Nuclear molecular configurations in heavy ion collisions

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(Received 9 June 1988)

The possibilities of nuclear molecular configurations in symmetric and asymmetric heavy ion collisions are investigated by choosing a proper parametrization for the molecule-like compound state. We treat the nucleus as a rotating liquid drop in terms of the elliptic lemniscatoid parametrization suggested by Royer and Remaud which describes all deformed shapes that occur during the process of fusion of two nuclei, starting from two touching spheres to a single one with the intermediate formation of a deep neck. Only one parameter (s for the symmetric and s_1 for the conditional asymmetric case) is needed to define all the shapes encountered in the process. The shape has a neck as long as s or s_1 is less than $1/\sqrt{2}$. In this parametrization, analytical expressions can be obtained for volume, surface, and moment of inertia while the Coulomb energy has to be evaluated numerically. The total energy of the system is calculated as a function of the parameter s or s_1, for various spins and if the stable shape corresponding to minimum energy is one with a neck, then it is concluded that the molecular configuration is possible. Our model yields results which are in good agreement with the experimental findings.

I. INTRODUCTION

The existence of nuclear molecular configurations in heavy ion collisions was first predicted by Bromley *et al.*[1] as a result of their experiments on the elastic scattering of ^{12}C by ^{12}C at energies just above the Coulomb barrier. When the reaction cross section was plotted as a function of incident energy, they observed that the curve exhibited peaks or resonances which they attributed to the formation of a molecule-like compound state by the two carbon nuclei. In many other cases of heavy ion reactions, such as $^{18}O+^{18}O$, $^{14}N+^{14}N$, $^{12}B+^{12}C$, $^{10}B+^{14}N$, and $^{12}C+^{13}C$, no evidence was found for the existence of prominent resonances in the energy range of a sub-Coulomb barrier. Even though there are certain systems which show the resonance structure, they do so only in selected reaction channels, and when the sum is taken over all channels the resulting total cross section exhibits a fairly smooth energy dependence. In contrast, in the reaction of ^{12}C on ^{12}C, at lower energies near the top of the Coulomb barrier, sharp, well-resolved resonances are observed in the total reaction cross section. An explanation as to why sharp resonances were observed in the Coulomb barrier energy range for some nuclei and absent in many other cases has been sought and discussed by a number of authors.[2–11]

Theoretically, Park *et al.*[11] have given a microscopic model considering the orbital motion of the outermost loosely bound nucleons around both nuclear centers generating a covalent binding. On the other hand, a macroscopic-microscopic model proposed by Leander and Larsson[12] using the Strutinsky shell correction method has led to the detection of secondary minimum in heavy ion potential near the interaction barrier in the case of ^{24}Mg in the (β,γ) plane. They have found different minima corresponding to different configurations at large deformation having axially symmetric and reflection asymmetric shape. It is doubtful whether one of the minima obtained in a one-center Nilsson model could really correspond to a molecular configuration. Thus there is some ambiguity as to which of the different minima corresponded to the molecular configuration.

The molecular model proposed by Chandra and Mosel[10] envisages a two-center harmonic oscillator with separation as an additional degree of freedom and has also utilized the Strutinsky shell correction prescription. They have obtained a secondary minimum in the form of a "shoulder" in the curve of interaction energy plotted as a function of the separation distance R for $^{12}C+^{12}C$ reaction assuming a nonaxially symmetric shape.

The model adopted by us is a simple macroscopic rotating liquid drop model (RLDM) which uses the elliptic leminiscatoid parametrization of Royer and Remaud.[13] It is a two-center parametrization incorporating the neck degree of freedom. In heavy ion collisions, the compound nuclei which are formed have large angular momentum as well as internal excitation. The latter may make the shell corrections vanish, and in such a context a macroscopic model can be taken to be valid. Even in the case where shell effects are appreciable the calculations have to start with a correct macroscopic description, which we feel is provided by the present model.

In this work, we have minimized, in the frame work of the RLDM, the total energy of the compound nucleus with respect to the relevant shape parameter for each angular momentum, and if the shape of higher stability is one with a neck it would imply the probable formation of a nuclear molecule. In this spirit, we have first investigated the possibilities of symmetrical molecular configurations at high spins in different heavy ion collisions.[14] We have further extended our calculation independently to certain asymmetric cases also[15] for which evidence[16] is

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available. Royer and Remaud[17] have given the asymmetric extension of the previous parametrization in connection with their discussion of fusion barriers. Let us now recall the important work by Broglia *et al.*[18] on the stability of a dinucleus system at high angular momentum. In contrast to their model our model deals with a merged dinucleus system and includes asymmetric cases also, but the proximity energy which is considered by them is not included in this work.

In Sec. II, we give a description of the model used and the expression for the various shape-dependent energy terms. The application of this model to symmetric cases is dealt with in Sec. III. The same has been extended to asymmetric case in Sec. IV.

II. DESCRIPTION OF THE MODEL

Royer and Remaud[13] had proposed a one-parameter family of shapes for the fusion of two identical nuclei using the elliptic lemniscatoid parametrization with a single parameter s which is the reduced neck diameter of the shape. This is capable of describing all possible intermediate deformation shapes encountered in the process of fusion of two nuclei, starting from two touching spheres and ending in a single sphere. We have extended this model to cover asymmetric cases also (see Fig. 1), and the equation to the axially symmetric and reflection asymmetric elliptic lemniscatoid in cylindrical coordinates is given by

$$\rho^2=\begin{cases}[(C_1^2s_1^2-2Z^2)+s_1^4C_1^4+4Z^2C_1^2(1-s_1^2)^{1/2}]/2 & \text{for } -C_1<Z<0\ , \\ [(C_2^2s_2^2-2Z^2)+s_2^4C_2^4+4Z^2C_2^2(1-s_2^2)^{1/2}]/2 & \text{for } 0<Z<C_2\ ,\end{cases} \tag{1}$$

where a is the neck radius and C_1 and C_2 are the elongations. The two parameters s_1 and s_2 are defined by

$$s_1=\frac{a}{C_1} \quad \text{and} \quad s_2=\frac{a}{C_2}\ .$$

The relation between s_1 and s_2 is given by[17]

$$s_2^2=\frac{s_1^2}{s_1^2+(1-s_1^2)\beta^2}\ ,$$

where

$$\beta=\frac{R_2}{R_1}\ ,$$

R_1 and R_2 being the initial radii of the two colliding nuclei. When s_1 increases from 0 to 1 the shape changes gradually from two touching spheres of radii R_1 and R_2 to a single one, the neck disappearing for values of $s_1>1/\sqrt{2}$.

Analytical expressions for the volume, surface area, and relative perpendicular moment of inertia (the moment of inertia in units of the moment of inertia of the equivalent sphere) of the two-center shape can be easily obtained and are given by

$$V=\frac{\pi}{24}\left[4\sum_{i=1,2}C_i^3+6\sum_{i=1,2}C_i^3s_i^2+3\sum_{i=1,2}C_i^3s_i^4(1-s_i^2)^{-1/2}\sinh^{-1}[2s_i^{-2}(1-s_i^2)^{1/2}]\right]\ , \tag{2}$$

$$s=4\pi R_0^2B_s=\pi\left[\sum_{i=1,2}C_i^2+\sum_{i=1,2}C_i^2(1-s_i^4)^{-1/2}s_i^4\sinh^{-1}[s_i^{-2}(1-s_i^4)^{1/2}]\right]\ , \tag{3}$$

where R_0 is the radius of the equivalent sphere and B_s the relative surface energy, and

$$J_\perp=\frac{1}{1024}\sum_{i=1,2}\frac{C_i^5s_i^2}{1-s_i^2}\left[\frac{112}{s_i^2}+8+30s_i^2-135s_i^4+(120s_i^4-135s_i^6)(1-s_i^2)^{-1/2}\sinh^{-1}\left[\frac{1-s_i^2}{s_i^2}\right]^{1/2}\right]\ . \tag{4}$$

For the relative Coulomb energy B_c there is no closed expression, and therefore it must be evaluated numerically. While Royer and Remaud[13] have used the method of Cohen and Swiatecki, we have adopted the Lawrence method as it lends itself easily for such computation in cylindrical coordinates discussed at length by Devanathan.[19] The procedure is to divide the deformed nucleus into thin discs and then calculate the interaction between any two discs. The Coulomb energy is the sum of the interaction energies between all such discs, taking care to avoid double counting. The expression arrived at involves six integrations but eventually can be reduced to three by the use of certain formulas involving Bessel functions and Watson's identity. The final expression for Coulomb energy E_c of the nucleus is given by

$$E_c=4\pi^2\sigma^2\int_{z_1}^{z_b}\rho_a^2dz_a\int_{z_1}^{z_2}\rho_b^2dz_b\int_0^1\frac{\sin^2\pi\omega d\omega}{|Z_b-Z_a|+[(Z_b-Z_a)^2+W^2]^{1/2}}\ ,$$

where

$$\omega=\frac{\phi}{W} \quad \text{and} \quad W^2=\rho_a^2+\rho_b^2-2\rho_a\rho_b\cos(\pi\omega)\ ,$$

and ρ is expressed as a function of Z. The triple integration involved in the above expression can be evaluated by using the 16-point Gaussian integration formula.

For the reflection symmetric case, we have $s_1=s_2=s$ and $C_1=C_2=C$. As s varies from 0 to 1, the lemnisca-

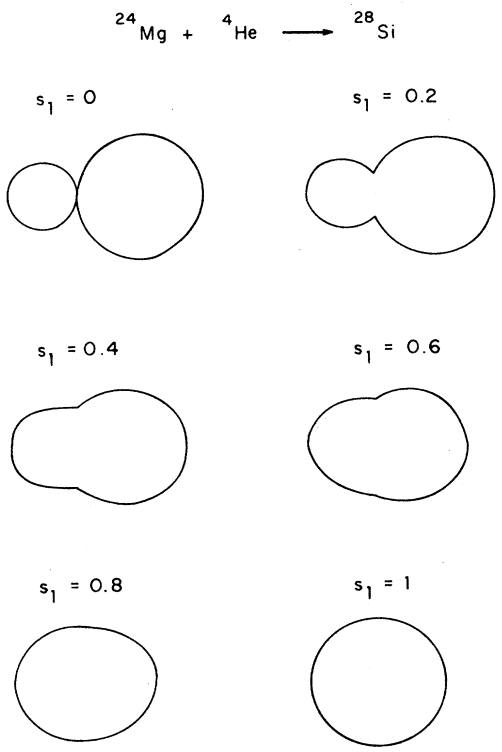

FIG. 1. The gradual shape changes from two touching spheres of radii R_1 and R_2 to a single sphere of radius R_0 as the reduced neck diameter of the left fragment s_1 varies from 0 to 1 in the conditional asymmetric collision $^{24}Mg+^{4}He\rightarrow{}^{28}Si$.

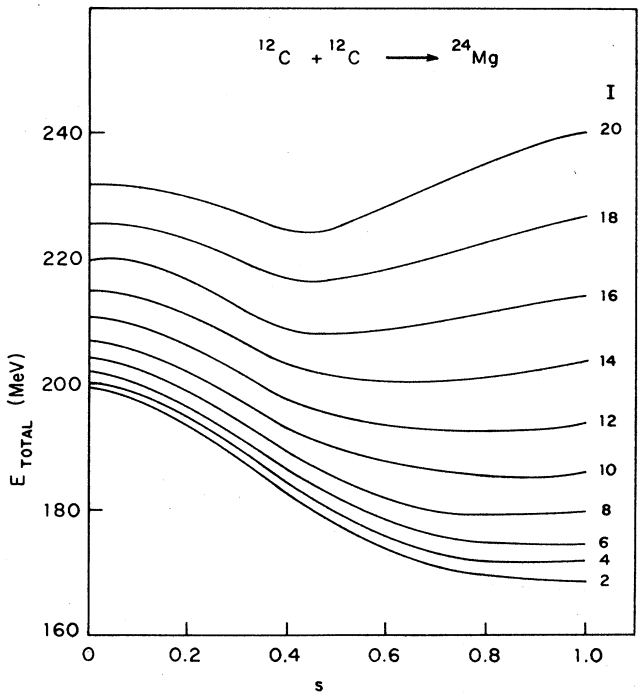

FIG. 2. Variation of the total energy E_{total} with the reduced neck diameter s for various spins I for the symmetric collision $^{12}C+^{12}C\rightarrow{}^{24}Mg$.

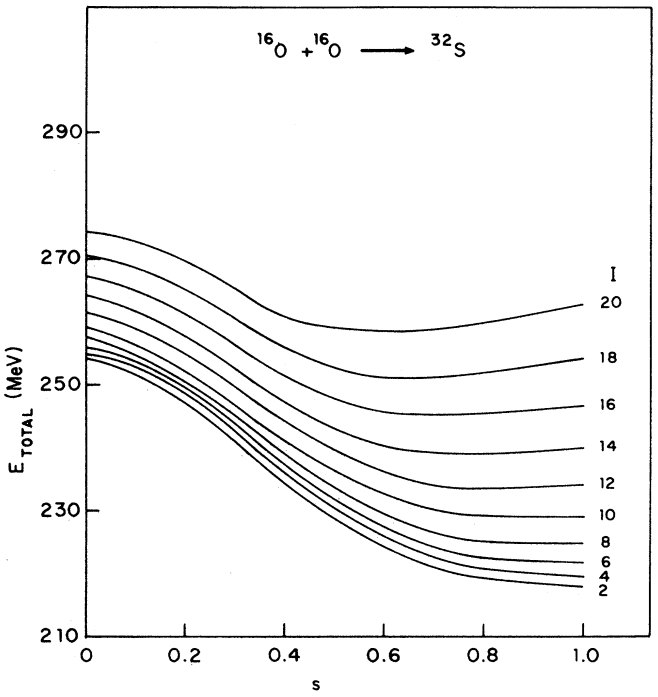

FIG. 3. Same as Fig. 2 for the case of $^{16}O+^{16}O\rightarrow{}^{32}S$.

toid varies from two touching spheres to a single compound nucleus with the intermediate formation of a deep neck. The neck disappears when $s > 1/\sqrt{2}$.

Knowing B_s, B_c, and $J_\perp$, the surface energy E_s, the Coulomb energy E_c and the perpendicular moment of inertia J are calculated as follows:

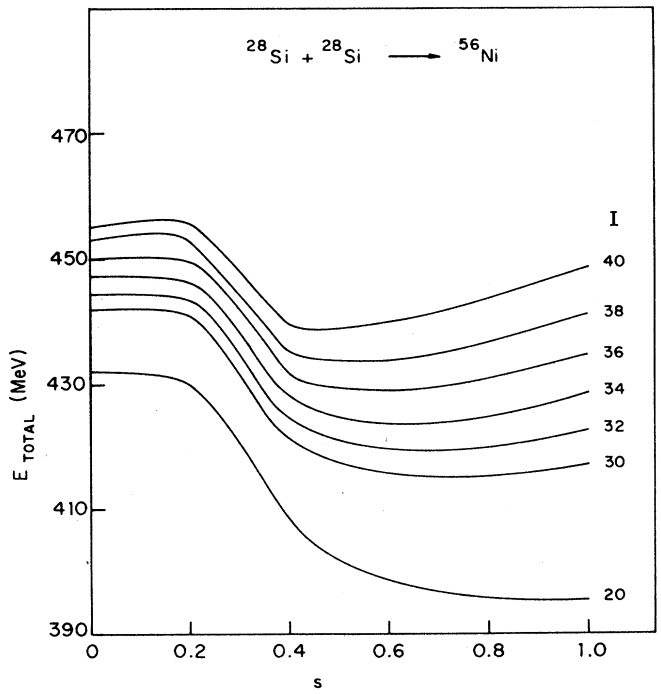

FIG. 4. Same as Fig. 2 for the case of $^{28}Si+^{28}Si\rightarrow{}^{56}Ni$.

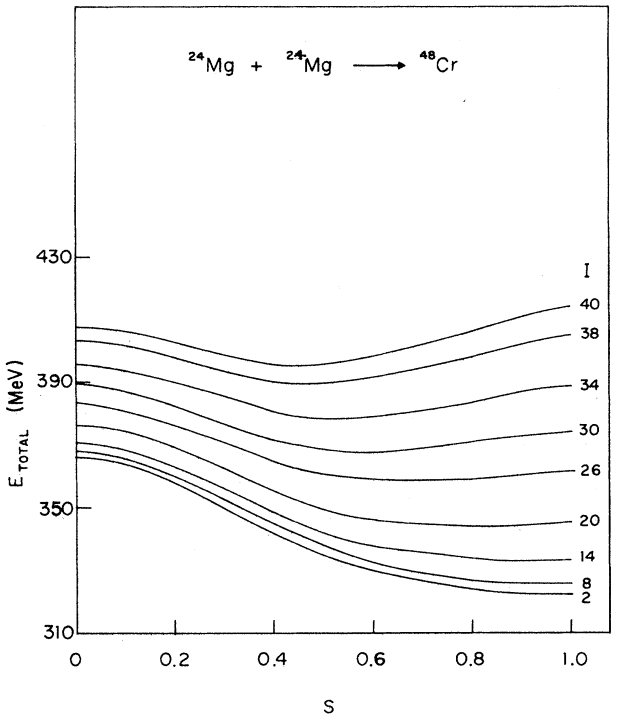

FIG. 5. Same as Fig. 2 for the case of $^{24}Mg + {}^{24}Mg \rightarrow {}^{48}Cr$.

$$E_s = B_s E_s^{(0)} = B_s a_s A^{2/3} ,$$

$$E_c = B_c E_c^{(0)} = B_c a_c Z^2 / A^{1/3} ,$$

and

$$J = J_\perp x \tfrac{2}{5} m_n A R_0^2 ,$$

where $a_s = 16$ MeV, $a_c = 0.7$ MeV, Z and A are the atomic number and mass number of the final nucleus, respectively, and m_n is the nucleon mass, and $E_s^{(0)}$ and $E_c^{(0)}$ are the surface and Coulomb energies of the equivalent spherical nucleus. The rotational energy E_R is given by

$$E_R = \frac{I^2}{2J} ,$$

I being the spin of the nucleus. The total energy is

$$E_{\text{total}}(s,I) = E_s(s) + E_c(s) + E_R(s,I) .$$

These calculations can be easily extended to the reflection asymmetric case also, E_{total} being calculated as a function of s_1 and I for a given β.

III. APPLICATION TO SYMMETRIC SYSTEMS

In order to investigate the possibilities of symmetric molecular configurations in heavy ion collisions, the following reactions have been considered:

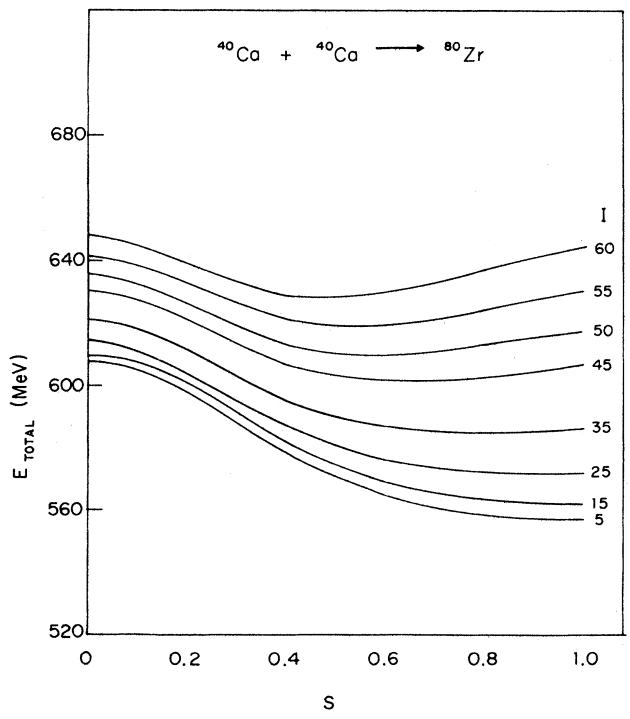

FIG. 6. Same as Fig. 2 for the case of $^{40}Ca + {}^{40}Ca \rightarrow {}^{80}Zr$.

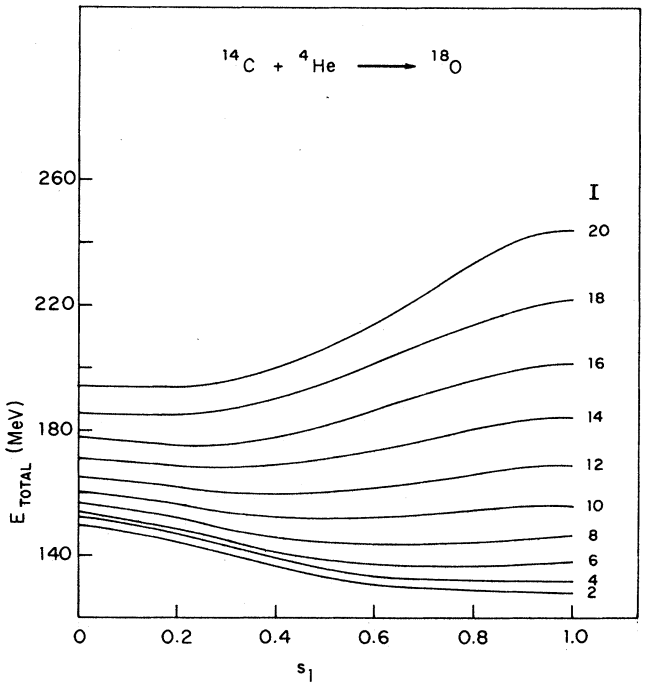

FIG. 7. Variation of the total energy E_{total} with the reduced neck diameter of the left fragment s_1 for various spins I for the conditional asymmetric collision: $^{14}C + {}^{4}He \rightarrow {}^{18}O$.

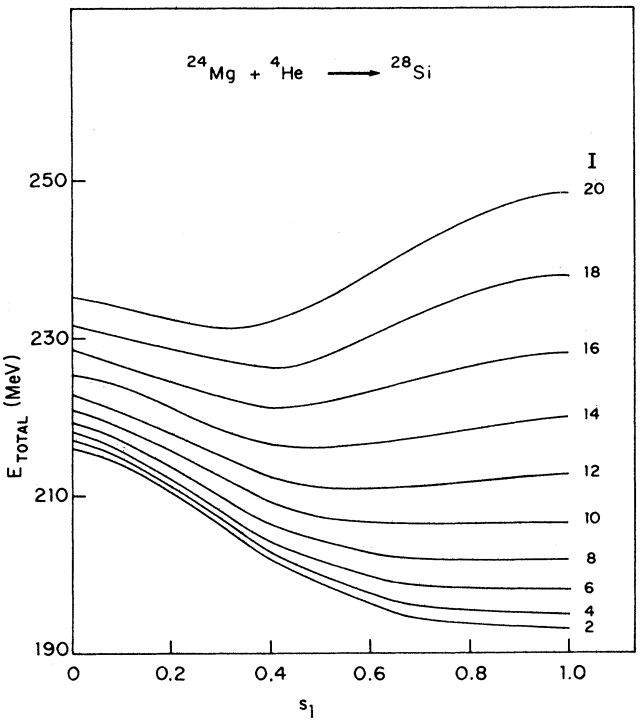

FIG. 8. Same as Fig.7 for the case of $^{24}Mg+{}^{4}He\rightarrow{}^{28}Si$.

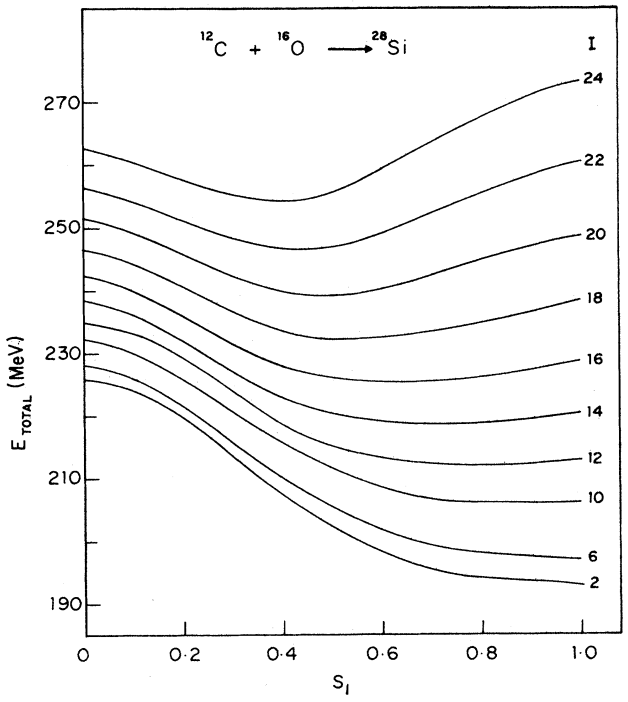

FIG. 9. Same as Fig.7 for the case of $^{16}O+{}^{12}C\rightarrow{}^{28}Si$.

$^{12}C+{}^{12}C\rightarrow{}^{24}Mg$,

$^{16}O+{}^{16}O\rightarrow{}^{32}S$,

$^{28}Si+{}^{28}Si\rightarrow{}^{56}Ni$,

$^{24}Mg+{}^{24}Mg\rightarrow{}^{48}Cr$,

$^{40}Ca+{}^{40}Ca\rightarrow{}^{80}Zr$.

The variation of total energy E_{total} with the reduced neck diameter s for various spins I in the above cases are shown in Figs. 2–6. From these figures, it is seen that ^{24}Mg is spherical ($s=1$) for $I=0-6$, deformed to a one-center ellipsoidal shape with $s=0.8$ for $I=8-12$, and acquires a dumbbell shape at $I=14-20$ ($s=0.6$) thereby indicating the possibility of molecular configurations at these spins. This does not happen up to spin $I=20$ in the case of ^{32}S, ^{48}Cr, ^{56}Ni, and ^{80}Zr indicating the absence of such molecular configurations up to this spin. In the case of ^{56}Ni molecular configurations have been experimentally found to occur[20] at $I=34-42$ which agrees with our result shown in Fig. 4. These results are thus in good agreement with the experimental findings.

IV. EXTENSION TO ASYMMETRIC SYSTEMS

We have considered the possibility of asymmetrical molecular configurations also in the reactions

$^{14}C+{}^{14}He\rightarrow{}^{18}O$,

$^{24}Mg+{}^{4}He\rightarrow{}^{28}Si$,

$^{16}O+{}^{12}C\rightarrow{}^{28}Si$,

and our results are shown in Figs. 7–9, which indicate the possible formation of ^{14}C-α- and ^{2}Mg-α-type molecules around a spin of $I=8$ and $I=12$ in the first two cases, respectively, thus generally agreeing with the finding of Suzuki *et al.* and Karekatte *et al.*[16] In the case of $^{16}O+{}^{12}C$, the molecular formation occurs at a spin of $I\geq 14$ similar to the case of $^{12}C+{}^{12}C$.

Thus it is seen that the Royer and Remaud type of two-center parametrization is highly suitable for studying molecular configurations in heavy ion collisions. The advantage of using this model is that the resulting equilibrium configuration can be readily tested for the presence of a neck in terms of the value of the parameter s, or s_1 as the case may be, thereby indicating the formation of nuclear molecules.

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