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Induced Tensor in Allowed Nuclear Beta Decay*

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The contribution of the "induced-tensor" term in allowed nuclear β decays is predicted in the impulse approximation. Current experimental results are compared with these predictions and additional experiments are suggested which could verify the existence of such an effect.

I. INTRODUCTION

In this note we wish to point out a result which is perhaps known to many, but which has not been previously stressed, concerning the so-called "induced-tensor" term in allowed nuclear β decay.[1]

We shall assume the conserved-vector-current (CVC) hypothesis[2] and the validity of the usual current-current interaction; then the β decay amplitude is given (for electron decay – modifications suitable for positron decay will be included at a later stage) by

$$T = \frac{G_V}{\sqrt{2}} \cos\theta_C \langle \beta_{p_2} | V_\mu(0) + A_\mu(0) | \alpha_{p_1} \rangle l^\mu , \qquad (1)$$

where G_V is the usual weak-coupling constant ($G_V m_p{}^2 \simeq 10^{-5}$), θ_C the Cabibbo angle, and l^μ the matrix element of the lepton current

$$l^\mu = \bar{u}(p)\gamma^\mu(1+\gamma_5)v(k).$$

Let p_1, p_2, p, k denote the four momenta of parent nucleus, daughter nucleus, electron, and neutrino and M_1, M_2, represent parent and daughter masses. We define also

$$P = p_1 + p_2; \quad q = p_1 - p_2 = p + k ;$$

$$M = \tfrac{1}{2}(M_1 + M_2) ; \quad \Delta = M_1 - M_2 .$$

Then to first order in recoil quantities the decay spectrum is

$$d\omega = \frac{|T|^2}{(2\pi)^5} F_-(Z,E)\left(1+\frac{3E-E_0-3\vec{p}\cdot\hat{k}}{M}\right) \times (E_0-E)^2 pEdEd\Omega_e d\Omega_\nu \,, \quad (2)$$

where $F_-(Z,E)$ is the usual Fermi function and accounts for the dominant Coulomb effects, E ($\vec{p}$) is the electron energy (momentum), $\hat{k}$ is a unit vector in the direction of the neutrino momentum, and E_0 is the maximum possible electron energy

$$E_0 = \Delta\frac{1+m_e^2/2M\Delta}{1+\Delta/2M}\,.$$

We write for the amplitude of a general allowed ($\Delta J = 0, \pm 1$; no) transition[3]

$$\langle\beta|V_\mu(0)+A_\mu(0)|\alpha\rangle l^\mu = \frac{1}{2M}aP\cdot l\delta_{JJ'}\delta_{MM'} - i\frac{1}{4M}C^{M'k;M}_{J'1;J}\epsilon_{ijk} \times[2bl_iq_j + i\epsilon_{ij\lambda\eta}l^\lambda(cP^\eta - dq^\eta)]\,, \quad (3)$$

where J, J' are the spins of the parent and daughter nuclei, respectively, and M, M' represent the initial and final components of nuclear spin along some axis of quantization. Here repeated Latin indices are summed from 1 to 3, while repeated Greek indices imply a four-vector contraction with the metric $g_{00} = -g_{11} = -g_{22} = -g_{33} = +1$. Using standard notation

$$a = g_V M_F\,,$$

$$c = g_A M_{GT}\,,$$

M_F, M_{GT} being the Fermi and Gamow-Teller matrix elements, b is the so-called "weak-magnetism" contribution which, between nuclear analog states, would be given by

$$b = A\left(\frac{J+1}{J}\right)^{1/2} M_F\mu_V\,,$$

where A is the mass number and μ_V is the isovector contribution to the magnetic moment measured in units of the proton magneton. The last coefficient d, usually called the induced tensor, is uniquely correlated with the existence of second-class currents if α, β are isotopic analogs.[4] On the other hand, if α, β are not members of a common isotopic multiplet, the presence of such terms is not forbidden by G-parity considerations even in the absence of a second-class interaction. d may then be considered as an alteration imposed by the presence of strong interactions on the usual axial-vector matrix element.

II. INDUCED TENSOR AND IMPULSE APPROXIMATION

It is of interest to ask how large this "induced" first-class contribution to d is expected to be. We can answer this question, at least within the context of the impulse approximation.[5] In order to relate the discussion to familiar results, we first discuss the vector current. For neutron decay, one finds

$$\langle p_{p_2}|V_\mu|n_{p_1}\rangle = \frac{1}{2M}\bar{u}(p_2)[g_V(q^2)P_\mu - ig_M(q^2)\sigma_{\mu\nu}q^\nu]u(p_1)\,, \quad (4)$$

where $g_V(0) = 1$ and $g_M(0) = \mu_p - \mu_n = 4.70$. In terms of Pauli spinors, correct to first order in p/M, we have

$$\langle p_{p_2}|V_\mu(0)|n_{p_1}\rangle l^\mu = \chi_p^\dagger\left[g_V(q^2)\left(l_0 - \frac{1}{2M}(\vec{p}_1+\vec{p}_2)\cdot\vec{l}\right) + ig_M(q^2)\frac{1}{2M}\vec{\sigma}\cdot\vec{q}\times\vec{l}\right]\chi_n\,, \quad (5)$$

which suggests the impulse-approximation prediction

$$V_0 = g_V(q^2)\sum_i \tau_i^+ e^{-i\vec{q}\cdot\vec{r}_i}\,,$$

$$\vec{V} = -i[g_V(q^2)\sum_i \tau_i^+\{e^{-i\vec{q}\cdot\vec{r}_i}, \vec{\nabla}_i\} + g_M(q^2)\sum_i \tau_i^+ e^{-i\vec{q}\cdot\vec{r}_i}\vec{\sigma}_i\times\vec{q}]/(m_p+m_n)\,. \quad (6)$$

Taking the lowest nonvanishing order in q we find

$$V_0 = g_V(0)\sum_i \tau_i^+\,,$$

$$\vec{V} = -i[g_M(0)\sum_i\tau_i^+\vec{\sigma}_i\times\vec{q} + g_V(0)\sum_i\tau_i^+\vec{L}_i\times\vec{q}]/(m_p+m_n) + \tfrac{1}{2}g_V(0)\sum_i\tau_i^+[H, \vec{r}_i\vec{q}\cdot\vec{r}_i]\,, \quad (7)$$

where H is the Hamiltonian, and the spin-orbit interaction has been neglected. If the last term is disregarded as being effectively second order in q,[6] then we have the usual impulse-approximation result

$$a = g_V \langle \beta \| \sum_i \tau_i^+ \| \alpha \rangle ,$$
$$b = [g_V \langle \beta \| \sum_i \tau_i^+ L_i \| \alpha \rangle + g_M \langle \beta \| \sum_i \tau_i^+ \sigma_i \| \alpha \rangle] \frac{2M}{m_p + m_n} , \tag{8}$$

where $|\alpha\rangle$, $|\beta\rangle$ represent the nuclear wave functions for the transition in question and terms of order E_0/M have been neglected.

We may treat the axial current in the same way assuming, for simplicity, the absence of a second-class interaction.[7] Then we have for neutron decay

$$\langle p_{p_2} | A_\mu(0) | n_{p_1} \rangle = \bar{u}(p_2) g_A(q^2) \gamma_\mu \gamma_5 u(p_1) , \tag{9}$$

where $g_A(0) \cong +1.23$ or in terms of Pauli spinors, correct to first order in q/M,

$$\langle p_{p_2} | A_\mu(0) | n_{p_1} \rangle l^\mu = \chi_p^+ g_A(q^2) \left[\vec{\sigma} \cdot \vec{1} - \frac{1}{2M} l_0 \vec{\sigma} \cdot (\vec{p}_1 + \vec{p}_2) \right] \chi_n , \tag{10}$$

which suggests the impulse-approximation result

$$A_0 = i g_A(q^2) \frac{\sum_i \tau_i^+ \{ e^{-i\vec{q}\cdot\vec{r}_i}, \vec{\sigma}_i \cdot \vec{\nabla}_i \}}{m_p + m_n} ,$$
$$\vec{A} = -g_A(q^2) \sum_i \tau_i^+ \vec{\sigma}_i . \tag{11}$$

Taking the lowest nonvanishing order in q and neglecting spin-orbit effects, we find

$$A_0 = g_A(0) \frac{i \sum_i \tau_i^+ \vec{\sigma}_i \times \vec{L}_i}{m_p + m_n} - \tfrac{1}{2} g_A(0) \sum_i \tau_i^+ [H, \vec{\sigma}_i \cdot \vec{r}_i q \cdot \vec{r}_i] ,$$
$$\vec{A} = -g_A(0) \sum_i \tau_i^+ \vec{\sigma}_i . \tag{12}$$

When the latter term in A_0 is dropped as before, we find the impulse-approximation predictions

$$c = g_A \left[\langle \beta \| \sum_i \tau_i^+ \sigma_i \| \alpha \rangle + \frac{\Delta}{m_p + m_n} \langle \beta \| i \sum_i \tau_i^+ \sigma_i \times L_i \| \alpha \rangle \right] , \tag{13}$$
$$d = g_A \frac{2M}{m_p + m_n} \langle \beta \| i \sum_i \tau_i^+ \sigma_i \times L_i \| \alpha \rangle ,$$

where terms of order E_0/M have been neglected. Thus a nonvanishing term is predicted by the impulse approximation:

$$d \cong A g_A \langle \beta \| i \sum_i \tau_i^+ \sigma_i \times L_i \| \alpha \rangle .$$

We note that the operator $i \sum_i \tau_i^+ \sigma_i \times L_i$ is found to vanish between analog states as required by G-parity considerations.[8]

III. EXPERIMENTAL VERIFICATION

It remains to suggest where a term of this type might be seen. Being a first-order recoil term, its contribution is generally of the order $E_0/m_p \simeq 10^{-2}$–10^{-3} compared with the usual allowed terms. However, its effect is more noticeable in a β-γ (or β-α) correlation experiment. Suppose a parent nucleus of spin J undergoes an allowed β decay to a daughter nucleus of spin J', which subsequently decays emitting either a photon (or an α particle) to a final nucleus of spin J''. We assume the parent nucleus to be unpolarized and we integrate over the neutrino direction, considering the spectrum in its dependence on the electron variables and on the direction of the photon (or α particle). The latter is characterized by a unit vector $\hat{K}$ along the direction of the photon (or α particle) momentum in the lab frame (rest frame of the parent). The spectrum is found to be[9]

$$d\omega = F_\mp(Z, E) \frac{G_V^2 \cos^2\theta_C}{(2\pi)^5} (E_0 - E)^2 p E dE d\Omega_e d\Omega_{\hat{K}} \left\{ f_1(E) + g(E) \frac{\hat{K}\cdot\vec{p}}{E} + \lambda^J_{J', J''} f_6(E) \left[\left(\frac{\hat{K}\cdot\vec{p}}{E} \right)^2 - \frac{1}{3} \frac{p^2}{E^2} \right] \right\} , \tag{14}$$

where the upper (lower) signs refer to electron (positron) decay, v^* is the velocity of the γ ray (or α particles) in the center-of-mass frame of the daughter,[10] and

$$f_1(E) = |a|^2 + |c|^2 - \frac{2}{3}\frac{E_0}{M}[|c|^2 \pm \mathrm{Re}\,c^*(b+d)] + \frac{2}{3}\frac{E}{M}(3|a|^2 + 5|c|^2 \pm 2\mathrm{Re}\,c^*b) - \frac{1}{3}\frac{m_e^2}{ME}[2|c|^2 \pm \mathrm{Re}\,c^*(2b+d)],$$

$$g(E) = \frac{2E_0}{3Mv^*}\left[-|a|^2 + \frac{|c|^2}{3}\left(1 - \frac{\lambda^J_{J',J''}}{10}\right)\right] - \frac{4E}{3Mv^*}\left[|a|^2 + \tfrac{5}{3}|c|^2\left(1 - \frac{\lambda^J_{J',J''}}{100}\right)\right], \tag{15}$$

$$f_6(E) = \frac{E}{20M}[|c|^2 \pm \mathrm{Re}\,c^*(b-d)] + \frac{1}{\lambda^J_{J',J''}}\left(|a|^2\frac{E(2E_0+E)}{2M^2v^{*2}} + |c|^2\frac{E[E(11 - \frac{4}{5}\lambda^J_{J',J''}) - 2E_0(1 - \frac{2}{5}\lambda^J_{J',J''})]}{6M^2v^{*2}}\right).$$

The terms in v^* reflect kinematic-shift effects associated with the transformation to the lab frame from the rest frame of the daughter nucleus, where the radiative decay is most simply characterized.

In (14) and (15) $\lambda^J_{J',J''}$ is a coefficient depending on the spins involved. If the α particle is in a state of orbital angular momentum 1 (e.g., $J'^{P'} = 1^-$; $J''^{P''} = 0^+$) then we find

$$\lambda^J_{J',J''} = \eta_{J,J'}\tau_{J',J''}(L=1) \tag{16a}$$

with

$$\eta_{J,J'} = \begin{cases} -J'/(2J'+3), & J' = J-1 \\ 1, & J' = J \\ -(J'+1)/(2J'-1), & J' = J+1 \end{cases} \tag{16b}$$

and

$$\tau_{J',J''}(L=1) = \begin{cases} 2(2J'+3)/J', & J'' = J'-1 \\ -2(2J'+3)(2J'-1)/J'(J'+1), & J'' = J' \\ 2(2J'-1)/(J'+1), & J'' = J'+1, \end{cases} \tag{16c}$$

while for electric or magnetic dipole photon emission

$$\lambda^J_{J',J''}(E1, M1) = \eta_{J,J'} \times [-\tfrac{1}{2}\tau_{J',J''}(L=1)].$$

Similarly, if the α particle is in a state of orbital angular-momentum 2 (e.g., $J'^{P'} = 2^+; J''^{P''} = 0^+$), we find

$$\lambda^J_{J',J''} = \eta_{J,J'}\tau_{J',J''}(L=2),$$

where $\eta_{J,J'}$ has been defined in (16) and

$$\tau_{J',J''}(L=2)\begin{cases} \frac{20}{7}(2J'+3)/J', & J'' = J'-2 \\ -\frac{10}{7}(2J'+3)(J'-5)/J'(J'+1), & J'' = J'-1 \\ -\frac{10}{7}(2J'+5)(2J'-3)/J'(J'+1), & J'' = J' \\ -\frac{10}{7}(2J'-1)(J'+6)/J'(J'+1), & J'' = J'+1 \\ \frac{20}{7}(2J'-1)/(J'+1), & J'' = J'+2, \end{cases} \tag{17}$$

while for electric or magnetic quadrupole photon emission

$$\lambda^J_{J',J''}(E2, M2) = \eta_{J,J'} \times \tfrac{1}{2}\tau_{J',J''}(L=2).$$

Similar expressions, of course, hold for higher L values and multipole radiations.

The important point is that the β-γ correlation

$$\alpha(E) = \lambda^J_{J',J''}f_6(E)/f_1(E) \tag{18}$$

is already of recoil order so that the d coefficient can make a substantial contribution. Unfortunately, the weak-magnetism term b is also present, and d may contain contributions from second-class currents. In order to eliminate these difficulties one may measure β-γ (or β-α) correlations on mirror pairs (e.g., Li^8, B^8; B^{12}, N^{12}; etc.). We find

$$\alpha_-(E)=\lambda^J_{J',J''}\frac{E}{20M}\left(1+\frac{b}{c}-\frac{d_{\rm I}}{c}-\frac{d_{\rm II}}{c}\right)+\frac{E[E(11-\frac{4}{5}\lambda^J_{J',J''})-2E_0(1-\frac{2}{5}\lambda^J_{J',J''})]}{6M^2v^{*2}},$$
$$\alpha_+(E)=\lambda^J_{J',J''}\frac{E}{20M}\left(1-\frac{b}{c}-\frac{d_{\rm I}}{c}+\frac{d_{\rm II}}{c}\right)+\frac{E[E(11-\frac{4}{5}\lambda^J_{J',J''})-2E_0(1-\frac{2}{5}\lambda^J_{J',J''})]}{6M^2v^{*2}}, \tag{19}$$

where we have separated d into first-class ($d_{\rm I}$) and second-class ($d_{\rm II}$) components.[11] Then

$$\alpha_-(E)-\alpha_+(E)=\frac{E}{10M}\left(\frac{b}{c}-\frac{d_{\rm II}}{c}\right)\lambda^J_{J',J''} \tag{20}$$

and is a measure of weak-magnetism plus second-class d contributions while

$$\alpha_-(E)+\alpha_+(E)=\frac{E}{10M}\left(1-\frac{d_{\rm I}}{c}\right)\lambda^J_{J',J''}+\frac{E[E(11-\frac{4}{5}\lambda^J_{J',J''})-2E_0(1-\frac{2}{5}\lambda^J_{J',J''})]}{3M^2v^{*2}} \tag{21}$$

becomes essentially a measurement of the quantity $d_{\rm I}/c$ – the "induced tensor."

Five cases appear amenable to experimental searches for this effect:

$$\begin{aligned}
&\text{(i) } Li^8\ (B^8) \longrightarrow Be^{8*}(2.90\text{ MeV})+e^-\ (e^+)+\bar\nu_e\ (\nu_e)\\
&\qquad\searrow He^4+\alpha(L=2),\quad \log ft=5.6\ (5.6)\\
&\text{(ii) } B^{12}\ (N^{12}) \longrightarrow C^{12*}(4.43\text{ MeV})+e^-\ (e^+)+\bar\nu_e\ (\nu_e)\\
&\qquad\searrow C^{12}+\gamma(E2),\quad \log ft=5.1\ (5.2)\\
&\text{(iii) } F^{20}\ (Na^{20}) \longrightarrow Ne^{20*}(1.63\text{ MeV})+e^-\ (e^+)+\bar\nu_e\ (\nu_e)\\
&\qquad\searrow Ne^{20}+\gamma(E2),\quad \log ft=5.0\ (5.0)\\
&\text{(iv) } Na^{24}\ (Al^{24}) \longrightarrow Mg^{24*}(4.12\text{ MeV})+e^-\ (e^+)+\bar\nu_e\ (\nu_e)\\
&\qquad\searrow Mg^{24*}(1.37\text{ Mev})+\gamma(E2),\quad \log ft=6.1\ (6.4)\\
&\text{(v) } Mg^{28}(P^{28}) \longrightarrow Si^{28*}(1.78\text{ MeV})+e^-\ (e^+)+\bar\nu_e(\nu_e)\\
&\qquad\searrow Si^{28}+\gamma(E2),\quad \log ft=4.9\ (4.7).
\end{aligned} \tag{22}$$

The first of these has been studied experimentally by Nordberg, Morinigo, and Barnes.[12] They give

$$\left.\begin{aligned}\alpha_-(E)&=(3.48\pm0.66)\times10^{-2}\\ \alpha_+(E)&=-(4.25\pm1.10)\times10^{-2}\end{aligned}\right\}\text{ for } E=11\text{ MeV},$$

which yields ($\lambda^J_{J',J''}=10$)

$$\begin{aligned}\alpha_-(E)-\alpha_+(E)&=\frac{E}{M}\frac{b-d_{\rm II}}{c}=(7.73\pm1.32)\times10^{-2},\\ \alpha_-(E)+\alpha_+(E)&=\frac{E}{M}\left(1-\frac{d_{\rm I}}{c}\right)+\frac{E(2E_0+E)}{M^2v^{*2}}\\ &=-(0.77\pm1.32)\times10^{-2}.\end{aligned} \tag{23}$$

Thus we find

$$\begin{aligned}(b-d_{\rm II})/Ac&=6.61\pm1.13,\\ d_{\rm I}/Ac&=1.68\pm1.13.\end{aligned} \tag{24}$$

In order to obtain an approximate theoretical estimate for these quantities we have calculated, using the Nilsson model, expected values of b/Ac, $d_{\rm I}/Ac$ for the nuclei suggested above in the impulse approximation. The daughter states were assumed to belong to pure $K=0$ rotational bands. Since the operators $\sum_i\tau_i^+\sigma_i$, $\sum_i\tau_i^+L_i$, $i\sum_i\tau_i^+\sigma_i\times L_i$ all have $|\Delta K|\le1$, they are unable, except for the mass-12 system, to connect ground states of the parent nuclei to $K=0$ band daughters without some sort of K band mixing in the parent, as evidenced by the relative smallness of the Gamow-Teller matrix elements indicated by (22). We have thus assumed – except for mass-12 – simple $K=1$ band mixing in order to estimate $b/Ac, d_{\rm I}/Ac$. The $K=1$ wave function is then connected to the $K=0$ daughter state by the operators we consider. The $K=1$ state considered to be mixed into the ground state is that state for which the projection of the extra nucleon minus the projection of the corresponding vacancy is equal to unity. For example, for Li^8 we assume an extra neutron to be in the Nilsson orbit having the form $|Njl\Omega\rangle=|1\tfrac{3}{2}1\tfrac{3}{2}\rangle$ in the limit $\beta\to0$ and a vacancy to exist in the $|1\tfrac{3}{2}1\tfrac{1}{2}\rangle$ Nilsson proton orbit (both with respect to Be^8), and we find[13]

$$\frac{d_{\rm I}}{Ac}\cong\frac{\langle\beta;1\tfrac{3}{2}1\tfrac{1}{2}|i(\sigma\times L)_-|\beta;1\tfrac{3}{2}1\tfrac{3}{2}\rangle}{\langle\beta;1\tfrac{3}{2}1\tfrac{1}{2}|\sigma_-|\beta;1\tfrac{3}{2}1\tfrac{3}{2}\rangle}.$$

Results of this calculation are given in Table I as a function of the deformation parameter β for Nil-

TABLE I. Values of b/Ac, d_I/Ac as a fraction of the deformation parameter, β, for the Nilsson wave functions given in column 1.

Be^8	β	0.0	0.1	0.2	0.3
$\lvert 1\frac{3}{2}1\frac{3}{2}\rangle\lvert 1\frac{3}{2}1\frac{1}{2}\rangle^*$	b/Ac	4.63	5.23	5.92	6.65
	d_I/Ac	0.00	0.73	1.59	2.48
C^{12}	β	0.0	−0.1	−0.2	−0.3
$\lvert 1\frac{1}{2}1\frac{1}{2}\rangle\lvert 1\frac{3}{2}1-\frac{1}{2}\rangle^*$	b/Ac	3.42	4.18	4.95	5.72
	d_I/Ac	1.50	1.48	1.99	2.73
Ne^{20}	β	0.0	0.1	0.2	0.3
$\lvert 2\frac{5}{2}2\frac{3}{2}\rangle\lvert 2\frac{5}{2}2\frac{1}{2}\rangle^*$	b/Ac	5.45	5.74	6.30	6.94
	$d_I\ Ac$	0.00	0.64	1.50	2.40
Mg^{24}	β	0.0	0.1	0.2	0.3
$\lvert 2\frac{5}{2}2\frac{5}{2}\rangle\lvert 2\frac{5}{2}2\frac{3}{2}\rangle^*$	b/Ac	5.45	6.10	6.80	7.53
	$d_I Ac$	0.00	0.80	1.67	2.56
Si^{28}	β	0.0	−0.1	−0.2	−0.3
$\lvert 2\frac{1}{2}0\frac{1}{2}\rangle\lvert 2\frac{5}{2}2\frac{3}{2}\rangle^*$	b/Ac	···	4.86	5.08	5.53
	d_I/Ac	···	1.25	1.88	2.64

sson particle-vacancy wave functions given in the first column. Note that the deformation parameter β is assumed to be the same for both parent and daughter nucleus, and that the mixing amplitude for the $|K=1\rangle$ states is not needed, since we are taking a ratio. It is hoped that although specific values of b, d_I, c are quite sensitive to the mixing parameter and to the nuclear wave functions, the ratios $b/Ac, d_I/Ac$ should not be. Thus in the mass-8 system, the predicted values for d_I/Ac, b/Ac are in agreement with the experimental results for reasonable values of the deformation: $\beta \simeq 0.2-0.3$.[14] Of course, the experimental value for d_I/Ac is not inconsistent with zero, so that improved experiments of the Be^8 system as well as the other suggested cases are required in order to verify the existence of an induced tensor.[15]

We note in passing that the agreement of $(b-d_{II})/Ac$ with the theoretical prediction for b/Ac argues against a large negative value for d_{II}/Ac in the system as would be suggested by an interpretation of the Wilkinson experiments in terms of second-class currents.[16] Recent work by Wilkinson and Alburger on the Be^8 system is also inconsistent with this interpretation.[17]

Finally, if second-class currents are assumed not to contribute in these decays so that $d_{II}=0$, an additional type of experiment suggests itself in order to determine the presence of d_I. For instance, in the mass-24 system, if the radiative width of the 4^+ analog state of Mg^{24} to the 4.1225-MeV, 4^+ state of Mg^{24} could be measured, then by the CVC hypothesis the weak-magnetism term b would be known.[18] A careful measurement of the β-γ correlation in Na^{24} alone would then be sufficient to determine the presence or absence of an induced-tensor coefficient.

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[1]See, e.g., J. N. Huffaker and E. Greuling, Phys. Rev. 132, 738 (1963); S. Weinberg, *ibid.* 112, 1375 (1958).

[2]R. P. Feynman and M. Gell-Mann, Phys. Rev. 109, 193 (1958).

[3]For positron decay we elect to reverse the sign in front of the d coefficient. For reference, we note that in terms of more familiar (covariant) notation we have (for electron decay)

$J=\frac{1}{2};\ J'=\frac{1}{2}$:

$$\langle\beta|V_\mu+A_\mu|\alpha\rangle=\bar{u}(p_2)\left[\frac{1}{2M}g_V P_\mu - i\frac{1}{2M}g_M\sigma_{\mu\nu}q^\nu + g_A\gamma_\mu\gamma_5 - i\frac{1}{2M}g_{II}\sigma_{\mu\nu}q^\nu\gamma_5\right]u(p_1)$$

$g_V=a;\ g_M=b/\sqrt{3};\ g_A=c/\sqrt{3};\ g_{II}=d/\sqrt{3}$

$J=0;\ J'=1$:

$$\langle\beta|V_\mu+A_\mu|\alpha\rangle=-\frac{1}{4M^2}F_M i\epsilon_{\mu\rho\lambda\eta}P^\rho q^\lambda\xi^{*\eta} + \xi_\mu^* F_A + \xi^*\cdot q P_\mu\frac{1}{4M^2}F_T\,,$$

$\sqrt{3}F_A=c-d\Delta/2M;\ \sqrt{3}F_T\cong d-c;\ \sqrt{3}F_M=b$

[4]B. R. Holstein and S. B. Treiman, Phys. Rev. C 3, 1921 (1971). To be more specific, such terms can arise from a second-class current with odd isotopic spin *or* from a first-class current with even isotopic spin, either of which would be anomalous on the present picture of weak interactions.

[5]C. W. Kim and H. Primakoff, Phys. Rev. 139, B1447 (1965).

[6]We note that $\langle\beta|[H,\vec{r}_i\vec{q}\cdot\vec{r}_i]|\alpha\rangle=-q_0\langle\beta|\vec{r}_i\vec{q}\cdot\vec{r}_i|\alpha\rangle$ so that such a term is formally of order q^2R^2, where R is the nuclear radius, but it tends to be considerably smaller, being essentially an isovector quadrupole term.

[7]Had we included a second-class interaction of the form

$$\langle p_{p_2}|A_\mu^{II}(o)|n_{p_1}\rangle=-i\frac{g_{II}}{2M}\bar{u}(p_2)\sigma_{\mu\nu}q^\nu\gamma_5 u(p_1)$$

and gone through the same procedure we would find no change in c, but the predicted value of d would become

$$d \cong A g_A \langle \beta \| i \textstyle\sum_i \tau_i^+ \sigma_i \times L_i \| \alpha \rangle + A g_{\text{II}} \langle \beta \| \textstyle\sum_i \tau_i^+ \sigma_i \| \alpha \rangle .$$

However, this prescription for dicussion of the *second-class* axial current is not unique. See, e.g., L. Wolfenstein, University of Michigan report, 1971 (to be published).

[8]For $|\alpha\rangle = |I=\frac{1}{2}, I_3=\frac{1}{2}\rangle$, $|\beta\rangle = |I=\frac{1}{2}, I_3=-\frac{1}{2}\rangle$, $U=e^{i\pi I_2}$, charge symmetry yields

$$\begin{aligned}\langle \beta | i \textstyle\sum_i \tau_i^+ \sigma_i \times L_i | \alpha \rangle &= \langle \beta | U^{-1} U i \textstyle\sum_i \tau_i^+ \sigma_i \times L_i U^{-1} U | \alpha \rangle \\ &= \langle \alpha | i \textstyle\sum_i \tau_i^- \sigma_i \times L_i | \beta \rangle \\ &= -\langle \beta | i \textstyle\sum_i \tau_i^+ \sigma_i \times L_i | \alpha \rangle^* .\end{aligned}$$

But time-reversal invariance requires

$$\langle \beta | i \textstyle\sum_i \tau_i^+ \sigma_i \times L_i | \alpha \rangle = \langle \beta | i \textstyle\sum_i \tau_i^+ \sigma_i \times L_i | \alpha \rangle^* .$$

Thus the matrix element must vanish. Similar reasoning applies for the case $I > \frac{1}{2}$.

[9]Similar expressions, for analog decays, are given in B. Holstein, Phys. Rev. C 4, 764 (1971), and in Ref. 4. These results are also valid for β-proton correlations as discussed in this reference.

[10]We have included terms of second order in E/Mv^* which can be important for β-α or β-proton correlations for which v^* can be quite small, but which can be neglected for β-γ angular correlations, where $v^*=1$.

[11]In principle c, b could also contain second-class contributions. However, b must be purely first class by the CVC hypothesis, and second-class contributions to c are small if the impulse approximation is good.

[12]M. E. Nordberg, F. B. Morinigo, and C. A. Barnes, Phys. Rev. 125, 321 (1962).

[13]We note that the reduced matrix elements for the operator $i\sigma \times L$ are given by

$$\begin{aligned}\langle J' L S \| i\sigma \times L \| J L S \rangle &= 0 , && J = J' \\ &= [L(2L+1)]^{\frac{1}{2}} , && J = J-1 \\ &= [(L+1)(2L+1)]^{\frac{1}{2}} , && J' = J+1 ,\end{aligned}$$

so that this operator connects only states of differing J.

[14]The measured quadrupole moments for Li^7, Be^9 suggest the value $\beta \simeq +0.4$.

[15]It is probably desirable in this regard to perform experiments on the lower-mass cases in order to eliminate effects due to second-forbidden terms of order $q^2 \tau_i^+ r_i^2$ which we have neglected. Such isovector quadrupole terms would generally have negligible effect on correlation experiments. However, since most decays in Eq. (22) are K forbidden, these corrections, which can involve $\Delta K = 2, 3$, may become important – especially for heavier nuclei as in mass 24 and 28, which involve $\Delta K = 4, 3$, respectively.

[16]D. H. Wilkinson, Phys. Letters 31B, 447 (1970), if interpreted in terms of second-class currents would imply, as discussed in Ref. 9 that $d_{\text{II}}/Ac \cong -6$ if the divergenceless hypothesis for the second-class axial current is adopted, as discussed by J. Delorme and M. Rho, Phys. Letters 34B, 238 (1971).

[17]D. H. Wilkinson and D. E. Alburger, Phys. Rev. Letters 26, 1127 (1971).

[18]This supposes that the $M1/E2$ ratio is already known from a γ-γ-correlation experiment. An alternate means of determining b is, of course, via measurement of the Al^{24} magnetic moment, which, combined with the already measured Na^{24}, yields the prediction

$$b = 6\sqrt{10}\,(\mu_{\text{Al}} - \mu_{\text{Na}}) .$$

See, however, Ref. 15.