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# Comparison of the Nuclear-Reaction Energy Scale with the Gamma-Ray Energy Scale*

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The excitation energy of the first excited state of $^{24}$Mg was measured with inelastic deuteron scattering using the energy of $\alpha$ particles from $^{210}$Po as a standard. The result is 1368.2 $\pm$ 0.5 keV. The average of recent measurements of the energy of $\gamma$ rays emitted by this state gives an excitation energy of 1368.67 keV based on the electron rest-mass energy. Thus the widely used polonium calibration energy is found to be consistent with the $\gamma$-ray energy standard to within 0.04%.

## I. INTRODUCTION

The purpose of this work is a direct comparison of the energy scale used for $\gamma$ rays with one widely used for particle energies. $\gamma$-ray energies are based on the energy equivalent of the electron mass, usually through the energy of the $^{198}$Hg(0.411 MeV) decay, whereas particle energies are based on decay energies of radioactive $\alpha$ emitters (usually $^{210}$Po), or threshold or resonance energies. These scales are presumed to be based on absolute measurements and are usually assumed to be consistent. There is, of course, much indirect evidence that they are consistent, because energy-level differences measured with particle reactions generally agree with those deduced from the energy of the $\gamma$-ray transition between the levels. Caution must be used in making such comparisons because often the $\gamma$-ray energies are based on assumed energy-level differences which have in turn been measured against $^{210}$Po, and often it is not clear what fundamental energy scale has been used in reported measurements. For these reasons it was considered desirable to make a direct comparison of the $^{210}$Po $\alpha$ energy with the $^{198}$Hg energy. Of course the $^{210}$Po $\alpha$ energy has been measured[1] absolutely and the $^{198}$Hg $\gamma$ ray is presumably known to very high accuracy[2] in terms of the fundamental constants. A direct comparison is then, in a sense, a check on the accuracy of these measurements, but the present interest is verification of the consistency of energy standards used for nuclear energy measurements.

Previous work in this laboratory[3] has shown that the $^{210}$Po $\alpha$ energy agrees with the $^7$Li$(p, n)^7$Be threshold energy commonly used and with the absolute determination of the RaC′ decay energy. Some question remained, however, about the comparison of the $\gamma$-ray energy scale and the $^7$Li$(p, n)$-$^7$Be threshold. A comparison of the energy scales may be made by measuring the excitation of a nuclear state with charged-particle analysis in terms of the $^{210}$Po energy and measuring the decay $\gamma$-ray energy in terms of the $^{198}$Hg decay energy.

For a precise comparison with the $^{210}$Po energy an excitation energy as near 5.3 MeV as possible is desired, but $\gamma$-ray measurements become difficult above 1 MeV. A good compromise may be made using the $^{24}$Mg state at 1.368 MeV. The $\gamma$-ray transition to the ground state has been very carefully measured in two laboratories. We summarize these measurements and then discuss our measurement of the excitation energy using charged-particle reactions.

In the accurate $\gamma$-ray energy measurements a secondary standard, namely a transition energy in $^{198}$Hg, was used. As the $^{198}$Hg transition was in turn measured[2] against the annihilation radiation, both measurements tie the $^{24}$Mg $\gamma$-ray energy to the electron rest mass. Changes in the fundamental constants of physics will change the resulting value but the changes will probably be less than 0.1 keV, which is smaller than the uncertainties in the present particle energy measurements.

External conversion was used with the Chalk River iron-free $\beta$ spectrometer by Murray, Graham, and Geiger[4] to compare the $^{24}$Mg(1.37 MeV) $\gamma$-ray energy, directly and through four "secondary calibration standard" $\gamma$ rays from $^{208}$Pb and $^{60}$Ni. The resulting level energy is found to be 1368.568 $\pm$ 0.044 keV. A curved-crystal spectrometer was used by Reidy[5] at Michigan to obtain the ratio of the $^{24}$Mg $\gamma$-ray energy to that of $^{198}$Hg. This measurement gave a level energy of $1368.77 \pm 0.11$ keV. Thus the measurements from the two laboratories agree within 0.2 keV. The unweighted average is 1368.67 with an uncertainty small compared to the present measurements.

## II. EXPERIMENTAL PROCEDURE

### A. Method

Particle energies were compared with the Notre Dame 50-cm broad-range spectrograph. High-quality deuteron beams were produced by the 4-MV accelerator, and a 7-MeV $^4$He beam was produced by the FN tandem accelerator. Stable operation was maintained for some time before a run was started and no machine parameters were changed during a run, so the energy stability was considerably greater than that calculated from the geometric resolution (0.08% for the deuteron beam).

Thin targets of magnesium, gold, or LiF were set in the reflection position. Polonium sources, prepared in the usual way by immersing a 0.5-mm-diam polished pure silver wire in a PoCl-HCl solution, were mounted on the target holder at the same height as the beam spot on the previously bombarded target, using a scanning microscope.

With the *fixed* magnetic field used, 3.35-MeV deutrons scattered at 116 or 115° from Mg fell near the upper end of the focal surface and deuterons leaving $^{24}$Mg in the 1.37-MeV state fell about $\frac{1}{8}$ of the way up the focal surface. Furthermore, the group of $\alpha$ particles from $^{210}$Po fell at the same position as the elastically scattered group, and deuterons elastically scattered from $^7$Li fell at the position of the inelastic group.

To determine the excitation energy ($E_x$) the three required quantities are the input energy, output energy, and scattering angle. With the simultaneous measurements of the elastic and inelastic groups, the result is insensitive to changes in input energy, relatively insensitive to changes in angle, and depends mainly on the *difference* in energy of the two groups. The crux of the measurements is then to convert the observed distance along the focal surface between the elastic and inelastic groups into an energy difference. By using a fixed magnetic field and placing a calibrating $\alpha$-particle group on the plate for each run, the problem of differential hysteresis in the magnetic field was virtually overcome.

The question remains, however, whether the difference in positions of the two groups is correctly converted to a difference in trajectory radius when the calibration curve obtained by *varying* the magnetic field (to give the $^{210}$Po $\alpha$ particles different trajectories) is used for a *fixed* field measurement. This is always assumed to be the case but we attempted to verify it to more than the usual precision by recording the deuteron group scattered from $^7$Li in each run.

Input energies calculated from these $^7$Li elastic groups disagreed slightly with those calculated from the $^{24}$Mg elastic group. Measurements of groups elastically scattered from other light elements indicated, however, that most if not all of this deviation arose from slight differences of the observation angle from the nominal angle. Targets with mass numbers ranging from 6 to 197 were used to scatter 2.5-MeV deuterons at 90°. The group from $^{197}$Au then fell near the plate position used for the elastically scattered particles in the data runs, whereas the group scattered from $^6$Li fell at the position of the inelastic group from the data runs. The results shown below demonstrate the validity of the calibration and justify the assumption of slight shifts in the scattering angle.

Runs were taken over a period of more than two years. During this time the spectrograph was completely realigned. The basic plate was extended so that the instrument could be used with the FN tandem beams as well as the 4-MV beam. The base plate and target chamber were releveled, a new angle scale inscribed, the second side of the plate holder adjusted to the focal surface, and new calibrations made. The second side of the plate holder was used for three of the runs. It is felt that these procedures have minimized systematic errors.

To guard against the possibility that a sharp resonance in the cross section might distort or displace the particle groups, a yield curve of the elastic and inelastic scattering was taken over a range of 40 keV in deuteron bombarding energy centered on the nominal energy used for the data runs.

The excitation energy was also measured using inelastic $\alpha$-particle scattering. One run was made with 7.08-MeV $\alpha$ particles at an observation angle of 82°. Under these conditions the inelastic group could be placed at the position of the $^{210}$Po $\alpha$ group. Input energy and angle were obtained by elastic scattering from $^{197}$Au and $^{12}$C. A second run with 5.48-MeV bombarding energy placed the elastic group from $^{197}$Au at the position of the $^{24}$Mg elastic group in the first run and the elastic group from $^{24}$Mg at the position of the inelastic group from the first run. Again energy and angle were obtained from the $^{197}$Au and $^{12}$C groups.

### B. Data Analysis

The fixed field used for all exposures in a given run was calculated from the observed position of the $\alpha$-particle group from the polonium source, using the most recent calibration to obtain the trajectory radius ($\rho$) from the position and using a value of 331.767 kG cm for the magnetic rigidity ($B\rho$) of the $^{210}$Po $\alpha$ particles. This field, which was found to agree with the NMR fluxmeter reading to within 0.05%, was used to find the $B\rho$ values of all particle groups. Energies were calculated from these $B\rho$ values using tables based on fundamental constants.[6] A bombarding energy and observation angle were then found which were consistent with the energy of deuterons scattered from both $^{24}$Mg and $^{7}$Li. Finally, these values were used together with the energy of the inelastically scattered deuterons to calculate the $Q$ value for the $^{24}$Mg$(d, d')^{24}$Mg*(1.37-MeV) reaction. In this calculation the relativistic mass of the excited $^{24}$Mg was used for the recoil mass. The negative of this $Q$ value is the desired excitation energy based on the energy of $\alpha$ particles from $^{210}$Po.

An alternative calculation was made to show that the method of calculating a magnetic field from the position of the calibration group properly corrected for differential hysteresis effects. In the alternate method the field ($B$) determined by the NMR fluxmeter was used and a difference in trajectory radius ($\Delta\rho$) calculated from the radius found from the calibration table, using the observed position of the given calibration groups and the radius found by dividing $B\rho$ for $^{210}$Po $\alpha$ particles by $B$. This $\Delta\rho$ was then used to correct the $\rho$ values for each deuteron group. Again, input energy and scattering angle were found from the $^{24}$Mg and $^{7}$Li elastic groups. Experience has shown that in recalibration the shape of the $\rho$-vs-position curve is generally maintained but the entire curve shifts by some $\Delta\rho$. The two methods of calculation gave identical results for the average of the 10 deuteron runs.

### III. RESULTS AND UNCERTAINTIES

The yield of elastically scattered deuterons over the range of 14 keV of bombarding energy used in the excitation-energy runs varied by ±3%. The yield of inelastically scattered deuterons varied by ±20%. The yield curves were extended over a 40-keV range but no large variations were found.

Figure 1 shows the data demonstrating that the calibration obtained with the $^{210}$Po $\alpha$ particles and varying field holds for a constant field. The points at plate distance 11 and 70 cm represent deuterons scattered from $^{6}$Li and $^{197}$Au, respectively. The polonium calibration was used to calculate a bombarding energy and scattering angle from the positions of these groups. Then the expected trajectory radii for groups scattered from $^{7}$Li, $^{12}$C, $^{16}$O, $^{19}$F, and $^{24}$Mg at this bombarding energy and angle were calculated. The differences between these calculated radii and those given by the calibration curve are plotted in the figure. The maximum deviation from the calibration curve was 0.009 cm. 11 of the 14 points lie within 0.004 cm, and the average deviation is 0.002 cm. This is of the order of 0.01% of the radius and is much smaller than the usual variations between calibration runs. The average scattering angle calculated from these runs was 6.5′ less than the nominal angle, suggesting that part of the 12′ average difference from nominal angle in the data runs arose from a zero offset.

An uncertainty of 0.007 cm in $\rho$ was assigned for each of the calibrations and an uncertainty of 5′ in the scattering angle. The latter is consistent with the fluctuations of +3.2′ to −4.8′ about the average value for the data runs.

The difference between the magnetic fields measured with the fluxmeter and those calculated from the position of the $^{210}$Po $\alpha$ groups in nine runs ranged from +0.75 to −3.41 G with an average of 1.5 G. In one run the difference was 14.6 G, which seems to indicate an error in the setting of this field. An uncertainty of 1.5 G was assigned to the field for a given run.

Because the deuterons elastically and inelastically scattered from $^{24}$Mg were simultaneously recorded, the measured excitation energy is insensitive to changes in input energy. The assigned uncertainty of 4 keV gives a negligible change in $E_x$.

Even though the Po source was positioned on the measured beam spot for each run an uncertainty of 0.025 cm was assigned to the position of the spectrograph object.

The largest known uncertainty comes in the determination of the difference in trajectory radii for the elastic and inelastic groups. A value of 0.015 cm was assigned to this.

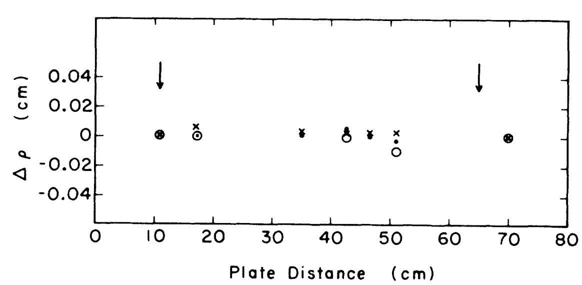


FIG. 1. Difference in trajectory radii obtained from the calibration curve and from elastic scattering of 2.5-MeV deuterons. The dots, crosses, and circles represent three different runs. Input energy and angle were adjusted to place the extreme points on the axis. Arrows show approximate plate positions for groups from data runs.

TABLE I. Assigned uncertainties and resulting variation in excitation energy.

| Quantity | Uncertainty | $\Delta E_x$ (keV) |
|---|---|---|
| Trajectory radius | 0.15 mm | 1.1 |
| Magnetic field | 1.5 G | 0.7 |
| Object position | 0.25 mm | 0.6 |
| Calibration | 0.5 keV | 0.4 |
| Reaction angle | 5′ | 0.2 |
| Input energy | 4 keV | 0.03 |

Table I displays the assigned uncertainties discussed above and the resulting uncertainties in $E_x$. Adding these in quadrature gives 1.5 keV as the internal error ($\Delta E_i$) for a given run.

The average value found for the excitation energy from 10 deuteron runs is 1368.2 keV and the standard deviation of the mean is 0.2 keV. The one run made with 7.0-MeV $^4He^{++}$ particles gave an excitation energy of $1369.1 \pm 1.5$ keV in excellent agreement with the more precise deuteron measurements. Thus our final value is 1368.2 $\pm$ 0.5 keV for the excitation energy of the $^{24}Mg$ first excited state, based on a value of 331.767 kG cm for the magnetic rigidity of the $\alpha$ particles from $^{210}Po$.

The results of these measurements agree with the $\gamma$-ray energies measured on what amounts to an absolute energy scale. This work then provides a direct verification of the consistency of the energy scale most frequently used for charged-particle measurements with that used for $\gamma$-ray measurements. The precision is better than 0.04% and corresponds to less than 2 keV in the $^{210}Po$ $\alpha$ energy. As accurate values for higher-energy $\gamma$ rays become available it may be possible to make an even more precise comparison of the energy scales.

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