PHYSICAL REVIEW C VOLUME 4, NUMBER 3 SEPTEMBER 1971

Spin-Orbit Coupling for Λ-^{4}He Scattering Using a One-Boson-Exchange Model for the ΛN Interaction*

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(Received 2 November 1970)

The polarization and cross sections for Λ-^{4}He elastic scattering are calculated for Λ incident energies 1–20 MeV. A Λ-^{4}He potential is constructed using a phenomenological ΛN Gaussian central potential and the empirical shape for ^{4}He, and a spin-orbit potential obtained from a one-boson-exchange (OBE) model for the ΛN interaction. As indicated recently by Downs, the OBE model for ΛN forces leads to strong antisymmetric spin-orbit terms due to vector-meson exchange. This means that the Λ-^{4}He potential depends only on part of the ΛN spin-orbit interaction, since it results from summing the latter interaction over the nucleon spin states. Parameters from three quite different OBE calculations are used to predict the Λ-^{4}He scattering, and these three sets of OBE parameters lead to remarkably similar predictions concerning the cross sections and polarization properties for this energy range. However, no Λ-^{4}He resonances are predicted, in contrast to recent Hartree-Fock calculations of Λ-^{4}He scattering. The polarization effects predicted are large, and experimental data concerning them would have interesting consequences for our knowledge of the origin of spin-orbit forces in the baryon-baryon system.

I. INTRODUCTION

Estimates of the central part of the ΛN interaction can be obtained from the cross sections measured for low-energy ΛN elastic scattering,[1-3] and from analyses of the binding energies and spins of the light Λ hypernuclei.[4,5] The spin-orbit part of the ΛN interaction is more difficult to determine. In the case of the NN interaction, the spin-orbit interaction becomes really important in NN scattering only at laboratory energies above about 150 MeV; below these energies, it produces small effects on the scattering cross sections, although some of its effects are clearly apparent in the patterns of nuclear energy levels and of nuclear properties.[6] For the ΛN interaction, the elastic scattering data available does not yield much information about the spin-orbit forces. It is difficult to produce Λ particles with both sufficient energy and sufficient intensity to allow the observation of a spin-orbit term in the ΛN interaction. For K^-p interactions at rest, the Λ production is copious but few of the Λ particles resulting have momentum above about 300 MeV/c. For K^-p interactions in flight, at higher energies, the relative yield of Λ particles is quite large, but the event rate in a hydrogen bubble chamber is necessarily rather low, and the Λ particles produced are distributed over a rather wide range of momenta.[7] A further complication is that, for Λp c.m. energy above 78 MeV, the ΣN channel becomes energetically available, and ΛN scattering must then be treated explicitly as a two-channel situation.[8]

In the p-shell hypernuclei, the ΛN spin-orbit force contributes directly to the binding energies, but it proves difficult to identify uniquely the contributions from the spin-orbit ΛN interaction, since there prove to be effects arising from ΛNN three-body forces which have the same characteristics as ΛN spin-orbit forces.[9] Recent analyses[10,11] of the effective ΛN spin-orbit force in the p-shell hypernuclei indicate a rather large ΛN spin-orbit effect, as would correspond to a ΛN spin-orbit force with magnitude somewhat larger than that known for the NN spin-orbit force in the p-shell nuclei. The sign found for this ΛN spin-orbit effect in these analyses is opposite that known for the NN spin-orbit effect; however, this conclusion depends on the rather detailed assumptions which have to be made concerning the existence and nature of ΛNN three-body forces, and so cannot yet be regarded as firmly established. It is clearly desirable that independent measurements should be obtained for the ΛN spin-orbit force, and this paper explores the possibility of using low-energy Λ-^{4}He elastic scattering to extract this information.

If the ΛN spin-orbit force is comparable in strength with the NN spin-orbit force, it is likely that measurements of Λ-^{4}He elastic scattering and polarization in the low-energy region (below the Λ energy, about 26-MeV laboratory kinetic energy, needed for ^{4}He breakup) could provide rather clear-cut information about the ΛN spin-orbit force. However, no data are yet available on the Λ-^{4}He scattering interaction.

To begin, it will be instructive to compare the Λ-^{4}He system with the well-known n-^{4}He system, which has a sharp $P_{3/2}$ resonance at neutron (lab) energy about 1 MeV, and a broad $P_{1/2}$ resonance at about 5 MeV.[12] Sack, Biedenharn, and Breit[13] achieved a reasonable fit to the n-^{4}He scattering phase shifts from 0–12 MeV using phenomenological central and spin-orbit potentials of Gaussian form. Their central potential had the form

$$V_c = U_c\, e^{-r^2/R^2}, \qquad (1.1)$$

with $U_c = -47.53$ MeV and $R = 2.30$ F. Neglecting the Pauli principle, this potential is strong enough to bind an s-wave nucleon by about 20 MeV. For comparison, Dalitz and Downs[14] have calculated a Λ-^{4}He Gaussian central potential to fit the Λ-^{4}He separation energy $B_\Lambda = 3.1$ MeV,[15] with range corresponding to the known ^{4}He size and to 2π exchange for the ΛN interaction. Their potential had $U_c = -43.81$ MeV, and range parameter $R = 1.565$ F.

For a first orientation, we calculated Λ-^{4}He elastic scattering phase shifts using the Dalitz-Downs central potential and the spin-orbit potential from the n-^{4}He analysis of Sack, Biedenharn, and Breit[13] [given by the form (1.1) with U_c replaced by $U_s = -5.85$ MeV for the $P_{3/2}$ state, and by $-2U_s = +11.7$ MeV for the $P_{1/2}$ state]. We found that the $P_{3/2}$ phase shift then reached a maximum of about 45°, for c.m. energy about 15 MeV, and that the $P_{1/2}$ phase shift was always small and negative below 8-MeV c.m. energy and did not exceed +4° above this energy. However, if the Λ-^{4}He spin-orbit potential were increased by a factor of about 4, then a low-energy $P_{3/2}$ resonance (at about 4-MeV c.m. energy) would result in Λ-^{4}He scattering. On the other hand, if the Λ-^{4}He spin-orbit potential had sign opposite that for the n-^{4}He system, the $P_{1/2}$ phase shift would reach about 70° in Λ-^{4}He scattering, the $P_{3/2}$ phase shift reaching a maximum of less than 20°; if the Λ-^{4}He spin-orbit potential were then doubled, a low-energy $P_{1/2}$ resonance would result for the Λ-^{4}He system. These order-of-magnitude estimates indicate that measurement of the polarization properties of Λ-^{4}He scattering would provide a sensitive test for the strength of the spin-orbit component of the ΛN interaction, even if the Λ-^{4}He spin-orbit forces are not strong enough to give rise to a Λ-^{4}He P-wave resonance.

To first order, the spin-orbit potential for Λ-^{4}He is the sum of the ΛN spin-orbit terms over the nucleons in the ^{4}He nucleus, averaged over the nucleon single-particle density in ^{4}He. Higher-order contributions to the Λ-^{4}He spin-orbit potential are expected to be small, as we shall discuss below. This contrasts with the n-^{4}He situation, where the NN tensor force arising from OPE provides, in second order, a large contribution to the N-^{4}He spin-orbit splitting.[16] These theoretical differences between Λ-^{4}He and N-^{4}He scattering will be discussed in Sec. II B.

Alexander, Gal, and Gersten[17] have investigated the possibility of using Λ-^{4}He scattering as a means to determine the ΛN spin-orbit force. They constructed ΛN spin-orbit potentials of Gaussian form with a range parameter appropriate to ω exchange, or to σ exchange (σ = isoscalar scalar meson assumed to have mass 400 MeV, whose exchange represents roughly the effects of the exchange of two pions with $I = 0$).[18] An upper limit on the strength U_s for these ΛN spin-orbit potentials was obtained from the remark that the expectation value of the ΛN spin-orbit potential between the Λ particle and a p-wave nucleon in the p-shell hypernuclei was unlikely to exceed 0.5 MeV.[10,19] Calculations were then made for the angular distribution $d\sigma(\Lambda\text{-}^4\text{He})/d\Omega$ and polarization $P_\Lambda(\theta)$ for a number of potential strengths $U_s(\Lambda N)$, for Λ-^{4}He c.m. kinetic energy of 13 MeV, to illustrate their sensitivity to U_s.

Gibson, Goldberg, and Weiss[20] and Gibson and Weiss[21] have also made calculations for the Λ-^{4}He system, considered as a five-particle system, within the Hartree-Fock approximation. The NN and ΛN potentials used in this work had the general form

$$U(r) = -W_1 e^{-(r_1/a_1)^2} + W_2 e^{-(r_2/a_2)^2}, \qquad (1.2)$$

consisting of a short-range repulsion together with an attraction of longer range. The same short-range repulsion was adopted for both NN and ΛN interactions, with parameters $W_2 = 145$ MeV and $a_2 = 0.82$ F, corresponding to a particular set of NN parameters discussed by Volkov.[22] The other NN parameters, $W_1^{NN} = 83.34$ MeV and $a_1^{NN} = 1.6$ F, were chosen to give the observed form factor[23] (up to $q^2 = 7$ F^{-2}) and binding energy for ^{4}He, in a four-particle Hartree-Fock calculation. The ΛN parameters, $W_1^{\Lambda N} = 85.8$ MeV and $a_1^{\Lambda N} = 1.21$ F, were chosen to give (in principle[24]) the observed B_Λ values for $^4_\Lambda$H and $^5_\Lambda$He in the appropriate Hartree-Fock approximation. Taking this ΛN central potential and the ^{4}He shape given by their Hartree-Fock calculations, Gibson, Goldberg, and Weiss[20] deduced an effective Λ-^{4}He central potential V_c (hereafter referred to as the GW potential). Gibson and Weiss[21] then added to this an *ad* *hoc* Λ-⁴He spin-orbit potential with shape given by

$$V_{LS}(r)=C_\Lambda\left(\frac{1}{r}\frac{dV_c}{dr}\right), \qquad (1.3)$$

where r denotes the Λ-⁴He separation, and went on to calculate $d\sigma/d\Omega$ and $P_\Lambda(\theta)$ for low-energy Λ-⁴He scattering as function of the sign and magnitude of the coefficient C_Λ in this spin-orbit potential. Some aspects of their calculations will be compared with our results in Sec. IV.

In this paper, we consider the spin-orbit component of the Λ-⁴He potential corresponding to the ΛN spin-orbit interaction given by the simplest one-boson-exchange (OBE) model for the baryon-baryon interaction. In this model, we assume that the baryon-baryon amplitude is given by the Born pole terms from the exchange of vector, pseudoscalar, and scalar mesons. We calculate the OBE amplitude for each meson exchange, and then make a nonrelativistic approximation to the scattering amplitude, following the procedures and conventions which have been stated clearly and systematically by Brown, Downs, and Iddings,[25] and finally identifying the Fourier transform of this amplitude with the OBE potential. The Λ-⁴He spin-orbit potential is then obtained by summing the ΛN spin-orbit potential over the nucleons in ⁴He, folding in the known density distribution for the nucleons in ⁴He.

In Sec. II, we discuss the OBE model for the ΛN interaction and the construction of the Λ-⁴He potential. In Sec. III, we review the various theoretical proposals which have been made concerning the values for the baryon-baryon-meson coupling constants. We use the predicted parameters appropriate to three different OBE models in our Λ-⁴He calculations, and in Sec. IV we discuss the cross sections $d\sigma/d\Omega$ and polarizations $P_\Lambda(\theta)$ obtained.

II. ΛN INTERACTION AND THE Λ-⁴He SPIN-ORBIT POTENTIAL

We assume that the Λ-⁴He interaction can be written as the sum of a central and a spin-orbit potential,

$$V(r)=V_c(r)+V_{LS}(r)\vec{L}\cdot\vec{S}_\Lambda, \qquad (2.1)$$

where $\vec{r}$ denotes the relative coordinate between the Λ particle and the ⁴He center of mass, $\vec{L}$ denotes the Λ-⁴He orbital angular momentum, and $\vec{S}_\Lambda$ is the Λ spin vector. We have already discussed above the central potential used by Dalitz and Downs.[14] This was based on the single-particle density distribution for a nucleon in ⁴He, given by

$$\rho(r_i)=(\pi a^2)^{-3/2}e^{-(r_i/a)^2}, \qquad (2.2)$$

where $\vec{r}_i$ denotes the relative coordinate between the ith nucleon and the ⁴He center of mass, and the value $a=1.175$ F is obtained from the electron scattering data[23] for ⁴He. Adopting a ΛN central potential of Gaussian form with intrinsic range equal to that for a Yukawa potential with range parameter $(2m_\pi)^{-1}$, the integration of the ΛN central potential over the nucleon distribution (2.2) leads to a Λ-⁴He central potential of the form (1.1), with $R=1.565$ F; the fit to the B_Λ value known for ${}^5_\Lambda$He requires $U_s=-43.8$ MeV.

A. OBE Model for the ΛN Interaction

OBE models have been quite successful in analyzing the nucleon-nucleon interaction[26-29] from 0–400 MeV; models with about 10 parameters have resulted in NN potentials which give good agreement with all the NN scattering and polarization data available. OBE models have also been used for the ΛN interaction[25,30,31]; comprehensive reviews of the OBE model for the ΛN interaction have been given recently by Downs[30] and, by Brown, Downs, and Iddings (BDI).[25]

Here we shall give a brief summary of the ΛN parameters appropriate to the OBE model, within the framework of SU(3) symmetry. For a given spin and parity, the bosons occur in SU(3) octet and singlet states. With SU(3) symmetry, the baryon-baryon-meson [$BBM(8)$] couplings for the boson octet $M(8)$ generally depend on two parameters, a coupling strength and a mixing parameter. For the singlet boson $M(1)$, the interaction $BBM(1)$ has the same coupling for all baryons. For a given meson octet, we write the mixing parameter $f=F/(F+D)$, and we denote the $T=0$, $\frac{1}{2}$, and 1 members of the octet by the suffices 0, $\frac{1}{2}$, and 1, respectively. The BBM couplings of relevance here may then be written as follows, in terms of f and the coupling amplitude G,

$$G_{\Lambda\Sigma 1}=\frac{2(1-f)G}{\sqrt{3}}, \qquad G_{\Sigma\Sigma 1}=2fG,$$

$$G_{NN0}=\frac{(4f-1)G}{\sqrt{3}}, \qquad G_{\Lambda\Lambda 0}=\frac{2(f-1)G}{\sqrt{3}},$$

$$G_{\Lambda N\frac{1}{2}}=-\frac{(1+2f)G}{\sqrt{3}}, \qquad G_{\Sigma N\frac{1}{2}}=(1-2f)G. \qquad (2.3)$$

We denote the coupling of the unitary singlet $M(1)$ as $G'_{BB0}\equiv G'$.

We now discuss the spin-orbit interactions resulting from scalar-, pseudoscalar-, and vector-meson exchanges. These interactions are obtained by first writing down the appropriate interaction Lagrangian, and then calculating the scattering amplitude for the exchange of one meson; the static potential quoted is then obtained from this amplitude, following the procedures discussed systematically and explicitly in BDI.[25]

Scalar-Meson (S) Exchange

To order $(m/M)^4$, a ΛN spin-orbit potential due to S_0 exchange has the form

$$V_{so}(S_0) = -\frac{G^S_{\Lambda\Lambda 0}G^S_{NN0}}{4\pi} m_S Y_1(m_S r)\frac{m_S^2}{4M_N M_\Lambda} \times\left[\left(\frac{M_N}{M_\Lambda}-\frac{m_S^2}{8M_N M_\Lambda}\right)\vec{\sigma}_\Lambda +\left(\frac{M_\Lambda}{M_N}-\frac{m_S^2}{8M_N M_\Lambda}\right)\vec{\sigma}_N\right]\cdot\vec{L}, \quad (2.4)$$

where we have used the notation

$$Y_n(x) = \left(-\frac{1}{x}\frac{d}{dx}\right)^n \frac{e^{-x}}{x}, \quad (2.5)$$

m_S denotes the mass of the $I=0$ scalar boson exchanged, and $\vec{L}$ denotes the relative orbital angular momentum within the ΛN system. Using the notation

$$\vec{S} = \tfrac{1}{2}(\vec{\sigma}_\Lambda + \vec{\sigma}_N), \quad \vec{S}^A = \tfrac{1}{2}(\vec{\sigma}_\Lambda - \vec{\sigma}_N), \quad (2.6)$$

the expression (2.4) may be rewritten

$$V_{so}(S_0) = -\frac{G^S_{\Lambda\Lambda 0}G^S_{NN0}}{4\pi} m_S Y_1(m_S r)\frac{m_S^2}{4M_N M_\Lambda} \times\left[\left(\frac{M_N^2+M_\Lambda^2-m_S^2/4}{M_N M_\Lambda}\right)\vec{S} -\left(\frac{M_\Lambda^2-M_N^2}{M_\Lambda M_N}\right)\vec{S}^A\right]\cdot\vec{L}. \quad (2.7)$$

We note the presence of an antisymmetrical spin-orbit interaction, an asymmetry arising here from the Λ-N mass difference, a possibility first pointed out by Downs and Schrils.[32]

These formulas hold equally for the exchange of S_0, the $I=0$ member of the scalar-meson octet, or of S', a unitary singlet meson. In the latter case, the coupling coefficients G^S_{NN0} and $G^S_{\Lambda\Lambda 0}$ are both to be replaced by G'_S, and m_S by $m_{S'}$. There is also a symmetric spin-orbit potential generated by $S_{\frac{1}{2}}$ exchange, with the form

$$V_{so}(S_{\frac{1}{2}}) = \frac{(G^S_{N\Lambda\frac{1}{2}})^2}{4\pi} P^x_{\Lambda N} m Y_1(mr) \times\frac{m^2}{2M_N M_\Lambda}\left(1-\frac{m^2}{8M_N M_\Lambda}\right)\vec{S}\cdot\vec{L}, \quad (2.8)$$

where $m=[m(S_{\frac{1}{2}})^2-(M_\Lambda-M_N)^2]^{1/2}$ and $P^x_{\Lambda N}$ is the Λ-N space-exchange operator.

Pseudoscalar-Meson (P) Exchange

The dominant noncentral interaction generated by $P(8)$ and $P(1)$ exchange is of tensor form, as is well known for the case of π exchange in the NN interaction. However, these ΛN tensor interactions are of much shorter range than that for the NN system, since they arise from the exchange of heavier mesons, the $K(494)$, $\eta(550)$, and $\eta'(962)$ mesons in place of $\pi(140)$.

Downs and Iddings[33] have pointed out that there is an antisymmetric spin-orbit potential generated by K exchange, in order $(m/M)^4$, given by the following form

$$V_{so}(K) = -P^x_{\Lambda N}\frac{(G^P_{\Lambda N\frac{1}{2}})^2}{4\pi} m Y_1(mr) \times\frac{m^2}{4M_\Lambda M_N}\frac{M_\Lambda^2-M_N^2}{M_\Lambda M_N}\vec{S}^A\cdot\vec{L}, \quad (2.9)$$

where $m=[m(K)^2-(M_\Lambda-M_N)^2]^{1/2}$, $P^x_{\Lambda N}$ is the Λ-N space-exchange operator and $G^P_{\Lambda N\frac{1}{2}}$ is the ΛNK coupling constant. This interaction has not been included in the present work.

Vector-Meson (V) Exchange

The Lagrangian for the BBV interactions involves two independent coupling forms, known as the electric and magnetic terms. We follow Sugawara and von Hippel (SVH)[29] and adopt the particular forms, for a BBV vertex representing absorption of a vector meson,

$$L^V_{int} = \bar{\psi}_B\left(G^E\frac{P_\nu}{2M}\phi^\nu - iG^M\gamma_5\epsilon_{k\lambda\mu\nu}\frac{P^k q^\lambda}{4\bar{M}^2}\gamma^\mu\phi^\nu\right)\psi_B, \quad (2.10)$$

where ϕ^ν denotes the wave function for the vector meson, and

$$P_\nu = (p'+p)_\nu, \quad (2.11a)$$

$$q_\nu = (p'-p)_\nu, \quad (2.11b)$$

p'_ν and p_ν denoting the initial and final baryon four momenta, respectively. The mass $\bar{M}$ occurs in expression (2.10) only for dimensional reasons; we choose to adopt the value $\bar{M}=\frac{1}{2}(M_\Lambda+M_N)=1027$ MeV. In the static limit, the interaction (2.10) reduces to the form

$$G^E\frac{M+M'}{2\bar{M}}\phi^0 - iG^M\frac{M+M'}{2\bar{M}}\vec{\sigma}\cdot\vec{q}\times\vec{\phi}, \quad (2.12)$$

which makes explicit the "electric" and "magnetic" character of these terms.

The spin-orbit potentials resulting from an $I=0$ vector-meson exchange consist of three terms, arising from electric (EE), magnetic (MM) and mixed (EM and ME) couplings. Correct to order $(m/\bar{M})^4$, these potentials are given by

$$V_{so}^{EE}(V_0)=\frac{G_{NN0}^{M}G_{\Lambda\Lambda 0}^{M}}{4\pi}mY_1(mr)\left(\frac{m}{2M}\right)^2 \times\left[\frac{M_\Lambda}{M_N}\left(1+\frac{m^2}{4M_NM_\Lambda}-\frac{m^2}{8M_\Lambda{}^2}\right)\vec{\sigma}_N +\frac{M_N}{M_\Lambda}\left(1+\frac{m^2}{4M_NM_\Lambda}-\frac{m^2}{8M_N{}^2}\right)\vec{\sigma}_\Lambda\right]\cdot\vec{L}\,, \tag{2.13}$$

$$V_{so}^{MM}(V_0)=\frac{G_{NN0}^{M}G_{\Lambda\Lambda 0}^{M}}{4\pi}mY_1(mr)\left(\frac{m}{2\overline{M}}\right)^4 \times\left(\frac{M_\Lambda}{M_N}\vec{\sigma}_\Lambda+\frac{M_N}{M_\Lambda}\vec{\sigma}_N\right)\cdot\vec{L}\,, \tag{2.14}$$

$$V_{so}^{EM}(V_0)=-\frac{G_{NN0}^{E}G_{\Lambda\Lambda 0}^{M}}{4\pi}mY_1(mr)\left(\frac{m}{\overline{M}}\right)^2\left(\frac{M_\Lambda+M_N}{2\overline{M}}\right) \times\left(1-\frac{m^2}{8M_N{}^2}-\frac{m^2}{8M_\Lambda^2}\right)\vec{\sigma}_\Lambda\cdot\vec{L}. \tag{2.15}$$

The potential $V_{so}^{ME}(V_0)$ is obtained by exchanging the labels Λ and N wherever they appear in expression (2.15).

We note that these spin-orbit interactions include both symmetric and antisymmetric terms. In $V_{so}^{EE}(V_0)$ and $V_{so}^{MM}(V_0)$, the antisymmetric term arises from the Λ-N mass difference, as we noted above for the cases of $V_{so}(S_0)$, $V_{so}(S')$, and $V_{so}(K)$, and is therefore of order $(M_\Lambda-M_N)/\overline{M}\approx 0.17$, relative to the symmetric term. However, in $V_{so}^{EM}(V_0)$ and $V_{so}^{ME}(V_0)$, the symmetric and antisymmetric terms are of comparable magnitude, depending on the relationships between the coupling constants which occur. As we shall discuss in Sec. III, the magnetic coupling $\Lambda\Lambda\omega$ and both electric and magnetic couplings $NN\phi$ are expected to be small. In this situation, all $V_{so}(\phi)$ are negligible, as are also $V_{so}^{MM}(\omega)$ and $V_{so}^{EM}(\omega)$; however, $V_{so}^{EE}(\omega)$ and $V_{so}^{ME}(\omega)$ are nonvanishing. To order $(m/\overline{M})^4$, the mixed coupling $V_{so}^{ME}(\omega)$ is proportional to $\vec{\sigma}_N\cdot\vec{L}$, so that the vector-exchange spin-orbit potential has a large antisymmetric component. However, we should note that the Λ-^{4}He spin-orbit potential involves an average of the ΛN spin-orbit potential over the nucleon spin states, so that this strongly nonsymmetric term $V_{so}^{ME}(\omega)$ will not contribute to the Λ-^{4}He potential under consideration in this paper.

K^* exchange also contributes to the ΛN spin-orbit potential. To order $(m/\overline{M})^4$, the result is given by[9]

$$V_{so}^{EE}(K^*)=-P_{\Lambda N}^{x}\frac{(G_{\Lambda N1/2}^{E})^2}{4\pi}mY_1(mr)\left(\frac{m^2}{\overline{M}m^*}\right)^2\frac{(M_N+M_\Lambda)^2}{8M_NM_\Lambda} \times\left[1+\frac{m^2}{(M_N+M_\Lambda)^2}-\frac{m^2}{8M_NM_\Lambda}\right]\vec{S}\cdot\vec{L}\,, \tag{2.16}$$

$$V_{so}^{MM}(K^*)=-P_{\Lambda N}^{x}\frac{(G_{\Lambda N1/2}^{M})^2}{4\pi}mY_1(mr)\left(\frac{m}{\overline{M}}\right)^2 \times\left[\frac{m^2}{8\overline{M}^2}\frac{(M_N+M_\Lambda)^2}{4M_\Lambda M_N}\vec{S}\cdot\vec{L}+\frac{M_\Lambda{}^2-M_N{}^2}{4\overline{M}^2}\vec{S}^A\cdot\vec{L}\right], \tag{2.17}$$

$$V_{so}^{EM}(K^*)=V_{so}^{ME}(K^*)=P_{\Lambda N}^{x}\frac{(G_{\Lambda N1/2}^{E}G_{\Lambda N1/2}^{M})}{4\pi}mY_1(mr)\left(\frac{m}{\overline{M}}\right)^2 \times\left(\frac{M_\Lambda+M_N}{2\overline{M}}\right)\left(1-\frac{m^2}{4M_NM_\Lambda}\right)\vec{S}\cdot\vec{L}\,, \tag{2.18}$$

where $m^*=m(K^*)$ and $m=[m^{*2}-(M_\Lambda-M_N)^2]^{1/2}$. We note that $V_{so}^{MM}(K^*)$ includes a small antisymmetric term; otherwise all the K^* spin-orbit contributions are symmetrical.

These calculations have neglected some nonstatic effects and short-range effects in the potential. In particular, we have omitted terms proportional to $\delta(r)$. The rationale for the omission of all such short-range terms is that we expect the existence of a strongly repulsive interaction at short distances ($r\leq 0.4$ F, typically) in all baryon-baryon interactions. This repulsive core in the ΛN interaction dominates over all other interactions in this inner region; it expels the ΛN wave function from this region, in consequence of which such short-range contributions have negligible expectation values, so that they may be omitted from our discussion. The origin of this hard-core repulsion is not yet understood; it is only clear that it must be present if any of the present calculations of the outer region of the baryon-baryon potentials are to make any physical sense.

B. Λ-⁴He Spin-Orbit Potential

We obtain this by summing the ΛN OBE spin-orbit potentials over the nucleons in ^{4}He and averaging over the density distribution (2.2). The BBM coupling constants to be used for the potentials given in Sec. II A will be discussed in Sec. III.

For the exchange of a boson of mass m and coupling-constant factors $C_{N\Lambda m}^{so}$ and $D_{N\Lambda m}^{so}$, the contribution to the Λ-^{4}He spin-orbit potential is

$$V_{LS}(r_\Lambda)\vec{S}_\Lambda\cdot\vec{L}_{\Lambda\alpha}=\sum_{i=1}^{4} m\int Y_1(m|\vec{r}_\Lambda-\vec{r}_i|)(\vec{r}_\Lambda-\vec{r}_i)\times\frac{M_N\vec{p}_\Lambda-M_\Lambda\vec{p}_N}{M_N+M_\Lambda}\cdot(C^{so}_{N\Lambda m}\vec{S}_\Lambda+D^{so}_{N\Lambda m}\vec{S}_i)\times\rho(r_i)d^3r_i\,. \tag{2.19}$$

In this expression (2.19) and henceforth, the vectors $\vec{r}_\Lambda$ and $\vec{r}_i$ are measured from the c.m. of the ^{4}He core, $\vec{S}_\Lambda$ and $\vec{S}_i$ are the Λ and ith nucleon spin vectors, respectively, and $(\vec{p}_\Lambda, \vec{p}_N)$ and (M_Λ, M_N) denote the momentum and mass of the particle specified. The terms proportional to $\vec{S}_i$ sum to zero, since $J=0$ for ^{4}He. In terms of the Λ-^{4}He c.m. momentum and the N_i momentum relative to the ^{4}He center of mass we may write

$$\frac{M_N\vec{p}_\Lambda-M_\Lambda\vec{p}_N}{M_N+M_\Lambda}=\frac{1}{4(M_N+M_\Lambda)}\times\left[(4M_N\vec{p}_\Lambda-M_\Lambda\vec{p}_\alpha)-\frac{M_\Lambda}{M_N}(4M_N\vec{p}_i-M_\Lambda\vec{p}_\alpha)\right]. \tag{2.20}$$

Since we know that the ^{4}He wave function is predominantly S state, the term $\vec{r}_i\times(\vec{p}_i-\frac{1}{4}\vec{p}_\alpha)$ vanishes identically; similarly, the term $\vec{r}_\Lambda\times(\vec{p}_i-\frac{1}{4}\vec{p}_\alpha)$ leads to zero after integration over $\vec{r}_i$. Next, the factor $(4M_N\vec{p}_\Lambda-M_\Lambda\vec{p}_\alpha)$ does not depend on the internal coordinates, so that we are left with the integration

$$\int Y_1(m|\vec{r}_\Lambda-\vec{r}_i|)(\vec{r}_\Lambda-\vec{r}_i)\rho(\vec{r}_i)d^3r_i=\vec{r}_\Lambda X\,, \tag{2.21}$$

where the coefficient X is given by

$$X(r_\Lambda)=\frac{1}{r_\Lambda^2}\int Y_1(m|\vec{r}_\Lambda-\vec{r}_i|)\vec{r}_\Lambda\cdot(\vec{r}_\Lambda-\vec{r}_i)\rho(\vec{r}_i)d^3r_i\,. \tag{2.22}$$

With the expressions (2.5) for $Y_1(x)$ and (2.2) for $\rho(\vec{r}_i)$, the integral (2.22) may be carried out explicitly, either directly or by the method indicated in Ref. 34 below, with the following result:

$$X(r)=-\frac{e^{-r^2/a^2}}{2(mr)^3}\left[\operatorname{erfc}(u_+)e^{u_+^2}(1-mr)-\operatorname{erfc}(u_-)e^{u_-^2}(1+mr)+\frac{4r}{a\sqrt{\pi}}\right], \tag{2.23}$$

where[35]

$$\operatorname{erfc}(x)=\frac{2}{\sqrt{\pi}}\int_x^\infty e^{-t^2}dt\,, \tag{2.24a}$$

$$u_\pm=\frac{ma}{2}\pm\frac{r}{a}\,. \tag{2.24b}$$

Since $\vec{L}_{\Lambda\alpha}$ has the form

$$\vec{L}_{\Lambda\alpha}=\vec{r}_\Lambda\times\frac{4M_N\vec{p}_\Lambda-M_\Lambda\vec{p}_\alpha}{4M_N+M_\Lambda}\,, \tag{2.25}$$

we conclude that

$$V_{LS}(r_\Lambda)=mC^{so}_{N\Lambda m}\frac{4M_N+M_\Lambda}{M_N+M_\Lambda}X(r_\Lambda)\,, \tag{2.26}$$

where we recall that r_Λ denotes the Λ-^{4}He separation.

Despite the $(r_{\Lambda N})^{-3}$ singularity in the ΛN spin-orbit potentials $Y_1(mr_{\Lambda N})$ considered, the Λ-^{4}He spin-orbit potential V_{LS} given by (2.26) is everywhere finite. For larger r_Λ, $r_\Lambda\gg a$ $(>1/m)$, expression (2.22) is clearly dominated by the Y_1 term, so that

$$V_{LS}(r_\Lambda)\xrightarrow[r_\Lambda\to\infty]{} m\,C^{so}_{N\Lambda m}\frac{4M_N+M_\Lambda}{M_N+M_\Lambda}e^{+(ma)^2/4}\frac{e^{-mr_\Lambda}}{(mr_\Lambda)^2}\,. \tag{2.27}$$

For small r_Λ, $V_{LS}(r)$ approaches a finite value as $r\to 0$. To discuss this in general for an arbitrary spin-orbit function $Y(mr_{\Lambda N})$, we replace the function Y_1 in expression (2.22) by Y and transform to the variable $\vec{s}=(\vec{r}_\Lambda-\vec{r}_i)$, which leads us to the forms

$$X(r_\Lambda)=\frac{1}{r_\Lambda^2}\int Y(ms)\vec{s}\cdot\vec{r}_\Lambda\rho(|\vec{r}_\Lambda-\vec{s}|)d^3s\,, \tag{2.28a}$$

$$=\frac{2\pi}{r_\Lambda}\int_0^\infty s^3Y(ms)ds\times\int_{-1}^{+1}\mu d\mu\rho((r_\Lambda^2-2r_\Lambda s\mu+s^2)^{1/2})\,. \tag{2.28b}$$

In the limit $r_\Lambda\to 0$, the expression (2.28b) approaches the limiting form

$$\lim_{r_\Lambda\to 0}X(r_\Lambda)=-\frac{4\pi}{3}\int_0^\infty Y(ms)\rho'(s)s^3ds+O(r_\Lambda^2)\,, \tag{2.29}$$

so that , even with the s^{-3} behavior of $Y_1(ms)$ near $s=0$, $X(0)$ is finite and $X'(0)=0$. With the Gaussian form (2.2) for ρ, the integration over μ can be carried out explicitly for expression (2.28) for arbitrary ΛN spin-orbit function $Y(ms)$, with the result

$$X(r_\Lambda)=\frac{2e^{-r_\Lambda^2/a^2}}{a^2r_\Lambda^{3/2}m^{7/2}}\int_0^\infty e^{-z^2/(ma)^2}Y(z)z^{5/2}I_{3/2}\left(z\frac{2r_\Lambda}{ma^2}\right)dz\,, \tag{2.30}$$

where I_ν denotes the modified Bessel function of order ν.

It is of interest to remark here [as a generalization of the relation (7) in work by Hughes and LeCouteur[36]] that the general function $X(r_\Lambda)$ given by the expressions (2.28) can be written conveniently in the form

$$X(r_\Lambda) = -\frac{1}{r_\Lambda}\frac{d}{dr_\Lambda}V(r_\Lambda)\,, \tag{2.31}$$

where $V(r_\Lambda)$ is given by the integral[34]

$$V(r_\Lambda) = \frac{1}{m^2}\int v(s)\rho(|\vec{r}_\Lambda - \vec{s}|)d^3s\,, \tag{2.32}$$

the function $v(x)$ within its integrand being explicitly given by the integral

$$v(x) = \int_x^\infty \alpha Y(\alpha)d\alpha\,. \tag{2.33}$$

When $Y(x)$ has the particular form $Y_1(x)$ given by (2.5) for $n=1$, then $v(x)$ is simply the Yukawa function $Y_0(x)$. The spin-orbit potential V_{LS} arising from n OBE processes, for a set of mesons with masses $\{m_j\}$ and coupling-constant factors $\{C^{so}_{N\Lambda m_j}\}$ with $j=1,2,\ldots n$, then corresponds to a function $V(r_\Lambda)$ given by (2.32) with

$$v(s) = \sum_{j=1}^{n} m_j C^{so}_{N\Lambda m_j} Y_0(m_j s)\,. \tag{2.34}$$

Now, if we were to attempt to calculate the Λ-^{4}He central potential $V_c(r_\Lambda)$ arising from this same set of OBE processes, treated in the simplest static approximation, the result would be given by the expression (2.32) evaluated for the function $v_c(s)$, where

$$v_c(s) = \sum_{j=1}^{n} m_j C^{c}_{N\Lambda m_j} Y_0(m_j s)\,, \tag{2.35}$$

the coupling-constant factors $C^c_{N\Lambda m}$ being those appropriate to the spin-independent central part of the OBE amplitude. Since the relationship between the coefficients $C^c_{N\Lambda m}$ and $C^{so}_{N\Lambda m}$ depends on the particular meson-exchange considered, there is no reason to expect that the derivative relationship (2.31) should hold generally between the Λ-^{4}He spin-orbit potential $V_{LS}(r_\Lambda)$ and the Λ-^{4}He central potential $V_c(r_\Lambda)$. In any case, such an attempt to calculate the central potential $V_c(r_\Lambda)$ from first principles would be unrealistic, owing to the very strong repulsion known to exist at very short distances ($\leq d \approx 0.4$ F) in the baryon-baryon interaction. In the present work, we prefer to adopt a phenomenological approach to $V_c(r_\Lambda)$, taking into account our knowledge of the ^{4}He shape and the range observed for the ΛN s-wave interaction, and fitting the potential strength to the B_Λ value observed for $^5_\Lambda$He.

We return now to the consideration of $V_{LS}(r_\Lambda)$ as given by the expressions (2.22) and (2.26). The $(r_{\Lambda N})^{-3}$ singularity in the ΛN spin-orbit potentials $Y_1(mr_{\Lambda N})$ is spurious, of course. In fact, the form of this potential for small $r_{\Lambda N}$ is determined by the behavior of the ΛN scattering amplitude for large values of the momentum transfer q. For very large q, the two-component approximation which has been adopted for the baryon spinor in the nonrelativistic reduction of the amplitude is no longer valid. For the ΛN system, in practice, some correction to the form $Y_1(mr_\Lambda)$ will be necessary, either by some more adequate calculation for the regime of large q (not yet convincingly achieved, since this is the regime where high-multiplicity meson systems and virtual baryonic pairs will play a significant role) or by some cutoff, appealing effectively to the existence of some strong (or even hard-core) repulsion of short range to exclude the ΛN system from this region of close approach. If the calculated ΛN spin-orbit potential is not cut off in some way, it will dominate the centrifugal barrier for small r; the $(r_{\Lambda N})^{-3}$ potential is simply too singular for use in the Schrödinger equation. Even with a simple cutoff, replacing $Y_1(mr_{\Lambda N})$ by zero for $r_{\Lambda N} \leq d$, this ΛN spin-orbit potential can still produce spurious bound states for some angular momentum channels if the cutoff radius d is not chosen sufficiently large. We have noted above that our Λ-^{4}He spin-orbit potential V_{LS} is well behaved even including this $(r_{\Lambda N})^{-3}$ singular term. However, we must make sure that the contributions to V_{LS} from the region $r_{\Lambda N} \leq d$ do not play a decisive role in the evaluation of V_{LS}, since the form (2.5) will certainly not be correct for this region; on the other hand, if the region $r_{\Lambda N} \leq d$ plays a minor role in the final values for $V_{LS}(r_\Lambda)$, then we can be sure that a proper treatment for $V_{so}(\Lambda N)$ will not modify $V_{LS}(r_\Lambda)$ significantly.

To regulate the behavior of $V_{so}(r_{\Lambda N})$ at small $r_{\Lambda N}$, we have used a subtraction in momentum space; we subtract from each OBE spin-orbit potential a term with a higher mass μ and with the coupling constant chosen to remove the $r_{\Lambda N}{}^{-3}$ singularity at small $r_{\Lambda N}$. For the two-body problem, this procedure is roughly equivalent to a cutoff, in that it suppresses $V_{so}(r_{\Lambda N})$ in the central region $r_{\Lambda N} < 1/\mu$. For example, at small $r_{\Lambda N}$ the symmetric spin-orbit term due to $I=0$ scalar-boson exchange has the form [cf. (2.4)]

$$\lim_{r_{\Lambda N}\to 0} V_{so}(S_0, r_{\Lambda N}) = \frac{-G^S_{\Lambda\Lambda 0}G^S_{NN0}}{4\pi}\frac{\vec{L}\cdot\vec{S}}{2\bar{M}^2}\left(1-\frac{m_S{}^2}{8\bar{M}^2}\right) \times\left(\frac{1}{r_{\Lambda N}{}^3}-\frac{m_S{}^2}{2r_{\Lambda N}}\right), \tag{2.36}$$

using the approximation $M_N \approx M_\Lambda \approx \bar{M}$ for purposes of illustration. The $r_{\Lambda N}{}^{-3}$ dependence in (2.31) is removed if we subtract a similar potential corresponding to mass μ, where $\mu > m_S$, and to a coupling constant G which satisfies the equation

$$\frac{G^2}{4\pi}\left(1-\frac{\mu^2}{8\bar{M}^2}\right) = \frac{G_{\Lambda\Lambda S}G_{NNS}}{4\pi}\left(1-\frac{m_S{}^2}{8\bar{M}^2}\right). \tag{2.37}$$

The analogous procedure was followed also for the vector mesons. We note that with this subtraction procedure[37] the singularity at the origin is reduced to $r_{\Lambda N}^{-1}$; this is evident from the form of (2.20). We use this cutoff procedure only in the spin-orbit forces, of course, and we find that the final results we obtain are not qualitatively sensitive to the value chosen for μ. This degree of insensitivity is due to the fact that the spin-orbit potential is effective only for states with $l_{\Lambda N} \geq 1$, so that the ΛN potential occurs always weighted by an additional factor of at least $r_{\Lambda N}^2$ (over and above the volume factor $r_{\Lambda N}^2 dr_{\Lambda N}$) which suppresses its contribution to V_{LS} from small values of $r_{\Lambda N}$. Indeed, we should emphasize here that the potential V_{LS} remains finite and well defined by these integral expressions even in the limit $\mu \to \infty$, i.e., for the case without cutoff, even though the ΛN spin-orbit potentials corresponding to this limit are quite unsatisfactory for calculations concerning the ΛN system itself.

The functions $V_{LS}(r)$ calculated for the Λ-^{4}He system have been plotted in Fig. 1 for three typical sets of OBE parameters appropriate for the ΛN interaction (as given in Table I and to be discussed in Sec. III) for two cases: (a) without cutoff, i.e., with $\mu = \infty$, and (b) with the arbitrary choice $\mu = 1500$ MeV. For case (a), the calculations were made using the expression (2.23) with the appropriate coefficient $(mC^{so}_{N\Lambda m})$ for each meson exchange included; in case (b), for each meson-exchange term, there was subtracted a corresponding term obtained by replacing m by μ in the potential and changing the coefficient of the potential in accord with the prescription (2.37). Figure 1 shows that this cutoff has quite a weak effect on the shape of the potential V_{LS}. All the calculated curves are quite well fitted by Gaussian forms $e^{-r_\Lambda^2/A^2}$ (to accuracy better than ±5% as far as $r_\Lambda \approx 2$ fm), the values for A being about 1.25 fm for $\mu = \infty$, and 1.28 fm for $\mu = 1500$ MeV. These parameters are naturally somewhat larger than the value $a = 1.175$ fm appropriate to $\rho(r)$, the largest being for the ΛN potential which is damped for small $r_{\Lambda N}$ by the cutoff μ. However, the magnitude of V_{LS} is quite strongly affected by the cutoff μ, the reduction factor being 0.70 for SVH, 0.63 for BDI, and 0.65 for the Deloff parameters. These conclusions are in good qualitative accord with the expectations discussed in Ref. 38, for the case of the ΛN force range small relative to the ^{4}He radius. We shall compare the results obtained for these two prescriptions concerning the ΛN spin-orbit potential in Sec. IV.

We conclude this section with a brief comparison between the discussion above and the situation for the N-^{4}He system. There are two complications for the latter case: (a) the great strength of the long-range tensor force effective in the NN system; and (b) the requirement of antisymmetry for the wave function of all the nucleons in the N-^{4}He system, which links the outer nucleon with those in the ^{4}He system. These complications result in large contributions to the N-^{4}He P-wave splitting beyond those which result from the sum of the two-body NN spin-orbit potentials. Sugie, Hodgson, and Robertson[16] have shown that these two effects, taken together, can account for about 30% of the

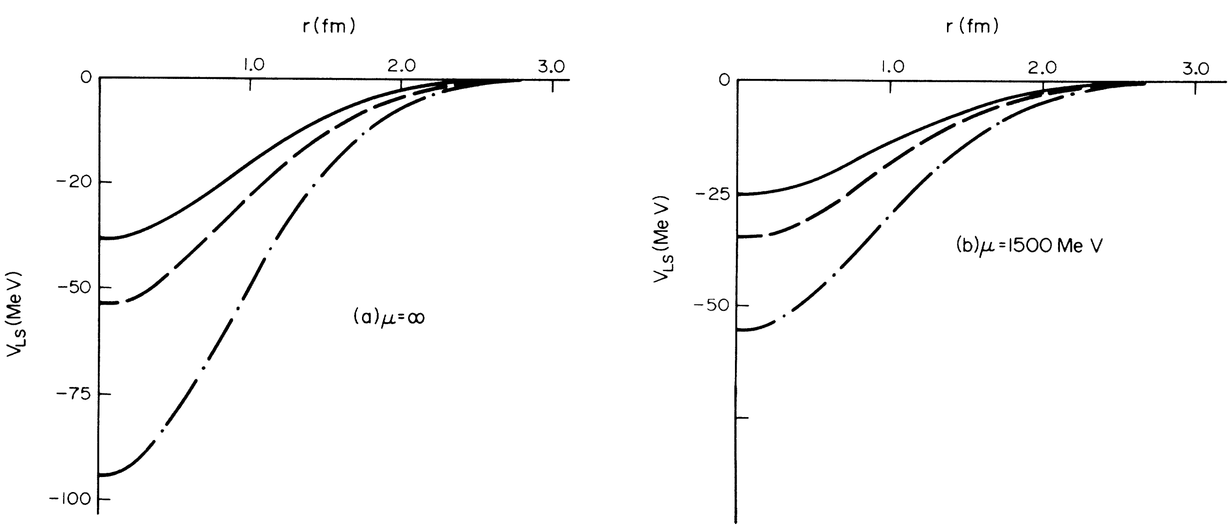

FIG. 1. The Λ-^{4}He spin-orbit force $V_{LS}(r)$(MeV) vs the Λ-^{4}He relative coordinate r. Solid line: SVH parameters; dot-dashed lines: BDI parameters; dashed line: Deloff parameters, used to determine the ΛN OBE spin-orbit interaction. (a) Without cutoff in the ΛN spin-orbit potential. (b) With cutoff, given by the subtracted mass $\mu = 1500$ MeV (as discussed in Sec. II B).

observed splitting. The ^{4}He ground state includes an admixture of D state to the predominant S state, due primarily to the (first-order) operation of the NN tensor force. When the N-^{4}He wave function is antisymmetrized with respect to all five nucleons, this NN tensor force contributes to the N-^{4}He P-wave splitting through its *exchange matrix element* between the P-wave nucleon and a product of the S state and D state of the ^{4}He wave function. Without the complete antisymmetrization of the N-^{4}He wave function, this tensor-force matrix element would vanish.

For the Λ-^{4}He system, there is no antisymmetrization between the Λ particle and the nucleons, since Λ and n are physically nonidentical particles. In any case, we have no reason to expect a strong tensor force in the ΛN system. This could arise most directly from single P exchange, K, η, and η' exchange being permitted; however, these particles are all rather massive and give rise only to rather short-range tensor forces (in comparison with $(m_\pi)^{-1}$ for the OPE process which generates the NN tensor force), which are correspondingly ineffective for the interaction of a Λ particle with nucleons in a nucleus. Hence, we expect the Λ-^{4}He spin-orbit potential to be given dominantly by the sum of the two-body ΛN spin-orbit forces, as we have assumed in this section.

TABLE I. The values of the OBE parameters used in this paper for the calculation of ΛN spin-orbit force. The parameters given are the electric and magnetic ρ couplings to nucleons, and the corresponding F-D mixing parameters f_V^E and f_V^M; the BBV unitary singlet couplings $G_V^{E'}$ and $G_V^{M'}$; the scalar mass and its coupling constant to nucleons, and the scalar F-D mixing parameter f_S. [Deloff does not specify the mixing parameter f_V^t explicitly. However, he interprets the ω meson as V_0, the $I=0$ member of the octet $V(8)$, and assumes the tensor coupling for NNV_0 to be zero. From (2.3), this requires $f_V^t=0.25$, the value we have adopted here for the Deloff parameter set. Deloff also omits the unitary singlet vector meson V', which is equivalent to assuming zero for all its baryonic couplings, in particular for its BBV' tensor coupling. With these assumptions, then, the tensor couplings $NN\omega$ and $NN\phi$ are zero for both the physical ω and the physical ϕ, a situation which is generally believed to hold rather well.]

Meson exchanged	Parameter	BDI	Deloff	SVH
Vector	$G_{NN1}^E/\sqrt{4\pi}$	1.27	1.78	1.0
	f_V^E	0.63	−0.84	1.0
	$G_{NN1}^M/\sqrt{4\pi}$	5.09	2.59	4.65
	f_V^M	0.43	−0.56	0.4
	$G_V^{E'}/\sqrt{4\pi}$	6.14	0.0	0.0
	$G_V^{M'}/\sqrt{4\pi}$	6.14	0.0	0.0
Scalar	$G_{NN0}^S/\sqrt{4\pi}$	2.48	5.25	3.95
	f_S	(−0.5)	−0.5	(−0.5)
	m_S (MeV)	490	820	560

III. OBE PARAMETERS

In principle, we could calculate both the central and spin-orbit Λ-^{4}He potentials from the ΛN OBE two-body potential. To calculate the Λ-^{4}He central potential V_c in this way would require a realistic treatment of the ΛN correlations in the Λ-^{4}He wave functions, and this potential V_c would almost certainly not lead to the B_Λ value observed for $^5_\Lambda$He. Instead, we have chosen to use for V_c the phenomenonological potential of Dalitz and Downs,[14] which is adjusted to fit this B_Λ value. We use this potential for Λ-^{4}He scattering for incident Λ energies up to 20 MeV. This assumes that the Λ-^{4}He central potential is not appreciably energy dependent for this energy range, and this appears a reasonable assumption, since we do not anticipate significant energy dependence for the ΛN potential itself over this energy range, nor do we expect any energy-dependent effects to arise from distortion of the α particle by the colliding Λ particle in this low-energy regime.

The baryonic couplings are not yet well established for the bosons which contribute dominantly to the ΛN spin-orbit force – the vector mesons ω, ϕ, and K^*, and the scalar mesons S_0 and possibly $S_{\frac{1}{2}}$ [generally known as $K_N(\sim1100)$]. In the present calculations, for the purpose of illustration, we have taken the ΛN parameters from three recent OBE models of the YN or baryon-baryon interactions:

(i) the single-channel ΛN effective potential of Deloff[31]

(ii) the two-channel YN potential of BDI,[25] and

(iii) the zero-parameter baryon-baryon potential of SVH.[32]

Deloff began by reproducing the Hamada-Johnston NN potential with OBE terms resulting from the exchange of pseudoscalar and vector octets, as well as of an assumed octet of scalar bosons. To fit the NN potential, he varied the masses and coupling constants of the scalar bosons and the coupling parameters for the $NN\eta$, $NN\omega$, and $NN\rho$ interactions, the pion coupling $NN\pi$ being assumed already well known. For the interaction BBV of a vector meson V with baryon B, Deloff used the vector (v) and tensor (t) coupling forms, with corresponding coupling parameters g and G; with the form for the BBV vertex representing the absorption of a vector meson given by

$$\mathfrak{L}_{\text{int}}^V=\bar{\psi}_B\left(g\gamma_\nu\phi^\nu+i\frac{G}{2m_V}\sigma_{\mu\nu}q^\mu\phi^\nu\right)\psi_B\,, \qquad (3.1)$$

where ϕ^v denotes the wave function for the vector meson and m_V denotes the vector-meson mass, rather than the coupling form (2.10) discussed in Sec. II above. The explicit relation between the coupling parameters (g, G) of (3.1) and (G^E, G^M) of (2.10) will be given below.

Having obtained the NN OBE parameters in this way, Deloff,[31] then considered the quadratic equations determined from them for the SU(3) mixing parameters for each octet (the mixing parameter f_V^t for the tensor coupling of the vector octet was fixed by the requirement that the tensor couplings should be zero for both $NN\omega$ and $NN\phi$). The assumption of SU(3) symmetry then fixed the coupling constants appropriate to the ΛN interaction. Of the eight solutions found for the mixing parameters f_P, f_V^v, and f_S, only one solution gave a qualitatively reasonable fit to the ΛN s-wave scattering data. This solution had the parameters $f_S = -0.62$, $(G^S_{NN0})^2/4\pi = 38$, $m_0 = 820$ MeV, and $m_1 = 910$ MeV, for the scalar boson octet; this large value for the coupling constant G^S_{NN0} is due in large part to the large mass values obtained for the scalar bosons. The resulting ΛN potential was somewhat too strong to give a satisfactory fit to the ΛN data. With a change to the mixing parameter $f_S = -0.5$ (for which $G^S_{N\Lambda\frac{1}{2}} = 0$) and a change of $(G^S_{NN1})^2/2\pi$ from 9.3 to 9.1 – these two changes together reduce $(G^S_{NN0})^2/4\pi$ to 27.3 – Deloff found that his potential gave good agreement with the experimental data on low-energy ΛN scattering.

The BDI calculations considered both ΛN and ΣN channels explicitly, so that they were in a position to compute the scattering, charge-exchange, and reaction processes for both channels. For the vector mesons, they treated the ω and ϕ mesons as SU(3) eigenstates, the ϕ being octet, the ω being unitary singlet, and so coupled universally to all the baryons. BDI also adopted the vector and tensor coupling form (3.1) for the vector-meson-baryon interactions BBV. The scalar meson was assumed to be a unitary singlet (denoted by σ), and so to have a universal coupling to the baryons. The BDI model constrained the ^1S Λp interaction to be more attractive than that for the $^3S_1$ state, and fitted the Λp scattering data from zero energy to the Σ production threshold. The adjustable parameters were a spin-independent hard-core radius, the scalar mass and coupling constant, and the coupling of ω to baryons. Small adjustments were also made in f_P and in the vector ρ coupling constant $g_{PP\rho}{}^2/4\pi$, to give the required spin dependence in the Λp interaction. In particular, the ρ coupling used was $g_{PP\rho}{}^2/4\pi = 0.32$, rather lower than the range 0.6–0.7 deduced by Sakurai[39] from a number of independent effects.

The final parameters given in the analyses by BDI and by Deloff are listed in Table I. For the BBV interactions, these are given in the (G^E, G^M) form corresponding to the interaction form (2.10), rather than the (g, G) form used by these authors for the interaction form (2.1). The general relation between them for the diagonal BBV interactions is readily obtained, with the general results[40]:

$$g = \frac{M_B}{\bar{M}} G^E - \frac{q^2}{4\bar{M}^2} G^M, \tag{3.2a}$$

$$\frac{2\bar{M}}{m_V} G = -G^E + \frac{M_B}{\bar{M}} G^M, \tag{3.2b}$$

for arbitrary momentum transfer q^2; for the nuclear-force calculations, the coupling amplitudes required are those for $q^2 = m_V{}^2$. These relations (3.2), for the particular value $q^2 = m_V{}^2$, have thus been used to obtain the parameters G^E, G^M, and f_V^E, f_V^M given in Table I from the parameters given in the original papers.

Three features of the BDI parameters deserve mention:

(i) The large ω coupling to baryons. This is in agreement with the OBE potential models for NN scattering.[27,41] However, it should be mentioned here that OBE calculations which fit the NN amplitudes directly,[42] by using dispersion relation, K-matrix, or other techniques to unitarize the Born terms generally obtain much smaller ω coupling $[(G^E_{NN\omega})^2/4\pi \approx 2\text{–}3$, compared with $(G^E_{NN\omega})^2/4\pi \approx 20$ for a potential model].

(ii) The scalar mass $m_S = 490$ MeV. This result is characteristic of many OBE analyses for the NN interaction, which require a low scalar mass between 400 and 550 MeV.[26,27] This low scalar mass is needed to provide central and spin-orbit potentials which fit the NN phase shifts, but it appears to disagree with the present indications that the $I = 0$ $\pi\pi$ s-wave phase shift shows a resonance for a mass value about 700 MeV, or perhaps even higher.[43–45] However, the width reported for this resonance appears to be very large ($\gg 100$ MeV),[44,45] and a recent OBE calculation for the NN interaction[46] also indicates a very large width ($\gtrsim 300$ MeV) for the scalar meson S_0. It is possible that the low mass $m_S \approx 500$ MeV required by the OBE calculations (which treat S_0 as a zero-width particle) results from the need to simulate a higher-mass particle with a very large width. It is also true that the S_0-exchange term is used to represent not only the terms arising from resonance $\pi\pi$ exchange, but also those arising from exchange of $I = 0$ s-wave $\pi\pi$ pairs with low mass, which give rise to relatively long-range attractions and which are correspondingly effective in binding.

(iii) The possibility of "double-counting." Using

an OPE potential in the two-channel Schrödinger equation sums up the ladder diagrams for pion exchange between (Λ, Σ) and N. The inclusion of an $I=0$ scalar boson exchange, since it is intended to account for all the $I=0$, $J^\pi = 0^+$ $\pi\pi$ exchange, both resonant and nonresonant, may duplicate some fraction of the effects already included in the double iteration of the $\Lambda\Sigma\pi$ vertex which gives $\Lambda \to \Sigma + \pi \to \Sigma + \pi + \pi$. How significant this "double counting" may be is not known. Essentially, this is an old problem, much discussed for the calculation of the NN potential,[47,48] that is, the question of what contribution the iterated OPE graph makes to the two-pion-exchange potential, but the problem arises here in a particularly acute form.

The third parameter set we have considered is that for the zero-parameter NN potential of SVH.[29] The SVH potential assumes that the electric and magnetic couplings of the BBV amplitude transform simply under SU(3) [cf. (2.3)]. The electric couplings are pure F type ($f_V^E = 1$) and are determined by the coupling $G_{NN\rho}^E$, known from other considerations; SVH actually choose $(G_{NN\rho}^E)^2/4\pi = 1.0$, somewhat larger than the values of Sakurai.[39] SVH also assume SU(6) coupling for the vector mesons, which requires $f_V^M = 0.4$ and leads to a definite ratio between the magnetic and electric couplings,

$$G_{NN\rho}^M / G_{NN\rho}^E = \tfrac{5}{3}(2M\mu_p/e) = 4.65\,. \qquad (3.3)$$

SU(6) coupling also implies that the physical ϕ does not couple with nucleons, and that the magnetic coupling of the physical ω with the Λ particle vanishes; thus $G_{\Lambda\Lambda\omega}^M = 0$. With this last relation, the mixed terms in $V_{so}(V_0)$ are strongly asymmetric, being proportional to $\vec{\sigma}_N \cdot \vec{L}$, up to terms of order $(m/M)^4$. However, this term gives no contribution to V_{LS} for the Λ-^{4}He system, since their sum over the nucleons of ^{4}He gives zero.

The SVH analysis for the NN interaction did not include scalar-boson exchange, but included multipion exchange by the inclusion of two additional channels, $N\Delta(1236)$ and $\Delta(1236)\Delta(1236)$, taking into account only the excitation of each of these channels by OPE (with some "unitarity suppression") from the NN channel. In fact, these two channels $N\Delta$ and $\Delta\Delta$ are both closed in the low-energy region of interest for the NN interaction. In place of the potential resulting from these off-diagonal couplings, we have included a scalar-boson exchange. With the mass $m_S = 560$ MeV, we varied the coupling constant G_{NN0}^S until the NN potential obtained with this scalar exchange and the vector and pseudoscalar exchanges of SVH fitted the Reid potential,[49] a phenomenological NN potential which reproduces the empirical NN phase shifts for $J \leq 2$ for the energy range 0–350 MeV. We found that $(G_{NN0}^S)^2/4\pi \approx 15$ gave reasonable agreement with the Reid potential, in particular for r outside the centrifugal barrier, in P and D states. We assumed that the scalar boson S_0 had equal coupling to N and Λ (i.e., either that S_0 is a unitary scalar, or that the mixing parameter f_S has the value -0.5); in Sec. IV we discuss the behavior of the Λ-^{4}He scattering as the scalar coupling $G_{\Lambda\Lambda 0}^S$ is varied.

The SVH parameters are given in Table I. The ω contribution is much smaller than in the Deloff or BDI analyses. On the other hand, the K^* exchange contribution is quite large, and we find that the resultant spin-orbit force using the SVH parameters is very similar to that with the other two parameters sets.[50]

IV. RESULTS AND DISCUSSION

Using the Λ-^{4}He central potential of (2.2) and the spin-orbit potentials from (2.4) and (2.13)–(2.18), we have calculated the differential cross sections, the polarization angular distributions, and the forward-backward asymmetry D for Λ-^{4}He elastic scattering from 1–20-MeV incident Λ kinetic energy where

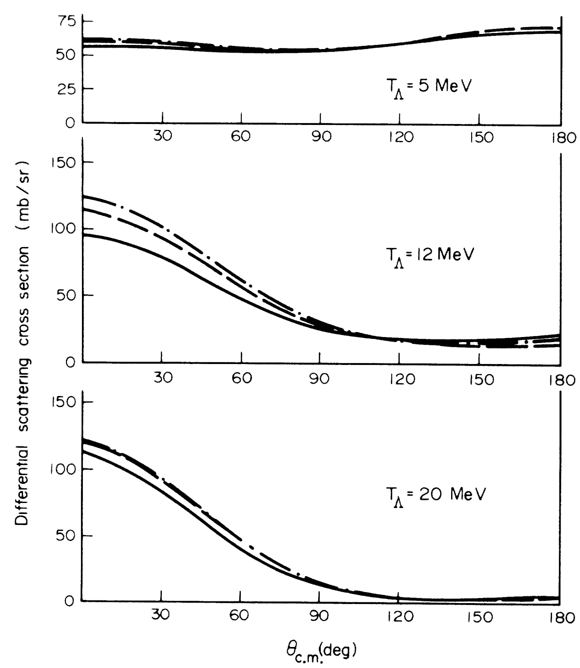

FIG. 2. Differential cross sections for Λ-^{4}He elastic scattering vs c.m. angle θ, for Λ lab kinetic energy T_Λ from 5–20 MeV. Solid line: SVH parameters; dot-dashed line: BDI parameters; dashed line: Deloff parameters; (All parameters with subtracted mass $\mu = 1500$ MeV).

$$D=\frac{\int_0^1 \frac{d\sigma}{d\Omega}d(\cos\theta)-\int_{-1}^0 \frac{d\sigma}{d\Omega}d(\cos\theta)}{\int_{-1}^1 \frac{d\sigma}{d\Omega}d(\cos\theta)}. \tag{4.1}$$

We calculated the Λ-^{4}He spin-orbit potentials V_{LS} corresponding to exchange of a scalar boson S_0 and of the vector bosons ω, ϕ, and K^*, using each of the three parameter sets listed in Table I. In Figs. 2–4 we plot the differential cross sections, polarizations, and D, respectively, for various values of the Λ incident kinetic energy T_Λ.

Since the K^* OBE potentials contain the space-exchange factor $P^x_{\Lambda N}$, the contribution of the K^* potential to V_{LS} for the Λ-^{4}He system should really be given by the integral form

$$\begin{aligned}V_{LS}(K^*;\vec{r}_\Lambda)\vec{L}_{\Lambda\alpha}\cdot\vec{S}_\Lambda\phi(\vec{r}_\Lambda)&=mC_{N\Lambda\frac{1}{2}}\int\psi_4^\dagger(\vec{r}_1,\vec{r}_2;\vec{r}_3,\vec{r}_4)Y_1(m|\vec{r}_\Lambda-\vec{r}_1|)\\&\times\Big[(\vec{S}_\Lambda+\vec{S}_1)\cdot(\vec{r}_\Lambda-\vec{r}_1)\times\frac{M_N\vec{p}_\Lambda-M_\Lambda\vec{p}_N}{M_N+M_\Lambda}\\&\times\psi_4(\vec{r}_\Lambda,\vec{r}_2;\vec{r}_3,\vec{r}_4)\phi(\vec{r}_1)d^3\vec{r}_1d^3\vec{r}_2d^3\vec{r}_3d^3\vec{r}_4\\&+3\text{ cyclic terms}\Big],\end{aligned} \tag{4.2}$$

the four terms corresponding to the interactions ΛN_i for $i=1$ to 4, in turn. Here ψ_4 denotes the ^{4}He spin-space wave function and ϕ denotes the wave function of the Λ particle relative to it. In order to simplify this expression and to lead to an algebraic expression for the potential V_{LS} (rather than an integral operator), we make the approximation of assuming that all ΛN spin-orbit interactions in this Λ-^{4}He system occur in relative P-wave states, so that we may replace the space-exchange operator $P^x_{\Lambda N}$ by -1. Since the spin-orbit interaction vanishes for ΛN relative S-wave states, the main error made in this approximation is that the spin-orbit interaction effectively adopted through this assumption has an incorrect sign for ΛN interactions in relative D-wave states; the importance of this error decreases rapidly with decreasing range for the ΛN interaction [i.e., with increasing mass m, the error involved being of order $1/ma$, where a denotes the scale parameter in the ^{4}He density distribution (2.2)]. Hence we use the expression (2.23) also for the K^* contribution to the spin-orbit force, after the replacement $P^x_{\Lambda N}=-1$ in the expressions (2.16)–(2.18).

The differential cross sections and polarizations are qualitatively the same for the three sets of

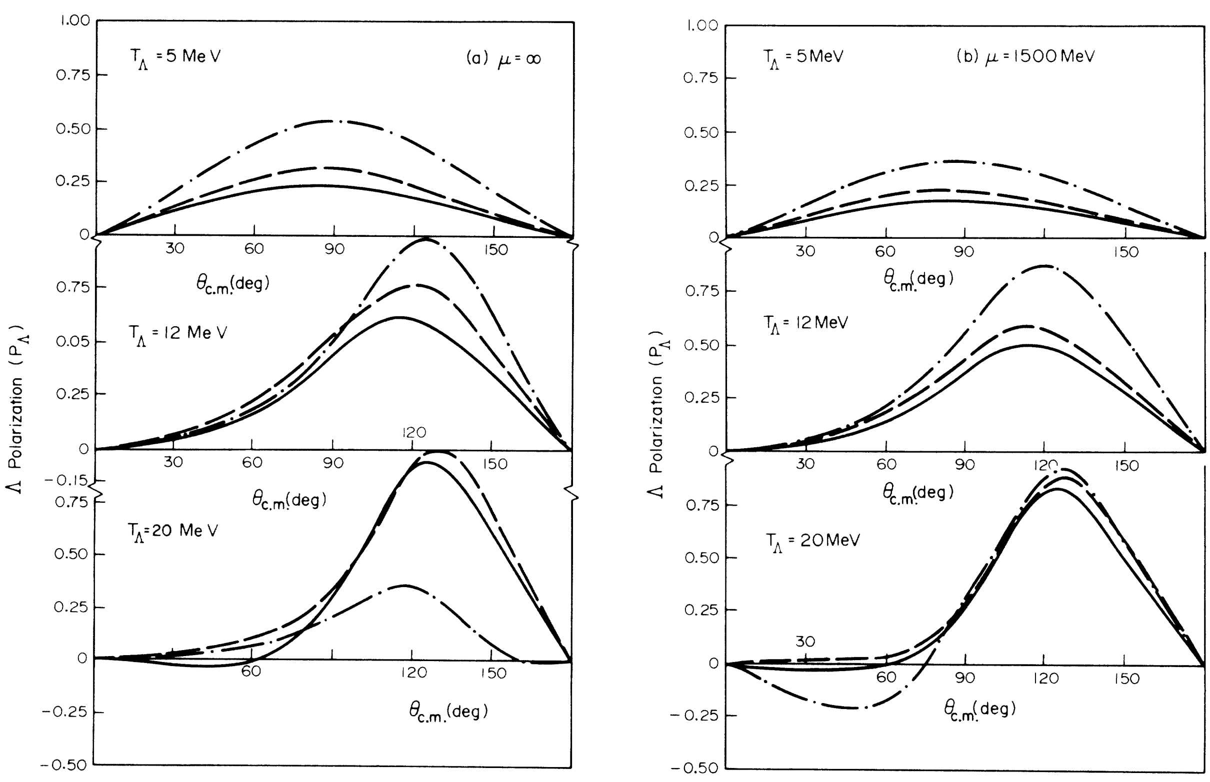

FIG. 3. Polarization $P_\Lambda(\theta)$ vs c.m. angle θ, for Λ-^{4}He elastic scattering with Λ lab energy T_Λ from 5–20 MeV, for two situations: (a) without cutoff, and (b) with cutoff given by subtracted mass $\mu=1500$ MeV. Solid line: SVH parameters; dot-dashed line: BDI parameters; dashed line: Deloff parameters.

parameters. The central force provides most of the differential cross section, and is substantially weaker than the corresponding central force in n-^{4}He scattering. The polarization curves given in Figs. 3(a) and (b) show considerable sensitivity to the precise choice of coupling parameters for the spin-orbit potential, and much greater sensitivity to the presence or absence of a cutoff in this potential. For example, at $T_\Lambda = 12$ MeV, the maximum polarization calculated for the Deloff potential falls from 0.77 to the value 0.57 when the cutoff $\mu = 1500$ MeV is introduced, whereas the SVH and BDI values are relatively little affected; at $T_\Lambda = 20$ MeV, the maximum polarization calculated for the BDI potential rises from 0.35 to 0.93 upon introduction of the cutoff $\mu = 1500$ MeV, whereas the SVH and Deloff values are much less affected. However, generally speaking, the calculated polarizations $P_\Lambda(\theta)$ have a remarkable qualitative similarity for all three parameter sets, considering the large differences in the input parameters of Table I. All three of the models used, with or without cutoff, predict large polarization for the Λ laboratory kinetic energies T_Λ between 10–20 MeV. As shown in Fig. 3, the polarization is positive at almost all points, rising to a maximum about 120° in the c.m. frame. With the Deloff and BDI parameters, the ω provides a major

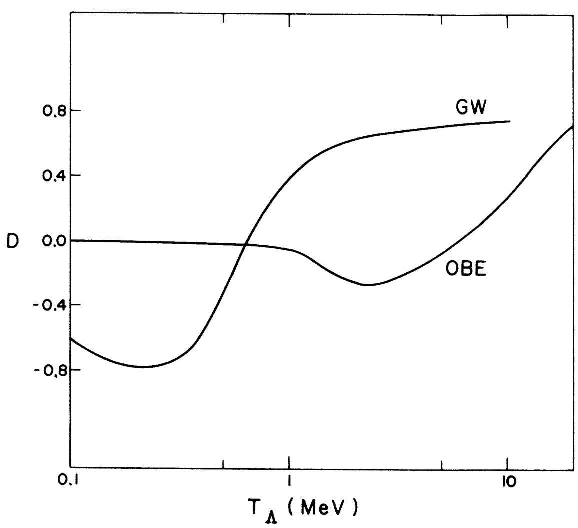

FIG. 4. Λ-^{4}He forward-backward asymmetry D [defined by (4.1)] vs Λ incident energy T_Λ, for lab energies 0.1–20 MeV. Our results (OBE) and the results of Gibson and Weiss with central force only (GW). The curve labeled OBE gives the asymmetry parameter D calculated as function of T_Λ for the SVH potential specified in Table I. The BDI and Deloff potentials give essentially the same D values for T_Λ below 5 MeV; for $T_\Lambda = 9$ MeV, the calculated values for D were 0.245 for BDI and 0.20 for Deloff, which lie close to the value 0.19 for SVH.

contribution to the spin-orbit force, while the K^* and S_0 give the largest spin-orbit terms in the SVH analysis.

We should emphasize here that the Λ-^{4}He spin-orbit potential comes from only a part of the ΛN spin-orbit potential. As remarked earlier, the summation over all four nucleons of ^{4}He gives zero for all ΛN spin-orbit terms proportional to $\vec{\sigma}_N \cdot \vec{L}$. For example, let us consider the terms (2.12)–(2.15) due to ω exchange, just to lowest order in $(m_\omega/\bar{M})^2$. It is convenient to express all of the coupling constants needed in terms of the electric $NN\rho$ coupling constants $G^E_{NN\rho}$. With the SVH parameters, these coupling constants are

$$G^E_{NN\omega} = 3G^E_{NN\rho}\,, \quad G^E_{\Lambda\Lambda\omega} = 2G^E_{NN\rho}\,, \quad G^M_{NN\omega} = (2.79)G^E_{NN\rho}\,, \quad G^M_{\Lambda\Lambda\omega} = 0\,. \tag{4.3}$$

For the purpose of illustration, let us also approximate $M_N \approx M_\Lambda \approx \bar{M}$ and take out the common factor

$$C_\omega(r) \equiv \left(\frac{m_\omega}{M}\right)^2 m_\omega Y_1(m_\omega r)\frac{(G^E_{NN\rho})^2}{4\pi}\,, \tag{4.4}$$

to give the net result for the ΛN system[51]

$$V_{so}(\omega) \approx C_\omega(r)[-2.58\vec{S}\cdot\vec{L} + 5.58\vec{S}^A\cdot\vec{L}\,]\,. \tag{4.5}$$

For the partial waves $^3P_2$ and $^3P_0$, the ω exchange contributes a weakly attractive spin-orbit force (i.e., a spin-orbit force which contributes a negative energy for positive $\vec{S}\cdot\vec{L}$; note that $\vec{S}\cdot\vec{L}$ takes the values +1 for the $^3P_2$ state and −2 for the $^3P_0$ state). For the partial waves $^3P_1$ and $^1P_1$, the spin-orbit potential $\vec{S}^A\cdot\vec{L}$ contributes an off-diagonal term linking these two states, whereas the $\vec{S}\cdot\vec{L}$ potential contributes again only in the diagonal terms, so that it is difficult to estimate the net effect of the two terms in expression (4.5). For Λ-^{4}He, on the other hand, the $\vec{S}\cdot\vec{L}$ and $\vec{S}^A\cdot\vec{L}$ terms contribute equally, since the $\vec{\sigma}_N\cdot\vec{L}$ terms average to zero, and the net ω-exchange spin-orbit coupling is then *repulsive*, being then proportional to $+3.58\vec{S}_\Lambda\cdot\vec{L}_{\Lambda\alpha}$ and due entirely to the electric $NN\omega$ coupling.

We should emphasize here that the spin-orbit potentials we have included in the discussion of Sec. II A are only the diagonal terms for the ΛN channel. For the one-channel approach of Deloff,[31] this is at least a consistent point of view. However, in a calculation of ΛN scattering including both ΛN and ΣN channels, a spin-orbit component for the *effective* ΛN *potential* can arise in other ways. For example, even if there were no diagonal ΛN spin-orbit interaction, the existence of a strong spin-orbit potential in the ΣN potential $V(\Sigma N \to \Sigma N)$ or in the off-diagonal potential $V(\Lambda N \to \Sigma N)$ would give rise to a splitting between the P phases for low-energy ΛN scattering and, in general, therefore, to some spin-orbit component in the effective ΛN potential for the low-energy region. The importance of these indirect effects in generating an effective ΛN spin-orbit potential is not yet known. To complicate this situation, there is also the possibility emphasized by Bodmer[52] for the Λ-^{4}He system that ΛN effective interactions which result from interactions passing through a ΣN intermediate state might be significantly suppressed for a Λ particle in interaction with ^{4}He, since the Λ-^{4}He system has $I=0$, which is not possible for the Σ-^{4}He system (although it is still possible for intermediate states Σ-^{4}He*, with $I=1$ excited states ^{4}He*). We shall not discuss these questions further here, since our purpose is primarily to illustrate the order of magnitude of the spin-orbit effects to be expected in low-energy Λ-^{4}He scattering, the factors which influence them, and their

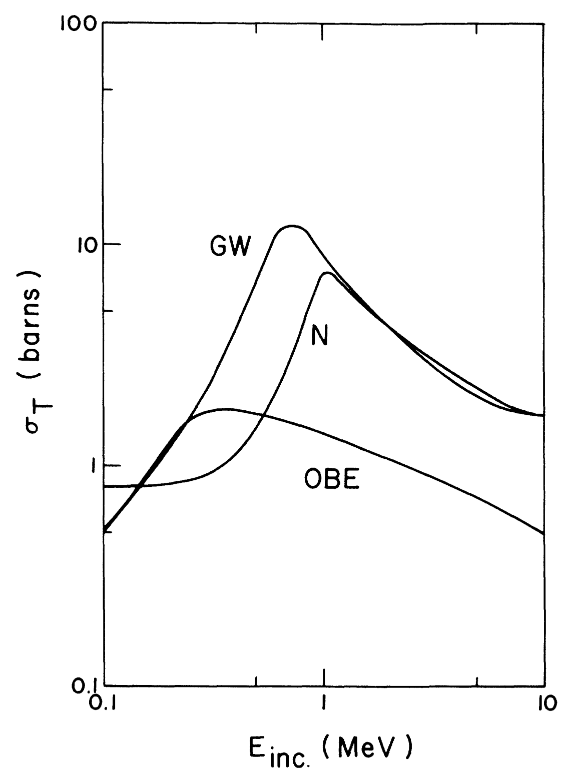

FIG. 5. Predicted total cross sections for Λ-^{4}He elastic scattering vs Λ lab energy $E_{inc.}$. Our results (OBE) and results of Gibson and Weiss with only central potential (GW) are given. For comparison the n-^{4}He total cross section (N) vs neutron incident lab energy is shown. (The curve OBE corresponds to the SVH parameters. The values calculated for BDI and Deloff parameters lie very close to this curve. For $T_\Lambda = 1$ MeV, the three calculated values agree to three significant figures; for $T_\Lambda = 5$ MeV, the BDI value lies 2% higher, the Deloff value only 0.5% higher; for $T_\Lambda = 9$ MeV, the BDI value lies 8% higher, the Deloff value only 1.5% higher.)

relationship with the ΛN interaction, rather than to attempt any definite calculation of the Λ-^{4}He spin-orbit potential.

In Fig. 5, we have plotted the calculated cross sections for Λ-^{4}He elastic scattering vs the incident laboratory energy E (the curve marked OBE in Fig. 5). For comparison, we have also plotted the cross sections calculated by Gibson and Weiss[21] for their central potential (GW), with no spin-orbit force ($C_\Lambda = 0$), and also the experimental n-^{4}He total cross sections, labeled N on Fig. 5. It is evident that the two predicted Λ-^{4}He total cross sections are qualitatively different in character. The reason for this difference is the fact that, with the GW potential which gives the correct B_Λ value for $^5_\Lambda$He in their Hartree-Fock calculations, Gibson and Weiss obtain a P-wave Λ-^{4}He resonance at a c.m. kinetic energy somewhat less than 1 MeV in their Hartree-Fock calculations for Λ-^{4}He scattering, even without any Λ-^{4}He spin-orbit interactions. As they remark, the addition of an attractive spin-orbit force ($C_\Lambda > 0$) will increase the Λ-^{4}He attraction in the $P_{3/2}$ state, whereas the addition of a repulsive spin-orbit force ($C_\Lambda < 0$) will increase this attraction in the $P_{1/2}$ state. In either case, the addition of quite a moderate spin-orbit force ($C_\Lambda \gtrsim 0.5$ for the $P_{3/2}$ case, or $C_\Lambda \lesssim -0.25$ for the $P_{1/2}$ case) would lead to the prediction[53] of a P-wave bound state $^5_\Lambda$He*, on the basis of their calculations.

Our approach and that of the GW calculations clearly lead to significantly different conclusions, which reflect primarily the much larger P-wave phase shifts given by their Hartree-Fock calculations. For example, the forward-backward asymmetry D shown in Fig. 4 is markedly different in the two calculations, and this asymmetry is dominantly an S-P interference effect. For energies of 1–8 MeV, our calculation using the Λ-^{4}He Gaussian central potential (1.1) and the SVH spin-orbit parameters (the case OBE in Fig. 4) gives small negative values for D, whereas the GW calculation[54] gives $D \approx 0.8$.

It is a question of interest to understand the dependence of the properties predicted for Λ-^{4}He scattering on the shape of the Λ-^{4}He central potential, in view of the qualitative difference between the predictions resulting from our calculations and those of Gibson and Weiss. At present, our indications are that the Λ-^{4}He scattering properties predicted have a relatively weak dependence on the shape of the Λ-^{4}He central potential, and we are inclined to attribute the larger part of this difference to approximations inherent in the Hartree-Fock approach to continuum scattering problems.[55]

With the BDI and SVH parameters, we assume that the S_0 meson couples to all baryons with equal strength. There is no particular reason to believe that the $T=0$ scalar boson is a unitary singlet; indeed, it is possible that the experimental candidates ϵ(700 MeV), δ(960 MeV), and K(~1020 MeV) may form an octet of scalar bosons.[44] If the S_0 were a member of an octet, its couplings to N and Λ would be related by the F-D mixing parameter f_S:

$$G^S_{\Lambda\Lambda 0}=\frac{2(1-f_S)}{1-4f_S}G^S_{NN0} \quad (4.6a)$$

$$G^S_{N\Lambda\frac{1}{2}}=\frac{1+2f_S}{1-4f_S}G^S_{NN0}\,. \quad (4.6b)$$

From (4.6) we see that the case $f_S=-0.5$ has the same effect for the ΛN system as if the S_0 meson were a unitary singlet; that is, $G_{NN0}=G_{\Lambda\Lambda 0}$ and $G_{N\Lambda\frac{1}{2}}=0$. Thus, for ΛN scattering in a single-channel model, there is no way to distinguish between the possibility of a scalar octet with $f_S=-0.5$ and the case of a unitary singlet S_0'.[56]

We have calculated the Λ-^{4}He polarization using the SVH parameters but varying the scalar mixing parameter f_S through the values $f_S=-0.5$, 0, +0.5, and +1.0. We have assumed that the $T=\frac{1}{2}$ scalar boson has mass $m(S_{\frac{1}{2}})=1.0$ BeV, and we have neglected the space-exchange character of the $S_{\frac{1}{2}}$-exchange potential, taking $P^x_{\Lambda N}=-1$, as discussed earlier in this section. The calculated polarization $P_\Lambda(\theta)$ is plotted in Fig. 6 for the case of a cutoff $\mu=1500$ MeV, for three values of the Λ incident energy T_Λ, and it will be seen that the polarization is sensitive to large changes in the scalar mixing parameter f_S. The case $f_S=-0.5$ is identical to the dot-dashed curves in Fig. 3(b), and is included for comparison. The polarization values range from −0.1 to +0.8 for T_Λ between 5–20 MeV. In all of the curves, the qualitative shape is the same but the maximum polarization for a given energy is rather different. For pure F-type coupling ($f_S=1$), there is no S_0 contribution to the Λ-α spin-orbit force, and the $S_{\frac{1}{2}}$ contribution is repulsive; for pure D-type coupling we have $G_{\Lambda\Lambda 0}=2G_{NN0}$, and a further attractive contribution from $S_{\frac{1}{2}}$ exchange; and for $f_S=0.5$ we have repulsive contributions from both S_0 and $S_{\frac{1}{2}}$ exchanges. As a consequence, the maximum polarization is considerably increased for $f_S=0$ and decreased for $f_S=0.5$ and 1, relative to the case $f_S=-0.5$. Of course, since scalar exchange provides a large part of the central OBE ΛN potential, these variations in f_S would also produce large changes in the ΛN central potential, but here we are considering only a phenomenological representation for this central potential.

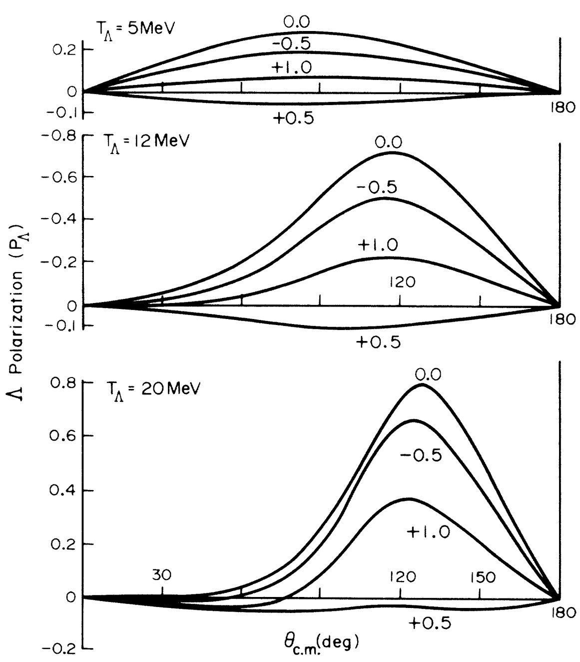

FIG. 6. Variation in Λ-^{4}He polarization with changes in the scalar F-D mixing parameter f_S, for the case with the SVH parameters used for vector couplings and the cutoff given by subtraction with mass $\mu=1500$ MeV. The polarization $P_\Lambda(\theta)$ is plotted vs c.m. angle θ for several values of the Λ lab kinetic energy, T_Λ.

In conclusion, we have taken a phenomenological central potential for the Λ-^{4}He interaction, and we have predicted the Λ-^{4}He spin-orbit potential from OBE models for the ΛN spin-orbit force. With this potential, we have calculated the cross sections and polarizations for Λ-α elastic scattering for Λ incident kinetic energies 1–20 MeV. We have used the parameters from three different OBE models which have been proposed for the ΛN interaction, both with and without a cutoff at short distances, reaching qualitatively the same results for each of these six cases. It was shown that (as was first pointed out by Downs[30]) large antisymmetric spin-orbit ΛN potentials can result from vector-meson exchange in the OBE approximation. The spin-orbit force effective in Λ-^{4}He scattering results from summing over the nucleons of ^{4}He, and this may differ significantly from the ΛN spin-orbit force. We compare our results with the predictions of Gibson and Weiss, who calculated Λ-^{4}He scattering using the Hartree-Fock approximation, and we show that the calculated total cross sections, forward-backward asymmetry, and polarization angular distributions are all significantly different for the two calculations. Measurements of the total Λ-^{4}He scattering cross sections for incident energies ≤5 MeV should distinguish between the predictions of the two models; alternatively, measurement of the forward-backward asymmetry between 1–10 MeV could resolve the differences in the two calculations. In both calculations, the Λ polarization effects predicted for Λ-^{4}He scattering in this energy range are large for spin-orbit interactions of strengths which appear reasonable in the light of OBE calculations of the spin-orbit component of the ΛN potential.

ACKNOWLEDGMENTS

The authors wish to thank Professor B. W. Downs and Professor C. K. Iddings for pointing out the antisymmetric spin-orbit forces present in pseudoscalar-meson exchange for the ΛN interaction, and Dr. M. S. Weiss for helpful correspondence concerning the predictions of the Hartree-Fock method for Λ-^{4}He scattering.

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*Work supported in part by the U. S. Atomic Energy Commission.

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[7]J. A. Kadyk, G. H. Trilling, G. Alexander, and P. Gaposchkin [University of California Radiation Laboratory Report No. UCRL-18805 Rev., 1969 (unpublished)], have used a platinum plate inside the LRL 25-in. hydrogen bubble chamber as a target for a 1.5-GeV/c K^- beam to increase the Λ production rate by about an order of magnitude, for Λ momenta between 300 and 1200 MeV/c. However, the statistics obtainable on Λp elastic scattering from such experiments are still several orders of magnitude below that needed for the observation of differential and polarization angular distributions with sufficient accuracy for the observation of spin-orbit effects (which would appear most clearly in the polarization data). All of the data available has recently been compiled conveniently by the Particle Data Group [O. Benary, N. Barash-Schmidt, L. Price, A. Rosenfeld, and G. Alexander, University of California Radiation Laboratory Report No. UCRL-20000 YN, January, 1970 (unpublished)].

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[10]J. M. Soper, A. Gal, and R. H. Dalitz, to be published. Preliminary accounts of this work are given in Ref. 5, and more recently by A. Gal, in *Proceedings of the International Conference on Hypernuclear Physics, Argonne National Laboratory, 1969,* edited by A. R. Bodmer and L. Hyman (Argonne National Laboratory, May, 1969), p. 348.

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$$V = \frac{4m + m_\Lambda}{m + m_\Lambda} (\pi c^2)^{3/2} \frac{c^2}{d^2} B \qquad \text{(i)}$$

for the strength of their spin-orbit potential (7). Here, in units of fm^2, d^2 is given by $(1.38 + c^2)$; since $c^2 = 0.14$ for ω exchange and 0.52 for σ exchange, the additional factor c^2/d^2 is quite substantial.

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[24]This ΛN interaction actually gives a rather low B_Λ value for ${}^4_\Lambda$H. As their tables show, $W_1^{\Lambda N}$ must be increased to 94.0 MeV to give $B_\Lambda = 2.07$ MeV for ${}^4_\Lambda$H, compatible with the empirical data. However, these parameters do reproduce the empirical B_Λ value for ${}^5_\Lambda$He, which is all that we require for the purpose of this paper.

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[34]When $v(s)$ has the Yukawa form $Y_0(ms)$, which corresponds to the form $m^2 Y_1(ms)$ for $Y(s)$, the integral (2.32) may be evaluated directly, with the result

$$V(r) = \frac{e^{-r^2/a^2}}{2mr}\left[\operatorname{erfc}(u_-)\,e^{u_-^2} - \operatorname{erfc}(u_+)\,e^{u_+^2}\right], \qquad \text{(i)}$$

where erfc(x) and $u_\pm$ are defined by Eqs. (2.24a) and (2.24b), respectively. The expression (2.23) for $X(r)$ can then be obtained from (i) by using the relation (2.31).

[35]We should like to mention that the numerical evaluation of this expression (2.23) involves strong cancellations between large terms, especially in the region of small r. The most reliable procedure is to use the continued fraction expression (7.1.14) given for $e^{z^2}\operatorname{erfc}(z)$ in *Handbook of Mathematical Functions*, edited by M. Abramowitz and I. A. Stegun, National Bureau of Standards, Applied Mathematics Series, No. 55 (U. S. Government Printing Office, Washington, D. C., 1964.) For small z (say $|z| \le 1.5$), Hastings approximations [Eq. (7.1.26), *ibid*.] gives more rapid convergence.

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[37]This method of regulating the spin-orbit potential is somewhat similar to that used by R. Bryan and B. L. Scott [Phys. Rev. 135, B434 (1964)] in their calculations for *NN* scattering. However, Bryan and Scott retained the ∇^2 terms in their expansion of the *NN* potential, and as a result, the definition of their regulator term was more directly related to the strength of the OBE coupling.

[38]If the ΛN spin-orbit potential $Y(s)$ has a small range parameter λ, the integral (2.28a) receives contributions only for a small domain for s, of linear dimension λ. If $\rho(|\vec{r}_\Lambda - \vec{s}|)$ is slowly varying over such a distance, then we may expand $\rho(|\vec{r}_\Lambda - \vec{s}|)$ about $s = 0$, thus

$$\rho(|\vec{r}_\Lambda - \vec{s}|) = \rho(r_\Lambda) - \vec{s}\cdot\vec{\nabla}\rho(r_\Lambda) + \tfrac{1}{2}(\vec{s}\cdot\vec{\nabla})^2\rho(r_\Lambda) - \tfrac{1}{6}(\vec{s}\cdot\vec{\nabla})^3\rho(r_\Lambda) + \cdots . \qquad \text{(i)}$$

Only the terms of (i) which are odd in $(\vec{s}\cdot\vec{\nabla})$ survive after the integration of (2.28a), giving the expression

$$X(r_\Lambda) = -\left[\frac{1}{3}\int Y(s)s^2 d^3s\right]\frac{1}{r_\Lambda}\frac{d\rho}{dr_\Lambda} - \left[\frac{1}{30}\int Y(s)s^4 d^3s\right]\frac{1}{r_\Lambda}\frac{d}{dr_\Lambda}\left(\frac{1}{r_\Lambda}\frac{d^2\rho}{dr_\Lambda^2}\right) + \cdots \qquad \text{(ii)}$$

in powers of λ/a. The static potentials discussed here are monotonic in $r_{\Lambda N}$ and of simple form, and the first term of (ii) represents quite a good approximation, since their range parameters are small, $a >> \lambda \approx 1/m$ and they are also strongly singular at $r_{\Lambda N} = 0$. This conclusion is well illustrated by the curves of Fig. 1(a) and (b). The second term of (ii) introduces an effective modification to a, increasing it to a value given approximately by $a/[1-\beta/(ma)^2]$ where the coefficient β depends on the form of the potential $Y(s)$.

[39]J. J. Sakurai, Phys. Rev. Letters 17, 1081 (1964).

[40]Authors discussing electromagnetic structure for the baryons have frequently used the amplitudes G^E and G^M defined by the equations [see, for example, S. Gasiorowicz, *Elementary Particle Physics* (John Wiley & Sons, Inc., New York, 1966), p. 437; and L. N. Hand, D. G. Miller, and R. N. Wilson, Rev. Mod. Phys. 35, 335 (1963)]

$$G^E = g + (q^2/2M_B)(G/m_V), \qquad \text{(i)}$$

$$G^M = g + 2M_B(G/m_V), \qquad \text{(ii)}$$

where g and (G/m_V) are generally denoted by F_1 and F_2. We note explicitly that the amplitudes G^E and G^M appropriate to the interaction (2.10) used here (and by SVH) differ from these combinations (i) and (ii) by the factors $(\bar{M}/M_B)(1-q^2/4M_B^2)^{-1}$ and $(\bar{M}/M_B)^2(1-q^2/4M_B^2)^{-1}$, respectively.

[41]Bryan and Scott, Ref. 37.

[42]See Ref. 26; in particular, A. Scotti and D. Y. Wong, Phys. Rev. 138, B145 (1965); S. Sawada, T. Ueda, W. Watari, and M. Yonezawa, Progr. Theoret. Phys. (Kyoto) 28, 991 (1962); R. Bryan and R. A. Arndt, Phys. Rev. 150, 1299 (1966).

[43]*Proceedings of a Conference on ππ and Kπ Interactions, at Argonne National Laboratory, 1969,* edited by F. Loeffler and E. Malumud (Argonne National Laboratory, Argonne, Illinois, May, 1969).

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[46]R. W. Stagat, F. Riewe, and A. E. S. Green, Phys. Rev. Letters 24, 631 (1970).

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[48]M. J. Moravcsik and H. P. Noyes, Ann. Rev. Nucl. Sci. 11, 95 (1961).

[49]R. Reid, Jr., Ann. Phys. (N.Y.) 50, 411 (1968).

[50]Since the K^* exchange term contains a factor $(-1)^l$, it is the odd-parity ΛN spin-orbit potential which is similar for all three of the parameter sets used.

[51]It is interesting to compare the complete ΛN spin-or-

bit potential from vector-meson exchange with the corresponding NN spin-orbit potential, using the SVH parameters for illustration. For this purpose, we also approximate $m(K^*) \approx m_\omega$ and $P^x_{\Lambda N} = -1$. The ΛN spin-orbit potential is then

$$V^{\Lambda N}_{so}(V) \approx C_\omega(r)\,(-17.8\vec{S}\cdot\vec{L} + 5.58\vec{S}^A\cdot\vec{L})\,, \quad \text{(i)}$$

whereas the NN spin-orbit potential is then

$$V^{NN}_{so}(V) \approx C_\omega(r)\,(-12.2\vec{S}\cdot\vec{L})\,. \quad \text{(ii)}$$

We note that the calculated spin-orbit interactions for the ΛN and NN systems have the same sign, contrary to the preliminary indications from the analysis of the Λ binding energies for the p-shell Λ hypernuclei (see Ref. 10).

[52]A. R. Bodmer, Phys. Rev. 141, 1387 (1966).

[53]Such a bound state, whether with $J^* = \frac{1}{2}$ or $J^* = \frac{3}{2}$, would undergo rapid $E1$ γ decay, ${}^5_\Lambda\text{He}^* \rightarrow {}^5_\Lambda\text{He} + \gamma$, to the ground state ${}^5_\Lambda$He. Since the energy of this γ ray would be large (of order 3 MeV), this γ-decay process would have a rate much larger than that for ${}^5_\Lambda\text{He}^*$ hypernuclear decay. The dipole moment is due to the ^{4}He charge, and is given by $[2M_\Lambda/(4M + M_\Lambda)]\langle f|\vec{r}|i\rangle$ where i, f denote the ${}^5_\Lambda\text{He}^*$ and ${}^5_\Lambda$He states, respectively, and $\vec{r}$ is the Λ-^{4}He separation vector.

[54]The curve labeled GW in Fig. 4 is for a purely central potential, but the inclusion of a spin-orbit potential still gives large positive values for D for energies 2–10 MeV for all the C_Λ values they consider.

[55]In papers now in preparation, we shall give a detailed discussion of these two questions: (i) the dependence of Λ-^{4}He scattering on the shape of the central potential V_c; and (ii) the approximations involved in the use of the Hartree-Fock method for light nuclear systems, especially for the scattering of a strongly interacting particle by a nucleus.

[56]We do not consider the added possibility of strong mixing between a $T = 0$ unitary singlet and octet member, which would further complicate the scalar-boson situation.

PHYSICAL REVIEW C VOLUME 4, NUMBER 3 SEPTEMBER 1971

Second-Class Currents and Analog Processes*

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(Received 13 April 1971)

Semileptonic processes between members of a common isotopic multiplet provide a nearly model-independent test for currents with anomalous or "second-class" G-parity properties. For such processes the implications of the presence of second-class currents are discussed for β decay, muon capture, and elastic neutrino scattering.

I. INTRODUCTION

Recent experiments by Wilkinson and Alburger on β decay rates of mirror transitions[1] have suggested the possibility that the $\Delta S = 0$ semileptonic weak current may contain a component which is anomalous under the G-parity operation.[2] Although it is conceivable that the Wilkinson effect may be due to differences in nuclear wave functions caused by electromagnetic interactions, it is important to determine to what extent such anomalous or "second-class" currents are known to be absent in weak processes. In this regard we have recently suggested analog β decay experiments,[3] since there exist for this case terms in the decay amplitude which can only be produced by a second-class current and which conversely must vanish in the absence of a second-class interaction.[4] Detection of such terms in the decay spectrum would then signal the presence of these currents in the semileptonic weak Hamiltonian.

In A (see Ref. 3) we examined nuclear β decays in which the parent nucleus was unpolarized with both electron and recoil directions being observed, transitions involving a polarized parent with only the final electron observed, and decays from an unpolarized parent into a daughter which subsequently decays electromagnetically, both the electron and photon being observed. The second-class interaction was assumed to involve only the axial current, and conserved vector current (CVC) and time-reversal invariance were assumed throughout.

In this paper we enlarge these considerations to include a more general type of β decay process, and we examine additional analog reactions associated with the semileptonic weak Hamiltonian. In Sec. II we relax the assumption of T invariance and consider the decay of a polarized parent with both electron and recoil observed in order to look for possible T-violating second-class effects as suggested by Kim and Primakoff[5] and also in order to examine additional tests for T-conserving second-class interactions. In Sec. III present experiments on analog muon capture are treated in order to see what limits on second-class terms are currently implied, and new experiments which may help to resolve the situation are suggested. Finally, Sec. IV discusses second-class terms in neutrino scattering on nucleons.