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Comment on the $^1S_0$ Nucleon-Nucleon Effective-Range Expansion Parameters*

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The most accurate nucleon-nucleon scattering experiments below 10 MeV are now consistent with the following values and uncertainties for the $^1S_0$ scattering lengths and effective ranges:

$$a^c_{pp} = -7.823 \pm 0.01 \text{ F}, \qquad r^c_{pp} = 2.794 \pm 0.015 \text{ F};$$

$$a_{nn} = -17 \pm 1 \text{ F}, \qquad r_{nn} = 2.84 \pm 0.03 \text{ F};$$

$$a_{np} = -23.715 \pm 0.015 \text{ F}, \qquad r_{np} = 2.73 \pm 0.03 \text{ F}.$$

Unless strong theoretical arguments can be advanced that *all* charge-dependent corrections to the parameters can be calculated accurately enough to prove that the values a_{nn}, r_{nn}, or r_{np} must be moved outside these limits, or strong experimental reasons given to believe that experiments leading to these results were significantly in error, it is argued that new calculations or experiments aimed at changing these values are likely to fail. This argument is supported by an analysis of the latest p-p experiments of Jarmie, Jett, Detch, and Hutson at 9.918 MeV, which reconfirm the one-pion-exchange shape effect and give $G^2_{\pi^0 p} = 15.3 \pm 2.4$ for the pion-nucleon coupling constant.

The corrections to the shape-independent effective-range approximation for nucleon-nucleon scattering in the $^1S_0$ state due to one-pion exchange (OPE) can be unambiguously predicted.[1-3] This prediction was confirmed using p-p scattering data below about 3 MeV in 1964.[4] If this shape effect is accepted, it removes a large systematic ambiguity in the projection of the effective-range plot to zero energy to determine a_{pp}, and the data then allow the very accurate determination of two parameters referring to the short-range nucleon-nucleon interaction in any model which has OPE as the longest-range component. The simplest interpretation of the charge-independence hypothesis is that this same nuclear model should predict both n-n and n-p scattering in the $^1S_0$ state, once the e^2/r Coulomb interaction is removed. This prediction fails, since it gives about −17 F for the nuclear scattering length, while it has been known since the early 1950's that the n-p scattering length is close to −23.7 F. Electromagnetic corrections corresponding to the extended charge and magnetic moment distributions of the nucleons measured by electron scattering change this prediction by less than a Fermi. In contrast to the highly sensitive parameters a_{nn} and a_{np}, the effective ranges are insensitive to small corrections; the p-p data below 3 MeV plus the simplest version of the charge-independence hypothesis requires[5] both r_{nn} and r_{np} to be about 2.84 F. If the $\pi^\pm$-π^0 mass difference in included in OPE, *and* some parameter (e.g., a charge-dependent splitting of the pion-nucleon coupling constants or a phenomenological parameter of the short-range interaction) is adjusted to fit the observed value of a_{np}, then the prediction for r_{np} falls to about 2.73 F with an estimated uncertainty[5] of only about 0.03 F.

The prediction of $r_{np} = 2.73 \pm 0.03$ F used to be in conflict with n-p total cross sections at 0.4926 and 3.205 MeV measured by Engelke, Benenson, Melkonian, and Lebowitz (EBML),[6] assuming all other experiments in the analysis correct. The other data[7] used determine a_{np}^t, ϵ_d, and a_{np}^s, and combined with EBML gave[5] $r_{np} = 2.44 \pm 0.11$ F. Remeasurement of the total neutron-hydrogen cross section for epithermal neutrons did not change this situation according to Houk and Wilson.[8] However, the discovery of an error in the evaluation of their experiment,[9] and a new measurement of the coherent neutron-hydrogen scattering length by Koester,[10] change a_{np}^t and a_{np}^s sufficiently to raise the values of r_{np} calculated[11] from the Columbia experiments to 2.646 ± 0.072 F. The situation has been still further improved by Davis and Barschall,[12] who have shown that the energy scale to which many of the neutron measurements have been referred is in error. Their revision does not change the (new) value of 2.66 ± 0.09 F obtained from the Columbia experiment at 0.4926 MeV, but their revision of the energy of 3.205 MeV down to 3.186 MeV raises the value of r_{np} calculated from that experiment to 2.77 ± 0.14 F. The case for an r_{np} close to 2.73 F is still further strengthened by the preliminary results of a new n-p total cross section measurement at 0.525 MeV by Simmons, Cramer, and Cranberg.[13] Thus there currently remains no significant discrepancy between this prediction from charge-independence and experiment. An quantitative theoretical explanation for the discrepancy between the predicted value of -17 F and the observed value of -23.7 F for a_{np} remains as elusive as ever.

Much less is known quantitatively about the n-n parameters. While some of the relevant experiments would probably appear anomalous if the n-n effective range were less than 2 or more than 4 F, none of them, even potentially, are within an order of magnitude of the accuracy needed to check the prediction for r_{nn}. Several attempts to measure a_{nn} have been pushed to the level of *statistical* uncertainty of one or two fermis, and while some measurements are consistent with -17 ± 1 F, some values fall more than a standard deviation away. However, in the opinion of this author, in no case is the three-particle theory used for the evaluation of the final state sufficiently under control to make any of these discrepancies troublesome.[13a] Until this theoretical situation is improved, it is probably best to consider these experiments as tests of certain approximations in final-state interaction theories using $a_{nn} = -17$ F as a calibration, rather than as "measurements" of a_{nn}.

Until recently, the situation with regard to p-p scattering has been more confusing. It proved possible to validate[14] the assumptions needed about the P waves in the previous analysis[4] of the data below 3 MeV by combining the value of A_{yy}/A_{xx} at 11.4 MeV measured by Catillon, Chapellier, and Garreta[15] with the accurate differential cross section measurement obtained by Johnston and Young[16] at 9.69 MeV; even the use of this spin-dependent information leaves the analysis ambiguous if $\vec{L}\cdot\vec{S}$ effects are not assumed to be small. There is strong theoretical reason to believe that the $\vec{L}\cdot\vec{S}$ interaction is of such short range that this must be so, but direct proof by spin-dependent experiments below 10 MeV appears hopeless with current techniques.[14] Slobodrian[17,18] distrusted the large amount of theoretical input needed to extract a_{pp} and r_{pp} from the data below 3 MeV, and also had reason to suspect that the method used to separate elastic scattering events from background in the 9.69-MeV experiment might be a source of systematic error.[18] His group therefore undertook accurate differential cross section measurements at 6.141, 8.097, and 9.918 MeV. Unfortunately, these experiments[19] failed to yield convincing results, as was demonstrated by MacGregor, Arndt, and Wright,[20] this author,[21] and on somewhat different grounds by Sher, Signell, and Heller (SSH).[22] Comparison with the corrected predictions from the 1964 analysis shown in Fig. 1 makes it clear that either these experiments, or the entire theory of the OPE shape correction, is wrong.[23] This figure also makes clear the fact that the main problem with p-p experiments is systematic rather than statistical error, and that large variations in the p-p parameters can be achieved by injudicious data selection and analysis.

Experiments at 9.69 and 9.918 MeV undertaken at Los Alamos by Jarmie, Brown, Hutson, and Detch[24] with the specific objective of providing an alternative to the Berkeley data produced the theoretically expected shape for the differential cross section as a function of angle, and hence the expected value for the central-force P-wave parameter Δ_c, but again failed to obtain a believable value for the absolute value of the cross section, and hence for δ_0^E: This situation was pointed out at the time of publication in an accompanying letter by Holdeman, Signell, and Sher.[25] This new discrepancy has finally been resolved by the discovery[26] that there had been an error in the measurement of one of the slit widths. Reevaluation[26] of the experiments at 9.69 and 9.918 MeV and a new experiment[26] at 13.6 MeV by Jarmie, Jett, Detch,

TABLE I. Accurate proton-proton electric $^1S_0$ phase shifts below 10 MeV, and the corresponding vacuum polarization phases and Foldy corrections; the phase shift used in the conventional effective-range expansion is obtained by $\delta_0^c = \delta_0^E + \tau_0 - \Delta_0$. The Foldy correction Δ_0 was computed by H. M. Lipinski for (a) the Hamada-Johnston potential, (b) a sum of three Yukawa potentials: OPE, intermediate-range attraction with mass and coupling constant adjusted to fit a and r, and short-range repulsion with the ω-meson mass and coupling constant adjusted to make δ_0 go negative at about 250 MeV, and (c) OPE plus a purely attractive Bargmann potential: $m_p V(r)/\hbar^2 = -2\beta^2(\beta^2-\alpha^2)/(\beta \cosh\beta r + \alpha \sinh \beta r)^2$. Model (c) gives a correction close to that for a single attractive Yukawa potential as quoted in Ref. 30. References are given in the caption to Fig. 1.

Lab energy (MeV)	δ_0^E (deg)	Reference Data	Analysis	$-\tau_0$ (deg)	$-\Delta_0$ (a)	(b)	(c)
0.38243	14.6110 ± 0.0115	BSB	GH	0.1332	0.1925	0.1926	0.1980
1.397	39.3213 ± 0.028	KDM	(NH)	0.0853	0.1935	0.1935	0.2056
1.855	44.3292 ± 0.023	KDM	(NH)	0.0802	0.1745	0.1745	0.1870
2.425	48.3553 ± 0.026	KDM	(NH)	0.0753	0.1544	0.1544	0.1667
3.037	51.0233 ± 0.040	KDM	(NH)	0.0713	0.1369	0.1368	0.1488
9.918	55.23 ± 0.13	JJDH	SH	0.0516	0.0683	0.0680	0.0774

and Huston give results in accord with theory; the two accurate phase shifts are plotted in Fig. 1. There is now every reason to believe that the value of 55.23 ± 0.13° for δ_0^E obtained by Signell and Holdeman[27] from the 9.918-MeV experiment plus five previously published values from experiments at 3 MeV and below are the most accurate $^1S_0$ phase shifts available below 10 MeV; these values are collected in Table I. A preliminary analysis by the author[28] of the newly evaluated data from Los-Alamos at 9.918 MeV gives a value of δ_0^E agreeing with that quoted to 0.01°, but with a somewhat larger error. Further checks will be made, but it appears unlikely that these values could be changed by as much as a standard deviation without invoking some bizzare assumptions.

The phase shifts in Table I still contain the long-range vacuum polarization effect. Model independent values of a_{pp}^E and r_{pp}^E can be extracted from them by using the modified effective-range expansion derived by Heller,[29] but we believe it more instructive to apply the Foldy[30] correction in order to obtain the phase shifts which would be produced by the nuclear and nonrelativistic electromagnetic interactions in the absence of vacuum polarization. These corrections have been computed for three models by H. M. Lipinski and are given in Table I. Since the effective-range expansion about $k^2 = 0$ diverges beyond 9.71 MeV due to the OPE branch cut, it would be a serious error to represent the shape correction phenomenologically by the terms $-Pr^3k^4 + Qr^5k^6$. Instead we use the approximation of Cini, Fubini, and Stanghellini (CFS)[2] for the OPE cut, with Wong-Noyes Coulomb correction,[31] which extends the radius of convergence to 38.8 MeV; explicitly:

$$\mathcal{C}^2 k \operatorname{ctn}\delta_0^c + 2k\eta h(\eta) = -1/a_{pp}^c + \tfrac{1}{2} r_{pp}^c k^2 - \frac{pk^4}{1+qk^2},$$

$$\eta = \frac{\alpha(m_p c^2 + T)}{[T(2m_p c^2 + T)]^{1/2}}, \quad k^2 = \frac{m_p c^2 T}{\hbar^2 c^2}, \quad \mathcal{C}^2 = \frac{2\pi n}{e^{2\pi n} - 1}, \tag{1}$$

$$h(\eta) = \sum_{s=1}^{\infty} \frac{\eta^2}{s(s^2+\eta^2)} - \gamma - \ln\eta,$$

where $\hbar c = 197.327\,891$ MeV F, $\alpha = 1/137.036\,02$, $m_p c^2 = 938.2592$ MeV, and T is the lab energy in MeV. The parameters p and q are not phenomenological constants, but are computed from the pion-nucleon coupling constant and pion mass, with Coulomb corrections,[31] according to the formulas

$$q = \frac{2 - \Gamma[\sqrt{2}\,(2f - \frac{1}{2}g) - 4/\mu a_{pp}^c + \mu r_{pp}^c]}{\mu^2\{1 - \Gamma[\frac{1}{2}\sqrt{2}\,(f - \frac{1}{2}g) - 1/\mu a_{pp}^c]\}},$$

$$p = (1 - \tfrac{1}{2} q\mu^2)(2\sqrt{2} f - 4/\mu a_{pp}^c - \mu r_{pp}^c),$$

$$\mu = \frac{m_{\pi^0} c^2}{\hbar c}, \quad m_{\pi^0} c^2 = 134.975 \text{ MeV}, \quad y = \frac{\alpha m_p}{m_{\pi^0}\sqrt{2}},$$

$$f = 2y\left(\frac{\pi}{2}\operatorname{ctn}\pi y + \gamma + y^2 \sum_s \frac{1}{s(s^2-y^2)} + \ln y\right),$$

$$g = \frac{m_{\pi^0}}{m_p}\,\alpha\left(\frac{\pi^2}{\sqrt{2}}\csc^2\pi y - \sqrt{2} - 2\sqrt{2}\sum_s \frac{1}{(s^2-y^2)^2}\right),$$

$$\Gamma = \frac{m_{\pi^0}}{4m_p} G^2(\sqrt{2}-1)^{-2y}. \tag{2}$$

TABLE II. Analysis of the six accurate p-p phase shifts below 10 MeV given in Table I, using the Coulomb-corrected Cini, Fubini, and Stanghellini shape dependence for fixed values of G^2; the Foldy correction used corresponds to a hard or stiff (Yukawa) repulsive core potential.

G^2	a_{pp}^c (F)	r_{pp}^c (F)	χ^2	p (F^3)	q (F^2)
10.0	−7.8192	2.7772	5.1609	0.4377	3.7339
12.0	−7.8216	2.7862	3.0732	0.5400	3.6037
14.0	−7.8243	2.7958	1.7932	0.6481	3.4642
16.0	−7.8272	2.8062	1.5863	0.7626	3.3146
18.0	−7.8304	2.8175	2.7952	0.8838	3.1538
20.0	−7.8337	2.8296	5.8800	1.0124	2.9803

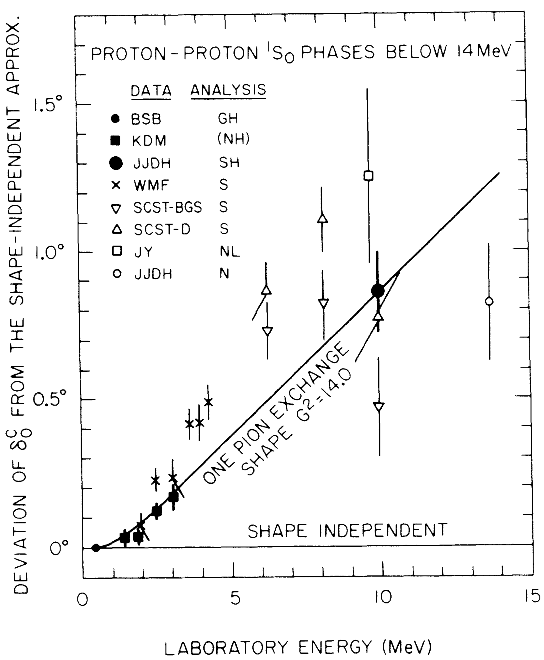

FIG. 1. Comparison of the predictions of Eq. (1) fitted to the data of KDM and BSB as reported in Ref. 23 ($a_{pp}^c = -7.8275$, $r_{pp}^c = 2.7937$, $p = 0.64788$, $q = 3.4619$, $G^2 = 14$) with the shape-independent result ($p = 0$) subtracted. References noted in the figure are:

●,	BSB:	J. E. Brolley, J. P. Seagrave, and J. G. Beery, Phys. Rev. 135, B1119 (1964).
	GH:	M. Gursky and L. Heller, Phys. Rev. 136, B1693 (1964).
■,	KDM:	D. J. Knecht, P. F. Dahl, and S. Messelt, Phys. Rev. 148, 1031 (1966).
	(NH):	The OPE corrections for the higher partial waves used by KDM in the analysis of their experiment were supplied by H. P. Noyes, and the vacuum polarization correction was computed by him from the formulas of GH.
●,	JJDH:	N. Jarmie, J. L. Jett, J. L. Detch, Jr., and R. L. Hutson, Phys. Rev. Letters 25, 34 (1970).
	SH:	P. Signell and J. Holdeman, as quoted in JJDH.
×,	WMF:	H. R. Worthington, J. M. McGruer, and D. E. Findley, Phys. Rev. 90, 899 (1953).
	S:	R. J. Slobodrian, Phys. Rev. Letters 21, 438 (1968).
▽,	SCST-BGS:	R. J. Slobodrian, H. E. Conzett, E. Schield, and W. F. Tivol, Phys. Rev. 174, 1122 (1968); BGS – background subtracted data,
△,	SCST-D:	D – discriminator data; clearly the two interpretations of the data are mutually incompatible, and at most one set should be used.
□,	JY:	L. H. Johnston and D. E. Young, Phys. Rev. 116, 989 (1959).
	NL:	H. P. Noyes and H. M. Lipinski, Phys. Rev. 162, 884 (1967).
○,	N:	Preliminary analysis by this author.

A more refined treatment using solutions of the Schrödinger equation for specific nuclear models including the long-range OPE interaction such as that carried out by Sher, Signell, and Heller[22] might appear desirable, but we doubt[32] that it would produce values of a_{pp}^c and r_{pp}^c in meaningful disagreement with the values quoted in the abstract. The result of this analysis using either the hard-core Hamada-Johnston potential or the stiff-core Yukawa potential with the ω-meson mass for the Foldy correction (a or b in Table I) is given in Table II. If instead we took the extreme view that the nuclear interaction is purely attractive at low energy and that the repulsion seen in the $^1S_0$ state above 250 MeV is due to a velocity-dependent effect (Foldy correction c in Table I), the scattering length decreases by only 0.0024 F and the effective range increases by only 0.0006 F; we believe this represents an upper limit for the model dependence of the Foldy correction.

If we assume that the pion-nucleon coupling constant is determined by other experiments, the results of this analysis can be summarized by

$$a_{pp}^c = -7.8243 \pm 0.0054 - 0.0014(G^2 - 14.0) \pm 0.0024 \text{ F},$$
$$r_{pp}^c = 2.7958 \pm 0.0080 + 0.0056(G^2 - 14.0) \pm 0.0006 \text{ F}, \qquad (3)$$

where the first error is statistical and the last is the upper limit for the model dependence due to the Foldy correction. If we use the same OPE shape correction in the Heller expansion,[29] we find $a_{pp}^E = -7.8146$, $r_{pp}^E = 2.7950$. It is hard to think of applications of these numbers, outside of the prediction of values of δ_0^E for direct comparison with experimental data, for which this much precision in the *nuclear* parameters a and r is of any use. If, as originally proposed by Cini, Fubini, and Stanghellini,[2] we use these experiments to measure the pion-nucleon coupling constant, Table II yields the value $G_{\pi^0 p}^2 = 15.29 \pm 2.38$. A decade of work has finally allowed the determination of this constant from p-p scattering data using the OPE singularity in the k^2 complex plane (in S waves) to an accuracy comparable with that obtained earlier from the corresponding singularity in $\cos\theta$ (in high partial waves).

The values of a_{pp}^c and r_{pp}^c obtained still agree with the values previously obtained from the data below 3 MeV[23] (-7.8275 ± 0.0049, 2.7937 ± 0.0065) and even with the preliminary values obtained in 1964[4] (-7.8259 ± 0.0048, 2.786 ± 0.014). Therefore the conclusions reached in 1965[5] as to the values of the n-n and n-p parameters required by charge independence still hold. The effects of the charge and current distributions can be included by fitting a nuclear model to the values of a and r derived from the data,[33] which will change the interaction parameters in the nuclear model compared to those which would be obtained under the assumption of point charges. Schneider and Thaler (ST)[34] found that these effects change the prediction for a_{nn} by -0.02 F; using charge and current distribution derived from more recent electron scattering results, including a portion of the electromagnetic interaction omitted by ST, and a different nuclear model, SSH[22] find the prediction for a_{nn} changed by $+0.31$ F compared to that for point charges. The effect on r_{nn} is only a few percent of the statistical uncertainty in r_{pp}, so can be ignored. Clearly the 6.7 F discrepancy between the predicted and observed values of a_{np} cannot be explained in this way. These calculations assume that the charge-current distribution follows the matter distribution given by the nonrelativistic wave function, whereas we know[5,35] that in the case of the reaction $n+p\rightarrow\gamma+d$ the meson exchange currents which are ignored in such a calculation contribute 10% of the observed cross section at threshold. The calculation becomes even more ambiguous if the strong interaction itself is highly nonlocal, which may very well be the case.[36] Thus, while we believe that it is still worth while to show that these effects are indeed small for conventional models, the quantitative significance of such calculations is highly uncertain. A much more detailed discussion of these problems which reaches much the same conclusion has been given by Breit *et al.*[37]

The obvious conclusion to be drawn from this survey of a decade of work on the determination of the effective-range expansion parameters is that further calculations or experiments are unlikely to increase our knowledge of them without some dramatic change in *both* the theoretical and the experimental situation. We have omitted the new 9.69-MeV data because it does not cover a wide enough angular range to be useful for a single-energy analysis, and the 13.6-MeV data because it might be beyond the range of quantitative reliability of the CFS formula; a repeat of the detailed analysis of all data below 30 MeV like that given by SSH[22] might be useful, but we doubt it will change the final results for a and r outside our errors. A purist might still like to see a highly accurate measurement of the absolute value of $\sigma_{pp}(90°)$ as a function of energy between 3 and 10 MeV in order to exhibit the shape effect in more detail. This approach would avoid the necessity of measuring Δ_c (which does not contribute at 90°), but still would necessarily require that the tensor force parameter for the P waves Δ_T be taken from theory; the prospect that any spin-dependent experiment could determine this tensor parameter to anything like the accuracy to which it is already known theoretically is extremely dim.[14] Such a 90° cross section experiment might well produce marginal discrepancies with Eq. (2), which in turn would give marginally useful restrictions on the shape of the two-nucleon interaction, but until the theory of strong interactions is under more control, this hardly seems worth the effort.

It remains to ask whether new p-p experiments below 10 MeV might give useful information not included in the $^1S_0$ parameters we have been discussing up to now. Existing experiments *do* determine very precise values for the P-wave central force combination of phase shifts Δ_c thanks to interference with the triplet Coulomb amplitude. SSH have already shown that this information can be put together with the experiments around 25 MeV to determine ^3P wave scattering lengths and effective ranges, but the uncertain experimental situation made these six parameters subject to considerable systematic uncertainty. This could now presumably be reduced, and could be reduced still further by constructing the analog of the CFS formula for the P waves. Since putting $1/a$ equal to zero in the usual N/D approach gives the correct threshold behavior for P waves, this should reduce the number of free parameters from six to three, but extensive checks against model calculations, similar to those already made by SSH to justify their particular effective-range formulas, would be needed before one could trust such three-parameter formulas. Only if it could be shown that additional values of Δ_c would improve the determination of these P-wave parameters would additional p-p differential cross section measurements as a function of angle be worth considering. Even if that were the case, the author of this comment would like to see some specific nuclear or elementary particle problem where this increased precision in the triplet-odd P waves is needed before he could encourage such experiments.

If no new nuclear information is likely to be forthcoming from p-p experiments below 10 MeV, the only other reason of which the author is aware for undertaking them is as a test of vacuum polarization; a specific proposal for new experiments near the Coulomb-nuclear interference minimum has been proposed for this purpose by Brolley.[38] A recent review of the tests of quantum electrodynamics at both low and high energy[39] shows complete agreement between theory and experiment to fantastically high accuracy, so it would be surprising indeed if this test failed. Expected modifications of the photon propagator due to muon and hadron pairs have, by the usual uncertainty principle argument, the same range as nuclear effects, so could not be disentangled without a complete theory of the strong interactions. Expected modifications from a convergence factor, if indefinite metric theories of finite quantum electrodynamics are followed, are necessarily of opposite sign, so would tend to cancel even these modifications, and in any case are of still shorter range. It is still true that vacuum polarization in p-p scattering is about the only place that the vacuum polarization correction to the photon propagator can be tested in a system containing only hadrons as physical particles,[40] and an attempt was made to check this test some years ago using the Wisconsin data.[41] The result is that the apparent strength of the vacuum polarization correction when treated as a free parameter deviates significantly from the theoretical prediction. Since the effect varies systematically with energy, it is probably an indication of an unknown systematic error in the Wisconsin data, and very unlikely to be evidence for a breakdown of quantum electrodynamics. It scarcely seems worth while to spend another ten years on those, or new, differential cross section measurements chasing that will-of-the-wisp. The experiment proposed by Brolley[38] would provide a more sensitive test, but also is many orders of magnitude too gross to be *expected* to show up a discrepancy with theory.

The earlier discussion of n-p experiments should make it clear that there may well still be systematic errors lurking in the low-energy neutron-proton scattering data. Continued attention to that problem is obviously desirable, but it seems unlikely that systematic error in these experiments can be driven down to the level where a test of the prediction for r_{np} to an accuracy of ±0.03 F would become believable. Any novel ideas here would be welcome, since three-nucleon calculations are quite sensitive to the value used for this parameter[42]; the same applies of course to r_{nn}, but experimental precision there looks close to impossible.

We conclude that the era when new information about nuclear force parameters could be derived directly from nucleon-nucleon scattering experiments below 10 MeV is drawing to a close, and that in the absence of novel experimental or theoretical ideas, the strenuous efforts which would be needed to improve the precision of the knowledge already obtained might better be directed to other objectives.

Extensive use has been made in this paper of unpublished calculations of the Foldy correction and charge-current distribution corrections using various potential models made by H. M. Lipinski in an earlier attempt to analyze the low-energy p-p data; this attempt was frustrated by the systematic errors discussed above. Extensive help with the more recent work was supplied by J. Post and W. Ross; a check on the assumption that charge-current distribution corrections affect only the nuclear parameters and not the values of a and r derived from the data was made by E. Zeiger and V. N. Athavale. The author is indebted to N. Jarmie for informing him of the essential result of Jarmie *et al.*[26] in a private discussion; he is sorry that this acknowledgment must[43] now take so impersonal a form.

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[13a]See *The Three-Body Problem in Nuclear and Particle Physics*, edited by J. S. C. McKee and P. M. Rolph (North-Holland Publishing Company, Amsterdam, The Netherlands, 1970) for discussion of this problem by several participants, and for references to the literature. While progress has been made by calibration of final state data on the n-n system to mirror states involving the p-p system, we have no guarantee that the nonrelativistic wave functions used in these theories have a unique significance inside the range of nuclear forces. It is an empirical question whether or not the two-body wave function probed by a third strongly interacting is independent of the energy or nature of the probe. This empirical question could be solved by a sufficiently de-

tailed analysis of three-body breakup experiments [cf. H. P. Noyes, Phys. Rev. Letters 25, 321 (1970)], but lacking such analysis, this author believes we should be cautious. Most careful analyses *do* support the n-n parameters recommended here.

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[32]Extensive unpublished numerical investigation of the effect of using this formula rather than the Hamada-Johnston potential for the energy variation below 3 MeV conducted by Noyes, Osborn, and Lipinski show that it might distort the results by close to one standard deviation in that energy region using available data. Now that the lever arm in the $k\,\mathrm{ctn}\delta$ plot has been extended by more than a factor of 3, thanks to the results near 10 MeV, this sensitivity disappears. The purist will probably still prefer to fit his nuclear model directly to the phase shifts given in Table I rather than to a_{pp}^c and r_{pp}^c derived in this way, but I have yet to discover any *experimental* way to invalidate the simpler treatment.

[33]This assumes that the charge-current distributions only change the interaction inside the range of nuclear forces, which is *a priori* probable because they are believed to arise from meson currents. There is a small effect due to the long-range part of the nonnuclear $1/r^3$ static magnetic dipole interaction, but this is detectable, if at all, only in the higher partial waves [cf. G. Breit and M. M. Ruppel, Phys. Rev. 127, 2123 (1967); and also Ref. 22]. That the effect of these corrections is approximately linear in energy over this energy range, and therefore only affects the interpretation of a and r as derived from the data rather than their numerical values, has been shown for the Hamada-Johnston potential by H. M. Lipinski, by the authors of Ref. 22 for a quite different nuclear model, and by E. Zeiger and V. N. Athavale for two uniformly charged spheres of appropriate radius; cf. also numerical tables in Nuovo Cimento 40B, 443 (1965) (Ref. 17).

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Redetermination of the Singlet Effective-Range Parameters of the Mongan Separable Potentials*

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The singlet effective-range parameters for the Mongan separable potentials are shown to be more than trivially in error. The implications of this result on triton calculations using these potentials are discussed.

The most extensive separable-potential fits to the two-nucleon data have been done by Mongan[1,2]; for convenience, we will refer to these as his "old" and "new" fits, respectively. The singlet effective-range parameters for these potentials are more than trivially in error; the discrepancies between the actual and reported values of these parameters have led other investigators to unearth spurious off-shell effects in three-nucleon calculations. In Table I, our determinations of the singlet effective-range parameters are compared with Mongan's for each of his potentials. The largest disparities are in the singlet effective ranges; these disparities can, in part, be traced to Mongan's use of the formula

$$r=(2/k_v)(1-1/ak_v)$$

for the calculation of the effective range r in terms of the scattering length a and the singlet anti-bound-state wave number k_v. This formula is quite unstable with regard to small errors in either a or k_v because of the numerical coincidence that $1/ak_v$ is close to unity (for the values $a=-23.678$ F and $k_v=-0.040\,045$ F^{-1}, $1/ak_v$ is 1.054 65) and because $2/k_v$ is quite large in magnitude (approximately -50). A simple analysis shows that if a fractional error α be made in a and β in k_v, then the magnitude of the fractional error in r is $(\alpha+\beta)/(ak_v-1)\approx 20(\alpha+\beta)$.

Differences in the effective ranges of the order of several tenths of a Fermi for singlet-S potentials indicate significant on-energy-shell variations among the potentials. With simple model calculations using attractive Yamaguchi-type potentials, Kharchenko, Petrov, and Storozhenko[3] have shown that the three-nucleon system reflects these variations quite strongly; for example, their results indicate that an increase of 0.4 F in the singlet effective range decreases the triton binding energy by roughly an MeV and that changing the scattering length from -23.7 to -17 F decreases the binding energy by about 0.3 MeV.

We have calculated the trinucleon binding energies for Mongan's singlet-S potentials (1) as the only nucleon-nucleon interaction and (2) taken together with his uncoupled case II (new) triplet-S potential; the results of these calculations are summarized in Table II. The singlet-S effective ranges are shown in the last column. We note that potentials that have similar effective ranges have nearly the same triton binding energies, rather independently of any functional differences of the separable-potential form factors.

In a paper on separable expansions of local potentials, Harms[4] compares the binding energy of model trinucleons in which the two-nucleon interaction exists only in the singlet-S state: With the Reid[5] soft-core singlet-S potential ($a=-17.1$ F, $r=2.8$ F), he obtains 1.02-MeV binding; with Mongan's type II (new) singlet-S potential ($a=-23.86$ F, $r=2.32$ F), he obtains 2.06-MeV binding. Harms