Phys. Rev. 133, B329 (1964).
[11]This convention is the same as Ref. 9, p. 430.
[12]M. L. Goldberger and K. M. Watson, *Collision Theory* (John Wiley & Sons, Inc., New York, 1964), p. 188.
[13]E. J. Konopinski, *The Theory of Beta Radioactivity* (Clarendon Press, Oxford, England, 1966).
[14]R. J. Blin-Stoyle, Phys. Rev. 120, 181 (1960).
[15]S. Wahlborn, Phys. Rev. 138, B530 (1964).
[16]Equation (15) is a special case of the more general result (26) of Sec. IV. This simple "branching-ratio" form results from the fact that all degrees of freedom associated with the β decay are summed, so that the β-decay width is independent of the magnetic quantum number M_B.
[17]Actually the efficiency of the observational apparatus is dependent on k. This efficiency will be folded in later. See Eq. (38).
[18]The proof here appears to depend excessively on the phase convention of this paper. This is not the case of course, since $F_{nn'}$ is bilinear in all states and their Hermitian conjugates.
[19]For estimates of β amplitudes, see Ref. 13, p. 201.
[20]See, for instance, Ref. 9, pp. 382 and 389.
[21]E. Feenberg and G. Trigg, Rev. Mod. Phys. 22, 399 (1950).
[22]This is developed very well in Ref. 9, Chaps. 6 and 7.
[23]H. Schopper, Nucl. Instr. Methods 3, 158 (1958).
[24]The estimate of Ref. 10 was $\lambda = 0.8$.

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Beta-Gamma Correlations and Matrix Elements for Some First-Forbidden Nonunique Beta Transitions

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Results of β-γ directional-correlation measurements in the nonunique first-forbidden β decay of the ground states of several odd-odd nuclei (Sb^{124}, Eu^{152}, Re^{186}, Tm^{170}, Rb^{84}) to the first 2^+ levels in the even-even daughter nuclei are presented. These measurements were made as a function of the β-ray energy using a small shaped-field magnetic spectrometer. The results, in combination with all other available experimental data on each β transition, have been analyzed to obtain the nuclear matrix element parameters. The analysis is based on the theoretical expressions given by Morita and Morita using exact electron radial wave functions which include finite nuclear size corrections. In addition, we have analyzed all available experimental data for the nonunique first-forbidden β decay of Re^{188} and Rb^{86}. A unique set of parameters was found for Re^{186}, Re^{188}, and Tm^{170} although the set for Tm^{170} suffers from a lack of adequate experimental information. For Rb^{86}, the matrix element sets depend on whether the shape of the β spectrum is statistical or not. For Rb^{84}, five distinct sets of parameters were found which fit all available data equally well. For both Sb^{124} and Eu^{152}, two sets of parameters were found that fit the data equally well; however, it is shown that a measurement of the β-circularly-polarized γ correlation as a function of energy can eliminate one set in each case. Appropriate discussions based on nuclear models are given.

I. INTRODUCTION

There are several examples of first-forbidden β decays which are anomalous in the sense that they are characterized by abnormally large *ft* values, deviations from the normal energy dependence of spectrum shapes, and anisotropic angular correlations. Such characteristics may be attributed to nuclear-structure effects which enter into the theoretical description of the β decay through the nuclear matrix elements. In a previous report[1] (hereafter referred to as I), a technique to extract numerical values for the nuclear matrix elements from an analysis of β-decay observables was presented. Such a study can serve as a useful tool in attempting to understand the structure of the nuclear states involved. As early as 1963 Matumoto *et al.*[2] studied the structure of low-lying 2^+ states in even-even nuclei via an analysis of the measured β-decay observables.

In this paper, we report a measurement of the β-γ directional correlations in the nonunique first-forbidden β decay of Sb^{124}, Eu^{152}, Tm^{170}, Re^{186}, and Rb^{84}. The details of the β-γ cascades for each case are given in Sec. IV. These data, in combination with all other available experimental results, were analyzed to obtain numerical estimates for the relevant nuclear matrix elements. In addition, all available data on the decay of Re^{188} and Rb^{86} were analyzed using similar techniques.

In recent years, a considerable amount of effort has been devoted to the experimental determination of nuclear matrix elements. In fact, one or more aspects of the β decay of the nuclei under investigation here have been studied previously. However, with the exception of Sb^{124}, Rb^{84}, and Rb^{86}, all previous analyses have been based on the theoretical expressions of Kotani[3] derived on the basis of the Konopinski-Uhlenbeck[4] approximation. In this approximation, nuclear finite size effects and also terms of order $(\alpha Z)^2$ in the expansion of the electron radial wave function are neglected. It was shown in I that the use of Kotani expressions may lead to incorrect matrix element values. With the use of high-speed computers one can perform the analysis without resorting to restrictive approximations. The electron radial wave functions which include nuclear finite size effects and finite de Broglie wavelength effects can be easily computed when required in the analysis. Tables of these wave functions are also available.[5]

Concurrently with the experimental work, theoretical calculations have been reported for all the nuclei under investigation here. However, in most cases these calculations were compared with the matrix element values obtained through the use of Kotani expressions. In addition, the matrix element values used in the comparison had either large error limits or there existed many different sets of values for a single decay because of a lack of adequate experimental information. In recent years, many new measurements have been made and, in addition, analysis techniques have improved to a point such that a comparison of available theoretical results with matrix elements obtained from new data and techniques is desirable.

To summarize, we have undertaken the present investigation with the following objectives in mind: (a) perform a careful measurement of the relevant β-γ directional correlations; (b) obtain a consistent set of matrix elements for all the nuclei under investigation by analyzing all available experimental data with the techniques described in I; and (c) compare the results of the analysis for each nucleus with available theoretical results.

In Sec. II we discuss the experimental details pertinent to the directional-correlation measurements. A brief review of the general formalism governing first-forbidden β decay and our method of analysis is presented in Sec. III. In Sec. IV the experimental and theoretical results and, finally, Sec. VI contains a brief summary of the results.

II. EXPERIMENTAL PROCEDURE

The β-γ directional-correlation data reported here were obtained by measuring coincidences between the appropriate γ ray detected by a conventional scintillation counter and β rays in a 5% momentum range selected by a 180° shaped-field magnetic spectrometer. A more detailed description of the equipment is found elsewhere.[6,7] An experimental arrangement of this type eliminates the troublesome corrections for γ-γ coincidences which are often present when a scintillation spectrometer is used to select the β energy, but has the disadvantage that solid angle corrections are not straightforward. In order to reduce the uncertainty in this respect, we have mapped the relative transmission across the 3-cm$\times$6-cm central defining aperture of the spectrometer by measuring the contribution of the total counting rate recorded by the β detector from each of the eighteen 1-cm$\times$1-cm areal elements of the defining aperture. The solid angle correction is obtained by numerical integration over this grid. Measurements taken at several β energies indicate that the correction factor is reliable to about 1% from 190 to 2500 keV. Other possible sources of systematic error discussed in Ref. 6 were found to be negligible. Effects of competing β-γ cascades (if any) at lower β-ray energies were evaluated experimentally. The electronic setup used to record the data has been described in detail elsewhere.[7]

The Rb^{84} source was obtained from the Nuclear Science and Engineering Corporation as a carrier-free source in the form of rubidium chloride. All other sources were obtained from the Oak Ridge National Laboratory. A small amount of Eu^{154} (~4%) was present in the Eu^{152} sample used in these measurements. All sources used were of the order of 0.1- to 0.3 mg/cm^2 thick and were mounted in 0.25-mil Mylar backings. The Sb^{124} source was prepared by liquid deposition of the chloride which was then converted *in situ* to the sulphide by exposure to H_2S gas. The Tm^{170} was in the form of $TmCl_3$. The Re^{186} was in the form of $HReO_4$.

III. THEORY AND METHOD OF ANALYSIS

The theoretical expressions for the first-forbidden β decay given by Morita and Morita[8] form the basis of our analysis. We use in the following the notation of Kotani for the matrix element parameters because it has been widely used. The correspondence between the notation used by Kotani and that used by Morita and Morita is given in I. The expressions given by Morita and Morita for the β-decay observables, in terms of the nuclear matrix element parameters specialized to $3^-(\beta)2^+(\gamma)0^+$ and $2^-(\beta)2^+(\gamma)0^+$ transitions are given by Fischbeck and Newsome[9] and in I,[10] respectively. The expressions for the $1^-(\beta)2^+(\gamma)0^+$ transitions are as follows:

(a) shape-correction factor,

$$C(W) = b_1 x^2 + b_2 y'^2 + b_3 u^2 + 2b_4 u y' - 2b_5 x y' + 2b_6 x u + b_7 , \tag{1}$$

(b) β-γ directional-correlation function,

$$\epsilon(W) = [b_8 x^2 - b_9 u^2 + b_{10}(2x - u + 3/\sqrt{6})y' + b_{11} x u + b_{12} x + b_{13} u - \tfrac{3}{2} b_{14}]/C(W) \tag{2}$$

(c) β-circularly-polarized γ correlation,

$$\begin{aligned} P_\gamma(W,\theta) = \cos\theta[& b_{15} x^2 - b_{16} y'^2 - b_{17} u^2 - b_{18} u y' \\ & - b_{19} x y' + b_{20} x u - b_{21} x - b_{22} y' - b_{23} u \\ & + b_{24}(\tfrac{4}{5} x - \tfrac{2}{5} u)(\tfrac{5}{2}\cos^2\theta - \tfrac{3}{2}) + b_{25}] \\ & \times \{C(W)[1 + \epsilon(W) P_2(\cos\theta)]\}^{-1} , \end{aligned} \tag{3}$$

where $y' = \xi' y$. The coefficients $b_i(W)$ contain combinations of the electron radial wave functions and are listed in the Appendix. The quantity $P_2(\cos\theta)$ is the second-order Legendre polynomial.

(d) The γ-ray angular distribution from oriented nuclei is given by

$$w(\theta) = 1 - \tfrac{3}{4} B_2 f_2 P_2(\cos\theta) , \tag{4}$$

where f_2 describes the orientation in the ground state of the parent nucleus. The quantity B_2 depends on the radiation emitted prior to the emission of the γ ray and therefore contains the nuclear matrix elements. The expression for B_2 used in the present analysis is given in the Appendix.

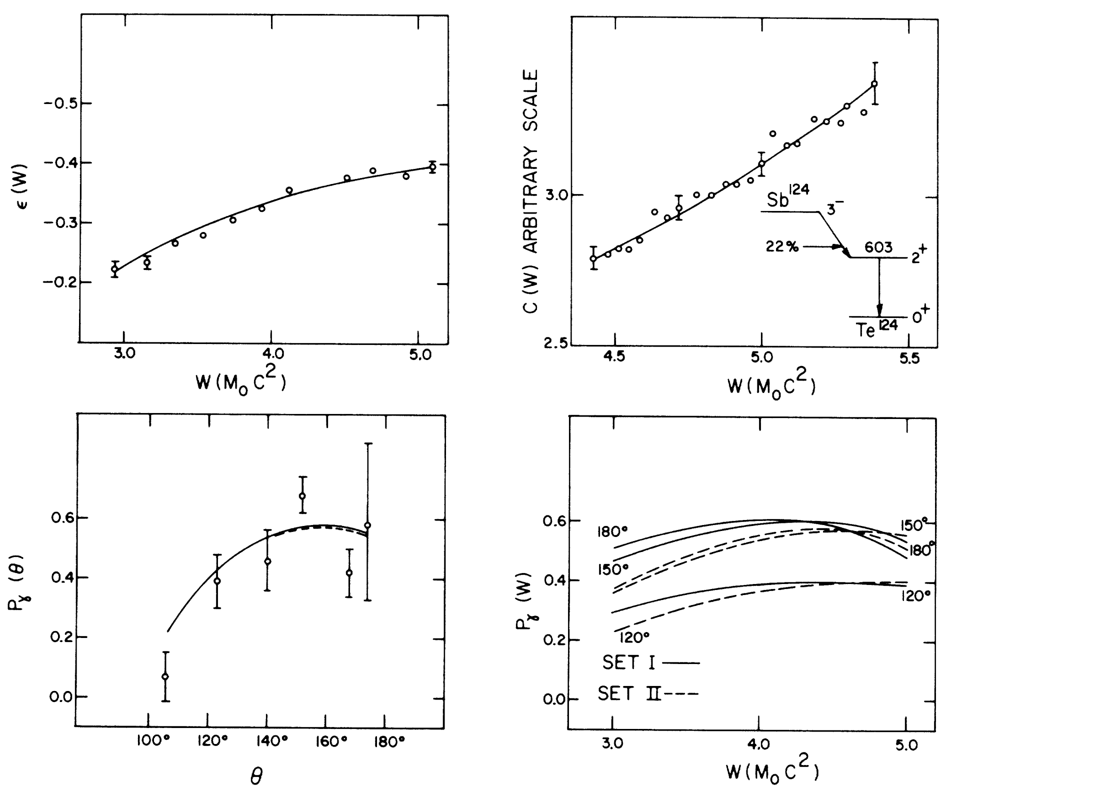


FIG. 1. The Sb^{124} data used in the present analysis. The solid and dashed curves through the data points represent the best-fit curves obtained for solution Sets I and II, respectively. The quantity $P_\gamma(W)$ was calculated using the best-fit solutions.

The method of analysis used here is identical to that described in I. Briefly, the matrix element parameters are determined by fitting all available data in the least-squared sense to the theoretical expressions of Morita and Morita. The program minimizes the weighted sum of the squares

$$\chi^2 = \sum \left[\frac{Q_{\text{calc}} - Q_{\text{exp}}}{\Delta Q_{\text{exp}}} \right]^2 , \tag{5}$$

where Q_{calc} refers to the value of the observable calculated from a given set of parameters, Q_{exp} is the corresponding experimental value, and ΔQ_{exp} is the experimental uncertainty in Q_{exp}. The summation is over all the experimental quantities being considered in a given case. In order to avoid serious errors, the polarization and nuclear orientation expressions are averaged over energy by numerical integrations before comparing to experimental results. No restrictions were placed on the magnitudes of the parameters during the minimization procedure. In particular, the relationship between certain matrix elements based on conserved vector current (CVC) or $V-A$ theory was not used because there is some doubt as to its applicability.[11]

The error assignments made in the present investigation are based on the parabolic error method described in detail in I.

IV. RESULTS

In this section we present the results of both the measurement of the β-γ directional correlation and the analysis of all available data to obtain nuclear matrix elements.

A. Sb^{124}

We have measured the β-γ directional correlation between the outer β-ray group (2.31 MeV) and the 603-keV $E2$ γ ray in the decay $Sb^{124} \rightarrow Te^{124}$. A partial decay scheme is shown in Fig. 1. The correlation results are given in Table I. Our results are in good agreement with previous measurements.[12-14]

TABLE I. Sb^{124} directional-correlation results.

W (m_0c^2)	$-\epsilon$	$\Delta\epsilon$ [a]
3.10	0.244	0.017
3.20	0.255	0.014
3.35	0.278	0.015
3.54	0.297	0.014
3.80	0.323	0.009
4.02	0.341	0.007
4.19	0.352	0.010
4.35	0.376	0.011
4.52	0.381	0.007
4.85	0.396	0.013
4.98	0.413	0.010
5.11	0.405	0.020

[a] $\Delta\epsilon$ is the error in the corresponding directional-correlation coefficient ϵ.

The correlation data presented in Table I together with the recent shape factor measurement of Hsue *et al.*[15] and the β-circularly-polarized γ correlation measurements of Alexander and Steffen[16] and Camp, Mann, and Bloom,[17] were analyzed to obtain the nuclear matrix elements. In the analysis, we have used the actual shape factor data points rather than the analytical fit obtained by Hsue and co-workers which is sensitive to the assumed end-point energy. Canty, Davidson, and Connor[18] and Nagarajan, Ravindranath, and Reddy[19] have reported somewhat different analytical expressions for the shape; however, inclusion of these measurements does not alter the results presented here.

Inclusion of all the aforementioned data in the search program yielded the two sets of matrix element parameters given in Table II. The fit to the data is shown in Fig. 1. These results are considerably different from those obtained by other authors using Kotani expressions. In addition, they are somewhat different from those obtained by other authors using the exact expressions, no doubt because we have used more recent shape factor and directional-correlation data.

The energy dependence of the circular polarization predicted by both sets of matrix element parameters is given in Fig. 1. It is evident that the ambiguity between the two sets can be eliminated by a measurement of this energy dependence. Numerical values for the matrix elements were obtained by methods similar to those described in I and are given in Table III. The corrected logft values obtained for solutions I and II are 10.61 and 10.9, respectively.

B. Eu^{152}

The β-γ correlation measurements were made on the outer β-ray group (1.485 MeV) and the 344-keV $E2$ γ ray in the decay $Eu^{152} \rightarrow Gd^{152}$. A partial decay scheme is shown in Fig. 2. The results of the measurements are given in Table IV. These results are in reasonable agreement with the measurements of Bhattacherjee and Mitra[20] and Alexander and Steffen,[21] but are somewhat larger than the measurements reported by Dulaney, Braden, and Wyley,[22] and substantially smaller than the measurements reported by Sunier, Debrunner, and Scherrer.[23]

In addition to the correlation data mentioned above, the shape factor measurements of Langer

TABLE II. Matrix element parameters.

Type		x	u	$\xi' y$	w	$\xi' v$
Sb^{124}	I	0.05 ±0.02	0.10 ±0.03	2.6 ±0.4	···	···
	II	0.56 ±0.03	0.05 ±0.03	8.5 ±0.6	···	···
Eu^{152}	I	0.08 ±0.02	0.18 ±0.03	3.6 ±0.6	···	···
	II	0.58 ±0.03	0.08 ±0.03	9.0 ±0.7	···	···
Re^{186} [a]		0.001 ±0.002	0.02 ±0.01	−0.14 ±0.04	···	···
Re^{188} [b]		−0.01 ±0.01	−0.052 ±0.005	0.31 ±0.06	···	···
Tm^{170}		0.02 ±0.02	0.03 ±0.02	−0.02 ±0.01	···	···
Rb^{86}	I [c]	−0.27 ±0.15	−0.70 ±0.20	−8.0 ±3.0	0.80 ±0.50	−8.0 ±2.0
	II	−0.05 ±0.03	0.06 ±0.02	0.04 ±0.01	1.0 ±0.5	−10.0 ±2.0
	III	−0.23 ±0.15	−0.73 ±0.20	−8.0 ±3.0	0.50 ±0.30	−7.0 ±2.0
Rb^{84}	I	−0.62 ±0.30	−0.15 ±0.08	4.5 ±2.0	−0.60 ±0.40	−3.0 ±1.0
	II	−0.17 ±0.09	0.26 ±0.12	1.2 ±0.5	−0.14 ±0.08	−0.2 ±0.1
	III	−0.16 ±0.08	0.26 ±0.12	1.0 ±0.5	−0.006 ±0.003	0.7 ±0.3
	IV	−0.63 ±0.30	−0.18 ±0.09	4.7 ±2.0	0.20 ±0.10	2.0 ±1.0
	V	−0.11 ±0.05	0.30 ±0.10	0.2 ±0.1	0.90 ±0.50	7.0 ±3.0

[a] The experimental value (Ref. 31) of the orientation parameter B_2 is 0.243 ± 0.007 whereas the value calculated with this solution set is 0.242.

[b] The experimental value (Ref. 31) of the orientation parameter B_2 is 0.147 ± 0.025 whereas the value calculated with this solution set is 0.146.

[c] Solutions I and II were obtained with a nonstatistical shape and solution III with a statistical shape.

and Smith[24] and the β-circularly-polarized γ correlation measurements of Alexander and Steffen[21] and Berthier, Lombard, and Sunier[25] were included in the analysis. We have used the actual shape factor data points rather than the analytical fit reported by Langer and Smith.

There is no reported analysis of the Eu^{152} using the exact expressions prior to this investigation. We were not able to obtain a unique solution for Eu^{152}; however, all of the randomly located starting points used in the search procedure converged to one of the two sets of matrix element parameters given in Table II. The curves through the data in Fig. 2 show the fits obtained. These results are considerably different from those obtained by other authors using approximate expressions.

The energy dependence of the β-circularly-polarized γ correlation predicted by both sets of matrix element parameters is given in Fig. 2. It is evident that the ambiguity between the two sets can be eliminated by a measurement of this energy dependence. Numerical values for the matrix elements are given in Table III. The corrected $\log ft$ values obtained for solutions I and II are 12.01 and 12.27, respectively.

C. Re^{186}

The β-γ correlation measurements were made on the inner β-ray group (939 keV) and 137-keV

TABLE III. Matrix element values.

Type		$\vert\int B_{ij}/\rho\vert$	$\vert\int \vec{r}/\rho\vert$	$\vert\int i\vec{\sigma}\times\vec{r}/\rho\vert$	$\vert\int i\vec{\alpha}\vert$	$\vert\int \vec{\sigma}\cdot\vec{r}/\rho\vert$	$\vert\int i\gamma_5\vert$
Sb^{124}	I	0.02 ±0.01	0.002 ±0.005	0.002 ±0.001	0.007 ±0.002	···	···
	II	0.014 ±0.005	0.020 ±0.005	0.0007 ±0.0007	0.007 ±0.002	···	···
Eu^{152}	I	$0.004\,^{+0.01}_{-0.004}$	0.0004 ±0.0004	0.0008 ±0.0004	0.003 ±0.0005	···	···
	II	0.003 ±0.002	0.003 ±0.001	$0.0\,^{+0.0007}_{-0.0}$	0.0004 ±0.0004	···	···
Re^{186}		0.60 ±0.08	$0.0007\,^{+0.001}_{-0.0007}$	0.01 ±0.005	0.002 ±0.001	···	···
Re^{188}		0.18 ±0.05	0.003 ±0.003	0.008 ±0.004	0.002 ±0.001	···	···
Tm^{170}		0.11 ±0.02	0.003 ±0.003	0.003 ±0.003	0.0004 ±0.0004	···	···
Rb^{86}	I [a]	0.3 ±0.1	0.08 ±0.05	0.2 ±0.1	0.04 ±0.04	0.2 ±0.1	0.03 ±0.02
	II	0.4 ±0.1	0.25 ±0.05	0.02 ±0.01	0.0003 ±0.0003	0.4 ±0.2	0.06 ±0.06
	III	0.3 ±0.1	0.07 ±0.03	0.18 ±0.08	0.03 ±0.03	0.14 ±0.05	0.03 ±0.02
Rb^{84}	I	0.8 ±0.3	0.6 ±0.2	0.13 ±0.05	0.07 ±0.04	0.6 ±0.3	0.04 ±0.02
	II	0.6 ±0.3	0.10 ±0.05	0.15 ±0.05	0.010 ±0.005	0.08 ±0.04	0.002 ±0.001
	III	0.6 ±0.3	0.10 ±0.05	0.15 ±0.05	0.010 ±0.005	0.003 ±0.001	0.006 ±0.003
	IV	0.8 ±0.3	0.6 ±0.2	0.15 ±0.05	0.07 ±0.04	0.16 ±0.07	0.03 ±0.01
	V	0.7 ±0.3	0.08 ±0.04	0.15 ±0.05	0.002 ±0.002	0.6 ±0.4	0.07 ±0.04

[a] Solutions I and II are for nonstatistical shape and solution III for statistical shape.

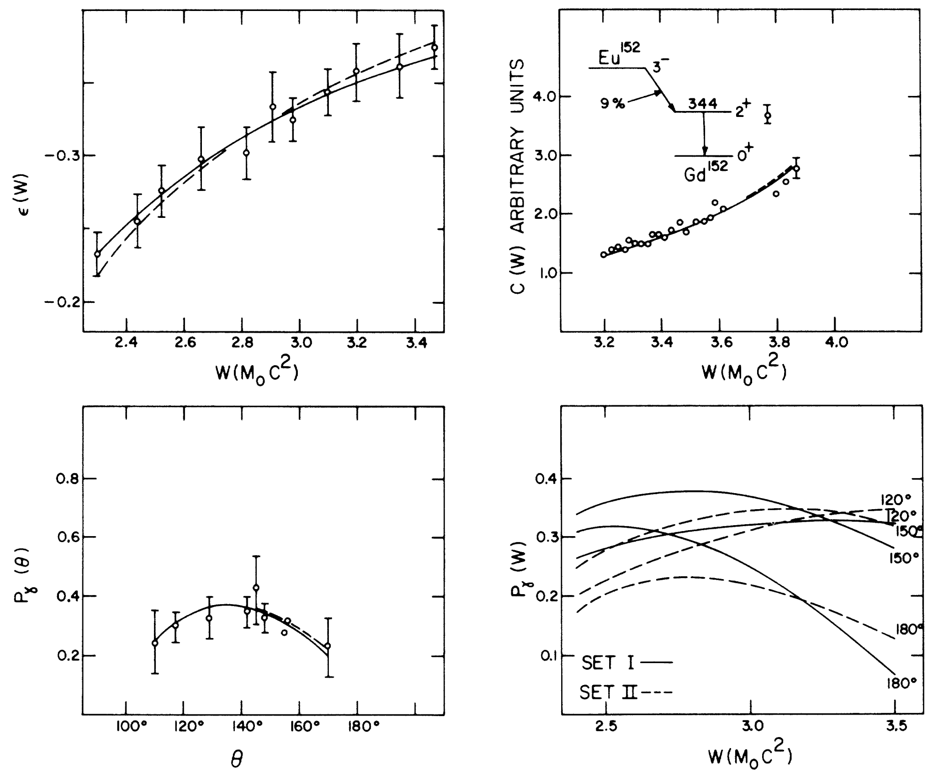


FIG. 2. The Eu^{152} data used in the present analysis. The solid and dashed curves through the data points represent the best-fit curves obtained for solution Sets I and II, respectively. The quantity $P_\gamma(W)$ was calculated using the best-fit solutions.

$E2\ \gamma$ ray in the decay $Re^{186} \rightarrow Os^{186}$. A partial decay scheme is shown in Fig. 3. The results of the measurement are given in Table V. The possible attenuation of the β-γ correlation due to the long lifetime of the 137-keV state is negligible as demonstrated by Trudel, Habib, and Ogata (THO).[26] These results are in good agreement with the measurements of Dulaney *et al.*[27] and Grenacs, Hess, and Rohmer,[28] but are somewhat smaller than the recent measurements of THO.

In addition to our correlation data, we have included in the analysis the shape factor measurements of Porter *et al.*,[29] the β-circularly-polarized γ correlation measurements of Delabaye,[30] and the measurements of the angular distribution of the 137-keV γ ray from oriented Re^{186} nuclei by Brewer and Shirley.[31] Porter *et al.* and more recently Andre and Liaud,[32] and Van der Werf, de Waard, and Beekhuis[33] have reported a nonstatistical shape. These three shape measurements are in agreement. A statistical shape measurement has been reported by THO.[26]

TABLE IV. Eu^{152} directional-correlation results.

W^{a}	$-\epsilon$	$\Delta\epsilon$
2.32	0.232	0.007
2.42	0.252	0.009
2.52	0.276	0.009
2.66	0.298	0.011
2.82	0.303	0.009
2.91	0.334	0.012
2.98	0.324	0.008
3.10	0.344	0.008
3.20	0.359	0.010
3.35	0.361	0.011
3.50	0.376	0.008

[a] Energy in units of m_0c^2.

Inclusion of all the aforementioned data in the search program, yielded only one satisfactory matrix element solution. The values of the parameters are listed in Table II. The curves shown in Fig. 3 were calculated from the parameters listed in Table II. In addition, we have calculated the β-circularly-polarized γ correlation as a function of energy. Figure 4 shows the $\chi^2(x_i)$ versus x_i curves obtained for our solution using the parabolic error method described in I. The curves not only define the error limits but exhibit the symmetry or nonsymmetry of the limits. We attempted an analysis using a statistical shape in accordance with the measurements of THO, but found that a statistical shape is inconsistent with the rest of the experimental data.

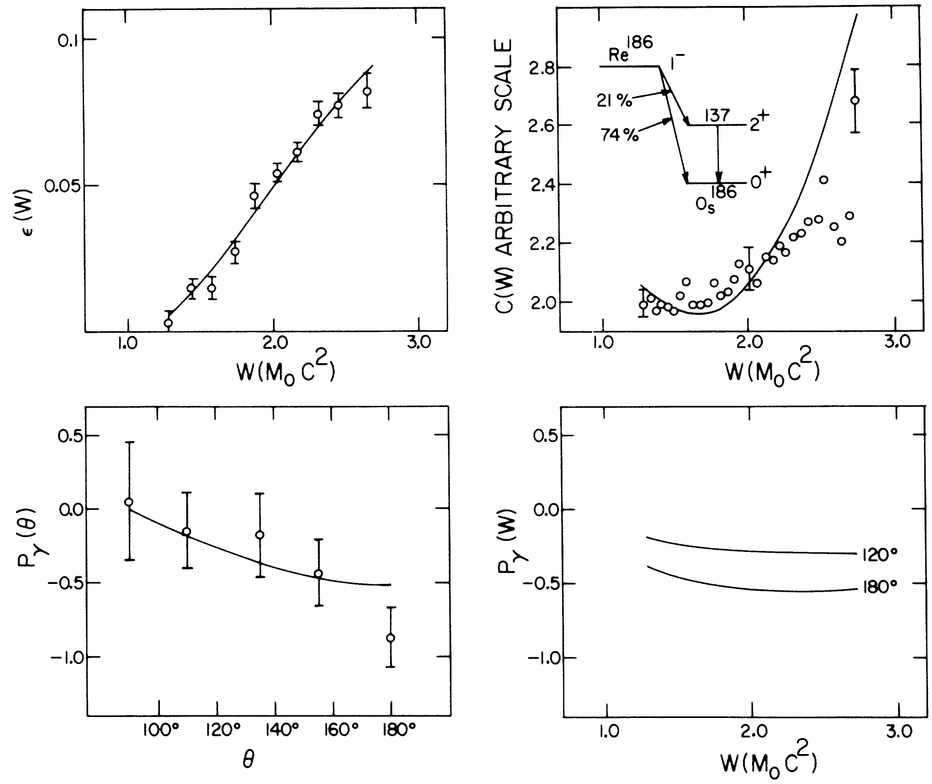


FIG. 3. The Re^{186} data used in the present analysis. The quantity $P_\gamma(W)$ was calculated using the solution set presented in Table II.

The matrix element parameters reported in Table II are not in agreement with those of Andre and Liaud[32] who used essentially the same method of analysis. We have no satisfactory explanation for this discrepancy.

Numerical values for the matrix elements are given in Table III. The corrected log*ft* value is 7.7.

D. Tm^{170}

The β-γ correlation measurements were made on the inner β-ray group (883 keV) and the 84-keV $E2$ γ ray in the decay $Tm^{170} \rightarrow Yb^{170}$. A partial decay scheme is shown in Fig. 5. The results of the measurement are given in Table VI. These results are in reasonable agreement with the measurements of Wyly, Braden, and Dulaney[34] and Runge.[35] Pfeifer and Runge[36] have investigated the effects of crystal fields and chemical bonds on the correlation due to the long lifetime of the intermediate state (~10^{-9} sec). They have also investigated the effects of internal bremstrahlung on the correlation due to the low γ-ray energy. They report that for the chemical form $TmCl_3$, the combined effects from the aforementioned sources is small. We have used $TmCl_3$ as our source material, and therefore expect our results are not perturbed. Moreover, our results are in good agreement with the results obtained by Runge.

In addition to our correlation data, we have included in the analysis the shape factor measurement of Van der Werf, de Waard, and Beekhuis.[33] No other experimental information is available for this transition. Runge's analysis of the available data included an additional parameter; the third-forbidden matrix element $\int(\vec{\alpha}\cdot\vec{r})\vec{r}$. On the basis of the Nilsson model, he showed that the transition is Λ-forbidden and requires the inclusion of the third-forbidden matrix element. His

TABLE V. Re^{186} directional-correlation results.

W[a]	ϵ	$\Delta\epsilon$
1.29	0.003	0.004
1.44	0.015	0.003
1.59	0.015	0.004
1.74	0.027	0.004
1.88	0.046	0.004
2.03	0.054	0.003
2.17	0.061	0.003
2.32	0.074	0.004
2.47	0.077	0.004
2.66	0.082	0.006

[a] Energy in units of m_0c^2.

results indicate that the third-forbidden matrix element is an order of magnitude larger than the normal first-forbidden matrix elements. Recently, Behrens and Bogdan[37] have investigated this decay theoretically and have shown that the inclusion of third-forbidden matrix elements does not improve their fit to experimental data (see Sec. V C).

Because of the lack of experimental information, we made no attempt to include the third-forbidden matrix element in our search procedure. With three pieces of information (including the ft value) and four parameters, the problem is already underconstrained. All of the randomly located starting points used in the search procedure converged to the single set of matrix element parameters given in Table II. However, it is by no means clear that this is a unique solution. The fit to the experimental data is shown by the curves in Fig. 5. We have also calculated the β-circularly-polarized γ correlation as a function of angle and energy using the parameters reported in Table II (see Fig. 5).

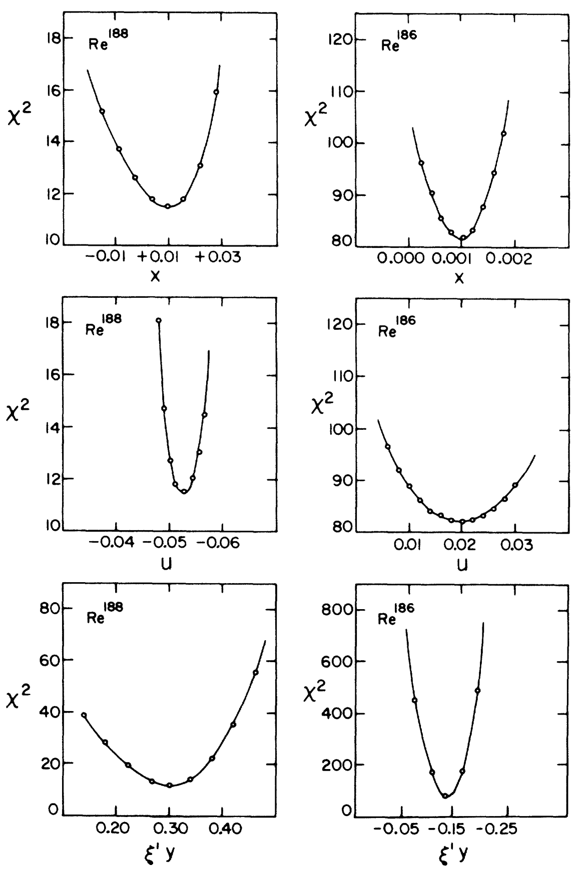


FIG. 4. Results of the parabolic error method study for the best-fit solutions to the Re^{186} and Re^{188} data (see text for details). The curves are best-fit parabolas through the lowest three points. These results were used to determine the errors quoted in Tables II and III.

We attempted an analysis which used the shape factor reported by Spejewski,[38] but found this shape to be inconsistent with the correlation data.

Numerical values for the matrix elements are given in Table III. The corrected logft value is 9.0.

E. Rb^{84}

The β-γ correlation measurements were made on the inner positron group (1790 keV) and the 880-keV $E2$ γ ray in the decay $Rb^{84} \rightarrow Kr^{84}$. A partial decay scheme is shown in Fig. 6. The results of the measurement are given in Table VII. These results are in good agreement with the recent measurement of de Beer *et al.*[39] but are somewhat smaller than the measurements reported by Simms *et al.*[40]

We have included in the analysis our directional-correlation results together with the shape factor measurement of Langer, Spejewski, and Wortman,[41] and the β-circularly-polarized γ correlation measurements of Boehm and Rogers.[42]

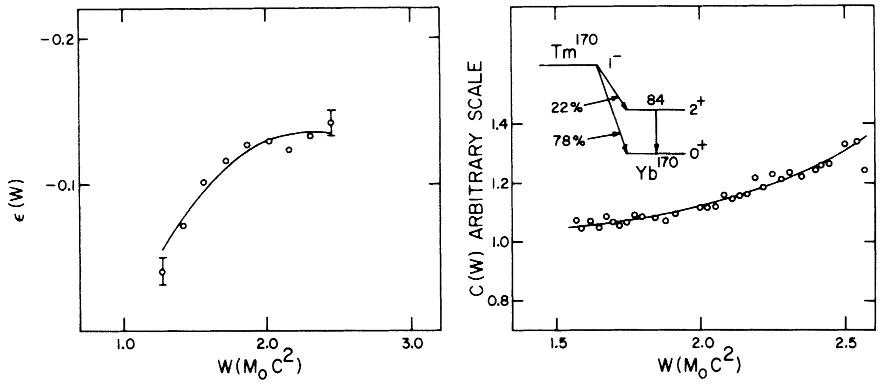
Because of the lack of experimental information, we were unable to find a unique solution for Rb^{84}. However, all randomly selected starting points of the search procedure converged to one of the five solution types given in Table II. The fit to the experimental data is shown in Fig. 6. We have also calculated the β-circularly-polarized γ correlation as a function of energy using the parameters reported in Table II (see Fig. 6).

The parameters listed in Table II are not in agreement with the results reported by Simms.[43] He reports only two solution sets which are different in sign and magnitude from any of our solutions. This somewhat surprising in view of the fact that his method of analysis is similar to ours, the only difference being the directional-correlation data used. In order to understand this discrepancy, we performed an analysis using his directional-correlation data; however, the results of this analysis were, within the error limits, identical to those given in Table II. It is not clear what is causing the discrepancy.

Numerical values for the matrix elements are given in Table III. The corrected logft value for the four solutions ranged from 7.5 to 7.8.

F. Re^{188}

The partial decay scheme for Re^{188} is given in Fig. 7. We have analyzed all available experimental data on the $1^-(\beta_{1962})2^+(\gamma_{155})0^+$ transition. The experimental data used in the analysis are shown in Fig. 7. The shape data were taken from the measurements of Van der Werf, de Waard, Beekhuis.[33] The β-γ directional-correlation data is that of Grenacs, Hess, Rohmer.[28] The β-circular-



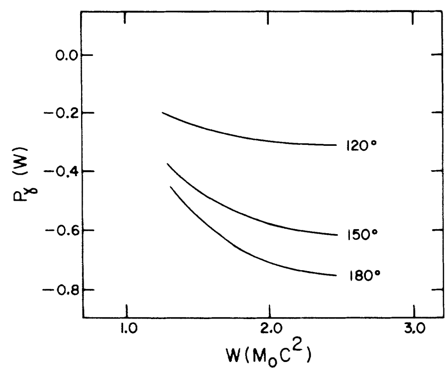


FIG. 5. The Tm^{170} data used in the present analysis. The quantity $P_\gamma(W)$ was calculated using the solution set present in Table II.

ly-polarized γ correlation was taken from the measurements of Gygax and Hess.[44] The orientation data is that due to Brewer and Shirley.[31]

Inclusion of all the aforementioned data in the search program yielded only one satisfactory matrix element solution which is given in Table II. Our results are in disagreement with those reported by Andre and Liaud[32]; however, this is not surprising since no polarization information was available at the time of their work. The β-circularly-polarized γ correlation as a function of energy calculated on the basis of the solution set reported in Table II is given in Fig. 7. A measurement of this energy dependence would test the validity of our results.

TABLE VI. Tm^{170} directional-correlation results.

W^a	$-\epsilon$	$\Delta\epsilon$
1.29	0.042	0.006
1.44	0.072	0.007
1.59	0.103	0.005
1.73	0.117	0.006
1.88	0.127	0.004
2.03	0.129	0.006
1.74	0.124	0.005
2.32	0.135	0.008
2.47	0.143	0.007

[a] Energy in units of m_0c^2.

Numerical values for the matrix elements are given in Table III. The corrected $\log ft$ value is 8.7.

G. Rb^{86}

The partial decay scheme for Rb^{86} is given in Fig. 8. We have analyzed all available experimental data on the $2^-(\beta_{700})2^+(\gamma_{1080})0^+$ transition. The experimental data used in the analysis are shown in Fig. 8. The shape data were taken from the measurements of Robinson and Langer[45] who reported a nonstatistical shape and also from the measurements of Spejewski,[38] and Thompson and Casper[46] who report a statistical shape. The β-γ directional-correlation data is that of Fischbeck and Wilkinson.[6] The angular dependence of the circular polarization was taken from the measurements of Viano *et al.*[47] and the energy dependence of the polarization is that obtained by Kneissl.[48]

The analysis of all the aforementioned data yielded a unique solution set for a statistical shape, and two solution sets for a nonstatistical shape. These results are given in Table II. The fit to the data is shown by the curves in Fig. 8. These results are in reasonable agreement with those reported by Viano *et al.*,[47] but are in disagreement with the results reported by Simms.[43] This disagreement is not surprising since we have used a completely different data set. Simms used the po-

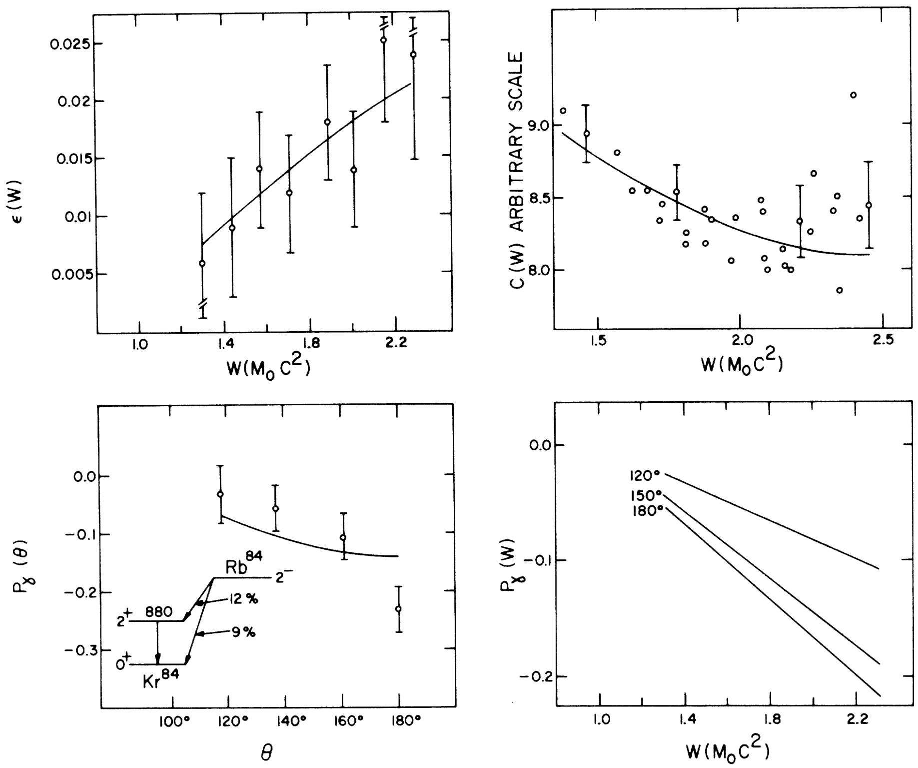


FIG. 6. The Rb^{84} data used in the present analysis. The curves through the data represent the fit for solution Set V. given in Table II; however, all solutions give similar results. The quantity $P_\gamma(W)$ is calculated using solution Set V.

larization measurements of Boehm and Rogers, whereas we used the more recent measurements of Viano *et al.* The measurement of the energy dependence of the polarization was not available at the time of Simms work. Finally, we have used slightly different β-γ directional-correlation data.

Numerical values for the matrix elements are given in Table III. The corrected $\log ft$ values for solutions I, II and III are 8.5, 8.2 and 8.6, respectively.

V. DISCUSSION

In the following subsections we discuss in order the $3^-(\beta)2^+(\gamma)0^+$ decays of Sb^{124} and Eu^{152}, the $1^-(\beta)2^+(\gamma)0^+$ decays of Tm^{170}, Re^{186}, and Re^{188}, and the $2^-(\beta)2^+(\gamma)0^+$ decays of Rb^{84} and Rb^{86}.

TABLE VII. Rb^{84} directional-correlation results.

W [a]	ϵ	$\Delta\epsilon$
1.29	0.006	0.006
1.44	0.009	0.007
1.59	0.014	0.005
1.73	0.012	0.005
1.88	0.018	0.005
2.03	0.014	0.005
2.17	0.025	0.007
2.32	0.024	0.009

[a] Energy in units of m_0c^2.

A. Sb^{124}

An inspection of the results in Table III shows that all the matrix elements are reduced in magnitude from the single-particle estimates[49] and that the rank 2 matrix element ($\int B_{ij}/\rho$) dominates the transition. A qualitative explanation for this latter effect has been given by Alexander and Steffen[16] who showed that the basic shell model transition is from a $h_{11/2}$ neutron to a $g_{7/2}$ proton configuration which forbids the rank 1 terms but allows the rank 2 terms. However, this forbiddeness does not account for the small value of $\int B_{ij}$. That an additional cancellation is occurring in this transition is suggested by the results for Sb^{122} reported in I. For Sb^{122}, $\int B_{ij}/\rho \approx 0.3$ which is an order of magnitude larger than the analogous Sb^{124} decay. The Sb^{124} 3^- state is essentially

$$|3^-\rangle = \alpha|(\nu h_{11/2}\pi d_{5/2})_{3^-}\rangle + \beta|(\pi g_{1/2})_{3^-}\rangle, \qquad (6)$$

while the Te^{124} first 2^+ state is essentially

$$|2^+\rangle = a|(g_{7/2})^2{}_{2+}\rangle + b|(d_{5/2})^2{}_{2+}\rangle + c|(d_{5/2}g_{7/2})_{2+}\rangle + \cdots \qquad (7)$$

Now the $\nu h_{11/2} \rightarrow \pi g_{7/2}$ transition allows the rank 2 matrix element $\int B_{ij}/\rho$. Kisslinger and Wu[50] have calculated $\int B_{ij}$ for this particular transition using a pairing force between pairs of like nucleons and a quadrupole force of equal magnitude between all nucleon pairs. They found that there is a cancellation between two major terms resulting in a value of 0.34 for $\int B_{ij}/\rho$ as compared to the shell-model prediction of 1.5. This is still an order of magnitude larger than the experimental results; however, as this was an approximate calculation, the result is encouraging. A qualitative explanation for the additional hindrance may be as follows:

The transition amplitude between the 3^- and 2^+ states can be written as [see Eqs. (6) and (7)]

$$\langle 3^- | B_{ij} | 2^+ \rangle = c\alpha \langle (h_{11/2} d_{5/2})_{3^-} | B_{ij} | (d_{5/2} g_{7/2})_{2^+} \rangle + a\beta \langle (g_{7/2} h_{11/2})_{3^-} | B_{ij} | (g_{7/2})^2{}_{2^+} \rangle , \quad (8)$$

since all other terms give zero contribution. However since β is small, hindrance can occur even though a is large provided c is small. A qualitatively similar situation exists for the transition $3^- \rightarrow 2_2^+$ (the second 2^+ state). This transition is also hindered, has large β-γ directional correlation[51] ($\epsilon \sim -0.2$), and the B_{ij} matrix element dominates. A possible explanation for this is that the $(g_{7/2})^2{}_{2^+}$ strength is spread over several of these low-lying states in Te^{124}.

Kisslinger and Wu also obtained order-of-magnitude results for the other three matrix elements by taking into account the contribution from neighboring shells. Their results together with our experimental results are compared in Table VIII. Both of our solution sets are in reasonable agreement with the predicted values. Kisslinger and Wu have suggested additional experiments to test the systematics of β decay in this region. The relevant experiments are: (1) a study of the decay $Sb^{122} \xrightarrow{\beta^+} Sn^{122}$ – in this case, they have predicted $\int B_{ij}$ to be five times larger than the corresponding decay $Sb^{122} \xrightarrow{\beta^-} Te^{122}$; (2) a study of Sb^{126} and Sb^{128} in order to determine the matrix elements. The decay of Sb^{126} to the 6^+ (1775 keV) level in Te^{126} has been recently studied by means β-γ correlations by Gupta.[52] He found that $\epsilon(\beta\text{-}\gamma) \sim -0.045 \pm 0.013$ for $E_\beta > 1.3$ MeV. This result, along with other information, suggests that the Sb^{126} ground state has $J^\pi = 7^-$. He showed that the $\int B_{ij}$ matrix element provides the dominant contribution to the decay and that small contributions of the rank 1 matrix elements are needed to quantitatively explain the measured ϵ.

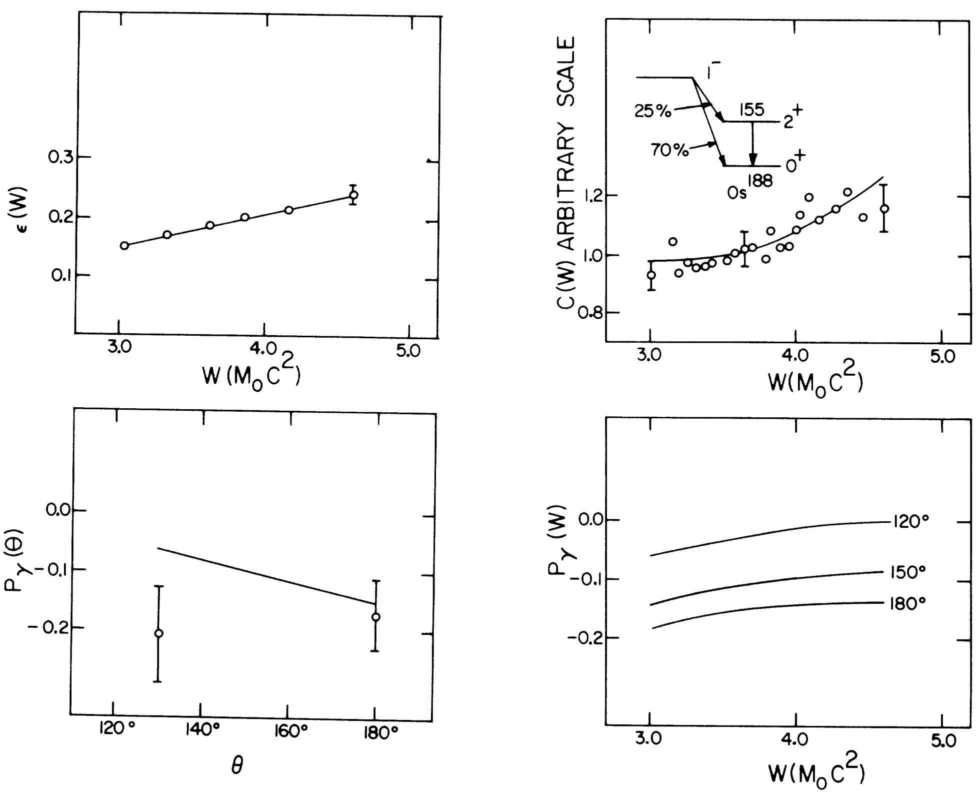


FIG. 7. The Re^{188} data used in the present analysis. The quantity $P_\gamma(W)$ was calculated using the solution set present in Table II.

B. Eu^{152}

The situation for Eu^{152} is similar to Sb^{124} in that the matrix elements are considerably smaller than single-particle estimates and the rank 2 matrix element dominates the transition. This is particularly true for Set I listed in Table III. A qualitative argument for this effect goes as follows: The ground state of Eu^{152} is characterized as a rotational state with $J^{\pi}(K)=3^-(3)$. The β transition is between this state and the first excited state of Gd^{152} which is described as a $J^{\pi}(K)=2^+(0)$ state. This is a K-forbidden transition because $\Delta K=3$ which violates the Bohr-Mottelson model selection rule $\Delta K \leq \lambda$ (where λ is the rank of the matrix element). An exactly similar situation occurs in the decay of Eu^{154}. The large $\log ft$ values for these transitions support the K assignments made here.

Bogdan and Lipnik[53] have investigated Eu^{152} assuming that K is not a good quantum number and thus describe the nuclear states as K-mixed states. In these calculations the ground state of Eu^{152} is treated as a mixture of $K=0, 1, 3$ configurations and the first excited states of Gd^{152} as a pure K=0 state. Under these assumptions they obtain expressions for the matrix elements which have as parameters the mixing coefficients and the relative phases of the Nilsson wave functions. Using our Set I solutions, and the $\log ft$ value they were able to obtain the ground-state wave function for Eu^{152}. The interesting thing is that these admixtures are very small compared to the $K=3$ intensity as seen from the wave function

$$|Eu^{152}, 3^-\rangle = (1-0.00978)^{1/2}|K=3, 3^-\rangle \pm 0.097|K=1, 3^-\rangle \pm 0.0145|K=0, 3^-\rangle .$$

In addition, they were able to show that the conclusions concerning Eu^{152} should be valid for Eu^{154}. A recent analysis of all available data for the decay $Eu^{154} \to Gd^{154}$ by Manthuruthil and Poirier[54] yielded results similar to the Eu^{152} results. Preliminary results of the analysis are presented in Table IX. These results tend to support the conclusions made by Bogdan and Lipnik. Finally, Bogdan and Lipnik were able to show that the relative dominance of $\int B_{ij}$ is due to cancellation effects between the rank 1 terms resulting from the configuration assumed for the Eu^{152} ground state.

The decay of the 3^- ground state of Tb^{160} to the first 2^+ state in Dy^{160} is expected to be similar to those of Eu^{152} and Eu^{154}. In fact, these three transitions have about the same hindrance. Recent work of Cipolla and Steffen[55] indicates the similarity of

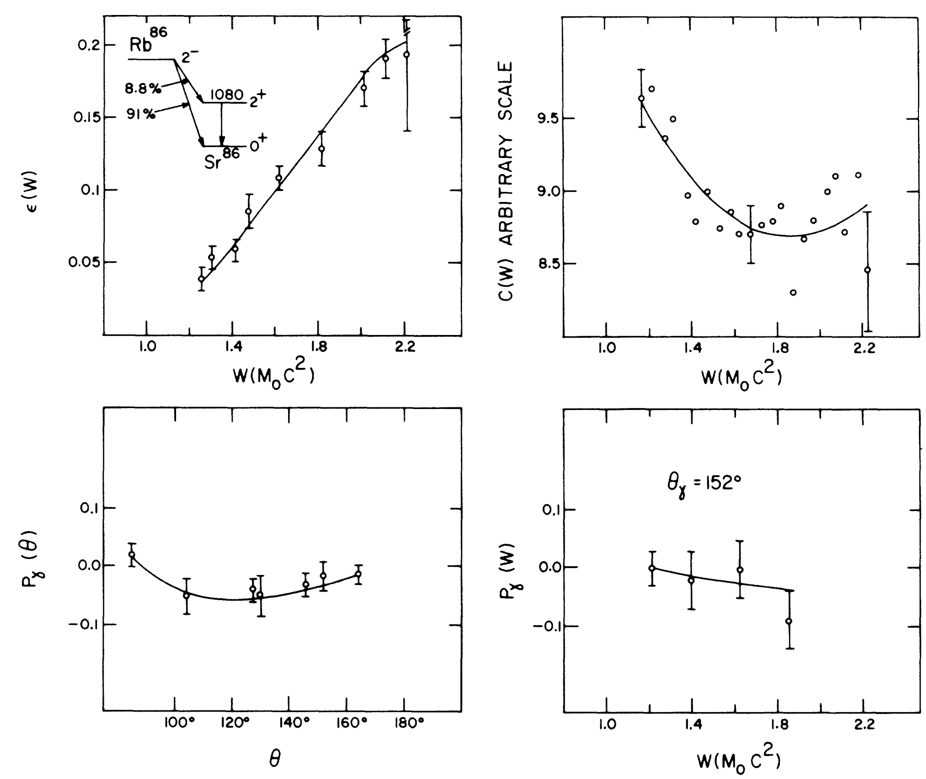


FIG. 8. The Rb^{86} data used in the present analysis. The curves through the data represent the fit for solution Set II given in Table II; however, solution Set I gives identical results. The fit to the $\epsilon(W)$, $P_\gamma(\theta)$, and $P_\gamma(W)$ data resulting from the statistical shape solution (Set III) are identical to those presented in the figure.

TABLE VIII. Predicted and observed values of the Sb^{124} matrix elements.

Matrix element	Theory	Experiment	
$\lvert\int \vec{r}/\rho\rvert$ [a]	$\sim 10^{-3}$	2.0×10^{-3}	2.0×10^{-2}
$\lvert\int \vec{\sigma}\times\vec{r}/\rho\rvert$	$\sim 10^{-2}$	2.0×10^{-3}	7.0×10^{-4}
$\lvert\int \vec{\alpha}\rvert$	$\sim 10^{-4}$	7.0×10^{-4}	7.0×10^{-4}

[a] The quantity ρ is the nuclear radius expressed in natural units.

the Tb^{160} decay to the Eu decays in the sense of K admixtures in the initial state, although another experimental matrix element set allowed by their data is at variance with this picture.

C. Tm^{170}

The logft value of 9.3 for this transition is rather large for a first-forbidden β decay, and, in addition, the matrix elements reported in Table III are significantly smaller than single-particle estimates. Since the transition takes place between a predominately $K=1$ state (there is evidence that the Tm^{170} ground state has a small admixture of $K=0$) and a $K=0$ state. This is not a K-forbidden decay. Runge[35] suggested an explanation based on the Λ-selection rule to account for the large logft value. According to the Nilsson model, the ground state of Tm^{170} has $\Lambda=2$ while the first excited state of Yb^{170} has $\Lambda=0$. Thus the transition corresponds to $\Delta\Lambda=2$ and is Λ forbidden. However, this description introduces the complication of requiring inclusion of the third-forbidden matrix elements. Runge obtained values of the matrix elements by including only one third-forbidden matrix element and invoking the relationship between $\int i\vec{\alpha}$ and $\int \vec{r}$ derived by Fujita[56] on the basis of CVC theory. He found that the correction matrix element $\int(\vec{\alpha}\cdot\vec{r})\vec{r}$ dominates the transition.

Bogdan *et al.*[57] have also investigated Tm^{170} and have shown that the Fujita relations are not valid for Tm^{170} β decay. They have calculated the matrix elements for Tm^{170} using Woods-Saxon wave functions and assuming the ground state of Tm^{170} to be a mixture of $K=1$ and 0 configurations.

They have used the ratio $\int i\vec{\alpha}/\int\vec{r}$ according to the procedure suggested by Damgaard and Winther[11] and found that the dominant matrix element is $\int\vec{r}$ in contrast to the conclusions drawn by Runge. However, these results were in poor agreement with experiment. Recently, Behrens and Bogdan[37] refined the calculations of the Tm^{170} observables. They have (a) included the exact radial dependence of the electron wave functions, (b) considered the third-forbidden matrix elements, and (c) included the deformed part of the Coulomb potential in the calculations. They found that the influence of the correction matrix element is small, but that the inclusion of the deformed part of the Coulomb potential improves agreement with experiment. The set of values which gives best agreement with experiment is: $\lvert\int\vec{r}/\rho\rvert=0.013$, $\lvert\int i\vec{\sigma}\times\vec{r}/\rho\rvert=0.144$, $\lvert\int B_{ij}/\rho\rvert=0.132$, $\lvert\int i\vec{\alpha}\rvert=0.012$. With the exception of $\lvert\int i\vec{\sigma}\times\vec{r}/\rho\rvert$, there is good agreement between these results and those presented in Table III. A measurement of the angular and energy dependence of the polarization is desirable in order to test the validity of the solutions presented here.

D. Re^{186} and Re^{188}

We treat Re^{186} and Re^{188} together since they have similar decay characteristics, matrix element values, and theoretical descriptions. It is clear from inspection of Table III that the matrix elements are reduced in magnitude and that $\int B_{ij}$ dominates the transition for both nuclei. Since both transitions connect states with $K=1$ and $K=0$, the K-selection rule is satisfied. In their initial work, Bogdan and co-workers[57] calculated the matrix elements for Re^{186} and Re^{188} using Woods-Saxon as well as Nilsson-type wave functions and included the ratio $\lvert\int i\vec{\alpha}/\int\vec{r}\rvert$ calculated according to the procedure suggested by Damgaard and Winther.[11] For both nuclei they find that $\int B_{ij}$ is the dominant matrix element in agreement with results presented here; however, the over-all agreement is poor. Recently Behrens and Bogdan[37] recalculated the β-decay observables for the Re isotopes using techniques similar to those described for Tm^{170}. The set of values for both Re^{186} and Re^{188} which gives best agreement with experiment is : $\lvert\int\vec{r}/\rho\rvert=0.001$, $\lvert\int i\vec{\sigma}\times\vec{r}/\rho\rvert=0.176$, $\lvert\int B_{ij}\rvert\rho\rvert=0.632$, $\lvert\int i\vec{\alpha}\rvert=0.008$. With the exception of $\lvert\int i\vec{\sigma}\times\vec{r}/\rho\rvert$, there is good agreement between these results and those presented in Table III. This is particularly true for Re^{186}. Finally, Behrens and Bogdan find that unlike Tm^{170} the agreement improves if one includes the correction matrix elements.

E. Rb^{84} and Rb^{86}

We treat Rb^{84} and Rb^{86} together since they have similar decay characteristic, matrix element values, and theoretical descriptions. Qualitatively, the matrix elements reported here for both nuclei

TABLE IX. Preliminary results for the Eu^{154} matrix element parameters.

Type	x	u	$\xi' y$
I	0.07 ± 0.04	0.12 ± 0.09	3.3 ± 1.4
II	0.42 ± 0.24	0.01 ± 0.04	6.8 ± 3.0

TABLE X. Rb^{86} matrix element parameters calculated in the Wahlborn model.

Phonon energy $\hbar\omega$ (MeV)	x	u	w
1.1	−0.02	−0.06	−0.06
1.5	−0.02	−0.05	−0.06
2.0	−0.02	−0.04	−0.05

can be understood on the basis of simple shell-model considerations. Among all the orbitals available in the 28–50 major shell, only $f_{5/2}$ and $g_{9/2}$ can couple to a 2^- state. Therefore it follows that only the transition $1f_{5/2} \rightleftarrows 1g_{9/2}$ can contribute to the first-forbidden β decay. However, this situation requires that only the matrix element $\int B_{ij}$ be nonzero. The fact that the remaining matrix elements are nonzero requires admixtures from the neighboring major shells. This picture is somewhat verified by the results for both Rb^{86} and Rb^{84}.

Wahlborn[58] has reported the results of extensive calculations concerning the decay of Rb^{84} and Rb^{86} within the framework of the shell model. He has introduced a particle-surface interaction in order to produce the required shell admixtures and has succeeded in obtaining expressions for the non-relativistic matrix element parameters, x, u, and w. The effect of the particle-surface interaction is to produce, in the 2^- state, admixed components of one phonon states (with energy $\hbar\omega$) coupled to the normal particle configuration. The expressions for x, u, and w are functions of the phonon energy $\hbar\omega$. Table X list values of the Rb^{86} paramaters for various values of the phonon energy. The lower bound on the phonon energy $\hbar\omega$ is the energy of the 2^+ state in Sr^{86}. The values for solution Set II presented in Table II are in reasonably good agreement with Wahlborn's results. The situation for Rb^{84} is similar. Wahlborn's calculations predict that the Rb^{84} matrix element parameters should be approximately a factor of five times as large as those for Rb^{86}. This is roughly what our results indicate if one uses the Rb^{86} solution Set II; however, quantitative agreement between theory and experiment is poor. Wahlborn concludes that the 2^+ states of Sr^{86} and Kr^{84} have a very complicated structure which cannot be understood on any simple particle or collective basis, but must follow some intermediate description.

TABLE XI. Rb^{84} and Rb^{86} matrix element parameters calculated by Kopytin and Batkin.

Matrix element parameter	Rb^{84}	Rb^{86}
x	−0.22	−0.003
u	0.83	0.04
w	1.1	0.04

Recently Kopytin and Batkin[59] reported extensive results for the reduced matrix elements of the various β-decay operators for the mass region $72 \leq A \leq 86$, using the theory of finite Fermi systems. The results of their calculation are presented in Table XI. In the case of Rb^{84}, our solution Set V is in good agreement with their results. However, in the case of Rb^{86}, our results require a large value of w contrary to their results. This is a surprising result since their model gives good quantitative agreement for all other studied nuclei in this mass region.

F. Matrix Elements and CVC Theory

The relation between the non-relativistic matrix element $\int \vec{r}$ and the relativistic matrix element $\int i\vec{\alpha}$ (the ratio $|\int i\vec{\alpha}/\int \vec{r}| = \Lambda$) has been of particular interest. Fujita[56] calculated Λ on the basis of conserved vector current (CVC) theory. His calculations were based on the Ahrens and Feenberg[60] approximation in which all nondiagonal Coulomb interaction matrix elements are neglected. Damgaard and Winther[11] demonstrated that this approx-

TABLE XII. Comparison of the ratio $|\int i\vec{\alpha}/\int \vec{r}|$.

Element		V − A	CVC	Exp
Sb^{124}	I	15	32	52^{+35}_{-18}
	II			15 ± 3
Eu^{152}	I	15	35	46^{+28}_{-17}
	II			16^{+2}_{-3}
Re^{186}		16	38	$140^{+\infty}_{-110}$
Re^{188}		16	38	$31^{+\infty}_{-19}$
Tm^{170}		15	36	$1^{+\infty}_{-1}$
Rb^{86}	I	11	26	30^{+60}_{-18}
	II			1^{+2}_{-1}
	III			35^{+100}_{-20}
Rb^{84}	I	5	18	8^{+12}_{-5}
	II			7^{+14}_{-4}
	III			6^{+14}_{-4}
	IV			8^{+12}_{-5}
	V			2^{+4}_{-1}

imation is not necessarily correct and, in certain cases, is likely to result in grave errors. In fact, the application of the ratio Λ as suggested in Ref. 11 has led to better agreement between experiment and theory[37, 57] for the nuclei Tm^{170}, Re^{186}, and Re^{188}. Recently, Fayans and Khodel[61] showed that the relation Λ derived by Damgaard and Winther is only correct when the interaction between quasiparticles of the nucleus (residual force) is neglected. If this interaction is taken into account, the results of the calculation can be changed considerably. They have calculated Λ for Bi^{210} decay and obtained different values depending upon which model was used.

From the previous discussions, it is clear that the theoretical prediction for the ratio Λ should not be used in the analysis for matrix elements. Instead, matrix elements should be determined independently so that the ratios obtained from the analysis can then be used to obtain information about the initial and final nuclear states.

Table XII lists the ratio Λ obtained from the present work for all nuclei under investigation. For some nuclei, there is a large discrepancy between Fujita's prediction and the experimental ratios. For Tm^{170}, Re^{186}, and Re^{188}, Bogdan *et al.*[57,37] have shown that this should be expected. A correct calculation of this ratio would be useful in order to make a more reliable comparison with experiment.

For those cases where a large discrepancy exists between our experimental value of Λ and the calculated value, we have attempted an analysis of the available data in which the theoretical value of the ratio Λ is introduced into the search procedure as a constraint. We allowed the program to search for solutions which satisfied the Fujita relationship within 50%. In no case did we find an acceptable solution which had a minimum χ^2 within a factor of 5 of our best-fit χ^2. However, in most cases the major contribution to the large value of χ^2 came from a poor fit to the directional-correlation data.

It is worth noting however that Fujita's estimate depends on the assumption that the nuclear states are of the shell-model type. Considering that the intermediate 2^+ states in all cases investigated here are of the collective type, the poor agreement with prediction is not surprising. It is interesting to note that the only experimental test of Fujita's relation available; namely, the study of the γ decay of the analog state[62] of the Ce^{141} ground state in Pr^{141}, is in agreement with Fujita's estimate. This result is supported by the β-decay work[63, 64] on Ce^{141}. What is clear is that the "CVC relation" is not model independent as expected, and that the relativistic matrix element $\int i\vec{\alpha}$ should be found from experiment.

VI. SUMMARY

Through a combined analysis of all available experimental data we have determined the first-forbidden β-decay matrix elements for Sb^{124}, Eu^{152}, Tm^{170}, Re^{186}, Re^{188}, Rb^{84}, and Rb^{86}. The analysis yielded unique solutions for Re^{186}, Re^{188}, and Rb^{86} (provided one assumes statistical shape for Rb^{86}). The two sets obtained for both Sb^{124} and Eu^{152} may be resolved by a measurement of the energy dependence of the β-circularily-polarized γ correlation. In the case of Tm^{170}, measurement of the polarization as a function of angle and/or energy is required before one can confidently accept the results presented here. A prediction of $P_\gamma(\theta, E)$ has been presented as a guide to future experiments. Improvement of the present results for Rb^{84} requires an accurate measurement of the polarization as a function of energy and angle.

All results obtained from the present investigation have been compared to available model calculations. For Sb^{124} it was found that inclusion of pairing and quadrupole forces results in cancellations which yield better agreement with experiment compared to pure shell models. Our results for Eu^{152} have been used to obtain the wave function for the ground state of Eu^{152}. For the nuclei Tm^{170}, Re^{186}, and Re^{188} calculations which used Woods-Saxon-type wave functions and included the deformed part of the Coulomb potential were in reasonable agreement with experiment. Inclusion of higher-order matrix elements in the calculations did not improve the results for Tm^{170} but did for Re^{186} and Re^{188}. Calculations within the framework of the shell model with a particle-surface interaction gave poor quantitative agreement with the experimental results for Rb^{84} and Rb^{86}. However, one important result of the calculations is that the 2^+ states in Kr^{84} and Sr^{86} were found to require a complicated description. It was also shown that a study of Rb^{84} and Rb^{86} could yield certain information about residual interactions when information is available from two or more neighboring nuclei. Another example of this situation is As^{74} and As^{76}. It would be very useful to determine the matrix elements for these nuclei; however, additional information is required. Such studies will provide a more extensive evaluation of the new calculations reported by Kopytin and Batkin.

In conclusion, we would like to emphasize the continuing need for accurate data on several β-decay observables, particularly those on the energy dependence of the β-γ circular polarization, and when feasible the angular distribution of electrons from oriented nuclei. Such information, when combined with a complete analysis similar to the one reported here, may be expected to provide a firm basis for comparing model-dependent calculations.

APPENDIX

Rationalized relativistic units $\hbar = m = c = 1$ are used. The electron and neutrino energies are denoted by E and q respectively and the electron momentum by p. The nuclear radius is expressed as $\rho = 0.4285\alpha A^{1/3}$ where α is the fine-structure constant. The quantities b in Eqs. (1)–(3) are defined as follows:

$$b_1 = \tfrac{1}{3}q^2L_0 - \tfrac{2}{3}qN_0 + 2L_1 + M_0 ,$$

$$b_2 = L_0 ,$$

$$b_3 = \tfrac{1}{6}q^2L_0 + \tfrac{2}{3}qN_0 + \tfrac{1}{2}L_1 + M_0 ,$$

$$b_4 = \tfrac{1}{3}qL_0 + N_0 ,$$

$$b_5 = \tfrac{1}{3}qL_0 - N_0 ,$$

$$b_6 = L_1 - M_0 ,$$

$$b_7 = \tfrac{1}{12}q^2L_0 + \tfrac{3}{4}L_1$$

$$b_8 = \tfrac{1}{2}L_1 - \tfrac{1}{3}qL_{12} + N_{12} ,$$

$$b_9 = \tfrac{1}{6}qL_{12} - \tfrac{1}{8}L_1 + \tfrac{1}{2}N_{12} ,$$

$$b_{10} = \tfrac{1}{2}L_{12} ,$$

$$b_{11} = \tfrac{1}{2}qL_{12} - \tfrac{1}{2}L_1 + \tfrac{1}{2}N_{12} ,$$

$$b_{12} = \frac{1}{2\sqrt{6}}(qL_{12} + 3L_1 - 3N_{12}) ,$$

$$b_{13} = -\frac{1}{2\sqrt{6}}(qL_{12} + \tfrac{3}{2}L_1 - 3N_{12}) ,$$

$$b_{14} = -\tfrac{1}{8}L_1 ,$$

$$b_{15} = -\tfrac{1}{3}q\vec{\mathrm{L}}_{12} - \tfrac{1}{3}q\vec{\mathrm{N}}_{11} + \vec{\mathrm{N}}_{12} + \Lambda_2 - m_1 ,$$

$$b_{16} = -\Lambda_1 ,$$

$$b_{17} = -\tfrac{1}{12}q^2\Lambda_1 + \tfrac{1}{6}q\vec{\mathrm{L}}_{12} - \tfrac{1}{3}q\vec{\mathrm{N}}_{11} + \tfrac{1}{2}\vec{\mathrm{N}}_{12} - \tfrac{1}{4}\Lambda_2 + m_1 ,$$

$$b_{18} = -\tfrac{2}{3}q\Lambda_1 - \vec{\mathrm{N}}_{11} + \tfrac{1}{2}\vec{\mathrm{L}}_{12} ,$$

$$b_{19} = \tfrac{2}{3}q\Lambda_1 - \vec{\mathrm{L}}_{12} - \vec{\mathrm{N}}_{11} ,$$

$$b_{20} = -\tfrac{1}{3}q^2\Lambda_1 + \tfrac{1}{2}q\vec{\mathrm{L}}_{12} + \tfrac{1}{2}\vec{\mathrm{N}}_{12} + \tfrac{6}{5}\Lambda_2 ,$$

$$b_{21} = \frac{1}{2\sqrt{6}}(-\tfrac{2}{3}q^2\Lambda_1 - qL_{12} + 3N_{12} - \tfrac{6}{5}\Lambda_2) ,$$

$$b_{22} = \frac{3}{2\sqrt{6}}\vec{\mathrm{L}}_{12} ,$$

$$b_{23} = -\tfrac{1}{3}q^2\Lambda_1 + q\vec{\mathrm{L}}_{12} + \tfrac{3}{5}\Lambda_2 + 3\vec{\mathrm{N}}_{12} ,$$

$$b_{24} = \frac{18}{\sqrt{6}}\Lambda_2 ,$$

$$b_{25} = \tfrac{5}{6}(\tfrac{1}{12}q^2\Lambda_1 + \tfrac{9}{20}\Lambda_2) .$$

The quantity B_2 in Eq. (4) is given by

$$B_2 = \frac{\beta_1 - \beta_2}{\beta_1 + \beta_2} ,$$

where β_1 and β_2 are defined in Paper I.

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[1] J. C. Manthuruthil and C. P. Poirier, Nucl. Phys. A118, 657 (1968).

[2] Z. Matumoto, M. Yamada, I.-T. Wang, and M. Morita, Phys. Rev. 129, 1308 (1963).

[3] T. Kotani, Phys. Rev. 114, 795 (1959).

[4] E. J. Konopinski and G. E. Uhlenbeck, Phys. Rev. 60, 308 (1941).

[5] C. P. Bhalla and M. E. Rose, Oak Ridge National Laboratory Report No. ORNL 3207, 1961 (unpublished).

[6] H. J. Fischbeck and R. G. Wilkinson, Phys. Rev. 120, 1762 (1960).

[7] R. F. Petry, Ph.D. thesis, Indiana University, 1963 (unpublished).

[8] M. Morita and R. S. Morita, Phys. Rev. 109, 2048 (1958).

[9] H. J. Fischbeck and R. W. Newsome, Jr., Phys. Rev. 133, B273 (1964).

[10] The over-all sign of the a_{26} coefficient given in the Appendix of Paper I (see Ref. 1) is incorrect. All signs should be reversed. The role of β^+ and β^- should be reversed in the discussion of the charge of the daughter nucleus on page 659. On page 665, the equation should read $C_A = 3.6 \times 10^{-12}$. An error in our original computer program makes the numerical values of the Au^{198} matrix elements reported in Paper I incorrect.

[11] J. Damgaard and A. Winther, Phys. Letters 23, 345 (1966).

[12] R. M. Steffen, Phys. Rev. 124, 145 (1961).

[13] S. K. Mitra, Nucl. Phys. 47, 293 (1963).

[14] J. W. Sunier, Helv. Phys. Acta 36, 429 (1963).

[15] S. T. Hsue, L. M. Langer, S. M. Tang, and D. A. Zollman, Nucl. Phys. 73, 379 (1965).

[16] P. Alexander and R. M. Steffen, Phys. Rev. 124, 150 (1961).

[17] D. C. Camp, L. G. Mann, and S. D. Bloom, Nucl. Phys. 73, 174 (1965).

[18] M. J. Canty, W. F. Davidson, and R. D. Conner, Nucl. Phys. 85, 317 (1966).

[19] T. Nagarajan, M. Ravindranath, and K. Venkata Reddy, Nucl. Phys. A134, 433 (1969).

[20] S. K. Bhattacherjee and S. K. Mitra, Phys. Rev. 126, 1154 (1962).

[21] P. Alexander and R. M. Steffen, Phys. Rev. 128, 1783 (1962).

[22] H. Dulaney, C. H. Braden, and L. D. Wyly, Phys. Rev. 117, 1092 (1960).

[23] J. W. Sunier, P. Debrunner, and P. Scherrer, Nucl.

Phys. 19, 62 (1960).
[24]L. M. Langer and D. R. Smith, Phys. Rev. 119, 1308 (1960).
[25]J. Berthier, R. Lombard, and J. W. Sunier, Compt. Rend. 252, 257 (1961).
[26]M. Trudel, E. E. Habib, and H. Ogata, Phys. Rev. C 1, 643 (1970).
[27]H. Dulaney, C. H. Braden, E. T. Patronis, and L. D. Wyly, Phys. Rev. 129, 283 (1963).
[28]L. Grenacs, R. Hess, and F. C. Rohmer, Helv. Phys. Acta 8, 374 (1965).
[29]F. T. Porter, M. S. Freedman, T. B. Novey, and F. Wagner, Phys. Rev. 103, 921 (1956).
[30]M. Delabaye, Ph.D. thesis, Louvain University, 1965 (unpublished).
[31]W. D. Brewer and D. A. Shirley, Nucl. Phys. A149, 392 (1970); J. D. Brewer, private communication.
[32]S. Andre and P. Liaud, Nucl. Phys. A121, 337 (1968).
[33]S. Y. Van der Werf, H. deWaard, and H. Beekhuis, Nucl. Phys. A134, 215 (1969).
[34]L. D. Wyly, C. H. Braden, and H. Dulaney, Phys. Rev. 129, 315 (1963).
[35]K. Runge, Z. Physik 183, 184 (1965).
[36]D. Pfeifer and K. Runge, Z. Physik 183, 195 (1965).
[37]H. Behrens and D. Bogdan, Nucl. Phys. A143, 468 (1970).
[38]E. H. Spejewski, Nucl. Phys. 82, 481 (1966).
[39]A. deBeer, J. H. Stainvenberg, B. Meindersma, and J. Blok, Nucl. Phys. A138, 457 (1969).
[40]P. C. Simms, A. Namenson, T. H. Wei, and C. S. Wu, Phys. Rev. 138, B777 (1965).
[41]L. M. Langer, E. H. Spejewski, and D. E. Wortman, Phys. Rev. 133, B1145 (1964).
[42]F. Boehm and J. Rogers, Nucl. Phys. 45, 392 (1963).
[43]P. C. Simms, Phys. Rev. 138, B784 (1965).
[44]F. Gygax and R. Hess, Helv. Phys. Acta 39, 209 (1966).
[45]R. L. Robinson and L. M. Langer, Phys. Rev. 112, 481 (1958).
[46]R. H. Thompson and K. J. Casper, Nucl. Phys. 72, 106 (1965).
[47]J. B. Viano, J. C. Renard, J. Menet, P. de Saintignon, A. Laverne, and P. Depommier, J. Phys. (Paris) 30, 763 (1969).
[48]U. Kneissl, Z. Naturforsch. 20a, 1364 (1965).
[49]The single-particle estimate for $\int B_{ij}/\rho$, $\int \vec{r}/\rho$, $\int \vec{\sigma}\times\vec{r}/\rho$, and $\int \vec{\sigma}\cdot\vec{r}/\rho \sim 1$.
[50]L. S. Kisslinger and C. S. Wu, Phys. Rev. 136, B1254 (1964).
[51]R. A. Amadori and K. S. R. Sastry, Bull. Am. Phys. Soc. 11, 409 (1966).
[52]N. K. Gupta, Ph.D. thesis, University of Massachusetts, Amherst, 1969 (unpublished).
[53]D. Bogdan and P. Lipnik, Nuovo Cimento 52B, 273 (1967).
[54]J. C. Manthuruthil and C. P. Poirier, private communication.
[55]C. J. Cipolla and R. M. Steffen, Nucl. Phys. A143, 404 (1970).
[56]J. I. Fujita, Phys. Rev. 126, 202 (1962).
[57]D. Bogdan, C. Protop, and I. Vata, Nucl. Phys. A119, 113 (1968).
[58]S. Wahlborn, Nucl. Phys. 58, 209 (1964).
[59]I. V. Kopytin and I. S. Batkin, Yadern. Fiz. 11, 342 (1970) [transl.: Soviet J. Nucl. Phys. 11, 192 (1970)]; I. S. Batkin and I. V. Kopytin, Yadern. Fiz. 11, 770 (1970) [transl.: Soviet J. Nucl. Phys. 11, 430 (1970)].
[60]T. Ahrens and E. Feenberg, Phys. Rev. 86, 64 (1952).
[61]S. A. Fayans and V. A. Khodel, Phys. Letters 30B, 5 (1969).
[62]H. Ejiri, P. Richard, S. Ferguson, R. Heffner, and D. Perry, Phys. Rev. Letters 21, 373 (1968).
[63]D. D. Hoppes, E. Ambler, R. W. Hayward, and R. S. Kaeser, Phys. Rev. Letters 6, 115 (1961).
[64]H. Beekhuis and R. J. Van Duinen, Nucl. Phys. A108, 382 (1968).