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PHYSICAL REVIEW C VOLUME 4, NUMBER 3 SEPTEMBER 1971

Spin Assignments in ^{20}F †

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Comparison of the results of the ^{18}O(^{3}He,p), ^{19}F(d,p), and ^{22}Ne(d,α) reactions leads to an assignment of $J^\pi = 1^+$ for the state at $E_x = 3.489$ MeV in ^{20}F. These results, together with previous information, lead to the identification of the 3.531-MeV state as the only 0^+ state predicted below 6 MeV. Recent information for the low-lying levels of ^{20}F is reviewed and compared with predictions of recent shell-model calculations.

I. INTRODUCTION

When the present studies were begun, roughly 10 years of experimental study[1-5] of ^{20}F had led to definite spin-parity assignments for only two states in ^{20}F. A study[2] of the γ-ray circular polarization following the β decay of the ground state of ^{20}F to the 2^+, first excited state of ^{20}Ne led to the assignment[2] $J^\pi = 2^+$ for the ^{20}F g.s. The β decay[1] of the ground state of ^{20}O to a state at 1.058 MeV yielded the assignment[1] $J^\pi = 1^+$ for the 1.058-MeV level of ^{20}F.

Many of the other levels of ^{20}F below $E_x = 4.3$ MeV have had limits placed on their spins and parities, mainly by angular correlation studies[3,4] and γ-decay systematics.[5] But at the time the present studies were begun, only the two states mentioned above had unique J^π assignments. During the course of the present investigations, another unique assignment was made. The 0.657-MeV state had been assigned $J = 1$ or 3 from ^{18}O-(^{3}He, $p\gamma$) angular correlation studies.[3,4] Furthermore, it was observed to be populated via $l = 2$ in the ^{19}F(d,p)^{20}F reaction.[6] Since the ground state of ^{19}F has $J^\pi = \frac{1}{2}^+$, this $l = 2$ assignment implied $J^\pi = 1^+$ or 2^+ if the neutron transfer was $d_{3/2}$ and $J^\pi = 2^+$ or 3^+ if the transfer was $d_{5/2}$. Finally, populating this state in the ^{19}F($\vec{d}$, p)^{20}F reaction with polarized deuterons established[7] that the reaction proceeded via $d_{5/2}$ transfer. Thus, the 0.657-MeV state of ^{20}F has[7] $J^\pi = 3^+$.

These known spins and parities are consistent with a deformed (prolate) picture of ^{20}F and with detailed shell-model calculations in a full s-d shell basis.[8] In the deformed picture, the lowest states arise from an odd neutron in the $K^\pi = \frac{3}{2}^+$ Nilsson orbit and the odd proton in the $K^\pi = \frac{1}{2}^+$ Nilsson orbit. These configurations then lead to a $K^\pi = 2^+$ rotational band which begins with the ground state, and a $K^\pi = 1^+$ band which supposedly starts with the 1.058-MeV state. The other available information for ^{20}F does not yet allow one to judge which of these two models gives better overall agreement with experiment. Perhaps when more experimental information is available, such a test can be made. [The in-band $B(E2)$ values would not appear to yield a sensitive test, since the shell model and rotational model give very similar $B(E2)$ predictions for in-band transitions.[8]]

Since the results of the present investigation will be compared below with recent shell-model calculations,[8] perhaps a brief discussion of those calculations is appropriate. A full description is contained in Ref. 8. These calculations were performed in a full s-d shell basis. The $A - 16$ nucleons were distributed among the $1d_{5/2}$, $2s_{1/2}$, and $1d_{3/2}$ orbitals. A number of realistic interactions were used,[8] four of which have been selected here for comparison.

The K+17O calculations[8] used a realistic effective Hamiltonian derived from the Hamada-Johnston potential by Kuo, and used by Kuo in calculations of ^{18}O, ^{18}F. (See Ref. 8 for further details.) In this interaction, the three single-particle energies were taken from the spectrum of ^{17}O: −4.15 MeV for $1d_{5/2}$, −3.28 MeV for $2s_{1/2}$, and +0.93 MeV for $1d_{3/2}$. The calculation contains 63 two-body matrix elements.

The K+12FP calculations[8] used the same Hamiltonian, but with twelve free parameters – the three

single-particle energies and nine two-body matrix elements involving only the $1d_{5/2}$ and $2s_{1/2}$ orbitals. All other two-body matrix elements were held fixed at the Kuo values.

The RIP, or "radial integral parametrization,"[8] contained 14 free parameters – the three single-particle energies, plus eleven linearly independent combinations of radial Talmi integrals.

The MSDI, or "modified surface-δ interaction,"[8] contained seven free parameters – the three single-particle energies together with surface-δ strengths for $T=0$ and $T=1$, and monopole strengths for $T=0$ and $T=1$.

In the MSDI calculation, the free paremeters were obtained[8] by minimizing the rms deviation to 41 pieces of experimental information for $A=17$ to 22: 29 excitation energies and 12 ground-state binding energies (with respect to ^{16}O). The RIP calculations fitted[8] these 41 pieces of information, together with the excitation energy of the 6^+ and 8^+ states in ^{20}Ne. In the K+12FP calculations 44 pieces of experimental information were fitted[8] – the above 43, together with the excitation energy of the first 5^+ level in ^{22}Na. Again, the reader is referred to Ref. 8 for full details.

Of these four interactions, the K+17O and K+12FP give a reasonably good over-all fit to the majority of available data in the $A=18-22$ nuclei.[8]

These shell-model calculations[8] for ^{20}F predict only one 0^+ state below an excitation energy of 5.9 MeV. For a variety of interactions, this 0^+ state is predicted[8] to lie between 2.0 and 3.5 MeV in excitation. The theoretical excitation energies for low-lying 0^+ and 1^+ states are summarized in Table I.

The lower-lying 0^+ state is predicted[8] to be populated with a large $l=0$ spectroscopic factor ($S_{th} \approx 0.6$) in the ^{19}F$(d,p)^{20}$F reaction. It is expected[8] to lie within 0.5 MeV of a 1^+ state which is also predicted[8] to be populated strongly via $l=0$ in (d,p). In a recent 16-MeV study of the ^{19}F(d,p) reaction,[6] four states below an excitation of 4.3

TABLE I. Predicted excitations of 0^+ and 1^+ states in ^{20}F. Excitation energies (MeV) for interactions which are described briefly in the text and fully in Ref. 8.

J^π	K+17O	K+12FP	RIP	MSDI
0^+	2.68	3.33	2.48	2.07
	5.91	6.06	6.63	6.80
1^+	0.92	1.34	0.65	0.52
	2.47	3.29	1.84	2.20
	3.75	4.09	4.38	3.57
	4.31	4.69	5.65	4.08

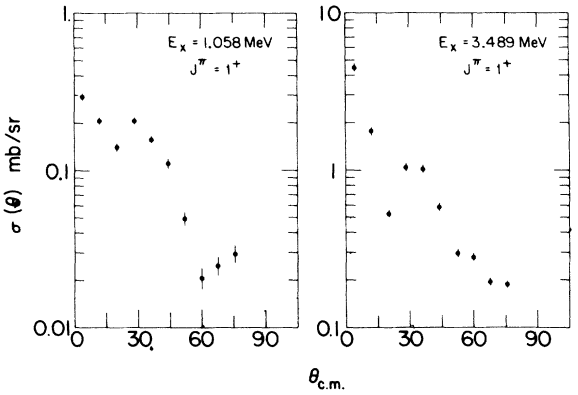

FIG. 1. Angular distributions for the reaction ^{18}O$(^3$He$,p)^{20}$F, at $E_{^3\text{He}}=18.0$ MeV, leading to a known 1^+ state at 1.058 MeV and to the state at 3.489 MeV which is assigned $J^\pi=1^+$ in the present work.

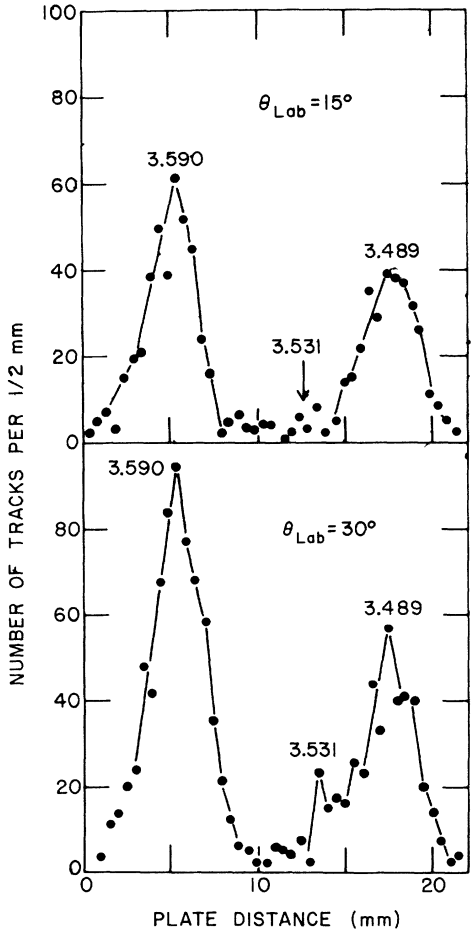

FIG. 2. Partial spectra for the reaction ^{22}Ne$(d,\alpha)^{20}$F obtained at $\theta_{lab}=15°$ (top) and 30° (bottom). The bombarding energy was 10.0 MeV. The weakness of the 3.531-MeV state relative to the 3.489-MeV state is apparent.

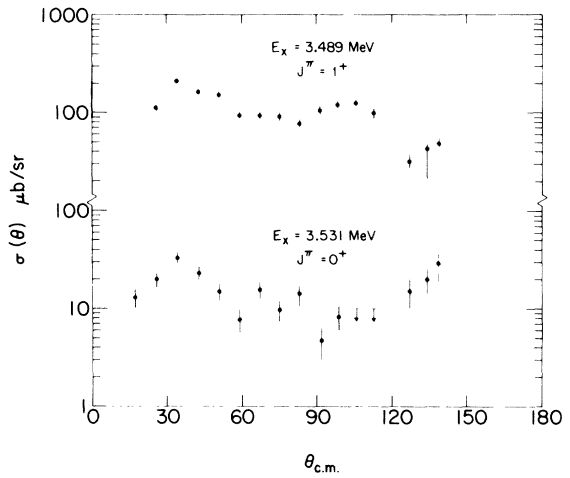

FIG. 3. Angular distributions for the reaction ^{22}Ne-$(d,\alpha)^{20}$F leading to the 3.489- and 3.531-MeV states in ^{20}F. The bombarding energy was 10.0 MeV. The two downward-pointing arrows in the 3.531-MeV angular distribution near $\theta_{c.m.}=110°$ reflect the fact that only an upper limit on the cross section was obtained at those angles.

MeV (viz., at $E_x = 1.058$, 3.489, 3.531, and 4.089 MeV) were observed[6] to be populated with $l=0$ components in their angular distributions. A fifth state, at 3.590 MeV, had[6] an apparent, though weak, $l=0$ component in its dominantly $l=2$ angular distribution. Of these five states, the state at 1.058 MeV is known[1] to have $J^\pi = 1^+$, and the state at $E_x = 4.089$ MeV has been assigned[6] 1^+ from its admixed $l=0+2$ transition in (d,p). Furthermore, the large $l=2$ component for the 3.590-MeV state forbids $J=0$ for that state. Thus, only the states at $E_x = 3.489$ and 3.531 MeV remain as candidates for the 0^+ state. The $l=0$ angular distributions observed in the ^{19}F$(d,p)^{20}$F reaction[6] limit the spin and parity of both of these states to 0^+ or 1^+. Since the second 0^+ state is predicted[8] to lie from 2.7 to 4.7 MeV above the first (Table I), it is likely that both the 3.489- and 3.531-MeV states have $J^\pi = 0^+$. Because of the restrictions discussed above, the state that is not 0^+ must be 1^+.

In order to determine which of these two states is the 0^+ state and which is the 1^+, we have performed the reactions ^{18}O(^{3}He, p)^{20}F and ^{22}Ne(d,α)-^{20}F. In a direct ^{18}O(^{3}He, p) reaction, a 0^+ state will be populated by a pure $L=0$ transition. $J^\pi = 1^+$ final states, however, may be populated by both $L=0$ and 2 ^{18}O(^{3}He, p) transitions. Hence, an admixed $L=0$ and 2 transition to a given state unambiguously identifies that state as having $J^\pi = 1^+$. Furthermore, a recent study of (^{3}He, p) reaction systematics[9] indicates that in the $A=18$ to 30 mass region all known $0^+ \rightarrow 1^+$ transitions exhibit an admixed $L=0+2$ angular distribution, rather than either pure $L=0$ or pure $L=2$.

On the other hand, a 0^+ state in ^{20}F cannot be directly excited by the ^{22}Ne(d,α) reaction, since the ground state of ^{22}Ne, the α particle, and the deuteron have spins and parities 0^+, 0^+, and 1^+, respectively. Even if the (d,α) reaction proceeds via a compound mechanism, 0^+ states would be more weakly populated than 1^+ states at similar excitation energies, because of the density-of-states factor.

II. RESULTS

The experiments were performed using a multiangle magnetic spectrograph and 18-MeV ^{3}He^{++}

TABLE II. States in ^{20}F which have unique spin and parity assignments.

E_x[a] (MeV)	J^π	Remarks	Theoretical E_x[b] (MeV) K+17O[b]	K+12FP[b]
0.0	2^+	$\beta\gamma$ circular polarization in decay to ^{20}Ne(first excited state) (Ref. 2)	0	0
0.657	3^+	$l=2$ in ^{19}F$(d,p)^{20}$F (Ref. 6) $J=1$ or 3 from ^{18}O(^{3}He, $p\gamma$)^{20}F (Refs. 3, 4) given $l=2$, ^{19}F$(\vec{d},p)^{20}$F implies $d_{5/2}$ transfer (Ref. 7)	0.61	0.92
1.058	1^+	β decay of ^{20}O (Ref. 1)	0.92	1.34
3.489	1^+	Pure $l=0$ in ^{19}F$(d,p)^{20}$F (Ref. 6) Admixed $L=0+2$ in ^{18}O(^{3}He, p)^{20}F (present work) Strong in ^{22}Ne$(d,\alpha)^{20}$F (present work)	2.47	3.29
3.531	0^+	Pure $l=0$ in ^{19}F$(d,p)^{20}$F (Ref. 6) Very weak in ^{22}Ne$(d,\alpha)^{20}$F (present work) γ decays only to a known 1^+ state (Ref. 5)	2.68	3.33
4.089	1^+	Mixed $l=0$ and $l=2$ in ^{19}F$(d,p)^{20}$F (Ref. 6)	3.75	4.09

[a] Excitation energies are from Ref. 6.

[b] From Ref. 8; interactions are fully described in Ref. 8.

and 10-MeV deuteron beams from the University of Pennsylvania tandem Van de Graaff accelerator. The ^{18}O target was prepared by oxidizing a Ca foil in water enriched to 97.4% in ^{18}O. The target thickness was approximately 30 μg/cm^2. The ^{22}Ne target consisted of Ne gas, enriched to 99.6% in ^{22}Ne, contained in a differentially pumped gas target.[10] The use of 185 μg/cm^2 Parylene[11] C exit windows allowed sufficient strength to maintain a cell pressure of 19.5 Torr (corresponding to a target thickness of 22.7 μg/cm^2) without seriously affecting the resolution of the exiting α particles. The experimental resolution obtained in the (d, α) study was approximately 24 keV full width at half maximum.

Figure 1 displays the angular distribution corresponding to the $(^3\mathrm{He}, p)$ transition to the 3.489-MeV state, together with that of the transition to the known[1] 1^+ state at 1.058 MeV. The absence of deep minima near 50° observed in $L=0$ transitions in this mass region is evidence for an $L=2$ component in the transitions. (The difference in the shapes of the two angular distributions at forward angles is due to the different relative amounts of $L=0$ and $L=2$ in the two distributions.) Thus the level at 3.489 MeV can be assigned spin and parity 1^+, and the state at 3.531 MeV remains as the only candidate for the low-lying 0^+ state. Unfortunately, the weakness of its cross section prevented the extraction of an angular distribution for the state at 3.531 MeV. This weakness of the $(^3\mathrm{He}, p)$ transition to the 0^+ state is not inconsistent with the suggested configuration of that state.[12] In addition, the experimental cross sections are proportional to $(2J_f+1)$ – leading to an extra factor-of-three reduction in the cross section of a 0^+ final state relative to a 1^+ final state. Thus, it is within expectations for the 0^+ state to be much weaker than the 1^+ state.

Perhaps even more convincing evidence of the $0^+, 1^+$ identifications is available from the (d, α) study. Figure 2 shows the portion of the (d, α) spectrum which is of interest. At forward angles, the state at 3.489 MeV is populated about 20 times more strongly than the 3.531-MeV state. As discussed above, the *direct* population of a 0^+ state by the ^{22}Ne(d, α) reaction is forbidden. Additional evidence for the $0^+, 1^+$ identifications comes from the (d, α) angular distributions for these two states, which are shown in Fig. 3. The angular distribution leading to the 3.531-MeV state exhibits near symmetry about 90°, with no forward peaking observed – both features of a compound process. The angular distribution of the state at 3.489 MeV, on the other hand, has a larger cross section at forward angles than at backward angles, with some indication of diffraction-like structure at forward angles. Both of these latter features are indicative of some direct component in the 3.489-MeV transition, along with the usual compound component.

In summary, two states (at $E_x = 3.489$ and 3.531 MeV) which were populated with large $l=0$ spectroscopic factors in the ^{19}F(d, p) reaction[6] are the only candidates for the sole 0^+ state predicted[8] to lie below 5.9 MeV in ^{20}F. The 3.489-MeV state has been observed to be populated via a mixture of $L=0$ and $L=2$ in the ^{18}O$(^3\mathrm{He}, p)$ reaction, and to be populated strongly in the ^{22}Ne(d, α) reaction with a possible $L=0$ direct-reaction component in its angular distribution. The 3.531-MeV state was weak in both reactions and was apparently formed via a compound process in the ^{22}Ne(d, α) reaction. These results allow the assignments $J^\pi = 1^+$ for the 3.489-MeV state and $J^\pi = 0^+$ for the 3.531-MeV state. These two states are most likely the 0^+ and 1^+ states which are predicted to lie within 40 keV of each other (K+12FP interaction in Table I) and to have similar configurations.

The present assignments bring to six the number of states in ^{20}F that have unique spin and parity assignments. These are listed in Table II, along with evidence[1-7] for those assignments. For all these states, a correspondence to a probable theoretical counterpart is indicated.

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PHYSICAL REVIEW C VOLUME 4, NUMBER 3 SEPTEMBER 1971

Reaction Matrix Elements and Structure Calculations with the Yale and Reid Potentials for the Ni Region*

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Employing the Yale and soft-core Reid free nucleon-nucleon potentials, the shell-model reaction matrix elements are calculated and tabulated for the Ni region. The matrix elements consist of the bare part and the renormalization effects due to the core polarization. The bare matrix elements are evaluated following the unitary-model approach of Shakin, Waghmare, Tomaselli, and Hull and the renormalization corrections are calculated in the same manner as by Kuo and Brown for a Ni^{56} core. As a test of these matrix elements, spherical shell-model spectra of Ni^{58}, Cu^{58}, Ni^{59}, Cu^{59}, Ni^{60}, Zn^{60}, and Cu^{60} are calculated. Quasiparticle calculations for the odd and even Ni isotopes are also performed. In general the calculated results are in good agreement with each other as well as with the observed data. It is concluded that different potentials which fit the same nucleon-nucleon scattering data give essentially the same results for effective-interaction and nuclear-structure calculations for the Ni region of the Periodic Table.

I. INTRODUCTION

In the last decade considerable effort had been spent in developing improved techniques suitable for calculating the spectra of finite nuclei from first principles using a nucleon-nucleon interaction which fits all the available elastic scattering data and the ground-state properties of the deuteron. One of the more successful approaches has been the one developed by Brueckner for nuclear-matter calculations and adopted by Kuo and Brown[1] for finite nuclei. The lowest-order term in the reaction matrix was first used in an effective-interaction calculation by Dawson, Talmi, and Walecka.[2] In their publications,[1,3] Kuo and Brown have used the excited configurations of the inert core to renormalize the matrix elements. The significance of the core-polarization diagram was first pointed out by Bertsch.[4]

Another approach, which has not received much attention, has been developed by Bell[5] and Villars.[6] In this method one introduces a unitary transformation, which operating on a shell-model wave function has the effect of introducing short-range correlations of the type discussed by Gomes, Walecka, and Weisskopf.[7] These correlations are produced by the Pauli principle as well as by the strongly repulsive character of the short-range part of the two-nucleon force.

Employing the unitary-model operator approach, Shakin, Waghmare, Tomaselli, and Hull[8,9] (SWTH) have calculated the effective interactions for the s-d shell region of the Periodic Table employing the Yale potential.[10] These interactions have been used by Pal and Stamp[11] and numerous other workers in describing the nuclear properties and spectra of the $N=Z$ nuclei. Though the treatment of Kuo and Brown and SWTH look different in appearance, the resulting equations look as if they had come out of a Brueckner treatment.

In the work of SWTH, the treatment of the tensor force also does not differ significantly except that no attempt is made to couple D waves with S waves in the short-range correlated wave function; the coupling is included entirely in the long-range part of the interaction. A special feature of the work of SWTH is the use of pseudopotentials to make the separation method applicable to P waves.

Employing the renormalized matrix elements of Hamada-Johnston (HJ) interaction, shell-model