,(p+k)^2)+(i\gamma\cdot p'+m)g_4(q^2,p'^2,(p+k)^2)]\gamma_5\left[\gamma_\mu-\frac{(2p+k)_\mu}{(p+k)^2-p^2}\gamma\cdot k\right] . \quad (2.47)$$

For later application it is important to note that the four divergence of the ΔM^i_μ becomes

$$k_\mu\Delta M^i_\mu = ie\epsilon^{ij3}\tau_j[\Gamma(q,p',p)-\Gamma(q-k,p',p)]$$
$$-e_N\tau_i[\Gamma(q,p',p)-\Gamma(q,p'-k,p)]$$
$$-\tau_i e_N[\Gamma(q,p',p+k)-\Gamma(q,p',p)] . \quad (2.48)$$

The three terms containing $\Gamma(q,p',p)$ cancel each other and drop out from the divergence. Consequently,

$$k_\mu\Delta M^i_\mu = -ie\epsilon^{ij3}\tau_j\Gamma(q-k,p',p)+e_N\tau_i\Gamma(q,p'-k,p)$$
$$-\tau_i e_N\Gamma(q,p',p+k) . \quad (2.49)$$

To conclude this section, it should be stressed that the minimal-substitution prescription alone cannot determine the whole current operator associated with the form factors because one can add arbitrarily terms which are divergence free. An example of such arbitrariness will be considered in Sec. IV.

III. PION PHOTOPRODUCTION

A. Basic equations

The pion photoproduction amplitude is calculated from the functional derivative

$$\langle T[\psi(x')\bar{\psi}(x)\phi_i(y)j_\mu(z)]\rangle$$
$$= -i\left[\frac{\delta}{\delta A^e_\mu(z)}\langle T[\boldsymbol{\psi}(x')\bar{\boldsymbol{\psi}}(x)\boldsymbol{\phi}_i(y)]\rangle\right]_{A^e\to 0} , \quad (3.1)$$

where the bold-faced $\boldsymbol{\psi}$, $\bar{\boldsymbol{\psi}}$, and $\boldsymbol{\phi}_i$ are the nucleon and pion field operators in the presence of the external field A^e, and the modified vertex function $\tilde{\Gamma}$ is defined by

$$\langle T[\boldsymbol{\psi}(x')\bar{\boldsymbol{\psi}}(x)\boldsymbol{\phi}_i(y)]\rangle$$
$$= -\int d^4\xi' d^4\xi\, d^4\eta\, \tilde{S}'_F(x'-x')$$
$$\times\tau_j\tilde{\Gamma}(\xi'\xi:\eta)\tilde{S}'_F(\xi-x)\tilde{\Delta}'^{ij}_F(y-\eta) . \quad (3.2)$$

The modified nucleon and pion propagators are given by

$$\tilde{S}'_F(x'-x)=\langle T[\boldsymbol{\psi}(x')\bar{\boldsymbol{\psi}}(x)]\rangle , \quad (3.3)$$

and

$$\tilde{\Delta}'^{ij}_F(y'-y)=\langle T[\boldsymbol{\phi}_i(y')\boldsymbol{\phi}_j(y)]\rangle . \quad (3.4)$$

The functional derivatives of these Green's functions define the electromagnetic vertex functions of nucleon and pion, respectively,

$$\left[\frac{\delta}{\delta A^e_\mu(z)}\tilde{S}'_F(x'-x)\right]_{A^e\to 0} = -\int d^4\xi' d^4\xi\, S'_F(x'-\xi')$$
$$\times\Gamma_\mu(\xi'\xi:z)S'_F(\xi-x) , \quad (3.5)$$

and

$$\left[\frac{\delta}{\delta A^e_\mu(z)}\tilde{\Delta}'^{ij}_F(y'-y)\right]_{A^e\to 0}$$
$$= -\int d^4\eta' d^4\eta\,\Delta'_F(y'-\eta')\Gamma^{\pi ij}_\mu(\eta'\eta:z)\Delta'_F(\eta-y) . \quad (3.6)$$

Using Eqs. (3.5) and (3.6), we perform the functional derivative in Eq. (3.1), and we get the pion photoproduction amplitude in the form

$$\langle T[\psi(x')\bar{\psi}(x)\phi_i(y)j_\mu(z)]\rangle$$
$$= i\int d^4\xi' d^4\xi\, d^4\eta S'_F(x'-\xi')M^i_\mu(\xi'\xi:\eta z)$$
$$\times S'_F(\xi-x)\Delta'_F(y-\eta) . \quad (3.7)$$

M^i_μ turns out to be divided into two parts,

$$M^i_\mu = M^{Bi}_\mu + \Delta M^i_\mu . \quad (3.8)$$

The first term is the sum of three terms which contain the isolated nucleon or pion pole,

$$M^{Bi}_\mu(x'x:yz)$$
$$= -\int d^4\xi' d^4\xi\,\Gamma_\mu(x'\xi':z)S'_F(\xi'-\xi)\tau_i\Gamma(\xi x:y)$$
$$-\int d^4\xi' d^4\xi\,\tau_i\Gamma(x'\xi':y)S'_F(\xi'-\xi)\Gamma_\mu(\xi x:z)$$
$$-\int d^4\eta' d^4\eta\,\Gamma^{\pi ij}_\mu(y\eta':z)\Delta'_F(\eta'-\eta)\tau_j\Gamma(x'x:\eta) , \quad (3.9)$$

and the second term is

$$\Delta M^i_\mu(x'x:yz)=\tau_i\left[\frac{\delta}{\delta A^e_\mu(z)}\tilde{\Gamma}(x'x:y)\right]_{A^e\to 0} . \quad (3.10)$$

Using the momentum-dependent local representation as described in Sec. II,

$$\Delta M^i_\mu(x'x:yz) = -i\delta(x'-x)\Delta M^i_\mu(x:yz) \quad (3.11)$$

and

$$\tilde{\Gamma}(x'x:y) = -i\delta(x'-x)\tilde{\Gamma}(x:y) , \tag{3.12}$$

one finds that Eq. (3.10) exactly corresponds to the interaction derived in the previous section.

B. Ward-Takahashi identity

Our next task is to drive the WT equation that must be satisfied by (3.7). Following the general prescription developed by Nishijima[6,7] we evaluate

$$\begin{aligned}(\partial/\partial z_\mu)\langle T[\psi(x')\bar{\psi}(x)\phi_i(y)j_\mu(z)]\rangle = &-\delta(x_0'-z_0)\langle T\{[\psi(x'),j_0(z)]\bar{\psi}(x)\phi_i(y)\}\rangle \\ &-\delta(x_0-z_0)\langle T\{\psi(x')[\bar{\psi}(x),j_0(z)]\phi_i(y)\}\rangle \\ &-\delta(y_0-z_0)\langle T\{\psi(x')\bar{\psi}(x)[\phi_i(y),j_0(z)]\}\rangle , \end{aligned} \tag{3.13}$$

where $j_\mu(z) = -\partial_z^2 A_\mu(z)$ is the electromagnetic current and $j_0(z) = -ij_4(z)$. Substituting the relations

$$[\psi(x'),j_0(z)]\delta(x_0'-z_0) = e_N\psi(x')\delta(x'-z) , \tag{3.14}$$

$$[\bar{\psi}(x),j_0(z)]\delta(x_0-z_0) = -\bar{\psi}(x)e_N\delta(x-z) , \tag{3.15}$$

$$[\phi_i(y),j_0(z)]\delta(y_0-z_0) = -ie\epsilon^{ij3}\phi_j(y)\delta(y-z) , \tag{3.16}$$

we obtain the WT equation

$$\begin{aligned}(\partial/\partial z_\mu)\langle T[\psi(x')\bar{\psi}(x)\phi_i(y)j_\mu(z)]\rangle = &-e_N\langle T[\psi(x')\bar{\psi}(x)\phi_i(y)]\rangle\delta(x'-z) \\ &+\langle T[\psi(x')\bar{\psi}(x)\phi_i(y)]\rangle e_N\delta(x-z) + ie\epsilon^{ij3}\langle T[\psi(x')\bar{\psi}(x)\phi_j(y)]\rangle\delta(y-z) . \end{aligned} \tag{3.17}$$

The Fourier transform of the photoproduction amplitude is defined as

$$M_\mu^i(x'x:yz) = \frac{-i}{(2\pi)^{12}}\int d^4p'd^4p\, d^4k\, e^{ip'\cdot(x'-y)+ip\cdot(y-x)+ik\cdot(y-z)}M_\mu^i . \tag{3.18}$$

In momentum space, we again suppress the arguments p', p, and k. The pion momentum is given by $q = p+k-p'$. From Eq. (3.17) we get

$$\begin{aligned}k_\mu M_\mu^i = &e_N\tau_i S_F'^{-1}(p')S_F'(p'-k)\Gamma(q,p'-k,p) \\ &-\tau_i e_N\Gamma(q,p',p+k)S_F'(p+k)S_F'^{-1}(p) \\ &-ie\epsilon^{ij3}\tau_j\Delta_F'^{-1}(q)\Delta_F'(q-k)\Gamma(q-k,p',p) , \end{aligned} \tag{3.19}$$

where

$$S_F'(x'-x) = \frac{-i}{(2\pi)^4}\int d^4p\, e^{ip\cdot(x'-x)}S_F'(p) \tag{3.20}$$

is the nucleon propagator and

$$\Delta_F'(y'-y) = \frac{-i}{(2\pi)^4}\int d^4q\, e^{iq\cdot(y'-y)}\Delta_F'(q) \tag{3.21}$$

is the pion propagator. For π^+ production, Eq. (3.19) becomes

$$\begin{aligned}k_\mu M_\mu^{\pi^+} = e[&\Delta_F'^{-1}(q)\Delta_F'(q-k)\Gamma(q-k,p',p) \\ &-\Gamma(q,p',p+k)S_F'(p+k)S_F'^{-1}(p)] , \end{aligned} \tag{3.22}$$

in precise agreement with the one derived by Kazes.[9]

Our last task is to prove that the interaction obtained in the previous section is indeed consistent with the WT relation (3.19). In momentum space M_μ^{Bi} becomes

$$\begin{aligned}M_\mu^{Bi} = &\tau_i\Gamma(q,p',p+k)S_F'(p+k)j_\mu(p+k,p) \\ &+j_\mu(p',p'-k)S_F'(p'-k)\tau_i\Gamma(q,p'-k,p) \\ &+j_\mu^{\pi ij}(q,q-k)\Delta_F'(q-k)\tau_j\Gamma(q-k,p',p) , \end{aligned} \tag{3.23}$$

where the nucleon and pion electromagnetic vertices are, respectively, given by

$$\Gamma_\mu(x'x:z) = \frac{-i}{(2\pi)^8}\int d^4p'd^4p\, e^{ip'\cdot(x'-z)+ip\cdot(z-x)}j_\mu(p',p) \tag{3.24}$$

and

$$\Gamma_\mu^{\pi ij}(y'y:z) = \frac{-i}{(2\pi)^8}\int d^4q'd^4q\, e^{iq'\cdot(y'-z)+iq\cdot(z-y)} \times j_\mu^{\pi ij}(q',q) . \tag{3.25}$$

Using the WT identities for the electromagnetic vertices of nucleon and pion,

$$k_\mu j_\mu(p',p) = e_N S_F'^{-1}(p')[S_F'(p)-S_F'(p')]S_F'^{-1}(p) , \tag{3.26}$$

$$k_\mu j_\mu^{\pi ij}(q',q) = -ie\epsilon^{ij3}\Delta_F'^{-1}(q')[\Delta_F'(q)-\Delta_F'(q')]\Delta_F'^{-1}(q) , \tag{3.27}$$

we find the four-dimensional divergence of M_μ^{Bi},

$$k_\mu M_\mu^{Bi} = \tau_i e_N \Gamma(q,p',p+k)[S_F'(p) - S_F'(p+k)]S_F'^{-1}(p) + e_N\tau_i S_F'^{-1}(p')[S_F'(p'-k) - S_F'(p')]\Gamma(q,p'-k,p) - ie\epsilon^{ij3}\tau_j \Delta_F'^{-1}(q)[\Delta_F'(q-k) - \Delta_F'(q)]\Gamma(q-k,p',p) \ . \quad (3.28)$$

On the other hand, the four divergence of the interaction ΔM_μ^i is given by Eq. (2.49) in Sec. II. Therefore, the sum of the isolated-pole contribution and the minimal-coupling interaction satisfies

$$k_\mu M_\mu^{Bi} + k_\mu \Delta M_\mu^i = -\tau_i e_N \Gamma(q,p',p+k) S_F'(p+k) S_F'^{-1}(p) + e_N \tau_i S_F'^{-1}(p') S_F'(p'-k)\Gamma(q,p'-k,p) - ie\epsilon^{ij3}\tau_j \Delta_F'^{-1}(q)\Delta_F'(q-k)\Gamma(q-k,p',p) \ . \quad (3.29)$$

One sees that the obtained divergence exactly reproduces the WT identity generalized to the photon-pion vertex.

IV. EXCHANGE CURRENTS

A. WT equation for *NN* interaction

Nishijima[6,7] derived WT equations for n-point Green's functions, and it is straightforward to apply his results to the two-nucleon system, as is done by Bentz.[12] The WT equation,

$$\begin{aligned}(\partial/\partial z_\mu)\langle T[\psi(x_1')\psi(x_2')\bar\psi(x_1)\bar\psi(x_2)j_\mu(z)]\rangle &= -e_N^{(1)}\langle T[\psi(x_1')\psi(x_2')\bar\psi(x_1)\bar\psi(x_2)]\rangle\delta(x_1'-z) + \langle T[\psi(x_1')\psi(x_2')\bar\psi(x_1)\bar\psi(x_2)]\rangle e_N^{(1)}\delta(x_1-z)\\ &\quad - e_N^{(2)}\langle T[\psi(x_1')\psi(x_2')\bar\psi(x_1)\bar\psi(x_2)]\rangle\delta(x_2'-z) + \langle T[\psi(x_1')\psi(x_2')\bar\psi(x_1)\bar\psi(x_2)]\rangle e_N^{(2)}\delta(x_2-z) \ , \quad (4.1)\end{aligned}$$

relates the divergence of the radiative Green's function to the nonradiative Green's function. The five-point Green's function is split into

$$\begin{aligned}\langle T[\psi(x_1')\psi(x_2')\bar\psi(x_1)\bar\psi(x_2)j_\mu(z)]\rangle &= -\langle T[\psi(x_1')\bar\psi(x_1)j_\mu(z)]\rangle S_F'(x_2'-x_2) - \langle T[\psi(x_2')\bar\psi(x_2)j_\mu(z)]\rangle S_F'(x_1'-x_1)\\ &\quad + \langle T[\psi(x_1')\bar\psi(x_2)j_\mu(z)]\rangle S_F'(x_2'-x_1) + \langle T[\psi(x_2')\bar\psi(x_1)j_\mu(z)]\rangle S_F'(x_1'-x_2)\\ &\quad + \langle T[\psi(x_1')\psi(x_2')\bar\psi(x_1)\bar\psi(x_2)j_\mu(z)]\rangle_c \ , \quad (4.2)\end{aligned}$$

where the first four terms are the electromagnetic interaction of noninteracting two nucleons, and the last term represents the connected part which defines the two-body current operator as

$$\langle T[\psi(x_1')\psi(x_2')\bar\psi(x_1)\bar\psi(x_2)j_\mu(z)]\rangle_c = i\int d^4\xi_1' d^4\xi_2' d^4\xi_1 d^4\xi_2 S_F'(x_1'-\xi_1')S_F'(x_2'-\xi_2') \times J_\mu(\xi_1'\xi_2';\xi_1\xi_2:z)S_F'(\xi_1-x_1)S_F'(\xi_2-x_2) \ . \quad (4.3)$$

The nonradiative Green's function is also decomposed into disconnected and connected parts,

$$\langle T[\psi(x_1')\psi(x_2')\bar\psi(x_1)\bar\psi(x_2)]\rangle = -S_F'(x_1'-x_1)S_F'(x_2'-x_2) + S_F'(x_1'-x_2)S_F'(x_2'-x_1) + \langle T[\psi(x_1')\psi(x_2')\bar\psi(x_1)\bar\psi(x_2)]\rangle_c \ , \quad (4.4)$$

and the latter defines the nucleon-nucleon interaction $\mathcal{V}$,

$$\langle T[\psi(x_1')\psi(x_2')\bar\psi(x_1)\bar\psi(x_2)]\rangle_c = \int d^4\xi_1' d^4\xi_2' d^4\xi_1 d^4\xi_2 S_F'(x_1'-\xi_1')S_F'(x_2'-\xi_2')\mathcal{V}(\xi_1'\xi_2';\xi_1\xi_2)S_F'(\xi_1-x_1)S_F'(\xi_1-x_2) \ . \quad (4.5)$$

Since the WT equation (4.1) holds true for the disconnected and connected parts separately, the connected part becomes, in momentum space,

$$\begin{aligned}k_\mu J_\mu &= -e_N^{(1)}S_F'^{-1}(p_1')S_F'(p_1'-k)\mathcal{V}(p_1'-k,p_2';p_1,p_2) + \mathcal{V}(p_1',p_2';p_1+k,p_2)e_N^{(1)}S_F'(p_1+k)S_F'^{-1}(p_1)\\ &\quad - e_N^{(2)}S_F'^{-1}(p_2')S_F'(p_2'-k)\mathcal{V}(p_1',p_2'-k;p_1,p_2) + \mathcal{V}(p_1',p_2';p_1,p_2+k)e_N^{(2)}S_F'(p_2+k)S_F'^{-1}(p_2) \ . \quad (4.6)\end{aligned}$$

Equation (4.3) can be calculated from the functional derivative

$$\langle T[\psi(x_1')\psi(x_2')\bar\psi(x_1)\bar\psi(x_2)j_\mu(z)]\rangle_c = -i\left[\frac{\delta}{\delta A_\mu^e(z)}\langle T[\psi(x_1')\psi(x_2')\bar\psi(x_1)\bar\psi(x_2)]\rangle_c\right]_{A^e\to 0} \ . \quad (4.7)$$

The nucleon-nucleon interaction $\tilde{\mathcal{V}}$ modified by the external field is defined by

$$\langle T[\psi(x_1')\psi(x_2')\bar{\psi}(x_1)\bar{\psi}(x_2)]\rangle_c=\int d^4\xi_1' d^4\xi_2' d^4\xi_1 d^4\xi_2 \tilde{S}'_F(x_1'-\xi_1')\tilde{S}'_F(x_2'-\xi_2') \times\tilde{\mathcal{V}}(\xi_1'\xi_2';\xi_1\xi_2)\tilde{S}'_F(\xi_1-x_1)\tilde{S}'_F(\xi_2-x_2) . \quad (4.8)$$

The direct evaluation of the functional derivative using Eq. (3.26) brings the total current operator into

$$J_\mu=J_\mu^{\rm ext}+J'_\mu . \quad (4.9)$$

The first term is the contribution of nucleon-pole terms in which the photon is attached to the external nucleon lines,

$$\begin{aligned}J_\mu^{\rm ext}(x_1'x_2';x_1x_2:z)=&\int d^4\xi' d^4\xi\, \Gamma_\mu(x_1'\xi':z)S_F'(\xi'-\xi)\mathcal{V}(\xi x_2';x_1x_2)\\ &+\int d^4\xi' d^4\xi\, \mathcal{V}(x_1'x_2';\xi' x_2)S_F'(\xi'-\xi)\Gamma_\mu(\xi x_1:z)\\ &+\int d^4\xi' d^4\xi\, \Gamma_\mu(x_2'\xi':z)S_F'(\xi'-\xi)\mathcal{V}(x_1'\xi;x_1x_2)\\ &+\int d^4\xi' d^4\xi\, \mathcal{V}(x_1'x_2';x_1\xi')S_F'(\xi'-\xi)\Gamma_\mu(\xi x_2:z) ,\end{aligned} \quad (4.10)$$

and the second term is

$$J'_\mu(x_1'x_2';x_1x_2:z)=-\left[\frac{\delta}{\delta A_\mu^e(z)}\tilde{\mathcal{V}}(x_1'x_2';x_1x_2)\right]_{A^e\to 0} . \quad (4.11)$$

In momentum space,

$$\begin{aligned}J_\mu^{\rm ext}=&-j_\mu(p_1',p_1'-k)S_F'(p_1'-k)\mathcal{V}(p_1'-k,p_2';p_1,p_2)-\mathcal{V}(p_1',p_2';p_1+k,p_2)S_F'(p_1+k)j_\mu(p_1+k,p_1)\\ &-j_\mu(p_2',p_2'-k)S_F'(p_2'-k)\mathcal{V}(p_1',p_2'-k;p_1,p_2)-\mathcal{V}(p_1',p_2';p_1,p_2+k)S_F'(p_2+k)j_\mu(p_2+k,p_2) .\end{aligned} \quad (4.12)$$

By the use of the WT equation (3.26) for the electromagnetic vertex function of the nucleon, we get

$$\begin{aligned}k_\mu J_\mu^{\rm ext}=&-S_F'^{-1}(p_1')[S_F'(p_1'-k)-S_F'(p_1')]e_N^{(1)}\mathcal{V}(p_1'-k,p_2';p_1,p_2)\\ &-\mathcal{V}(p_1',p_2';p_1+k,p_2)e_N^{(1)}[S_F'(p_1)-S_F'(p_1+k)]S_F'^{-1}(p_1)\\ &-S_F'^{-1}(p_2')[S_F'(p_2'-k)-S_F'(p_2')]e_N^{(2)}\mathcal{V}(p_1',p_2'-k;p_1,p_2)\\ &-\mathcal{V}(p_1',p_2';p_1,p_2+k)e_N^{(2)}[S_F'(p_2)-S_F'(p_2+k)]S_F'^{-1}(p_2) .\end{aligned} \quad (4.13)$$

Subtracting the divergence of $J_\mu^{\rm ext}$ from that of the total current J_μ, we get the constraint on the current J'_μ,

$$\begin{aligned}k_\mu J'_\mu=&-e_N^{(1)}\mathcal{V}(p_1'-k,p_2';p_1,p_2)\\ &+\mathcal{V}(p_1',p_2';p_1+k,p_2)e_N^{(1)}\\ &-e_N^{(2)}\mathcal{V}(p_1',p_2'-k;p_1,p_2)\\ &+\mathcal{V}(p_1',p_2';p_1,p_2+k)e_N^{(2)} .\end{aligned} \quad (4.14)$$

B. On-shell nucleons

In order to compare with the result of Gross and Riska,[4] we put the external nucleons on their mass shell, and use the pion-nucleon vertex function

$$\Gamma=i\gamma_5 g_1(q^2,-m^2,-m^2)=i\gamma_5 g f(q) . \quad (4.15)$$

The minimal coupling we have derived is now reduced to the contact interaction

$$\Delta M_\mu^i=ieg\epsilon^{ij3}\tau_j\frac{(2q-k)_\mu}{q^2-(q-k)^2}i\gamma_5[f(q)-f(q-k)] , \quad (4.16)$$

which induces two exchange current processes. In the first process, the contact interaction on nucleon 1,

$$\Delta M_\mu^{(1)i}=ieg\epsilon^{ij3}\tau_j^{(1)}\frac{(q'+q)_\mu}{q'^2-q^2}i\gamma_5^{(1)}[f(q')-f(q)] , \quad (4.17)$$

emits the pion with momentum q' and isospin index i which is to be reabsorbed by nucleon 2. In the second process, the pion with momentum q and isospin index j emitted by nucleon 1 is reabsorbed by the contact interaction on nucleon 2,

$$\Delta M_\mu^{(2)j}=ieg\epsilon^{ij3}\tau_i^{(2)}\frac{(q'+q)_\mu}{q'^2-q^2}i\gamma_5^{(2)}[f(q')-f(q)] . \quad (4.18)$$

The pionic exchange current, on the other hand, is induced by the process in which the pion with momentum q and isospin index j absorbs photon, thereby becoming the pion with momentum q' and isospin index i. The electromagnetic vertex of pion has the structure

$$j_\mu^{\pi ij}(q',q)=-ie\epsilon^{ij3}\Gamma_\mu^\pi(q',q) . \quad (4.19)$$

The total pion-exchange current operator becomes

$$J'_\mu = ieg^2[\tau^{(1)}\times\tau^{(2)}]_3 i\gamma_5^{(1)} i\gamma_5^{(2)} \left[\Gamma_\mu^\pi(q',q)\Delta_F(q)\Delta_F(q')f(q)f(q') - (q'+q)_\mu \frac{f(q')-f(q)}{q'^2-q^2}\Delta_F(q')f(q') - (q'+q)_\mu \frac{f(q')-f(q)}{q'^2-q^2}\Delta_F(q)f(q) \right] , \tag{4.20}$$

where

$$\Delta_F(q) = 1/(q^2+\mu^2) \tag{4.21}$$

is the free pion propagator and μ is the pion mass.

The most general form of the electromagnetic vertex of the pion is[7]

$$\Gamma_\mu^\pi(q',q) = A(k^2,q'^2,q^2)\left[(q'+q)_\mu - \frac{k\cdot(q'+q)}{k^2}k_\mu \right] + B(k^2,q'^2,q^2)(q'+q)_\mu . \tag{4.22}$$

Evaluating its divergence, we find

$$k_\mu \Gamma_\mu^\pi(q',q) = B(k^2,q'^2,q^2)(q'^2-q^2) . \tag{4.23}$$

Since the WT identity (3.27) holds both for renormalized and unrenormalized operators, it places the constraint

$$B(k^2,q'^2,q^2) = 1 , \tag{4.24}$$

if one uses the free propagator Δ_F instead of the renormalized Δ'_F. For on-shell pion, it is customary to employ a parametrization

$$A(k^2,-\mu^2,-\mu^2)+1 = F_\pi(k^2) . \tag{4.25}$$

The three terms in Eq. (4.20) can be put together into

$$B(k^2,q'^2,q^2)\Delta_F(q)\Delta_F(q')f(q)f(q') - \frac{f(q')-f(q)}{q'^2-q^2}[\Delta_F(q')f(q')+\Delta_F(q)f(q)] = -\frac{\Delta(q')-\Delta(q)}{q'^2-q^2} = \Delta(q)\Delta(q')\frac{\Delta^{-1}(q')-\Delta^{-1}(q)}{q'^2-q^2} , \tag{4.26}$$

where $\Delta(q)$ is the pion propagator regularized by the pion-nucleon form factors,

$$\Delta(q) = \frac{f(q)^2}{q^2+\mu^2} . \tag{4.27}$$

The current operator (4.20) is now cast in the form

$$J'_\mu = ieg^2[\tau^{(1)}\times\tau^{(2)}]_3 i\gamma_5^{(1)} i\gamma_5^{(2)} \times \Delta(q)\Delta(q')\Gamma_\mu^{\pi R}(q',q) , \tag{4.28}$$

where

$$\Gamma_\mu^{\pi R}(q',q) = A^R\left[(q'+q)_\mu - \frac{k\cdot(q'+q)}{k^2}k_\mu \right] + B^R(q'+q)_\mu \tag{4.29}$$

and

$$A^R = \frac{A}{f(q)f(q')} . \tag{4.30}$$

The effect of the minimal coupling associated with the pion-nucleon vertex is to modify the form factor $B=1$ into

$$B^R = \frac{\Delta^{-1}(q')-\Delta^{-1}(q)}{q'^2-q^2} . \tag{4.31}$$

If we put $A^R = F_\pi(k^2)-1$, we could reproduce the result of Gross and Riska[4] for $F_0(k^2)=1$. They derived this equation by assuming the WT identity for the dressed vertex of pion, Eq. (4.29). Indeed from

$$k_\mu \Gamma_\mu^{\pi R}(q',q) = \Delta^{-1}(q') - \Delta^{-1}(q) , \tag{4.32}$$

we find B^R in agreement with Eq. (4.31). Our approach justifies the use of the WT equation for the renormalized vertex function. However, note that A^R is completely undetermined in the approach in which the WT relation alone is used. Our approach reveals that A^R is given by

$$[F_\pi(k^2)-1]/f(q)f(q')$$

rather than $F_\pi(k^2)-1$.

The four-divergence of the exchange current (4.28) is given by

$$k_\mu J'_\mu = ie[\tau^{(1)}\times\tau^{(2)}]_3[v(q')-v(q)] , \tag{4.33}$$

where

$$v(q) = -g^2 i\gamma_5^{(1)} i\gamma_5^{(2)}\Delta(q) . \tag{4.34}$$

This equation coincides with (4.14) if we make the replacement

$$\mathcal{V}(p_1'-k,p_2';p_1,p_2)=\mathcal{V}(p_1',p_2';p_1+k,p_2)=\boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}^{(2)}v(q') , \quad (4.35)$$

$$\mathcal{V}(p_1',p_2'-k;p_1,p_2)=\mathcal{V}(p_1',p_2';p_1,p_2+k)=\boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}^{(2)}v(q) . \quad (4.36)$$

In this way the requirement of the WT equation for the nucleon-nucleon interaction is fulfilled.

As is emphasized in Sec. II, the minimal-coupling interaction is undetermined up to four-divergence-free terms. We can generalize Eq. (4.16) as

$$\Delta M_\mu^i = ieg\epsilon^{3ij}\tau_j i\gamma_5 \frac{f(q)-f(q-k)}{q^2-(q-k)^2} \times \left\{ F_0(k^2)(2q-k)_\mu - [F_0(k^2)-1]\frac{k\cdot(2q-k)}{k^2}k_\mu \right\} . \quad (4.37)$$

The added term, which is proportional to an arbitrary form factor $F_0(k^2)-1$, is a conserved-current interaction, and therefore does not affect the WT equation. The modified exchange-current operator becomes

$$J'_\mu = ieg^2[\boldsymbol{\tau}^{(1)}\times\boldsymbol{\tau}^{(2)}]_3 i\gamma_5^{(1)} i\gamma_5^{(2)} \left\{ \Gamma_\mu^\pi(q',q)\Delta_F(q)\Delta_F(q')f(q)f(q') - F_0(k^2)(q'+q)_\mu \frac{f(q')-f(q)}{q'^2-q^2}[\Delta_F(q')f(q')+\Delta_F(q)f(q)] + [F_0(k^2)-1]k_\mu \frac{f(q')-f(q)}{k^2}[\Delta_F(q')f(q')+\Delta_F(q)f(q)] \right\} . \quad (4.38)$$

We again bring this into the form of (4.28) with B^R unchanged and with A^R given by

$$A^R = \frac{1}{f(q)f(q')}[A+1-F_0(k^2)]+B^R[F_0(k^2)-1] . \quad (4.39)$$

If one sets $A=F_\pi(k^2)-1$, the parametrization of Gross and Riska,[4]

$$\gamma = \frac{A^R}{A} = \frac{1}{f(q)f(q')}\left[1-\frac{1-F_0(k^2)}{1-F_\pi(k^2)}\right] + B^R\left[\frac{1-F_0(k^2)}{1-F_\pi(k^2)}\right] , \quad (4.40)$$

is essentially reproduced except for the factor $1/f(q)f(q')$ in front of the first term.

C. Off-shell nucleons

We now turn to the exchange current for off-shell nucleons. The current operator becomes

$$J'_\mu = \tau_j^{(1)}\Gamma^{(1)}(q,p_1',p_1)\Delta_F'(q)j_\mu^{\pi ij}(q',q) \times\Delta_F'(q')\tau_i^{(2)}\Gamma^{(2)}(-q',p_2',p_2) + \Delta M_\mu^{(1)i}\Delta_F'(q')\tau_i^{(2)}\Gamma^{(2)}(-q',p_2',p_2) + \tau_j^{(1)}\Gamma^{(1)}(q,p_1',p_1)\Delta_F'(q)\Delta M_\mu^{(2)j} , \quad (4.41)$$

where

$$q=p_1-p_1'=p_2'-p_2-k , \quad (4.42)$$

$$q'=p_1-p_1'+k=p_2'-p_2 . \quad (4.43)$$

using the four divergence of the minimal coupling, Eq. (2.49),

$$k_\mu\Delta M_\mu^{(1)i} = -ie\epsilon^{ij3}\tau_j^{(1)}\Gamma^{(1)}(q'-k,p_1',p_1) + e_N^{(1)}\tau_i^{(1)}\Gamma^{(1)}(q',p_1'-k,p_1) - \tau_i^{(1)}e_N^{(1)}\Gamma^{(1)}(q',p_1',p_1+k) , \quad (4.44)$$

$$k_\mu\Delta M_\mu^{(2)j} = ie\epsilon^{ij3}\tau_i^{(2)}\Gamma^{(2)}(-q-k,p_2',p_2) + e_N^{(2)}\tau_j^{(2)}\Gamma^{(2)}(-q,p_2'-k,p_2) - \tau_j^{(2)}e_N^{(2)}\Gamma^{(2)}(-q,p_2',p_2+k) , \quad (4.45)$$

and the WT equation for the pion vertex, Eq. (3.27), we obtain the constraint Eq. (4.14) with

$$\mathcal{V}(p_1'-k,p_1;p_2',p_2) = -\boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}^{(2)}\Gamma^{(1)}(q,p_1'-k,p_1) \times\Delta_F'(q')\Gamma^{(2)}(-q,p_2',p_2) , \quad (4.46)$$

$$\mathcal{V}(p_1',p_1+k;p_2',p_2) = -\boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}^{(2)}\Gamma^{(1)}(q,p_1',p_1+k) \times\Delta_F'(q')\Gamma^{(2)}(-q,p_2',p_2) , \quad (4.47)$$

$$\mathcal{V}(p_1',p_1;p_2'-k,p_2) = -\boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}^{(2)}\Gamma^{(1)}(q,p_1',p_1) \times\Delta_F'(q)\Gamma^{()}(-q,p_2'-k,p_2) , \quad (4.48)$$

$$\mathcal{V}(p_1',p_1;p_2',p_2+k) = -\boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}^{(2)}\Gamma^{(1)}(q,p_1',p_1) \times\Delta_F'(q)\Gamma^{(2)}(-q,p_2',p_2+k) . \quad (4.49)$$

We again find that the sum of the pionic current and the minimal-coupling terms satisfies the WT equation for the nucleon-nucleon interaction. It is clear that the divergence of the one-pion exchange current in the most general form is free from the pion electromagnetic form factor.

D. Electromagnetic form factor of the nucleon

The electromagnetic vertex function of the nucleon contained in the isolated-pole term (4.12) has the most general form

$$j_\mu(p',p)=iF_1\gamma_\mu-iF_2\sigma_{\mu\nu}k_\nu+F_3k_\mu+(i\gamma\cdot p'+m)(iF_4\gamma_\mu-iF_5\sigma_{\mu\nu}k_\nu+F_6k_\mu)$$
$$+(iF_7\gamma_\mu-iF_8\sigma_{\mu\nu}k_\nu+F_9k_\mu)(i\gamma\cdot p+m)+(i\gamma\cdot p'+m)(iF_{10}\gamma_\mu-iF_{11}\sigma_{\mu\nu}k_\nu+F_{12}k_\mu)(i\gamma\cdot p+m)\ . \tag{4.50}$$

(Different parametrizations can be found in the literature.[8–11]) The twelve form factors are functions of p'^2, p^2, and $k^2=(p'-p)^2$. Using the fully dressed nucleon propagator

$$S_F'(p)=i\gamma\cdot pG(p^2)+mF(p^2)\ , \tag{4.51}$$

the WT identity (3.26) imposes the four constraints

$$F_1+2mF_4+k^2F_6+(p^2+m^2)F_{10}=e_NG(p'^2)\ , \tag{4.52}$$

$$F_1+2mF_7-k^2F_9+(p'^2+m^2)F_{10}=e_NG(p^2)\ , \tag{4.53}$$

$$F_4-F_7-k^2F_{12}=0\ , \tag{4.54}$$

$$k^2F_3-(p'^2+m^2)F_4+(p^2+m^2)F_7$$
$$=e_Nm[F(p'^2)-F(p^2)]-e_Nm[G(p'^2)-G(p^2)]\ . \tag{4.55}$$

As long as the form factors are consistent with these constraints, the current divergence does not contain them. The choice of Gross and Riska[4] corresponds to

$$F_3=F_5=F_8=F_{10}=F_{11}=F_{12}=0$$

and $G=F=1$. The above conditions lead to $F_4=F_7=0$ and

$$F_6=-F_9=-(F_1-e_N)/k^2\ .$$

Therefore

$$j_\mu(p',p)=i(F_1-e_N)\left[\gamma_\mu-\frac{k_\mu}{k^2}\gamma\cdot k\right]$$
$$+ie_N\gamma_\mu-iF_2\sigma_{\mu\nu}k_\nu\ . \tag{4.56}$$

We can further rewrite this as

$$j_\mu(p',p)=i(F_1+2mF_2)\gamma_\mu-F_2(p'+p)_\mu$$
$$+(i\gamma\cdot p'+m)\left[-iF_2\gamma_\mu-\frac{F_1-e_N}{k^2}k_\mu\right]$$
$$+\left[-iF_2\gamma_\mu+\frac{F_1-e_N}{k^2}k_\mu\right](i\gamma\cdot p+m) \tag{4.57}$$

in agreement with Berends and West.[13] As is pointed out by Naus and Koch,[14] their choice is too restrictive to be realistic, but their conclusion that the current divergence is independent of the electromagnetic form factors remains true.

V. SUMMARY

We have proposed a method of constructing an electromagnetic current operator associated with the pion-nucleon vertex function. Our basic tool is the minimal-substitution prescription. We have transformed the nonlocal vertex function into a momentum-dependent local operator and replaced the nucleon momentum operators by the gauge-invariant ones. The resulting minimal coupling interaction is proved to be consistent with the WT identity derived for the pion photoproduction. We also derived the WT equation for the two-nucleon system and examined the validity of our current operator for the one-pion exchange current. We found that the minimal coupling introduced into the pion-nucleon vertex is an essential ingredient which is needed for the WT equation to be valid. We showed that our result is consistent with that of Gross and Riska.[4]

Our result is also in conformity with Nishijima[6,7] who stressed that the WT equation holds true for composite particles as well as for elementary particles. Although a current operator is influenced by the electromagnetic form factors of participating particles, its four divergence is free from these form factors and takes the form as if the composite particles are structureless pointlike ones. This fact was already found by Berends and West[13] in pion electroproduction, and more recently by Gross and Riska[4] in nuclear exchange currents. Our result supports the conclusion that there arise no constraints on electromagnetic form factors to be used in hadronic interactions.

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