bit potential from vector-meson exchange with the corresponding NN spin-orbit potential, using the SVH parameters for illustration. For this purpose, we also approximate $m(K^*) \approx m_\omega$ and $P^x_{\Lambda N} = -1$. The ΛN spin-orbit potential is then

$$V^{\Lambda N}_{so}(V) \approx C_\omega(r)\,(-17.8\vec{S}\cdot\vec{L} + 5.58\vec{S}^A\cdot\vec{L})\,, \qquad \text{(i)}$$

whereas the NN spin-orbit potential is then

$$V^{NN}_{so}(V) \approx C_\omega(r)\,(-12.2\vec{S}\cdot\vec{L})\ . \qquad \text{(ii)}$$

We note that the calculated spin-orbit interactions for the ΛN and NN systems have the same sign, contrary to the preliminary indications from the analysis of the Λ binding energies for the p-shell Λ hypernuclei (see Ref. 10).

[52]A. R. Bodmer, Phys. Rev. 141, 1387 (1966).

[53]Such a bound state, whether with $J^* = \frac{1}{2}$ or $J^* = \frac{3}{2}$, would undergo rapid $E1$ γ decay, ${}^5_\Lambda\text{He}^* \to {}^5_\Lambda\text{He} + \gamma$, to the ground state ${}^5_\Lambda$He. Since the energy of this γ ray would be large (of order 3 MeV), this γ-decay process would have a rate much larger than that for ${}^5_\Lambda\text{He}^*$ hypernuclear decay. The dipole moment is due to the ^{4}He charge, and is given by $[2M_\Lambda/(4M + M_\Lambda)]\langle f|\vec{r}|i\rangle$ where i, f denote the ${}^5_\Lambda\text{He}^*$ and ${}^5_\Lambda$He states, respectively, and $\vec{r}$ is the Λ-^{4}He separation vector.

[54]The curve labeled GW in Fig. 4 is for a purely central potential, but the inclusion of a spin-orbit potential still gives large positive values for D for energies 2–10 MeV for all the C_Λ values they consider.

[55]In papers now in preparation, we shall give a detailed discussion of these two questions: (i) the dependence of Λ-^{4}He scattering on the shape of the central potential V_c; and (ii) the approximations involved in the use of the Hartree-Fock method for light nuclear systems, especially for the scattering of a strongly interacting particle by a nucleus.

[56]We do not consider the added possibility of strong mixing between a $T = 0$ unitary singlet and octet member, which would further complicate the scalar-boson situation.

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Second-Class Currents and Analog Processes*

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Semileptonic processes between members of a common isotopic multiplet provide a nearly model-independent test for currents with anomalous or "second-class" G-parity properties. For such processes the implications of the presence of second-class currents are discussed for β decay, muon capture, and elastic neutrino scattering.

I. INTRODUCTION

Recent experiments by Wilkinson and Alburger on β decay rates of mirror transitions[1] have suggested the possibility that the $\Delta S = 0$ semileptonic weak current may contain a component which is anomalous under the G-parity operation.[2] Although it is conceivable that the Wilkinson effect may be due to differences in nuclear wave functions caused by electromagnetic interactions, it is important to determine to what extent such anomalous or "second-class" currents are known to be absent in weak processes. In this regard we have recently suggested analog β decay experiments,[3] since there exist for this case terms in the decay amplitude which can only be produced by a second-class current and which conversely must vanish in the absence of a second-class interaction.[4] Detection of such terms in the decay spectrum would then signal the presence of these currents in the semileptonic weak Hamiltonian.

In A (see Ref. 3) we examined nuclear β decays in which the parent nucleus was unpolarized with both electron and recoil directions being observed, transitions involving a polarized parent with only the final electron observed, and decays from an unpolarized parent into a daughter which subsequently decays electromagnetically, both the electron and photon being observed. The second-class interaction was assumed to involve only the axial current, and conserved vector current (CVC) and time-reversal invariance were assumed throughout.

In this paper we enlarge these considerations to include a more general type of β decay process, and we examine additional analog reactions associated with the semileptonic weak Hamiltonian. In Sec. II we relax the assumption of T invariance and consider the decay of a polarized parent with both electron and recoil observed in order to look for possible T-violating second-class effects as suggested by Kim and Primakoff[5] and also in order to examine additional tests for T-conserving second-class interactions. In Sec. III present experiments on analog muon capture are treated in order to see what limits on second-class terms are currently implied, and new experiments which may help to resolve the situation are suggested. Finally, Sec. IV discusses second-class terms in neutrino scattering on nucleons.

II. ANALOG β DECAY

We consider the reactions

$$\alpha \to \beta + e^+(e^-) + \nu_e\ (\bar{\nu}_e)\ ,$$

where α, β are members of a common isotopic multiplet. Our notation is the same as employed in A, but we review it here for completeness. Let p_1, p_2, p, k denote, respectively, the four momenta of the parent nucleus α, daughter nucleus β, electron, and neutrino, with M_1, M_2 being parent, daughter masses. We define

$$P = p_1 + p_2, \quad q = p_1 - p_2 = p + k\ ,$$
$$M = \tfrac{1}{2}(M_1 + M_2), \quad \Delta = M_1 - M_2\ .$$

The β decay amplitude is assumed to be given by the standard current-current form

$$T = \frac{G_V}{\sqrt{2}} \cos\theta_C \langle\beta|\, V_\mu(0) + A_\mu(0)\,|\alpha\rangle\, l^\mu + \text{H.c.}\ , \qquad (1)$$

where l^μ represents the lepton current

$$l^\mu = \bar{u}_l(p)\gamma^\mu(1+\gamma_5)v_l(k)\ .$$

G_V ($\simeq 10^{-5} m_p^{-2}$) is the weak-coupling constant and θ_C ($\simeq 15°$) is the Cabibbo angle.

If j is the (common) spin of the nuclei, m and m' are the projections of parent and daughter spins in some direction, and J_i are the components of the angular momentum operator in nuclear-spin space, we can write, correct to first order in recoil and assuming CVC

$$\langle\beta|\, V_\mu^{(\dagger)} + A_\mu^{(\dagger)}\,|\alpha\rangle = \frac{1}{2M} aP\cdot l \langle jm'|1|jm\rangle - \frac{1}{4M}\frac{1}{\sqrt{j(j+1)}}\langle jm'|[J_i, J_j]|jm\rangle \times (2b l_i q_j + ic\epsilon_{ij\mu\nu} l^\mu P^\nu \mp id\epsilon_{ij\mu\nu} l^\mu q^\nu), \qquad (2)$$

where the upper (lower) sign preceding d refers to electron (positron) decay. According to conventional notation

$$a = g_V M_F\ ,$$
$$c = g_A M_{GT}\ ,$$
$$b = A[(j+1)/j]^{1/2} M_F \mu_V\ ,$$

where M_F, M_{GT} are the Fermi and Gamow-Teller matrix elements, A is the mass number, and μ_V is the isovector magnetic moment. The form factors a, b, c must arise from first-class currents, while d can only be produced by a second-class term in the interaction when α, β are assumed to be analog states.[4]

Suppose the parent nucleus has net polarization $\mathcal{P} = \langle m\rangle/j$ and orientation parameter

$$\Lambda_j = [-3\langle m^2\rangle/j(j+1)] + 1\ .$$

If

$$E_0 = \Delta \frac{1 + m_e^2/2M\Delta}{1 + \Delta/2M}$$

represents the maximum electron energy, the spectrum in electron and neutrino variables is found to be

$$d\omega = F_\mp(Z,E)\frac{G_V^2\cos^2\theta_C}{(2\pi)^5}(E_0 - E)^2 pE\,dE\,d\Omega_e d\Omega_\nu \Bigg(h_1(E) + \frac{\vec{p}\cdot\hat{k}}{E} h_2(E) + \left[\left(\frac{\vec{p}\cdot\hat{k}}{E}\right)^2 - \frac{1}{3}\frac{p^2}{E^2}\right] h_3(E)$$
$$+ \mathcal{P}\left[\frac{\hat{n}\cdot\vec{p}}{E} h_4(E) + \frac{\hat{n}\cdot\vec{p}}{E}\frac{\hat{k}\cdot\vec{p}}{E} h_5(E) + \hat{n}\cdot\hat{k}\, h_6(E) + \hat{n}\cdot\hat{k}\frac{\hat{k}\cdot\vec{p}}{E} h_7(E) + \frac{\hat{n}\cdot\vec{p}\times\hat{k}}{E} h_8(E) + \frac{\hat{n}\cdot\vec{p}\times\hat{k}}{E}\frac{\hat{k}\cdot\vec{p}}{E} h_9(E)\right]$$
$$- \Lambda_j \Bigg\{ \left(\frac{\hat{n}\cdot\vec{p}\,\hat{n}\cdot\hat{k}}{E} - \frac{1}{3}\frac{\vec{p}\cdot\hat{k}}{E}\right) h_{10}(E) + \left(\frac{\hat{n}\cdot\vec{p}\,\hat{n}\cdot\hat{k}}{E} - \frac{1}{3}\frac{\vec{p}\cdot\hat{k}}{E}\right)\frac{\hat{k}\cdot\vec{p}}{E} h_{11}(E) + \left[(\hat{n}\cdot\hat{k})^2 - \frac{1}{3}\right] h_{12}(E)$$
$$+ \left[\left(\frac{\hat{n}\cdot\vec{p}}{E}\right)^2 - \frac{1}{3}\frac{p^2}{E^2}\right] h_{13}(E) + \hat{n}\cdot\hat{k}\frac{\hat{n}\cdot\vec{p}\times\hat{k}}{E} h_{14}(E) + \frac{\hat{n}\cdot\vec{p}}{E}\frac{\hat{n}\cdot\vec{p}\times\hat{k}}{E} h_{15}(E) \Bigg\} \Bigg)\ , \qquad (3)$$

where E, p are the energy and momentum of the electron, $\hat{k}$ is a unit vector in the direction of the neutrino momentum, $\hat{n}$ represents a unit vector in the direction of polarization, and the $h_i(E)$ are real functions of E and the form factors a, b, c, d:

$$h_1(E) = |a|^2 + |c|^2 - \frac{2E_0}{3M}[|c|^2 \pm \text{Re}\,c^*(b+d)] + \frac{2E}{3M}(3|a|^2 + 5|c|^2 \pm 2\,\text{Re}\,c^*b) - \frac{1}{3}\frac{m_e^2}{ME}[2|c|^2 \pm \text{Re}\,c^*(2b+d)], \qquad (4a)$$

$$h_2(E) = |a|^2 - \tfrac{1}{3}|c|^2 + \frac{2E_0}{3M}[|c|^2 \pm \text{Re}\,c^*(b+d)] - \frac{4E}{3M}(3|c|^2 \pm \text{Re}\,c^*b)\ , \qquad (4b)$$

$$h_3(E) = \frac{E}{M}(-3|a|^2 + |c|^2)\ , \qquad (4c)$$

$$h_4(E)=\left(\frac{j}{j+1}\right)^{1/2}\left[2\,\mathrm{Re}\,a^*c-\frac{E_0}{M}\mathrm{Re}\,a^*(c\pm b\pm d)+\frac{E}{M}\mathrm{Re}\,a^*(7c\pm b\pm d)\right]$$
$$\mp\frac{1}{j+1}\left\{|c|^2-\frac{E_0}{2M}[|c|^2\pm\mathrm{Re}\,c^*(d+b)]+\frac{E}{2M}[5|c|^2\mp\mathrm{Re}\,c^*(d-3b)]\right\}, \tag{4d}$$

$$h_5(E)=\left(\frac{j}{j+1}\right)^{1/2}\left[-\frac{E}{M}\mathrm{Re}\,a^*(5c\mp d\pm b)\right]\pm\frac{1}{j+1}\left\{\frac{E}{2M}[5|c|^2\mp\mathrm{Re}\,c^*(d-b)]\right\}, \tag{4e}$$

$$h_6(E)=\left(\frac{j}{j+1}\right)^{1/2}\left[2\,\mathrm{Re}\,a^*c+\frac{E}{M}\mathrm{Re}\,a^*(5c\mp d\pm b)-\frac{m_e^2}{ME}\mathrm{Re}\,a^*(c\pm b)\right]$$
$$\pm\frac{1}{j+1}\left\{|c|^2-\frac{E_0}{M}[|c|^2\pm\mathrm{Re}\,c^*(d+b)]+\frac{E}{2M}[7|c|^2\pm\mathrm{Re}\,c^*(d+3b)]-\frac{m_e^2}{2ME}[|c|^2\pm\mathrm{Re}\,c^*(d+b)]\right\}, \tag{4f}$$

$$h_7(E)=\left(\frac{j}{j+1}\right)^{1/2}\left[\frac{E_0}{M}\mathrm{Re}\,a^*(c\pm d\pm b)+\frac{E}{M}\mathrm{Re}\,a^*(-7c\mp d\mp b)\right]$$
$$\pm\frac{1}{j+1}\left\{\frac{E_0}{2M}[|c|^2\pm\mathrm{Re}\,c^*(d+b)]-\frac{E}{2M}[7|c|^2\pm\mathrm{Re}\,c^*(d+b)]\right\}, \tag{4g}$$

$$h_8(E)=\mp\left(\frac{j}{j+1}\right)^{1/2}\left[2\,\mathrm{Im}\,a^*c-\frac{E_0}{M}\mathrm{Im}\,a^*(c\pm d\pm b)+2\frac{E}{M}\mathrm{Im}\,a^*(3c\pm b)\right]\mp\frac{1}{j+1}\left[\frac{E_0}{2M}\mathrm{Im}\,c^*(d+b)-\frac{E}{M}\mathrm{Im}\,c^*d\right], \tag{4h}$$

$$h_9(E)=\pm\left(\frac{j}{j+1}\right)^{1/2}\times 6\frac{E}{M}\mathrm{Im}\,a^*c, \tag{4i}$$

$$h_{10}(E)=|c|^2-\frac{E_0}{2M}[|c|^2\pm\mathrm{Re}\,c^*(d+b)]+\frac{E}{M}(3|c|^2\pm\mathrm{Re}\,c^*b), \tag{4j}$$

$$h_{11}(E)=-3|c|^2\frac{E}{M}, \tag{4k}$$

$$h_{12}(E)=\frac{E_0-E}{2M}[|c|^2\pm\mathrm{Re}\,c^*(d+b)], \tag{4l}$$

$$h_{13}(E)=\frac{E}{2M}[|c|^2\pm\mathrm{Re}\,c^*(d-b)], \tag{4m}$$

$$h_{14}(E)=\frac{E_0-E}{2M}\mathrm{Im}\,c^*(d+b), \tag{4n}$$

$$h_{15}(E)=\frac{E}{2M}\mathrm{Im}\,c^*(d-b). \tag{4o}$$

Equation (3) reduces to the results of A when $\hat{k}$ is averaged over or when $\mathcal{P}, \Lambda$ are set to zero, but it also yields new phenomena interesting in their own right. h_8, h_9, h_{14}, h_{15} are seen to be T violating. The latter two cannot contribute to spin-$\frac{1}{2}$ decay ($\Lambda=0$), for which experiments to search for possible time-reversal noninvariance have been conducted.[6] The most recent results on Ne^{19} are about at the level where possible T-violating second-class terms as suggested by Kim and Primakoff ($\mathrm{Im}\,d\neq 0$) might begin to appear.[7] In order to isolate a second-class T-violating term, a measurement of the energy dependence of the asymmetry can be employed. Since present results require $\mathrm{Im}\,a^*c\ll 1$, which implies via the CVC hypothesis that $\mathrm{Im}\,c^*b\ll 1$, the energy-dependent term is essentially determined only by $\mathrm{Im}\,c^*d$. Also measurements of the T-violating correlations involving orientation

$$\hat{n}\cdot\hat{k}\,\hat{n}\cdot\vec{p}\times\hat{k}, \quad \hat{n}\cdot\vec{p}\,\hat{n}\cdot\vec{p}\times\hat{k}$$

are sensitive primarily to $\mathrm{Im}\,c^*d$.

A number of tests for T-conserving second-class terms are seen in addition. If one believes that the Wilkinson effect is completely due to second-class currents, the observed slope of

$$\frac{ft(e^+)}{ft(e^-)}-1 \text{ vs } E_0^+ + E_0^-$$

reveals that $d/Ac\sim -6$ for the nuclei considered in Ref. 1.[8] If one assumes the validity of this result for other nuclei as well, as given by the impulse approximation,[9] and that $b/Ac\sim +4$,[10] then measurement of the sign of

$$cb+cd\sim\begin{cases}+4Ac^2 & \text{if } d=0\\ -2Ac^2 & \text{if } d/Ac=-6\end{cases}$$

is sufficient to suggest the presence or absence of the second-class term. Such a combination appears, for example, in the orientation term $(\hat{n}\cdot\hat{k})^2-\frac{1}{3}$. Unfortunately, it is the combination

$cb-cd$ which appears in the electron orientation terms and which survives when the neutrino is unobserved. For $cb-cd$ the magnitude must be measured in order to draw conclusions about the presence of d. Of course, all such recoil terms are quite small and difficult to detect. However, the careful measurements in the T-violation experiments and in the test of the CVC hypothesis[11] suggest that these effects should be observable.

Finally, in Appendix B we extend the results of A on dipole $\beta\gamma$ correlations to quadrupole $\beta\gamma$ correlations and to p- and d-wave $\beta\alpha$ or βp correlations, which could also be useful in seeking second-class effects, since such correlations are dependent on the combination $cb-cd$.

III. ANALOG MUON CAPTURE

Aside from β decay other opportunities to study properties of the semileptonic weak Hamiltonian come from neutrino scattering and muon capture processes. We first consider the latter. In order to minimize model dependence, we again restrict consideration to cases in which both parent and daughter nucleus in the reaction

$$\mu^- + N_1 \rightarrow \nu_\mu + N_2$$

are members of a common isotopic multiplet. In addition, we must require that nucleus N_1 be stable in order that targets may be constructed. Since all suitable analog ground states except for He^3 and p are unstable e^+ emitters with rather brief lifetimes, it is sufficient to restrict our attention to He^3, p, both of which are $J=I=\frac{1}{2}$. Thus[12]

$$\begin{aligned} T=&\frac{G_V}{\sqrt{2}}\cos\theta_C \bar{u}(k)\gamma^\mu(1+\gamma_5)u(p) \\ &\times\bar{u}(p_2)\Big[g_V(q^2)\gamma_\mu-\frac{i}{2M}\sigma_{\mu\nu}q^\nu g_M(q^2)+g_A(q^2)\gamma_\mu\gamma_5 \\ &-g_P(q^2)q_\mu\gamma_5+\frac{i}{2M}\sigma_{\mu\nu}q^\nu\gamma_5 g_{\rm II}(q^2)\Big]u(p_1)\,, \end{aligned} \tag{5}$$

where

$$\begin{aligned} q^2&=-m_\mu^2\left[1-2\frac{M_2-M_1}{m_\mu}+\frac{M_2-M_1-m_\mu}{M_1+m_\mu}\left(1-\frac{M_2-M_1}{m_\mu}\right)\right] \\ &=\begin{cases}-0.88m_\mu^2 & p-n \\ -0.96m_\mu^2 & He^3\text{-}H^3\end{cases} \end{aligned}$$

is the momentum transfer, and all form factors are real if T-invariance holds.[13] We must for muon capture, of course, include the induced pseudoscalar term $g_P(q^2)$.

In terms of Pauli spinors and in the approximation that the muon is at rest, we have

$$\begin{aligned} T=&\chi_2^\dagger\chi_\nu^\dagger(1-\vec{\sigma}\cdot\hat{k}) \\ &\times[G_V 1\cdot 1'-G_A\vec{\sigma}\cdot\vec{\sigma}'-G_P\vec{\sigma}\cdot\hat{k}\vec{\sigma}'\cdot\hat{k}]\chi_\mu\chi_1, \end{aligned} \tag{6}$$

where the unprimed operators act on the lepton spinors and the primed operators act in the hadron-spin space. Here

$$\begin{aligned} G_V&=g_V(q^2)\left(1+\frac{k_0}{E_2+M_2}\right)-g_M(q^2)\frac{m_\mu k_0}{2M(M_2+E_2)} \\ G_A&=g_A(q^2)+\frac{k_0}{E_2+M_2}g_V(q_2)+g_M(q^2)\frac{k_0}{2M}\left(1+\frac{k_0-m_\mu}{E_2+M_2}\right) \\ &\quad-g_{\rm II}(q^2)\left[\frac{m_\mu-k_0}{2M}-\frac{k_0^2}{2M(E_2+M_2)}\right] \\ G_P&=\frac{k_0}{E_2+M_2}\left[g_A(q^2)-g_V(q^2)-m_\mu g_P(q^2)\right] \\ &\quad-\frac{k_0}{2M}\left\{[g_{\rm II}(q^2)+g_M(q^2)]\left(1+\frac{k_0}{E_2+M_2}\right)\right. \\ &\quad\left.+g_{\rm II}(q^2)\frac{m_\mu}{E_2+M_2}\right\} \end{aligned} \tag{7}$$

with

$$k_0=m_\mu\frac{M_1}{M_1+m_\mu}\left(1+\frac{M_1^2-M_2^2+m_\mu^2}{2m_\mu M_1}\right)$$

being the neutrino energy.

The muon is captured from a $1s$ orbital state and if the initial muonic-nucleus density matrix is

$$\rho=\tfrac{1}{4}$$

so that both muon and nucleus are unpolarized and a statistical mixture of triplet and singlet hyperfine states obtains, we find

$$\Gamma=R[|G_V|^2+2|G_A|^2+|G_A+G_P|^2] \tag{8}$$

with

$$R=\frac{G_V^2\cos^2\theta_C Z_1^3 m_\mu^3\alpha^3}{2\pi^2}C_1(A)\frac{E_2+M_2}{2E_2}\frac{k_0^2}{1+k_0/E_2}\times\left(\frac{M_1}{M_1+m_\mu}\right)^3,$$

where Z_1 is the charge of the parent nucleus, α is the fine-structure constant, and $C_1(A)$ is a correction factor for the nonpoint nature of the nucleus, given by Kim and Primakoff as[14]

$$C_1(p)=1.00 \quad C_1(He^3)=0.965\,.$$

The spin-independent density matrix $-\rho=\frac{1}{4}-$ is known to be an adequate approximation for He^3 capture, but for gaseous hydrogen, where atomic collisions can induce spin exchange, essentially all capture takes place from the hyperfine singlet state so that[15]

$$\rho=\tfrac{1}{4}(1-\vec{\sigma}\cdot\vec{\sigma}')$$

and

$$\Gamma = R\,|G_V + 3G_A + G_P|^2 \ .$$

We may now examine the experimental data on such processes in order to see if second-class currents are consistent with present results. Only one experiment exists in gaseous hydrogen, the others being in liquid, where molecular effects cloud the theoretical interpretation. This experiment yields[16]

$$\Gamma = 651 \pm 57 \ \text{sec}^{-1},$$

which corresponds to $|G_V + 3G_A + G_P|^2 = 23.0 \pm 2.0$ The q^2 dependence of g_V and g_M is given by elastic electron scattering experiments[17]

$$g_V(q^2 = -0.88 m_\mu^2) = 0.98 \ ,$$

$$g_M(q^2 = -0.88 m_\mu^2) = 3.60 \ ,$$

while $g_A(0)$ is known from the ft value in neutron β decay[18]:

$$g_A(q^2 = 0) = 1.23 \pm 0.01 \ .$$

The momentum dependence of g_A is uncertain although elastic neutrino scattering experiments on complex nuclei suggest that[19]

$$\frac{g_A(q^2)}{g_A(0)} = \frac{1}{(1 - q^2/m_A{}^2)^2}$$

is not unreasonable with $m_A \sim 1$ GeV.

The value of $g_P(q^2)$ is even less clear. The only experimental results for $g_P(q^2)$ come from muon capture, but, as seen from Eq. (7), in the presence of a second-class axial current it is not the pseudoscalar coupling constant alone but rather $g_P + (1/m_\mu)g_{\text{II}}$ which is determined in this fashion. The only way in which to unambiguously evaluate g_P is to measure g_{II} independently (e.g., from its effects on the spectrum in polarized $n \to pe^-\bar{\nu}_e$ decay as calculated in A) and to compare this value with $g_P + (1/m_\mu)g_{\text{II}}$ as deduced from muon capture.

Of course, there exists a theoretical prediction for g_P involving partially conserved axial-vector current (PCAC)[20] which yields

$$g_P(q^2) = \frac{2M}{m_\pi^2 - q^2} g_A(q^2) \left\{ 1 - \frac{m_\pi^2}{q^2}\left[1 - \frac{2F_\pi g_{\pi 21}(q^2)}{2Mg_A(q^2)}\right]\right\} . \tag{9}$$

If one assumes, based on the impulse approximation, that[21]

$$1 - \frac{2F_\pi g_{\pi 21}(q^2)}{2Mg_A(q^2)} = 0$$

then

$$\frac{m_\mu g_P(q^2)}{g_A(q^2)} = \frac{2Mm_\mu}{m_\pi^2 - q^2} , \tag{10}$$

which for proton capture becomes

$$m_\mu g_P(q^2 = -0.88 m_\mu^2)/g_A(q^2 = -0.88 m_\mu^2) = 6.8 .$$

Frazier and Kim have suggested a linear extrapolation of $g_{\pi 21}(q^2)$ into the timelike region using the Goldberger-Treiman[22] and mass-shell values as known, which gives

$$m_\mu g_P(q^2 = -0.88 m_\mu^2)/g_A(q^2 = -0.88 m_\mu^2) = 7.2 .^{23}$$

However, this correction is within the 10% uncertainty typically appended to PCAC predictions.

The value of $g_{\text{II}}(q^2)$ is, of course, not known. As discussed in the previous section, the Wilkinson experiments have suggested that[9] $d(0)/Ac(0) \sim -6$. which implies in our case $g_{\text{II}}(q^2)/g_A(q^2) \sim -6$.

Defining $g_{\text{II}}(q^2)/Ag_A(q^2) = \tau$ and $m_\mu g_P(q^2)/Ag_A(q^2) = \sigma$, the experimental results for hydrogen muon capture determine $\sigma + \tau$ as a function of $g_A(q^2)$, as shown in Fig. 1. The value of $g_A(q^2)$ suggested by the most recent neutron-lifetime measurements and by the tentative neutrino scattering results is 1.21, but this should be considered uncertain by at least 5%. Current experiments are seen to be consistent with the PCAC predicition alone, but they do not rule out a sizable negative value for τ as suggested by Ref. 1. More stringent limits on both $g_A(q^2)$ and the capture rate are needed in order to resolve this ambiguity.

For He^3 capture, there exist two recent experimental numbers[24,25]:

$$\Gamma = 1505 \pm 46 \ \text{sec}^{-1},$$

$$|G_V|^2 + 2|G_A|^2 + |G_A + G_P|^2 = 4.82 \pm 0.15 ;$$

$$\Gamma = 1465 \pm 67 \ \text{sec}^{-1},$$

$$|G_V|^2 + 2|G_A|^2 + |G_A + G_P|^2 = 4.69 \pm 0.21 .$$

From elastic electron scattering on tritium and He^3,[26]

$$g_V(q^2 = -0.96 m_\mu^2) = 0.81, \quad g_M(q^2 = -0.96 m_\mu^2) = -14.10,$$

while the value of $g_A(0)$ given by tritium β decay is[27]

$$g_A(0) = -1.215 \pm 0.025 \ ,$$

which gives in the impulse approximation[22]

$$g_A(q^2 = -0.96 m_\mu^2) \cong \frac{g_A^{np}(q^2 = -0.96 m_\mu^2) g_V^{np}(q^2 = 0)}{g_A^{np}(q^2 = 0) g_V^{np}(q^2 = -0.96 m_\mu^2)} \times \frac{g_M(q^2 = -0.96 m_\mu^2)}{g_M(q^2 = 0)} g_A(q^2 = 0) \cong -1.04 .$$

However, it is difficult to assess the accuracy of this prediction.

For $g_P(q^2)$ the impulse approximation, Eq. (10), yields again

$$\tfrac{1}{3} m_\mu g_P(q^2 = -0.96 m_\mu^2) = 6.8 g_A(q^2 = -0.96 m_\mu^2) \ .$$

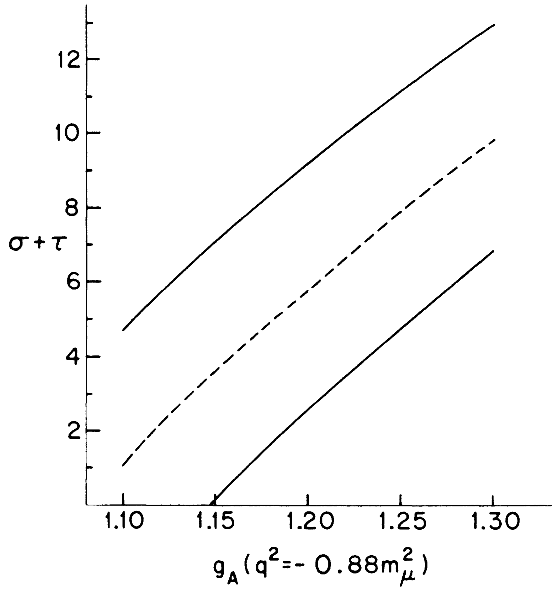

FIG. 1. $\sigma+\tau$ versus $g_A(q^2=-0.88m_\mu^2)$ for muon capture on gaseous hydrogen; $\Gamma=651\pm 57\ \mathrm{sec}^{-1}$.

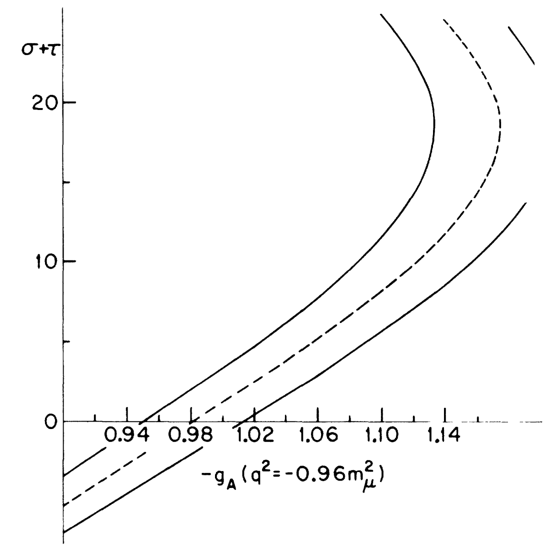

FIG. 2. $\sigma+\tau$ versus $g_A(q^2=-0.96m_\mu^2)$ for muon capture on He^3; $\Gamma=1465\pm 67\ \mathrm{sec}^{-1}$.

Kim and Frazier, based on PCAC and a theoretical value for $g_{\pi H^3 He^3}(q^2=-0.96m_\mu^2)$ suggest that a more realistic estimate is[23]

$$\tfrac{1}{3}m_\mu g_P(q^2=-0.96m_\mu^2)=4.0g_A(q^2=-0.96m_\mu^2)\ .$$

In He^3 capture $\sigma+\tau$ versus $g_A(q^2=-0.96m^2)$ is given in Figs. 2 and 3 for the two experiments. With the assumed value $g_A(q^2)\cong -1.04$ and no second-class currents ($\tau=0$) the impulse-approximation prediction, $\sigma=6.8$, is seen not to be favored, while the Frazier-Kim result, $\sigma=4.0$, is in basic agreement with the data. However, due to uncertainty in both $g_A(q^2)$ and $g_P(q^2)$ a large negative value for τ cannot be ruled out and is even suggested if one accepts the impulse approximation for σ.

If either target or muon is polarized, this uncertainty due to $g_A(q^2)$ can perhaps be reduced. An experimental difficulty in this case is that the spin-orbit interaction partially depolarizes the muon as it cascades down to the $1s$ state from its original atomic capture level, $(n,l)\gg 1$, and both the muon and nucleus may be depolarized via spin-spin forces.[28,29] If there is nuclear polarization $P\hat{n}$ and muon polarization $P'\hat{n}$ when the $1s$ shell is initially reached, then the density matrix at time of nuclear capture may be represented by[30]

$$\rho=\tfrac{1}{4}[1\cdot 1'+\tfrac{1}{2}(P+P')(\vec{\sigma}\cdot\hat{n}+\vec{\sigma}'\cdot\hat{n})+PP'\vec{\sigma}\cdot\hat{n}\vec{\sigma}'\cdot\hat{n}]\ . \tag{11}$$

We find then

$$\frac{d^2\omega}{d\hat{k}}=\frac{R}{4\pi}\{|G_V|^2+2|G_A|^2+|G_A+G_P|^2-2PP'\mathrm{Re}[G_A^*(G_A+G_V+G_P)-\tfrac{1}{3}G_P^*(G_A-G_V)]$$
$$-\tfrac{1}{2}(P+P')|G_V-G_A-G_P|^2\hat{n}\cdot\hat{k}+2PP'\mathrm{Re}G_P^*(G_A-G_V)[(\hat{n}\cdot\hat{k})^2-\tfrac{1}{3}]\}\ . \tag{12}$$

If $\Gamma_\pm^{(1)}$ represents those events with $1>\pm\hat{n}\cdot\hat{k}>\frac{1}{2}$ and $\Gamma_\pm^{(2)}$ those with $\frac{1}{2}>\pm\hat{n}\cdot\hat{k}>0$,

$$\frac{\Gamma_+^{(1)}+\Gamma_+^{(2)}-\Gamma_-^{(1)}-\Gamma_-^{(2)}}{\Gamma_+^{(1)}+\Gamma_+^{(2)}+\Gamma_-^{(1)}+\Gamma_-^{(2)}}=-\tfrac{1}{4}(P+P')\frac{|G_V-G_A-G_P|^2}{|G_V|^2+2|G_A|^2+|G_A+G_P|^2-2PP'\mathrm{Re}[G_A^*(G_A+G_V+G_P)-\frac{1}{3}G_P^*(G_A-G_V)]}$$
$$\equiv -\tfrac{1}{4}(P+P')A(g_A(q^2),\sigma+\tau)\ , \tag{13}$$

$$\frac{\Gamma_+^{(1)}+\Gamma_-^{(1)}-\Gamma_+^{(2)}-\Gamma_-^{(2)}}{\Gamma_+^{(1)}+\Gamma_+^{(2)}+\Gamma_-^{(1)}+\Gamma_-^{(2)}}=\tfrac{1}{4}PP'\frac{2\,\mathrm{Re}G_P^*(G_A-G_V)}{|G_V|^2+2|G_A|^2+|G_A+G_P|^2-2PP'\mathrm{Re}[G_A^*(G_A+G_V+G_P)-\frac{1}{3}G_P^*(G_A-G_V)]}$$
$$\equiv\tfrac{1}{4}PP'B(g_A(q^2),\sigma+\tau)\ . \tag{14}$$

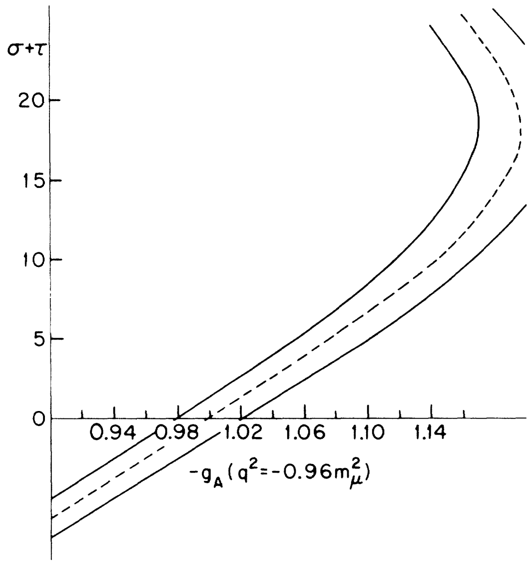

FIG. 3. $\sigma+\tau$ versus $g_A(q^2=-0.96m_\mu{}^2)$ for muon capture on He^3; $\Gamma=1505\pm 46\ sec^{-1}$.

Figure 4 shows the neutrino asymmetry A versus $g_A(q^2)$ for values of $\sigma+\tau$ suggested by Fig. 2. It is seen that a careful measurement of A – which may be done with either nuclear polarization *or* muon polarization – serves to provide rather strong limits on $g_A(q^2)$ and, through Figs. 2 and 3, on $\sigma+\tau$.

Figure 5 shows the double-polarization factor B versus $\sigma+\tau$ for various values of $g_A(q^2)$. B is seen to be extremely sensitive to $\sigma+\tau$ and almost independent of $g_A(q^2)$, as could have been guessed from its proportionality to G_P. Unfortunately this measurement is experimentally very demanding, since both nuclear and muon polarization are required, and the coefficient $\frac{1}{4}PP'$ which multiplies B is most likely small.

We conclude that current muon-capture experiments on hydrogen and He^3 do not enable us to conclude anything definite about the possible presence of second-class currents, because of uncertainties in our knowledge of $g_A(q^2)$ and $g_P(q^2)$. However, experiments utilizing polarization effects can perhaps eliminate some of this uncertainty.

IV. ANALOG NEUTRINO SCATTERING

Finally, we comment briefly on elastic neutrino scattering as a probe for information about the possible presence of second-class currents. We restrict our attention here to the reactions

(a) $\nu_l(k)+n(p_1)\rightarrow l^-(p)+p(p_2)$,

(b) $\bar{\nu}_l(k)+p(p_1)\rightarrow l^+(p)+n(p_2)$,

since other analog transitions should be very unlikely except in the very forward direction.

Neutrino scattering on nucleons has been discussed by several authors.[31] Letting $P=p_1-p_2$, $q=p_1-p_2$, $n=p+k$ we write

$$\frac{d\sigma}{dq^2}=-\frac{G_V{}^2\cos^2\theta_C}{8\pi\epsilon^2\times 4M^2}W_{\mu\nu}(P,q)\tau^{\mu\nu}(q,n), \tag{15}$$

where $W_{\mu\nu}(P,q)$ is the spin-averaged hadronic tensor

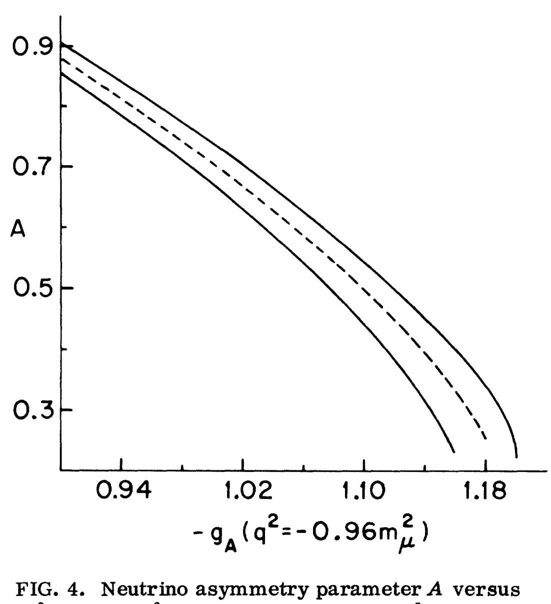

FIG. 4. Neutrino asymmetry parameter A versus $g_A(q^2=-0.96m_\mu{}^2)$ for muon capture on He^3 with $\sigma+\tau$ taken from Fig. 3. PP' has been taken as $\frac{1}{12}$.

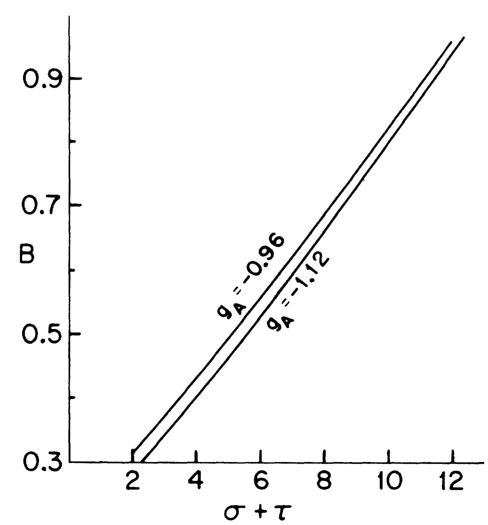

FIG. 5. Double-polarization correlation parameter B versus $\sigma+\tau$ for muon capture on He^3. PP' has been taken as $\frac{1}{12}$.

$$\frac{1}{4M^2}W_{\mu\nu}(P, q)=\tfrac{1}{2}\sum_{s_1,s_2}\langle N'_{p_2,s_2}|V_\mu+A_\mu|N_{p_1,s_1}\rangle \times\langle N'_{p_2,s_2}|V_\nu+A_\nu|N_{p_1,s_1}\rangle^*, \tag{16}$$

and $\tau_{\mu\nu}(q, n)$ is the lepton tensor

$$\tau_{\mu\nu}(q, n)=n_\mu n_\nu - q_\mu q_\nu + g_{\mu\nu}(q^2-m_l^2)\pm i\epsilon_{\mu\nu\alpha\beta}q^\alpha n^\beta, \tag{17}$$

and ϵ is the incident neutrino energy in the lab frame. $W_{\mu\nu}(P, q)$ is conventionally written in the form[32]

$$\begin{aligned}W_{\mu\nu}(P, q)=&-g_{\mu\nu}W_1(q^2)+P_\mu P_\nu W_2(q^2)\\&+i\epsilon_{\mu\nu\alpha\beta}P^\alpha q^\beta W_3(q^2)+q_\mu q_\nu W_4(q^2)\\&+(P_\mu q_\nu+q_\mu P_\nu)W_5(q^2)\\&+i(P_\mu q_\nu-q_\mu P_\nu)W_6(q^2),\end{aligned} \tag{18}$$

where $W_i(q^2)$ are real functions of the nucleon form factors and q^2. If only first-class (or only second-class) currents contribute to $V_\mu+A_\mu$, so that $G(V_\mu+A_\mu)G^{-1}=\pm(V_\mu-A_\mu)$ one finds, using CPT invariance,

$$W_{\mu\nu}(P, q)=\begin{cases}W_{\mu\nu}(P, -q) & VV, AA \text{ terms } [W_i(q^2), i\neq 3]\\ -W_{\mu\nu}(P, -q) & VA, AV \text{ terms } [W_3(q^2)]\,.\end{cases}$$

The opposite behavior obtains for first-class–second-class interference terms. Then

$$W_i(q^2)=\begin{cases}(\text{1st class})^2+(\text{2nd class})^2 & i=1, 2, 3, 4\\ \text{1st class}\times\text{2nd class} & i=5, 6\,.\end{cases}$$

Defining the n-p matrix element as in Eq. (A1) this is borne out by the explicit form of the W_i functions given in Appendix A.

Hence detection of $W_5(q^2)$ or $W_6(q^2)$ would be sufficient to verify the existence of second-class currents. However, since

$$q^\mu\tau_{\mu\nu}(q, n)=m_l^2(n-q)_\nu,$$

the proportionality of these effects to the lepton mass makes them correspondingly difficult to detect experimentally. Instead, a careful analysis of the q^2 dependence of $W_2(q^2)$, as measured say via

$$\frac{d\sigma_{\nu,\bar\nu}}{dq^2}\xrightarrow[\epsilon\,\text{large}]{}-\frac{G_V^2\cos^2\theta_C}{32\pi M^2\epsilon^2}(P\cdot n)^2W_2(q^2) \tag{19}$$

can for $-q^2\sim 4M^2$ yield information about $g_{\rm II}(q^2)$ provided the q^2 dependence of $g_A(q^2)$ is known. While the latter may be found from $W_3(q^2)$, e.g., as measured by comparison of reactions (a) and (b) via

$$\frac{d\sigma_\nu}{dq^2}-\frac{d\sigma_{\bar\nu}}{dq^2}=\frac{G_V^2\cos^2\theta_C}{8\pi M^2\epsilon^2}P\cdot n[q^2W_3(q^2)-m_l^2W_5(q^2)], \tag{20}$$

it is clear that definite conclusions may be difficult to reach in this fashion. With this in mind, we suggest here an alternative method to check for second-class currents involving polarization effects. These experiments are admittedly futuristic. However, they yield a most conclusive test for the presence of such currents, and they provide interesting theoretical effects.

In the presence of spin dependence, the hadronic tensor takes on a much more complex form involving terms such as

$$(S_\mu^+q_\nu+q_\mu S_\nu^+)W_7(q^2)+i\epsilon_{\mu\nu\alpha\beta}q^\alpha S_+^\beta W_8(q^2)+\cdots.$$

If both initial and final spin are included and $S_\pm = s_1\pm s_2$, one finds using CPT invariance

$$W_{\mu\nu}(P, q, S_+, S_-)=\begin{cases}W_{\mu\nu}(P, -q, -S_+, S_-) & \begin{array}{l}VV, AA \text{ terms: (1st class)}^2+\text{(2nd class)}^2\\ VA, AV \text{ terms: 1st class}\times\text{2nd class}\end{array}\\ -W_{\mu\nu}(P, -q, -S_+, S_-) & \begin{array}{l}VA, AV \text{ terms: (1st class)}^2+\text{(2nd class)}^2\\ VV, AA \text{ terms: 1st class}\times\text{2nd class}\,.\end{array}\end{cases}$$

Thus, for example, $W_7(q^2)$ is purely a first-class–second-class interference term, while $W_8(q^2)$ is not.

If the target nucleon is polarized but the final spin is unobserved, the differential cross section can be written as

$$\frac{d\sigma_\nu}{dq^2}=-\frac{G_V^2\cos^2\theta_C}{32\pi M^2\epsilon^2}[a_1+\mathcal{P}(a_2^t s_1\cdot P+a_3^t s_1\cdot n+a_4^t\epsilon_{\mu\alpha\beta\gamma}s_1^\mu P^\alpha q^\beta n^\gamma)], \tag{21}$$

where $\mathcal{P}$ is the degree of initial polarization, s_1 is a four vector describing the polarization direction $-s_1\cdot p_1=0$, $s_1^2=-1$, and a_i are real functions of momenta and of the nucleon form factors. Likewise, if the final nucleon polarization is measured after ν scattering from an unpolarized nucleon target, we define

$$\frac{d\sigma_\nu}{dq^2}=-\frac{G_V^2\cos^2\theta_C}{64\pi M^2\epsilon^2}(a_1+a_2^f s_2\cdot P+a_3^f s_2\cdot n+a_4^f\epsilon_{\mu\alpha\beta\gamma}s_2^\mu P^\alpha q^\beta n^\gamma), \tag{22}$$

where s_2 describes the final nucleon polarization. Then it is shown in Appendix A that if the mass of the lepton is neglected, the absence of second-class currents requires that

$$a_i^t = a_i^f, \quad i = 2, 3, 4 .$$

In the limit $m_l^2 \to 0$, ϵ large

$$\begin{aligned}
a_2^{t,f} &\cong (P\cdot n)^2 \frac{1}{M}\mathrm{Re}(g_A g_M^* \mp g_V g_{\mathrm{II}}^*)\,, \\
a_3^{t,f} &\cong 2MP\cdot n\left[2\,\mathrm{Re}g_A^*\left(g_V + \frac{q^2}{4M^2}g_M\right) \mp 2\,\mathrm{Re}g_{\mathrm{II}}^*(g_V+g_M)\frac{q^2}{4M^2}\right], \\
a_4^{t,f} &\cong -P\cdot n\frac{1}{M}[\,\mathrm{Im}(g_V g_M^* \mp g_A g_{\mathrm{II}}^*)]\,.
\end{aligned} \tag{23}$$

Thus any inequality of a_i^t and a_i^f measured at the same momentum transfer and energy provides proof for the presence of second-class terms. Comparison of the transverse polarization terms tests for T-violating second-class currents.

Of course, to the extent that the other form factors are known, measurement of either a_i^t or a_i^f independently can indicate the necessity for a second-class term. Such effects are much more prominent here than in the spin-averaged case, since in the latter they appear only in order q^2/M^2, m_l^2/M^2 relative to the leading contributions. Unfortunately these measurements belong to future generation experiments.

ACKNOWLEDGMENT

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APPENDIX A

Defining the nucleon form factors as

$$\begin{aligned}
\langle p_{p_2,s_2}|V_\mu|n_{p_1,s_1}\rangle &= \bar{u}(p_2)\left[\gamma_\mu g_V(q^2) - q_\mu g_S(q^2) - i\frac{1}{2M}\sigma_{\mu\nu}q^\nu g_M(q^2)\right]u(p_1) \\
\langle p_{p_2,s_2}|A_\mu|n_{p_1,s_1}\rangle &= \bar{u}(p_2)\left[\gamma_\mu\gamma_5 g_A(q^2) - q_\mu\gamma_5 g_P(q^2) - i\frac{1}{2M}\sigma_{\mu\nu}q^\nu\gamma_5 g_{\mathrm{II}}(q^2)\right]u(p_1)\,,
\end{aligned} \tag{A1}$$

we find

$$\begin{aligned}
W_1(q^2) &= 4M^2\left\{|g_A(q^2)|^2 - \frac{q^2}{4M^2}[\,|g_A(q^2)|^2 + |g_V(q^2)+g_M(q^2)|^2]\right\}, \\
W_2(q^2) &= |g_V(q^2)|^2 + |g_A(q^2)|^2 - \frac{q^2}{4M^2}(|g_M(q^2)|^2 + |g_{\mathrm{II}}(q^2)|^2)\,, \\
W_3(q^2) &= -2\,\mathrm{Re}g_A^*(q^2)[g_V(q^2)+g_M(q^2)]\,, \\
W_4(q^2) &= -|g_V(q^2)+g_M(q^2)|^2 - |g_A(q^2)+2Mg_P(q^2)|^2 + (4M^2-q^2)[\,|g_s(q^2)|^2 + |g_P(q^2)|^2]\,, \\
\left.\begin{array}{l} W_5(q^2) = 2M\mathrm{Re} \\ W_6(q^2) = 2M\mathrm{Im}\end{array}\right\} & \left\{-g_S^*(q^2)\left[g_V(q^2) + \frac{q^2}{4M^2}g_M(q^2)\right] + \frac{1}{2M}g_{\mathrm{II}}^*(q^2)\left[g_A(q^2) + \frac{q^2}{2M}g_P(q^2)\right]\right\},
\end{aligned} \tag{A2}$$

which are seen to obey the rules given in Sec. IV.

Suppose polarization effects are included. If we neglect the lepton mass, so that $q^\mu\tau_{\mu\nu}(q,n)=0$, the most general form for $W_{\mu\nu}(P, q, S_+, S_-)$ including either target polarization information or polarization of the scattered nucleon (but not both) can be written in terms of the tensors

$$
\begin{aligned}
&\text{(i)}\quad P_\mu S_\nu^\pm + P_\nu S_\mu^\pm ,\\
&\text{(ii)}\quad i(P_\mu \epsilon_{\nu\alpha\beta\gamma} S_\pm^\alpha P^\beta q^\gamma - \epsilon_{\nu\alpha\beta\gamma} S_\pm^\alpha P^\beta q^\gamma P_\nu),\\
&\text{(iii)}\quad i\epsilon_{\mu\nu\alpha\beta} P^\alpha S_\pm^\beta ,\\
&\text{(iv)}\quad i\epsilon_{\mu\nu\alpha\beta} q^\alpha S_\pm^\beta ,\\
&\text{(v)}\quad g_{\mu\nu} S_\pm \cdot P ,\\
&\text{(vi)}\quad P_\mu P_\nu S_\pm \cdot P ,\\
&\text{(vii)}\quad i\epsilon_{\mu\nu\alpha\beta} P^\alpha q^\beta S_\pm \cdot P ,
\end{aligned} \tag{A3}
$$

where only the time-reversal-conserving terms have been written, for simplicity. These tensors have as coefficients products of nucleon form factors and momenta. However, the momenta can only be of the form P^2, q^2 both of which are even under the operation $P \to P$, $q \to -q$, since $P \cdot q = M_1^2 - M_2^2 \cong 0$. Thus is is sufficient to consider the behavior of the above tensors alone under the spin- and momentum-reversal operation given in Sec. IV. Since (ii), (iii), (iv), (vii) are VV, AA terms while (i), (v), (vi) involve VA interference, we find

$$
\text{(i), (ii), (iv), (v), (vi), (vii)} = \text{I, II} \qquad \text{(iii)} = \text{II, I} ,
$$

where by (i) = I, II we mean that the coefficient of $P_\mu S_\nu^+ + S_\mu^+ P_\nu$ must be of the form (1st class)^2 + (2nd class)^2, while that of $P_\mu S_\nu^- + S_\mu^- P_\nu$ must be 1st class × 2nd class.

When these forms are contracted with the lepton tensor and the lepton mass is neglected, e.g.,

$$
(i\epsilon_{\mu\nu\alpha\beta} q^\alpha S_+^\beta \,\text{I} + i\,\epsilon_{\mu\nu\alpha\beta} q^\alpha S_-^\beta \,\text{II})\tau^{\mu\nu}(q, n) = 2q^2(S_+ \cdot n\,\text{I} + S_- \cdot n\,\text{II}) + O(m_l^2),
$$

it is seen that terms in $S_+ \cdot n$, $S_+ \cdot P$ must be of the form (1st class)^2 + (2nd class)^2, while those in $S_- \cdot n$, $S_- \cdot P$ must be of the form 1st class × 2nd class. Similarly, if T violation is included, results are found for $\epsilon_{\mu\alpha\beta\gamma} S_\pm^\mu P^\alpha q^\beta n^\gamma$ as given in Sec. IV.

APPENDIX B

In A we showed that in the β decay of an unpolarized parent nucleus N_1 of spin j to an analog daughter which subsequently decays electromagnetically via a dipole transition (either $E1$ or $M1$) to a final nucleus N_3 of spin j' the spectrum in electron and photon variables is

$$
d\omega = F_\mp(Z, E)\frac{G_V^2 \cos^2\theta_C}{(2\pi)^5}(E_0 - E)^2 pE\, dE\, d\Omega_e\, d\Omega_\gamma \left\{ f_1(E) + g(E)\frac{\hat{K}\cdot\vec{p}}{E} + \lambda_{j,j'} f_6(E)\left[\left(\frac{\hat{K}\cdot\vec{p}}{E}\right)^2 - \frac{1}{3}\frac{p^2}{E^2}\right]\right\}, \tag{B1}
$$

where

$$
\begin{aligned}
f_1(E) &= |a|^2 + |c|^2 - \frac{2}{3}\frac{E_0}{M}[|c|^2 \pm \mathrm{Re}\,c^*(b+d)] + \frac{2}{3}\frac{E}{M}(3|a|^2 + 5|c|^2 \pm 2\,\mathrm{Re}\,c^* b) - \frac{1}{3}\frac{m_e^2}{ME}[2|c|^2 \pm \mathrm{Re}\,c^*(2b+d)],\\
g(E) &= \frac{2}{3}\frac{E_0}{M}\left[-|a|^2 + \tfrac{1}{3}|c|^2\left(1 - \frac{\lambda_{j,j'}}{10}\right)\right] - \frac{4}{3}\frac{E}{M}\left[|a|^2 + \tfrac{5}{3}|c|^2\left(1 - \frac{\lambda_{j,j'}}{100}\right)\right],\\
f_6(E) &= \frac{E}{20M}[|c|^2 \pm \mathrm{Re}\,c^*(b-d)],
\end{aligned} \tag{B2}
$$

$$
\lambda_{j,j'} = \begin{cases} -(2j-1)/(j+1) & j' = j+1 \\ (2j+3)(2j-1)/j(j+1) & j' = j \\ -(2j+3)/j & j' = j-1 , \end{cases} \tag{B3}
$$

and $\hat{K}$ is a unit vector in the direction of the photon momentum. Thus a measurement of the $\beta\gamma$ correlation function $f_6(E)$ provides information about the combination $cd - cb$ which, provided c, b are known, gives the size of the second-class term d.

On the other hand, besides photon decay, many daughter nuclei decay with the emission of an α particle[33]

$$
(\mathrm{Na}^{20} \to \underset{\substack{\downarrow \\ \mathrm{O}^{16} + \alpha}}{\mathrm{Ne}^{20}} + e^+ + \nu_e)
$$

or of a proton

$$
(\mathrm{Ne}^{17} \to \underset{\substack{\downarrow \\ \mathrm{O}^{16} + p}}{\mathrm{F}^{17}} + e^+ + \nu_e) .
$$

The structure of the spectrum in these decays is identical to that in Eq. (B1) (where now $\hat{K}$ represents a unit vector in the direction of the α or p momentum) with, however, a redefinition of $g(E)$ and of $\lambda_{j,j'}$. $g(E)$ is replaced by $g(E)/v^*$, where v^* is the velocity ($v^* = 1$ for γ) of the α particle or proton as measured in the rest frame of the daughter. For $\lambda_{j,j'}(N_1 \to N_3)$ suppose the α particle or proton possesses a well-defined angular momentum in the rest frame of the daughter nucleus (e.g., in the example above, $L=2$ for $Ne^{20} \to O^{16} + \alpha$ and $L=1$ for $F^{17} \to O^{16} + p$). Then $\lambda_{j,j'}(N_1 \to N_3) = \lambda_{j,j'}(L)$, where

$$L=0 \quad \lambda_{j,j'} = 0,$$

$$L=1 \quad \lambda_{j,j'} = \begin{cases} 2(2j-1)/(j+1) & j'=j+1 \\ -2(2j+3)(2j-1)/j(j+1) & j'=j \\ 2(2j+3)/j & j'=j-1, \end{cases}$$

$$L=2 \quad \lambda_{j,j'} = \begin{cases} +20(2j-1)/7(j+1) & j'=j+2 \\ -10(2j-1)(j+6)/7(j+1)j & j'=j+1 \\ -10(2j+5)(2j-3)/7(j+1)j & j'=j \\ -10(2j+3)(j-5)/7(j+1)j & j'=j-1 \\ +20(2j+3)/7j & j'=j-2, \end{cases}$$

where j' is the spin of the final nucleus in α decay and is the vector sum of the spin of the final nucleus and the proton spin for a delayed proton transition. In some situations L can have several values, and in these cases $\lambda_{j,j'}(N_1 \to N_3)$ will be an appropriate linear combination of $\lambda_{j,j'}(L)$. Of course, the lowest value of L is generally predominant because of angular momentum barriers.

We note finally that just as for $E1$ or $M1$ photon emission

$$\lambda_{j,j'}(E1, M1) = -\tfrac{1}{2}\lambda_{j,j'}(L=1),$$

we have for quadrupole emission

$$\lambda_{j,j'}(E2, M2) = +\tfrac{1}{2}\lambda_{j,j'}(L=2),$$

where j' is the spin of the final nucleus.

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[1]D. H. Wilkinson, Phys. Letters 31B, 447 (1970). See, however, D. H. Wilkinson and D. E. Alburger [Phys. Rev. Letters 26, 1127 (1971)] which suggests that the effect in the previous reference may be caused by electromagnetism rather than by second-class currents. This makes it all the more important to perform experiments on analog decays, where much of the uncertainty due to nuclear physics may be ameliorated.

[2]S. Weinberg, Phys. Rev. 112, 1375 (1958).

[3]B. R. Holstein and S. B. Treiman, Phys. Rev. C 3, 1921 (1971), hereafter referred to as A.

[4]As pointed out in A, this is not completely correct. Firstly, electromagnetic effects have been neglected. Secondly, such terms can arise from a second-class current with odd isotopic spin *or* from a first-class current with even isotopic spin. However, since either of these are anomalous in the present picture of weak decays, their detection would be of interest and we refer to any such term as "second class" for convenience.

[5]C. W. Kim and H. Primakoff, Phys. Rev. 180, 1502 (1969).

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[7]F. P. Calaprice, private communication. If the second-class term scales with mass number, as suggested by the impulse approximation, such terms might be of order $E_0/m_p \sim 4\times10^{-3}$.

[8]Our results are seen to predict

$$\frac{ft^+}{ft^-} - 1 \cong -\frac{2}{3}\frac{(E_0^+ + E_0^-)}{m_p}\frac{d}{Ac}$$

$$= (\text{according to Ref. 1})\,\frac{4}{3}\frac{(E_0^+ + E_0^-)}{m_e} \times 1.7\times10^{-3}.$$

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[12]We could include for completeness an induced scalar term. However, we are assuming the second-class current to be axial.

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