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Nuclear Structure of the $N = 82$ Isotones*

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The nuclear structure of the $N = 82$ isotones Ba^{138}, La^{139}, Ce^{140}, Pr^{141}, and Nd^{142} has been studied using the single-proton-transfer reactions (He^3, d) and (d, He^3) on La^{139}, Ce^{140}, and Pr^{141} targets. Distorted-wave analysis of the angular distributions has been used to determine angular momentum transfers and spectroscopic factors for states up to about 3-MeV excitation. The experimental data are compared with recent shell-model calculations, and in particular with the results of calculations using a new simplifying coupling scheme, the pseudo spin-orbit coupling scheme.

I. INTRODUCTION

From the point of view of the nuclear shell model there is considerable evidence that systems with 50 and 82 nucleons form particularly stable systems, that both $N = 82$ and $Z = 50$ are "good" closed shells. Thus, there is reason to expect that the properties of the low-lying states of the $N = 82$ isotones should be explainable in terms of configurations of the remaining $(Z - 50)$ protons. These excess protons should be filling, predominantly, the $1g_{7/2}$, $2d_{5/2}$, $2d_{3/2}$, $3s_{1/2}$, and $1h_{11/2}$ single-particle orbits.

Proton-transfer reactions such as (He^3, d) and (d, He^3) are particularly well suited for studying the nature of the low-lying levels of such nuclei, since they either add or take away a proton from the target nucleus and thus should excite strongly only those states, where the proton configurations have a good overlap with those in the ground state of the target nucleus. These reactions are selective as to the character of the state they populate, and as a result the total number of states excited is relatively small. Further, the nature of the reaction mechanism is sufficiently well understood that it is possible to extract from the experimental data much of the spectroscopic information necessary for a theoretical study of the nuclear structure in this region of the Periodic Table. Such information includes the energy of the states, their spins and parities, and the intrinsic strengths (spectroscopic factors) of the various transitions which in turn provide information on the occupation probabilities of the shell-model orbits.

In the experimental work reported here the low-lying levels of Ba^{138}, La^{139}, Ce^{140}, Pr^{141}, and Nd^{142} have been investigated by means of the (He^3, d) and/or the (d, He^3) reaction. Additional proton-transfer data are now available[1] on the same family of $N = 82$ isotones for I^{135}, Cs^{137}, Pm^{143}, and Eu^{145}, as well as some data from (n, γ), β decay, (γ, γ') and inelastic proton, deuteron, and α-particle scattering.

A recent shell-model calculation[2] taking into account the complete $g_{7/2}$, $d_{5/2}$ configurations of the $(Z - 50)$ protons with some excitation into the $s_{1/2}$, $d_{3/2}$ orbits has met with considerable success in explaining the data on the positive-parity levels. Recent calculations using quasiparticle techniques

and the Tamm-Dancoff approximation,[3-5] may be of particular relevance for the study of the single-proton-transfer reactions to final states in odd nuclei. Of even greater interest here is the fact that the $N = 82$ nuclei, in which the active protons are filling predominantly the $g_{7/2}$, $d_{5/2}$ single-particle orbits, form a prime example of the usefulness of a new simplifying coupling scheme, the pseudo spin-orbit coupling scheme.[6-8] A comparison between these calculations and the experimental data is given in Sec. IV.

II. EXPERIMENTAL

Deuterons and $(He^3)^{++}$ ions were accelerated to 28.90 and 44.19 MeV, respectively, using the University of Michigan 83-in. sector-focused cyclotron. The momenta of the particles emitted in the reactions were determined by magnetic analysis, and the particles themselves were detected either in nuclear emulsions or in solid-state position-sensitive detectors placed at the image surface of the magnetic analysis system. The experimental facilities have been described in detail elsewhere.[9]

Lanthanum, cerium, and praseodymium are rare-earth metals characterized by high electronegativity and oxidation potential, and to prevent oxidation of thin targets and the resulting structural weakness, the materials were first evaporated onto thin carbon substrates, and then a second thin layer of carbon evaporated onto the metal. Targets of La^{139} (isotopic abundance of 99.91%), Pr^{141} (100%), and natural cerium (88.5% of Ce^{140}) were prepared from the metal while targets enriched to 99.7% in Ce^{140} were prepared by evaporation of CeO_2. The thickness, ranging from 50 $\mu g/cm^2$ to 1.2 mg-cm^2, was determined by measuring the energy lost by 5.48-MeV α particles from Am^{241} in traversing the targets. The revised range-energy tables of Boujot and Williamson[10] were used to convert the energy loss to target thickness. These revised tables eliminate the 25% discrepancy for lead noted in earlier work[11] and agree with the range-energy tables of Whaling[12] to within 5%. An additional uncertainty in target thickness arises from estimating accurately the amount of oxygen contaminant. If completely oxidized, oxygen would comprise 20% of the total weight. Since for thick targets the oxidation is confined to the surface, this uncertainty is estimated to be less than 10%. The thickness of a number of targets was also determined by cutting out and weighing pieces of known area using an electrobalance. While the two techniques agreed to within 10%, the weighing technique is considered less reliable, because of the difficulty of obtaining pieces of accurately known area.

The determination of the beam energy and the solid angle of the magnetic analysis system, the use of nuclear emulsions and position-sensitive particle detectors at the image surface of the magnetic analysis system, the accuracy of the beam-current integration and counting statistics, and matters of line shape and background subtraction have been discussed elsewhere.[9,11,13] Taking these factors into account, the uncertainties are estimated to be less than 25% for the absolute cross sections, less than 10% for the relative cross sections, and less than 25 keV for the excitation energies up to 4.2 MeV.

The measured energy resolution, which varied from 30 to 50 keV in these measurements, was due primarily to target thickness.

III. EXPERIMENTAL RESULTS

Excitation spectra and angular distributions measured in the (d, He^3) and (He^3, d) reactions on La^{139}, Ce^{140}, and Pr^{141} are shown in Figs. 1 through 15.

The assignment of the orbital angular momentum transfer for each transition was made on the basis of the fit of the differential cross section computed by distorted-wave analysis (DWA) to the measured angular distributions at forward angles, where the signature for each angular momentum transfer is most distinct. A number of the observed distributions are best fit by an admixture of two or more l values. This is to be expected for the target nuclei La^{139} and Pr^{141}, which have nonzero ground-state spins, since final states of specific J^π can often be reached by two different angular momenta transfers. For those few cases, where states were not resolved, the l values were determined by fitting the experimental angular distribution to a weighted sum of two predicted angular distributions and adjusting the weighting factor until the best fit to the data was obtained.

Spectroscopic strengths have been deduced from fits to the first one or two maxima, since these are most consistently reproduced by the DWA calculations.

A. Distorted-Wave Analyses

Spectroscopic factors were extracted from the experimental data using the University of Colorado distorted-wave code DWUCK written by Kunz.[14] The optical-model parameters used in the calculation are listed in Table I and are those of Newman, Becker, and Preedom[15] for the deuteron potentials and Gibson *et al.*[16] for the proton potentials. For the (d, He^3) calculations, a lower cutoff of 8 F, also found necessary in other work,[17-19] was used. Correspondence between measured cross sections

TABLE I. Optical-model parameters.

Parameter		Deuteron [a]	Deuteron [b]	Helium 3	Bound proton
V_0	(MeV)	100.4	102.6	175.0	c
r_0	(fm)	1.119	1.05	1.14	1.24
a	(fm)	0.814	0.924	0.723	0.65
r_c	(fm)	1.30	1.30	1.4	1.25
V_{so}	(MeV)	7.0	6.8	0.0	
W_0	(MeV)	0.0	0.0	17.4	
$4W_D$	(MeV)	63.2	62.0	0.0	
r_i	(fm)	1.244	1.27	1.60	
a_i	(fm)	0.861	0.801	0.81	
					$\lambda_p = 25$ [d]

[a] Parameter set used for the (d, He^3) reactions.
[b] Parameter set used for the (He^3, d) reactions.
[c] Adjusted to reproduce proton separation energy.
[d] Spin-orbit coupling of λ_p times the Thomas term was used.

and spectroscopic factors was made using the relations

$$\left(\frac{d\sigma}{d\Omega}\right)_{(He^3,d)} = 4.42\frac{2J_f+1}{2J_i+1}S\sigma_{lj}(\theta)$$

and

$$\left(\frac{d\sigma}{d\Omega}\right)_{(d,He^3)} = 2.95S\sigma_{lj}(\theta),$$

where $\sigma_{lj}(\theta)$ are the cross sections calculated by the distorted-wave code, and the numerical factors are normalizations due to Bassel.[20] In the cases of (He^3, d) reactions on odd-A targets, where the spins, J_f, of the residual nuclei are not known, the quantity $(2J_f+1)S_{lj}$ is extracted. Standard least-squares techniques were used to compare measured and computed cross sections.

B. $Ce^{140}(d, He^3)La^{139}$ Reaction

The spectrum (Fig. 1) shows strong transitions to the ground state and 0.166-MeV state of La^{139} and four weak transitions between 1.0 and 2.0 MeV.

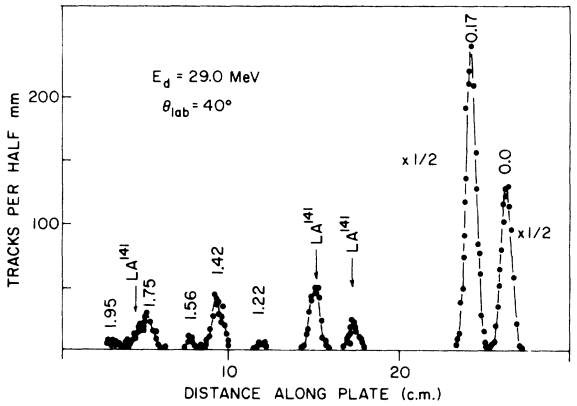

FIG. 1. The spectrum measured for the $Ce^{140}(d, He^3)La^{139}$ reaction.

Several weak transitions observed with the natural cerium target were identified as levels in La^{141} by using an enriched Ce^{140} target. The measured angular distributions are shown in Fig. 2.

The ground state is populated by an $l_p = 4$ momentum transfer corresponding to the removal of a $1g_{7/2}$ proton from the $J^\pi = 0^+$ ground state of Ce^{140}. The 0.166-MeV state is populated by an $l_p = 2$ transfer and is identified with the removal of a $2d_{5/2}$ proton. This assignment is in accord with the shell model and with the assignment $J^\pi = \frac{5}{2}^+$ from γ-decay studies.[21] The four weak states at 1.22, 1.42, 1.56, and 1.95 MeV are populated by $l_p = 0, 5, 2$, and 2 momentum transfers, respectively, and appear to be the same states excited in the $Ba^{138}(He^3, d)La^{139}$ reaction.[1] Spectroscopic strengths were determined under the assumption

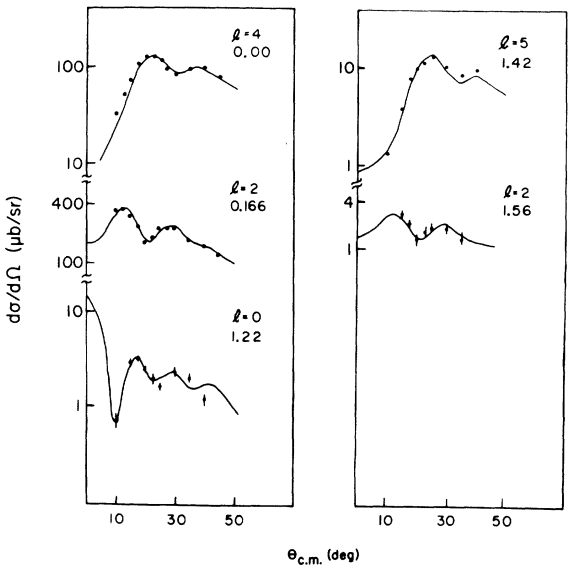

FIG. 2. The angular distributions measured for five levels in $Ce^{140}(d, He^3)La^{139}$.

TABLE II. Summary of results for levels of La^{139} excited in the $Ce^{140}(d, He^3)La^{139}$ reaction.

E_x (MeV)	n, l, j	$C^2S_{l,j}$
0.00	$1g_{7/2}$	5.65
0.166	$2d_{5/2}$	1.53
1.22	$3s_{1/2}$	0.018
1.42	$1h_{11/2}$	0.75
1.56	$2d_{3/2}$	0.043
1.95	$2d_{3/2}$	0.017

that these transitions correspond to the removal of $3s_{1/2}$, $1h_{11/2}$, and $2d_{3/2}$ protons. The possibility that the 1.56- or 1.95-MeV states are populated by removal of a $2d_{5/2}$ proton cannot be precluded by the present measurements, but in either case the spectroscopic strengths are small and have little effect on the conclusions to be drawn from this study.

The spectroscopic information is tabulated in Table II; the spectroscopic strengths were normalized to 8, which is the number of protons outside the $Z = 50$ core. It is clear that the valence protons in Ce^{140} predominantly occupy the $1g_{7/2}$ and $2d_{5/2}$ single-particle orbits, a fact that will be used in analyzing the pickup reactions on the neighboring odd isotones La^{139} and Pr^{141}. These results are in essential agreement with those obtained recently by Wildenthal, Newman, and Auble at 40 MeV.[1]

C. $La^{139}(d, He^3)Ba^{138}$ Reaction

The ground-state spin and parity of La^{139} is $\frac{7}{2}^+$, hence a number of angular momentum couplings between the target nucleus and transferred proton are possible, and it can be expected that the $1g_{7/2}$ and $2d_{5/2}$ strengths will be fragmented into several states in the residual even-even nucleus Ba^{138}. The likelihood of fragmentation coupled with the small population of the $1h_{11/2}$, $3s_{1/2}$, and $2d_{3/2}$ orbits in Ce^{140} suggest the observed pickup strength from those single-particle orbits will be small.

The spectrum measured at a laboratory angle of 20° (Fig. 3) shows seven strong transitions below 2.5 MeV together with several very weak transitions between 2.5 and 3.1 MeV. The rather broad group at 2.44 MeV is due to the unresolved levels at 2.44 and 2.47 MeV. The angular distributions for the seven strong transitions are shown in Fig. 4; the solid curves are the result of a distorted-wave calculation using a radial cutoff of 8.0 F.

The spectroscopic factors (Table III) are normalized such that their sum would equal 6.33. It was felt that a normalization to 7.0 (the number of valence protons) would be too large, in view of the absence of any $1h_{11/2}$ strength and the presence of the weak transitions for which no spectroscopic factors were derived.

The excitation energies determined in these measurements are in good agreement with those measured in the $Ba^{137}(n, \gamma)Ba^{138}$ reaction[22] and the β decay of Cs^{138}.[23] The state at 2.09 MeV was not observed in the (n, γ) and β-decay measurement.

D. $Pr^{141}(d, He^3)Ce^{140}$ Reaction

The spectrum recorded at a laboratory angle of 25° (Fig. 5) shows some 12 states between 0.0 and 4.0 MeV of excitation. The inset shows the region of excitation from 2.0 to 2.6 MeV studied with a relatively thin ($\simeq$80 μg/cm^2) target. It is clear that a state at 2.52 MeV is being populated in addition to the stronger state at 2.54 MeV and that two states are being populated near 2.1 MeV.

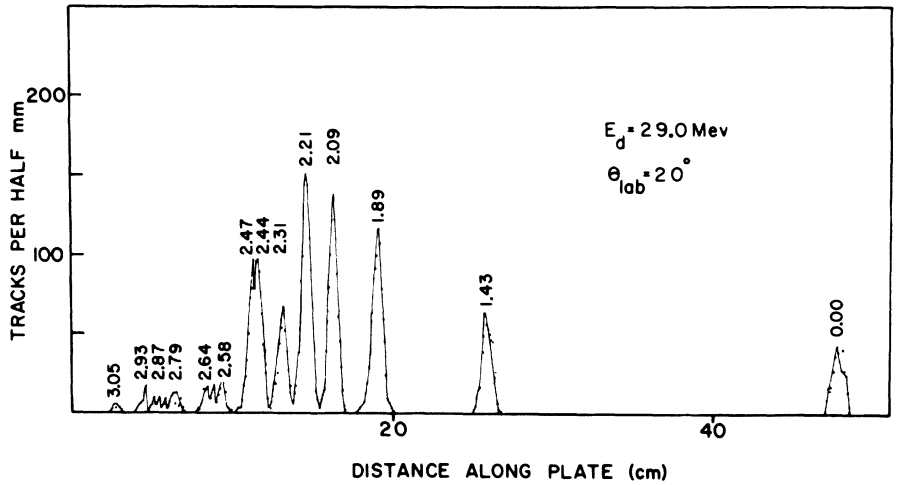

FIG. 3. The measured spectrum for $La^{139}(d, He^3)$-Ba^{138}

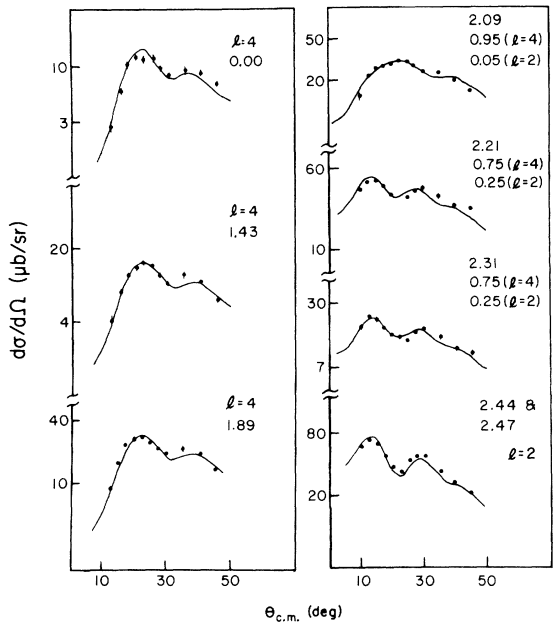

FIG. 4. The angular distributions measured for the seven relatively strong levels observed in the reaction $La^{139}(d, He^3)Ba^{138}$.

Angular distributions (Fig. 6) were measured for all states except those at 2.90 and 3.13 MeV. The solid curves are the result of local, zero-range distorted-wave calculations using a lower cutoff of 8.0 F. The spectroscopic information is summarized in Table IV. In determining the spectroscopic factors it was assumed that all the $l=2$ strength resulted from the pickup of a $2d_{5/2}$ proton. The occupation probabilities for Pr^{141} should be similar to those for Ce^{140}, where the occupation of the $2d_{3/2}$ orbit was determined to be small.

TABLE III. Summary of results for levels of Ba^{138} excited in the $La^{139}(d, He^3)Ba^{138}$ reaction.

E_x (MeV)	n, l, j	$C^2S_{l,j}$
0.00	$1g_{7/2}$	0.420
1.43	$1g_{7/2}$	0.720
1.89	$1g_{7/2}$	1.64
2.09	$1g_{7/2}$	1.49
	$2d_{5/2}$	0.08
2.21	$1g_{7/2}$	0.77
	$2d_{5/2}$	0.26
2.31	$1g_{7/2}$	0.39
	$2d_{5/2}$	0.13
2.44 [a]	$2d_{5/2}$	0.45
2.47 [a]	$2d_{5/2}$	

[a] Unresolved doublet.

These results agree well with previous measurements, the most comprehensive being the study of the decay of La^{140} to levels in Ce^{140}.[24] The l values given in Table IV are consistent with the spin assignments made in the decay scheme study. The level observed at 3.00 MeV and populated by an $l=2$ transition has recently been observed in the decay of Pr^{140} to Ce^{140}.[25] If this transition is the result of the pickup of a $d_{5/2}$ proton, the allowable spins and parity of the 3.00-MeV level are 0^+, 2^+, and 4^+.

Application of the sum rules of Macfarlane and French for pickup reactions suggests that the average number of particles in each subshell is 6.17 for the $1g_{7/2}$ and 1.94 for $2d_{5/2}$, in good agree-

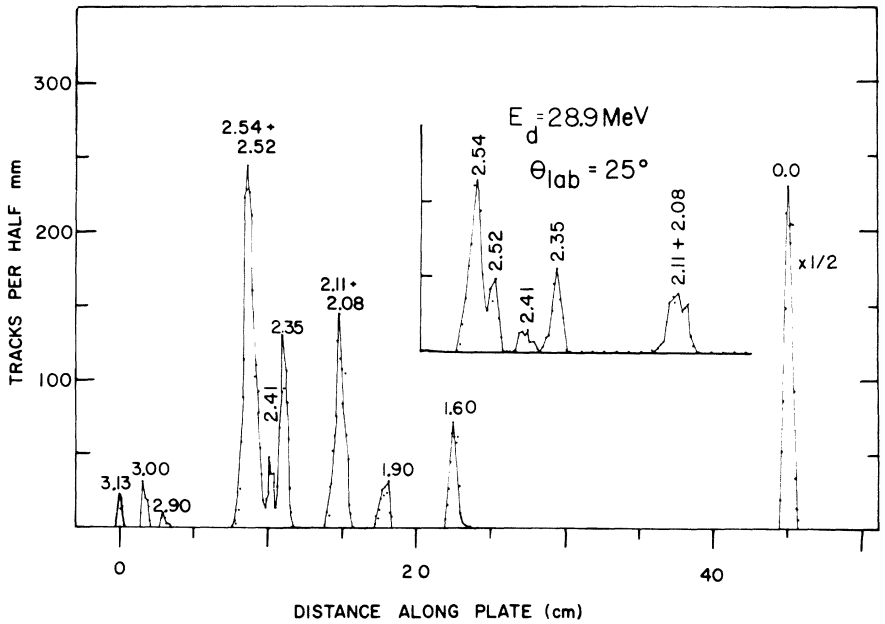

FIG. 5. The $Pr^{141}(d, He^3)$-Ce^{140} spectrum.

ment with the expectation that the nine valence protons in Pr^{141} will be occupying mainly these two levels with only a small probability of occupying higher subshells. These occupation probabilities are consistent with those measured in the (d, He^3) reaction on the neighboring even isotones Ce^{140} and Nd^{142}.[1, 19]

E. $Ce^{140}(He^3, d)Pr^{141}$ Reaction

The deuteron spectrum (Fig. 7) shows six strong transitions in the range of excitation from 0.0 to 4.0 MeV with the states at 1.60 and 1.64 MeV being only partially resolved. If there were no fragmentation of the levels only five transitions would be expected corresponding to adding a proton to each of the subshells between $Z = 50$ and $Z = 82$. The measurements indicate that the $3s_{1/2}$ strength is split almost evenly between the states at 1.30 and 1.64 MeV.

The angular distributions measured for the strong transitions are shown in Fig. 8. The solid lines are the results of a local zero-range distorted-wave calculation using the optical-model parameters given in Table I. The distorted-wave curves for this and the two following (He^3, d) reactions were averaged over the 4° acceptance angle of the spectrograph. The principal effect of this was to dampen somewhat the oscillations in the $l = 0$ and $l = 2$ angular distributions. The states at 1.60 and 1.64 MeV were analyzed as a single angular distribution with the percentages of $l = 2$ and $l = 0$ strength being extracted as discussed earlier.

The results are in essential agreement with those of Wildenthal, Newman, and Auble.[1] The states of Pr^{141} have also been studied by the proton-pickup reaction $Nd^{142}(d, He^3)Pr^{141}$ by Baer and Bardwick.[19] In the pickup reaction only five states were observed and there was no evidence for the fragmentation of the $3s_{1/2}$ strength. Beery, Kelly, and McHarris[26] have studied the decay of Nd^{141} to levels in Pr^{141} and Moreh and Nof[27] have studied Pr^{141} using the (γ, γ') reaction. The present results are consistent with these studies, but because the proton stripping reaction is more selective in the type of state populated, many fewer states are observed.

The spectroscopic information, summarized in Table V, indicates that the $2d_{3/2}$ and $3s_{1/2}$ subshells are almost completely unoccupied. This is in agreement with the results of the proton-pickup reactions, which indicate that the occupation probability for those subshells is small. The spectroscopic factor for the $1h_{11/2}$ transition is 0.6, whereas a value close to unity was expected as a consequence of the results of the pickup reaction $Ce^{140}(d, He^3)La^{139}$. This may result from the

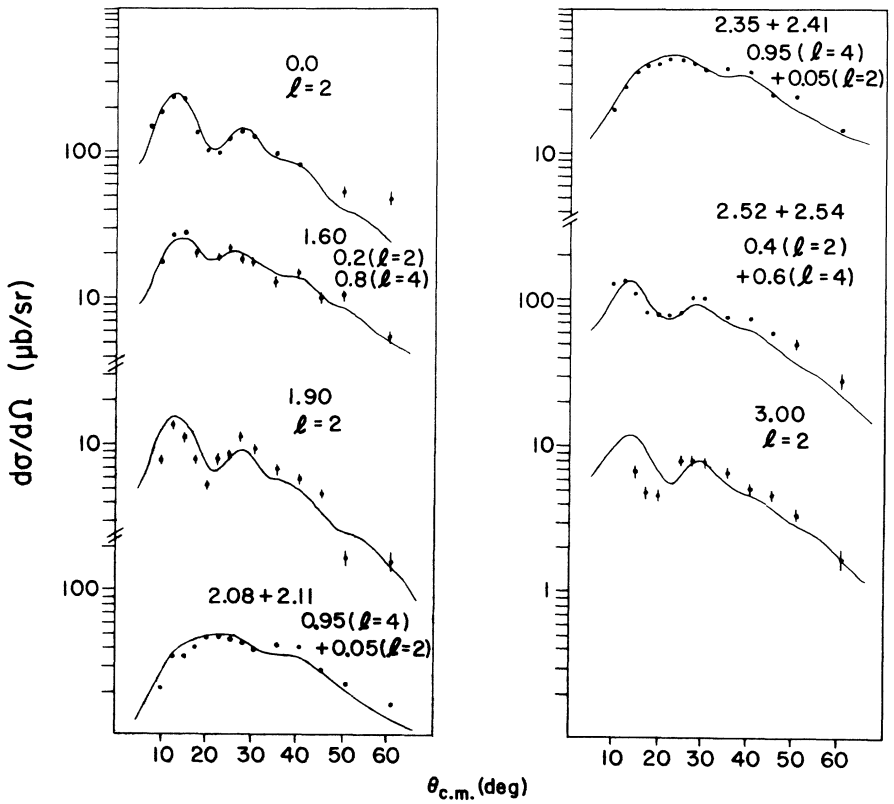

FIG. 6. Angular distributions measured for levels in $Pr^{141}(d, He^3)Ce^{140}$.

$1h_{11/2}$ single-particle state mixing with the $\frac{11}{2}^-$ state formed by coupling the odd $2d_{5/2}$ proton in the ground state of Pr^{141} with a 3^- core excitation. Some mixing seems to be present since the 1.11-MeV $\frac{11}{2}^-$ state in Pr^{141} is excited by inelastic α-particle scattering.[28] Configurations more complicated than the two-state model discussed in Ref. 28 are needed to explain the missing $1h_{11/2}$ strength. Below an excitation energy of 4.0 MeV in Ce^{140}, no $l=4$ transitions with spectroscopic strength ≥ 0.1 are observed in the present experiment.

F. $La^{139}(He^3, d)Ce^{140}$ Reaction

The 28 states below an excitation energy of 4.36 MeV shown in the deuteron spectrum (Fig. 9) are populated by adding a proton to one of the five available subshells. The large number of states is to be expected due to the coupling of the angular momentum of the transferred proton to that of the odd $g_{7/2}$ proton in La^{139}. Above 4.36-MeV excitation a nearly continuous background of weak transitions is observed.

The states below 2.6-MeV excitation in Ce^{140} are observed in both the pickup and stripping reactions and are formed by either $l=2$ or $l=4$ momentum transfer. Thus it may be assumed that these are predominantly $(1g_{7/2}2d_{5/2})$ in nature. States involving a $2d_{3/2}$ configuration would not be expected to appear with any appreciable strength in the pickup reaction.

Due to the expected large fragmentation of the $1h_{11/2}$ strength, it is possible that some of the $1h_{11/2}$ strength may have been missed in analyzing the data. Meaningful angular distributions could not be measured for some of the very weak states.

Angular distributions measured for the 21 strongest transitions are shown in Figs. 10–12. The solid lines are again the result of local zero-range distorted-wave calculations. When a mixture of l values is indicated, the percentages refer to the ratio of the spectroscopic strengths for the two l values.

Only two $l=4$ transitions, to the ground and 2.63-MeV states, were observed in these measure-

TABLE IV. Summary of results for levels of Ce^{140} excited in the $Pr^{141}(d, He^3)Ce^{140}$ reaction.

E_x (MeV)	n, l	$C^2S_{l,j}$
0.0	$2d$	0.62
1.60	$2d$	0.12
	$1g$	0.39
1.90	$2d$	0.07
2.08	$2d$	0.12
2.11	$1g$	2.20
2.35	$2d$	0.12
2.41	$1g$	2.35
2.52	$2d$	0.83
2.54	$1g$	1.24
3.00	$2d$	0.07

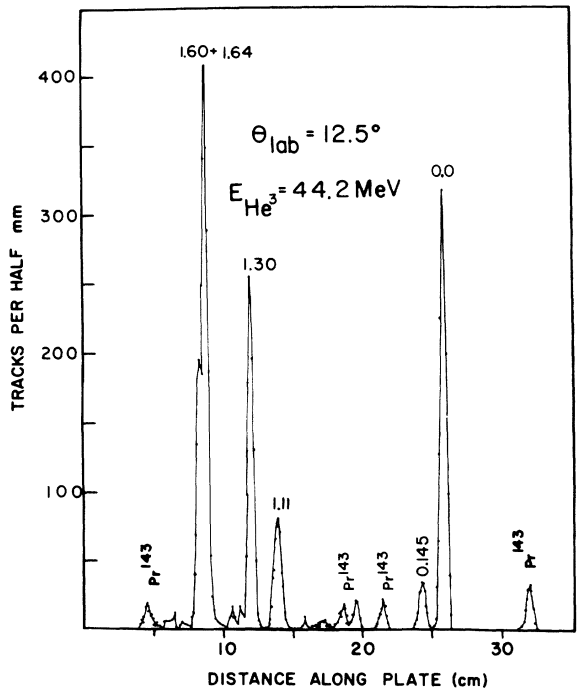

FIG. 7. The $Ce^{140}(He^3, d)Pr^{141}$ spectrum. No other strong transitions are observed up to 4-MeV excitation.

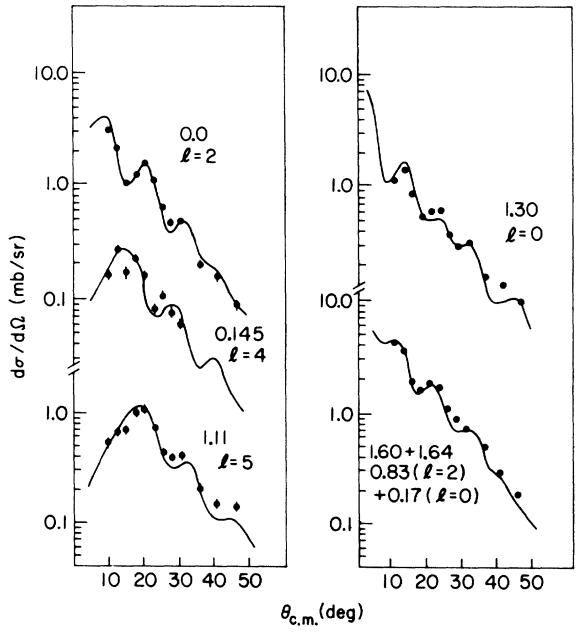

FIG. 8. Angular distributions measured for levels in $Ce^{140}(He^3, d)Pr^{141}$.

TABLE V. Summary of results for levels of Pr^{141} excited in the $Ce^{140}(He^3, d)Pr^{141}$ reaction.

E_x (MeV)	n, l, j	$C^2S_{l,j}$
0.0	$2d_{5/2}$	0.46
0.145	$1g_{7/2}$	0.31
1.11	$1h_{11/2}$	0.60
1.30	$3s_{1/2}$	0.49
1.60	$2d_{3/2}$	0.93
1.64	$3s_{1/2}$	0.41

ments. This is in agreement with the expectation, based on the results of the pickup experiments, that the transition strengths for the nearly full $1g_{7/2}$ subshells would be small. The possibility of the admixture of a small amount of $g_{7/2}$ strength to some of the $l = 2$ transitions cannot be ruled out, since the $l = 2$ cross section would completely mask the $l = 4$ contribution. It is somewhat surprising that no transition to the 1.90-MeV 0^+ state was detected. That transition would require a pure $l = 4$ momentum transfer.

The spectroscopic information is summarized in Table VI. The angular momentum transfers determined in this reaction are compatible with those determined in the $Pr^{141}(d, He^3)Ce^{140}$ reaction and also with the spin assignments made in the study of the decay of La^{140} to levels in Ce^{140}.[24]

G. $Pr^{141}(He^3, d)Nd^{142}$ Reaction

The deuteron spectrum (Fig. 13) shows 20 states below 4.5-MeV excitation and, again, a large number of states was expected because of the many possible couplings of the transferred proton to the odd $d_{5/2}$ proton in Pr^{141}.

The angular distributions measured for 16 states below an excitation energy of 4.0 MeV are shown in Figs. 14 and 15. The solid curves are the results of local zero-range distorted-wave calculations. Many of the positive-parity states observed in this reaction can be populated by two or more angular momentum transfers although in some cases two or more states being populated are unresolved.

The spectroscopic information is summarized in Table VII. Since in general J_f is not known, it is possible to extract only $(2J_f + 1)S$ for each of the observed transitions. Application of the sum rules of Macfarlane and French[29] yield the average number of holes in each subshell.

A 3^- state at an excitation energy of 2.09 MeV in Nd^{142} has been observed in inelastic proton and deuteron scattering by Christensen and Yang.[30] However, the state at 2.09 ± 0.02 MeV populated in the (He^3, d) reaction must be of positive parity, since it is formed by the transfer of two units of orbital angular momentum. No evidence was found for a state at 1.97 MeV with spin and parity equal to 4^+ as suggested by Raman.[31] The fact that such a state is not populated in the (He^3, d) reaction is something of a mystery.

No angular distributions characteristic of the transfer of a $g_{7/2}$ proton have been observed. This is not too surprising since the $g_{7/2}$ subshell is nearly filled in Pr^{141} and the coupling of a $g_{7/2}$ proton with a $d_{5/2}$ proton produced six states with spins ranging from 1^+ to 6^+. Thus, the small amount of $g_{7/2}$ strength would be spread over several levels and would be difficult to observe. A possible candidate for a state populated by a $g_{7/2}$ transition is the weak state at 2.20 MeV. No attempt was made to fit a theoretical angular distri-

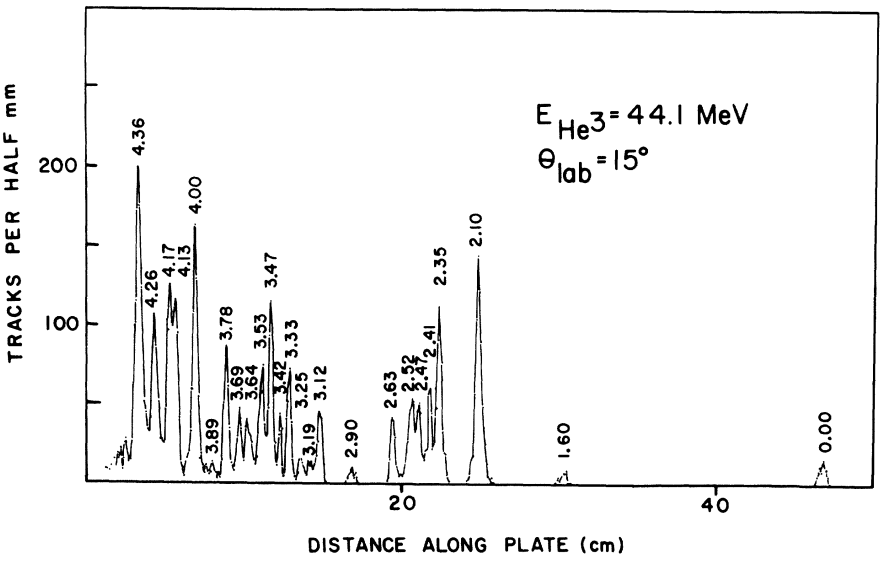

FIG. 9. The $La^{139}(He^3, d)$-Ce^{140} spectrum.

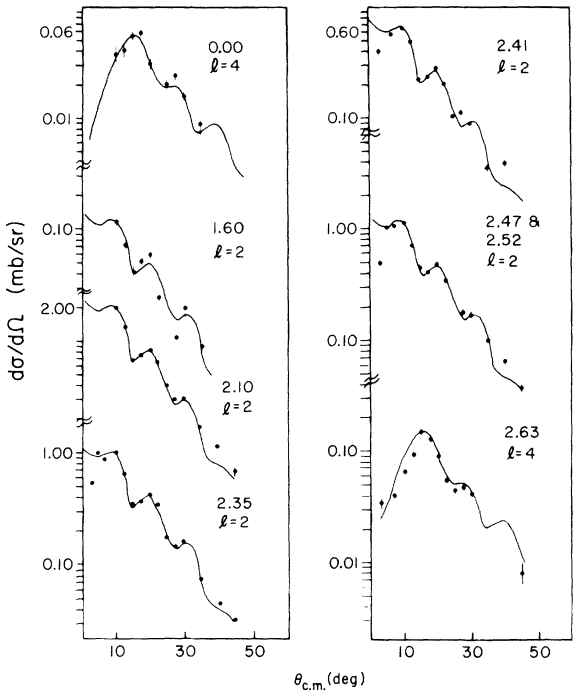

FIG. 10. Measured angular distributions for levels in $La^{139}(He,d)Ce^{140}$. See Figs. 11 and 12 for additional angular distribution measurements in this reaction.

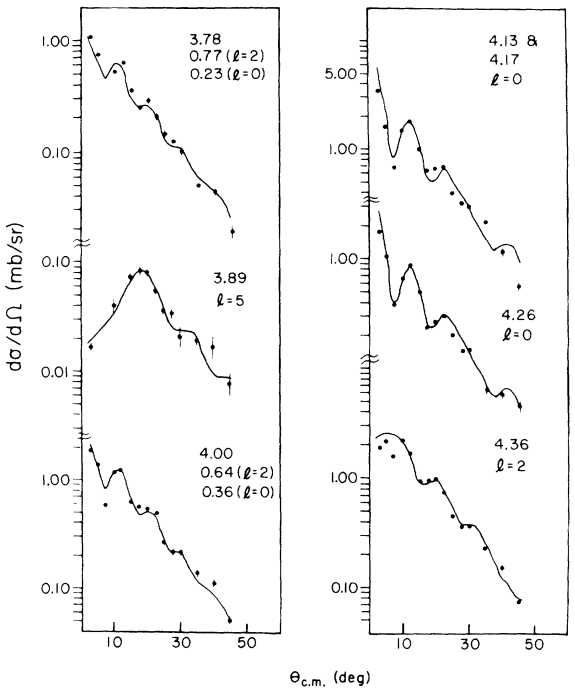

FIG. 12. Angular distributions for $La^{139}(He^3,d)Ce^{140}$.

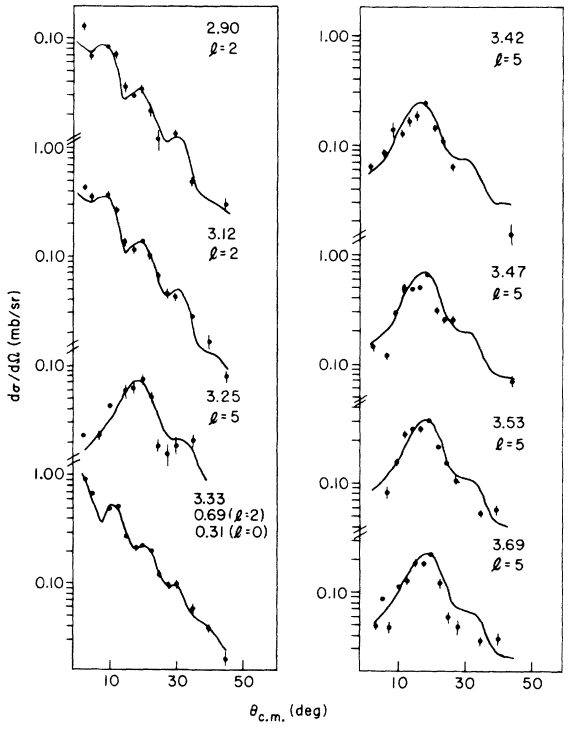

FIG. 11. Angular distributions for $La^{139}(He^3,d)Ce^{140}$.

TABLE VI. Summary of results for levels of Ce^{140} excited in the $La^{139}(He^3, d)Ce^{140}$ reaction.

E_x (MeV)	n, l, j	$(2J_f+1)C^2S_{l,j}$
0.00	$1g_{7/2}$	6.49
1.60	$2d_{5/2}$	0.83
2.10	$2d_{5/2}$	14.00
2.35	$2d_{5/2}$	6.97
2.41	$2d_{5/2}$	4.53
2.47	$2d_{5/2}$	4.13
2.52	$2d_{5/2}$	3.63
2.63	$1g_{7/2}$	17.20
2.90	$2d_{5/2}$	0.783
3.12	$2d_{5/2}$	3.40
3.25	$1h_{11/2}$	4.17
3.33	$3s_{1/2}$	2.70
	$2d_{3/2}$	1.24
3.42	$1h_{11/2}$	13.40
3.47	$1h_{11/2}$	35.90
3.53	$1h_{11/2}$	20.60
3.69	$1h_{11/2}$	12.50
3.78	$2d_{3/2}$	4.72
	$3s_{1/2}$	1.43
3.89	$1h_{11/2}$	4.45
4.00	$2d_{3/2}$	5.55
	$3s_{1/2}$	3.20
4.13 [a]	$3s_{1/2}$	7.33
4.17 [a]	$3s_{1/2}$	
4.26	$3s_{1/2}$	3.70
4.36	$2d_{3/2}$	17.80

[a] Unresolved doublet.

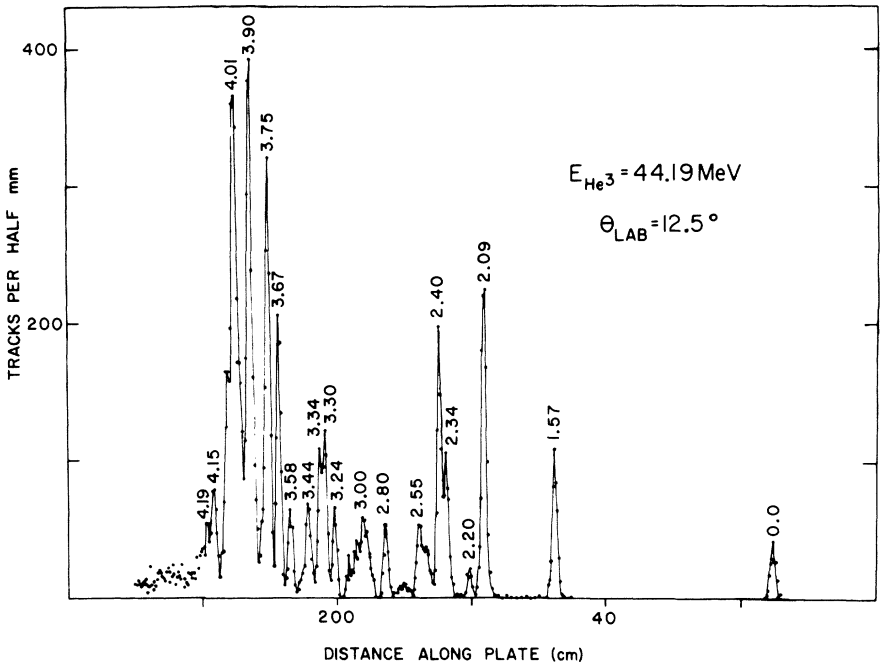

FIG. 13. The $Pr^{141}(He^3,d)$-Nd^{142} spectrum.

bution to it because of the poor statistics and the rather structureless angular distribution.

H. Occupation Probabilities

The sum rules of Macfarlane and French[29] have been applied to determine either the average number of particles in a given subshell (pickup reactions) or the average number of holes in a subshell (stripping reactions). These values were then divided by the appropriate $(2J+1)$ to yield the normalized occupation probabilities. Table VIII lists U_j^2 and V_j^2 for the three target nuclei La^{139}, Ce^{140}, and Pr^{141}. It is clear that there are rather significant uncertainties (±30%) in some of these values since for a given value of j, $U_j^2 + V_j^2 = 1$. Generally a more accurate value of the occupation probability is obtained by measuring it directly in the pickup reaction. This is especially

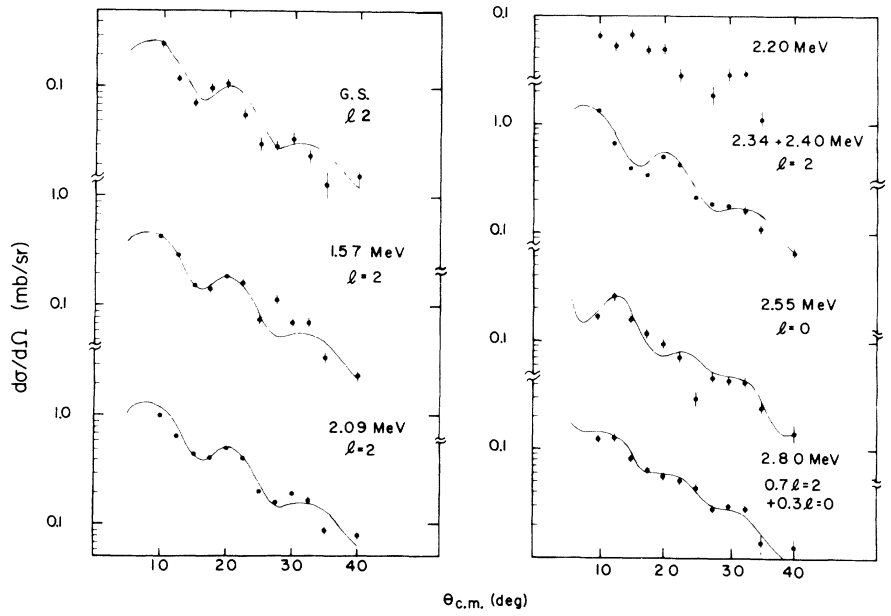

FIG. 14. Angular distributions for $Pr^{141}(He^3,d)Nd^{142}$. See Fig. 15 for additional angular distributions.

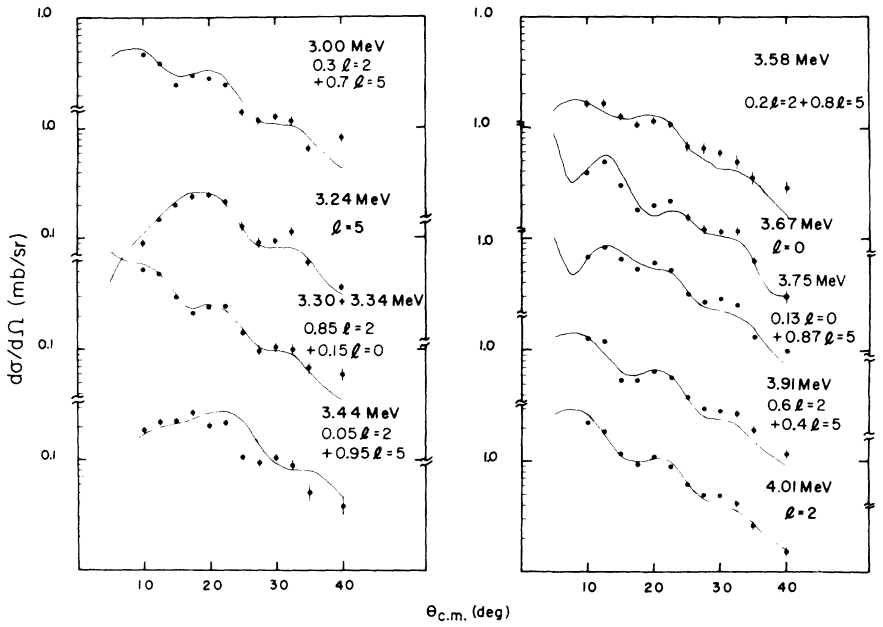

FIG. 15. Angular distributions for $Pr^{141}(He^3,d)Nd^{142}$.

so for $V_{11/2}{}^2$ in Ce^{140}. The rather large value of $(1-U_{11/2}{}^2)$ is presumably the result of not seeing all of the stripping strength (because of fragmentation) rather than an abnormally large occupation of the $1h_{11/2}$ orbit. Wildenthal, Newman, and Auble[1] have found that distorted-wave calculations for $N=82$ nuclei give more consistent results for pickup and stripping when they reduce the radius of the spin-orbit potential; in particular the spectroscopic factors for $1h_{11/2}$ stripping transitions are 15% larger.

The value of $U_{1/2}{}^2$ for Pr^{141} is somewhat too low and $U_{3/2}{}^2$ is somewhat too large. Part of this discrepancy can be attributed to difficulties in the

TABLE VII. Summary of results for levels of Nd^{142} excited in the $Pr^{141}(He^3, d)Nd^{142}$ reaction.

E_x (MeV)	n, l	$(2J_f+1)C^2S_{l,j}$
0.0	2d	2.10
1.57	2d	3.13
2.09	2d	7.80
2.34		
2.40	2d	7.37
2.55	3s	1.50
2.80	3s	0.31
	2d	0.72
3.00	2d	3.20
	1h	7.60
3.24	1h	12.60
3.30	3s	0.63
3.34	2d	3.50
3.44	2d	0.63
	1h	10.70
3.58	2d	1.00
	1h	4.00
3.67	3s	2.40
3.75	3s	2.60
	1h	17.50
3.90	2d	10.30
	1h	6.00
4.01	3s	2.50
	2d	14.00

TABLE VIII. Occupation probabilities measured for the $N=82$ isotones.

	j	$U_j{}^{2a}$	$V_j{}^{2b}$
La^{139}	$\frac{7}{2}$	0.37	0.68
	$\frac{5}{2}$	0.80	0.15
	$\frac{11}{2}$	0.99	...
	$\frac{1}{2}$	1.0	...
	$\frac{3}{2}$	0.96	...
Ce^{140}	$\frac{7}{2}$	0.31	0.70
	$\frac{5}{2}$	0.46	0.255
	$\frac{11}{2}$	0.60	0.063
	$\frac{1}{2}$	0.90	...
	$\frac{3}{2}$	0.93	...
Pr^{141}	$\frac{7}{2}$	0.	0.77
	$\frac{5}{2}$	0.57	0.32
	$\frac{11}{2}$	0.81	...
	$\frac{1}{2}$	0.70	...
	$\frac{3}{2}$	1.38	...

[a] Determined by the (He^3, d) reactions.
[b] Determined by the (d, He^3) reactions.

distorted-wave analysis and part to the increased uncertainty in extracting the value of S from transitions, where mixtures of $l = 0$ and $l = 2$ momentum transfers are allowed.

A further test of the reliability of the distorted-wave calculations is obtained by comparing the values of S obtained in the ground-state transitions for the two reactions $A(d, He^3)B_{g.s.}$ and $B(He^3, d)A_{g.s.}$. The two spectroscopic strengths should be equal, since one reaction is the inverse of the other. Four such comparisons can be made using the six ground-state spectroscopic strengths reported here and the spectroscopic strengths determined by Wildenthal, Newman, and Auble[1] in the reactions $Ba^{138}(He^3, d)La^{139}$ and $Nd^{142}(d, He^3)Pr^{141}$. When the transferred proton is a $g_{7/2}$ proton, with $j = (l - \frac{1}{2})$, the stripping strength is approximately 20% greater than the inverse pickup strength. However, when a $2d_{5/2}$ is involved, with $j = (l + \frac{1}{2})$, the stripping strength is approximately 20% less than the pickup strength. Reducing the radius of the spin-orbit potential in the distorted-wave calculations, as suggested by Wildenthal, would presumably bring the values into closer agreement.

IV. SHELL-MODEL CALCULATIONS

A. Pseudo Spin-Orbit Coupling Scheme

Wildenthal[2] has carried out a detailed shell-model calculation for the $N = 82$ isotones, using a modified-surface δ interaction, which yields excitation energies and spectroscopic factors in good agreement with the available experimental results. The calculation makes use of the Oak Ridge–Rochester shell-model code,[32] based on the conventional j-j coupling scheme. In order to keep the dimensionality of the shell-model space within bounds, it has been truncated in the following way in Wildenthal's calculation: The $(Z - 50)$ valence protons are restricted to the $1g_{7/2}$ and $2d_{5/2}$ orbits; additional configurations in which a single proton is excited to either the $2d_{3/2}$ or $3s_{1/2}$ orbit are also included. Even with this somewhat limited space, dimensions of the shell-model matrices may range as high as 300×300. Inclusion of the full $g_{7/2}, d_{5/2}, d_{3/2}, s_{1/2}, h_{11/2}$ shell-model space for the $(Z - 50)$ protons is beyond the scope of present computer technology. This illustrates the difficulty encountered by conventional shell-model calculations, a difficulty associated with the explosion of the dimensionality of the shell-model matrices in the j-j coupling scheme. Calculations using quasiparticle techniques and the Tamm-Dancoff approximation furnish a means of truncating the size of the shell-model space. Such calculations may give reliable results for states of predominant one-quasiparticle character.[3,4] States of predominant 2- and 3-quasiparticle character, however, must be expected to be influenced by admixtures of 4- and 5-quasiparticle states. The inclusion of such states again leads to relatively large matrices.[33] In addition, this technique is beset by all the difficulties and the additional calculational labors associated with the BCS approach.

It may therefore be fruitful to see whether a new simplifying coupling scheme can be used to truncate efficiently the size of the shell-model matrices. The pseudo spin-orbit coupling scheme[6-8] is such a scheme. The 82-neutron nuclei with $A \leq 142$, in which the protons are filling predominantly the $g_{7/2}, d_{5/2}$ single-particle levels, furnish a particularly good testing ground for this scheme, since calculations for these nuclei will involve mainly a single pseudo spin-orbit doublet. For a system of nucleons in a pair of nearly degenerate levels such as $g_{7/2}, d_{5/2}$, [or $(l+2)_{j+1}, l_j$ in general], it is possible to assign to the single nucleon a pseudo spin $b = \frac{1}{2}$ and a pseudo orbital angular momentum $c = 3$, (or $l + 1$ in general), such that the vector coupling $b + c = j$ yields the quantum numbers j of the doublet. The physical significance of this assignment comes from the fact that the many-nucleon total pseudo spin (B) and total pseudo orbital angular momentum (C) are approximately good quantum numbers for a system of real nucleons, provided the single-particle splitting of the $(l+2)_{j+1} l_j$ pseudo spin-orbit doublet is not too large. (Although the actual separation of the $g_{7/2}$ and $d_{5/2}$ levels is approximately 1 MeV, this is sufficiently small compared with two-body interaction energies of about 2.5 MeV in these nuclei.)

It is the aim of the present investigation to see whether a highly approximate calculation, based on this pseudo spin-orbit coupling scheme, can rival the much more detailed and precise calculations of Wildenthal. The motivation therefore is to determine to what extent the pseudo spin-orbit coupling scheme allows us to reduce the dimension of the basis space and give a simplified treatment of certain shell-model matrix elements without sacrificing too much of the success of Wildenthal's calculation. The hope is that similar simplified shell-model calculations may be performed for nuclei requiring much richer configurations. The procedure also makes it easier to include the effects of excitations into the higher $1h_{11/2}$, $2d_{3/2}$, and $3s_{1/2}$ orbits of the 82-neutron nuclei. For this reason the surface δ interaction, used by Wildenthal, is chosen as the two-body interaction.

The advantages of the pseudo spin-orbit coupling scheme[6-8] are the following:

(1) In the approximation in which the $(l+2)_{j+1}, l_j$ doublet is considered degenerate, and the two-body interaction is approximated by the surface δ interaction, the total pseudo spin (B) and the total pseudo orbital angular momentum (C) as well as the pseudo $(l+1)$-shell seniority (v), are good quantum numbers. [Note that $J = B + C$, with J = real total angular momentum. The seniority v is the total (mixed configuration) seniority associated with the irreducible representations of the rotation group in $2l+1$ dimensions, in our case a pseudo f-shell seniority quantum number].

(2) One of the characteristic features of the two-particle spectrum of the surface δ interaction is that for each even J only one state, corresponding to one specific superposition of two particle states, is depressed in energy. In even nuclei the low-lying $J \neq 0$ states are the states with $v = 2$, $B = 0$, and C = even. The quantum number $2B$ serves as a generalized seniority number which counts the number of nucleons not in favored pairs. For the higher-seniority numbers B becomes more important than v in ordering the levels.[6] The simplicity of the (B, C) scheme can perhaps best be illustrated by examining the structure of a state such as the lowest $J = 2$ state in the eight-proton nucleus Ce^{140}. In the (B, C) scheme this state is described in zeroth approximation as a single state in which the pseudo f-shell nucleons are coupled to $v = 2$, $B = 0$, and $C = 2$. The equivalent description of the same state in the conventional j-j coupling scheme involves a superposition of eight almost equally important components of the type $(g_{7/2})^{n'}{}_{J'}(d_{5/2})^{n''=8-n'}{}_{J''}$ with $J', J'' = 0, 2$; or $2, 0$; or $\frac{7}{2}, \frac{5}{2}$.

(3) The single-particle energy splitting $d_{5/2}$-$g_{7/2}$ can be described by a one-body pseudo spin-orbit coupling term in the Hamiltonian. The selection rules for its matrix elements ($|\Delta B| \leq 1, |\Delta C| \leq 1$; $B, C\ 0 \leftrightarrow 0$), and its one-body character lead to considerable simplification in the calculation, since it is the primary pseudo spin breaking term in the Hamiltonian. In addition, its matrix elements diagonal in v are zero for the $\frac{1}{2}$ full shell ($Z = 57$ for the pseudo f shell), and relatively small for nuclei near the $\frac{1}{2}$ full shell, an effect which contributes to the goodness of the quantum numbers B and C even in nuclei where the single-particle splitting $\epsilon_{j+1} - \epsilon_j$ is relatively large.

(4) The calculations of shell-model matrices are complicated, mainly because of the large number of states with high seniority. States with $v \geq 4$ in even nuclei, and states with $v \geq 5$ in odd nuclei are important mainly insofar as they are admixed into states of lower v. The very precise location of states with $v \geq 4$ may therefore not be necessary. Exact calculations[6] for degenerate pseudo spin-orbit doublets show that states with $v \geq 4$ are given to very good approximation if the two-body interaction in such states is replaced with a simplified interaction, the so-called generalized pairing interaction in which the effective interaction strength for the favored $J \neq 0$ pairs is independent of J (but differs from the pairing strength for $J = 0$). Matrix elements diagonal in v for states with $v \geq 4$ can thus be given to very good approximation by the eigenvalues of this generalized pairing interaction (GPI) which have a very simple form. For even nuclei, e.g.

$$E^{\mathrm{GPI}}_{v,B} - E^{\mathrm{GPI}}_{v=0,B=0} = G\frac{(2c+1)}{(2c+3)}\left[\frac{v}{4}(4c+4-v) + B(B+1)\right],$$

with $c = 3$ for the $g_{7/2}, d_{5/2}$ doublet.

B. Basis States for the Pseudo Spin-Orbit Coupling Scheme

Since the pseudo spin-orbit coupling scheme gives an economical description of the shell-model space, and since the GPI gives a good approximation for the spectroscopically less important high-seniority states, the simplified shell-model calculations were based on the following approximations. If the states of interest are made up predominantly of components with $v \leq v_{\max}$, then:

(a) diagonal matrix elements of states with $v \geq (v_{\max} + 2)$ are approximated by the eigenvalues of the GPI;

(b) symmetry-breaking terms (matrix elements off diagonal in B and C) are included only for matrix elements connecting states with $v \leq v_{\max}$ to states with $v' \leq (v_{\max} + 2)$. If the two-body interaction is the surface δ interaction, the only symmetry-breaking terms within a single pseudo spin-orbit doublet arise from the single-particle Hamiltonian;

(c) for matrix elements with v and v' *both* $\leq v_{\max}$, the full shell-model Hamiltonian is used. In the present calculation the two-body interaction is approximated by the surface δ interactions.

In the 82-neutron nuclei the $v = 1$ states associated with the $3s_{1/2}, 2d_{3/2}$ doublet fall in the very rich region of $v = 3$ states for the $1g_{7/2}, 2d_{5/2}$ doublet. The fragmentation of the $s_{1/2}$ and $d_{3/2}$ single-particle strengths will therefore be a sensitive function of the exact position of these $v = 3$ states. These will be influenced not only by admixtures of $v = 5$ states, which would be included by (b) above, but should also be a rather sensitive function of the exact form of the two-body interactions. The use of the surface δ interaction may therefore be of questionable validity for this problem. For this reason calculations for the odd nuclei were restricted to states with predominant $v = 1$ com-

ponents of the $g_{7/2}, d_{5/2}$ doublet. For the even isotones the situation is much more favorable, and the present calculations concentrate on those states below about 2.5 MeV which are primarily $v=0$ or $v=2$ in character. Approximation (b) above thus excludes states with $v \geq 6$ from the basis. Calculations were performed for the following three truncations of the shell-model space:

Type (1). The n protons outside the $Z=50, N=82$ core were permitted to fill only the $g_{7/2}, d_{5/2}$ doublet.

Type (2). In addition to the configurations $(g_{7/2}, d_{5/2})^n$, one proton was permitted in either the $s_{1/2}$ or $d_{3/2}$ orbit; but in this case the remaining $n-1$ protons in the $(g_{7/2}, d_{5/2})^{n-1}$ configuration were restricted to $v_{n-1}=1$. This is the truncation of the shell-model space, which most closely approximates that used by Wildenthal.

Type (3). Since excitations of pairs out of the $g_{7/2}, d_{5/2}$ levels must be expected to be important, a third truncation of the shell-model basis included, in addition to states of the above configurations, states with two protons in the $1h_{11/2}$ orbit or in the $3s_{1/2}, 2d_{3/2}$ orbits. For these excitations, the remaining $n-2$ protons of the $(g_{7/2}, d_{5/2})^{n-2}$ configurations were coupled to $v_{n-2}=0$. The $h_{11/2}$ single-particle level was taken at an energy ϵ, above the center of gravity of the $g_{7/2}, d_{5/2}$ doublet, and the $s_{1/2}$ and $d_{3/2}$ single-particle levels were assumed degenerate at an energy ϵ_0 above the $g_{7/2}, d_{5/2}$ center of gravity. The two-body matrix elements between the basis states added for calculations of type (2) and (3) and the basis states for the pure $g_{7/2}, d_{5/2}$ configurations of type (1) were calculated using the surface δ interaction. Since the added states are therefore connected to states of pure $g_{7/2}, d_{5/2}$ configurations by the strong part of our shell-model Hamiltonian, their inclusion has a strong effect on the $v=2$ states.

Because of the approximations (a), (b), and (c), above, our truncation of the shell-model space, particularly for calculations of type (3), corresponds in general philosophy to that known in calculations with quasiparticle techniques by the name of second Tamm-Dancoff approximation,[33] (inclusion of $v=4$ states). However, the present approach is a strict shell-model approach in which calculations have been simplified as much as possible. The actual calculation of matrix elements in the pseudo spin-orbit coupling scheme is very similar to that used in the LS coupling scheme of atomic spectroscopy. Fractional parentage expansions are used. The coefficients of fractional parentage (cfp's) needed for the $g_{7/2}, d_{5/2}$ calculation are those of the atomic f shell. For example, the term in the single-particle Hamiltonian which removes the degeneracy of the $g_{7/2}$ and $d_{5/2}$ levels can be expressed in terms of a one-body pseudo spin-orbit coupling term $b \cdot c$ by

$$H_{\text{s.p.}} = \epsilon_{7/2} n_{7/2} + \epsilon_{5/2} n_{5/2} = n\left[\tfrac{1}{2}\bar{\epsilon} + \frac{1}{2(2c+1)}\Delta\right] + \frac{2}{(2c+1)}\Delta\left(\sum_{i=1}^{n} b_i c_i\right),$$

with $c=3$ for the pseudo f shell. Here, $n = n_{7/2} + n_{5/2}$ is the number of protons filling the $g_{7/2}, d_{5/2}$ doublet; $\bar{\epsilon} = \frac{1}{2}(\epsilon_{7/2} + \epsilon_{5/2})$, and $\Delta = (\epsilon_{7/2} - \epsilon_{5/2})$. Using a fractional parentage expansion, matrix elements of this term in the $|v\alpha(B,C)\rangle$ scheme for a pure $g_{7/2}, d_{5/2}$ configuration are

$$\begin{aligned}
&\langle nv'\alpha(B'C')J|H_{\text{s.p.}}|nv\alpha(BC)J\rangle \\
&= n\left(\bar{\epsilon} + \frac{\Delta}{14}\right)\delta_{vv'}\delta_{\alpha\alpha'}\delta_{BB'}\delta_{CC'} + n12\Delta W(BCB'C';1J)\left[\frac{(2B+1)(2C+1)(2B'+1)(2C'+1)}{14}\right]^{1/2} \\
&\times \sum_{\substack{v_{n-1}\alpha_{n-1} \\ B_{n-1}C_{n-1}}} [(-1)^{B'-B-J+\frac{1}{2}+B_{n-1}+C_{n-1}} W(B\tfrac{1}{2}B'\tfrac{1}{2};B_{n-1}1)W(C3C'3;C_{n-1}1) \\
&\quad \times \langle n-1 v_{n-1}\alpha_{n-1}(B_{n-1}C_{n-1});(\tfrac{1}{2}3)]\!]nv\alpha(BC)\rangle \langle n-1 v_{n-1}\alpha_{n-1}(B_{n-1}C_{n-1});(\tfrac{1}{2}3)]\!]nv'\alpha'(B'C')\rangle].
\end{aligned}$$

The labels α are needed to distinguish between states of the same v, B, and C, whenever a specific pair of angular momenta (B,C) can occur more than once for a given v. The W coefficients are Racah coefficients.

C. Numerical Results

The energy eigenvalues and eigenvectors were obtained by diagonalization of the shell-model matrices which, in the truncation adopted here, were never larger than 23×23. The strength factor G for the surface δ and GPI, and the separation Δ between the $g_{7/2}$ and $d_{5/2}$ single-particle states were adjusted to give approximate fits to the observed splittings of the two $v=1$ states in the odd nuclei and the separation of the $v=0$ and $v=2$ states in the even nuclei. The values $G=0.38$ MeV and $\Delta = (\epsilon_{7/2} - \epsilon_{5/2}) = -1.00$ MeV were used in

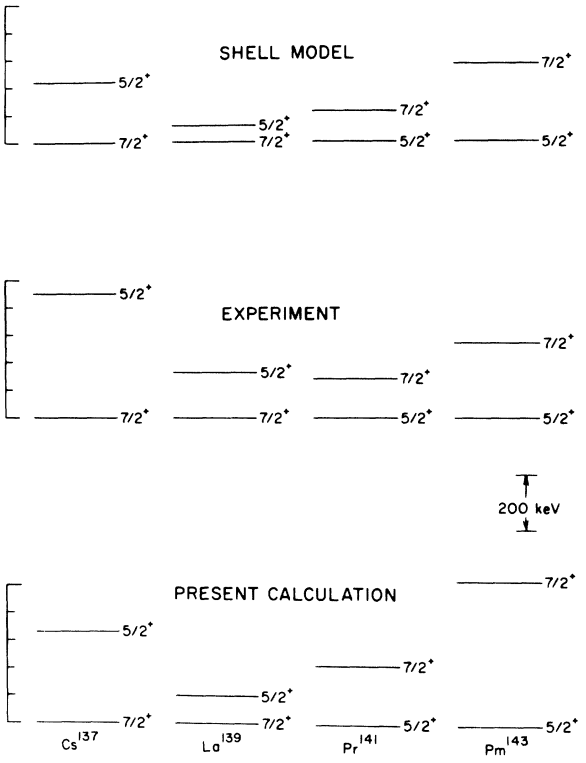

FIG. 16. Comparison of the energies of the $v=1$ states from the shell-model calculation of Wildenthal (See Ref. 2), experiment, and the present calculation, namely the pseudo spin-orbit calculation (type 1).

the calculations of type 1 and type 2. In the type-3 calculation, where more states are included in the basis, the interaction strength G had to be reduced to 0.20 MeV, but Δ was kept fixed at -1.00 MeV. The $h_{11/2}$ and $s_{1/2}, d_{3/2}$ single-particle-energy parameters, ϵ_1 and ϵ_0, were chosen somewhat arbitrarily to be 2.5 MeV. The results of the calculation for even nuclei were insensitive to their actual value, with variations in G being able to compensate for changes in ϵ_0 and ϵ_1 to a large extent.

1. *Results for Odd-N Nuclei*

Figure 16 shows a comparison of the results of the present calculation of the energies of the $v=1$ states in the truncated basis space of type 1 with the experimentally observed energies of the lowest $J=\frac{5}{2}, \frac{7}{2}$ states and the results of the detailed shell-model calculation of Wildenthal.[2]

The agreement between the present calculation and the experimental results is comparable to that of other calculations.[2-4] The agreement is poorest for Pm^{143}. With 11 protons outside the $Z=50$ core, however, excitations out of the $(g_{7/2}, d_{5/2})$ basis space become more important. For this reason, more weight was given to the lighter nuclei in the choice of the single-particle-energy parameter Δ.

From the viewpoint of single-nucleon-transfer reactions the most interesting states in the odd nuclei are the $v=1$, $J=\frac{5}{2}, \frac{7}{2}$ states and the $v=1$

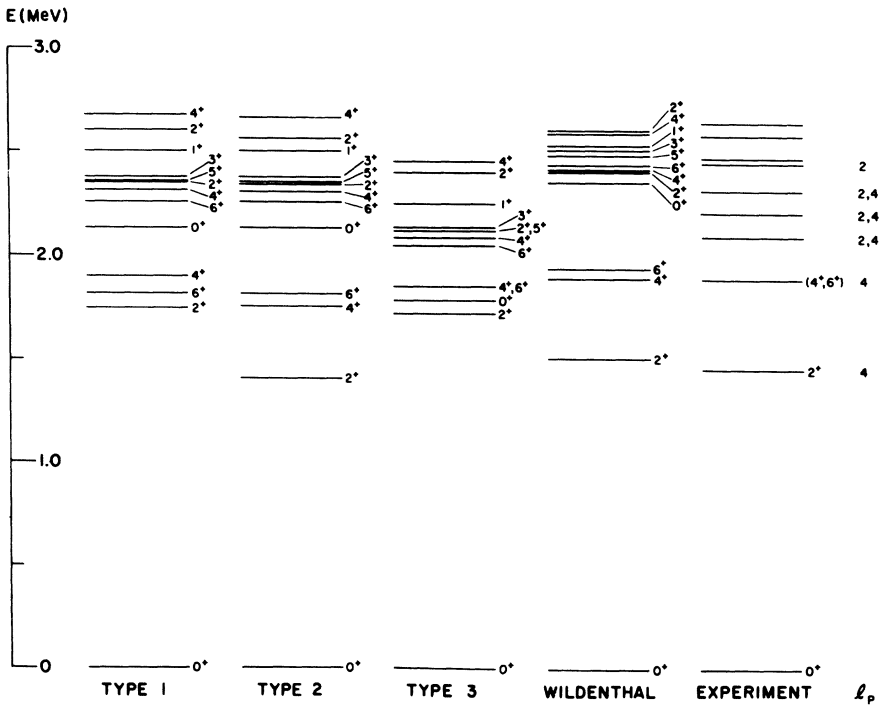

FIG. 17. Calculated and observed states in Ba^{138}. A description of the types of calculation is given in the text.

states associated with the $3s_{1/2}$, $2d_{3/2}$ doublet. These latter states are admixed with the $v = 3$ states of the $1g_{7/2}, 2d_{5/2}$ doublet, since they lie at approximately the same energy. Attempts to calculate the energies and the precise mixing of the $v = 1$ states of the $3s_{1/2}, 2d_{3/2}$ doublet and the $v = 3$ states of the $1g_{7/2}, 2d_{5/2}$ doublet in the truncated shell-model space of type 2 met with little success. Indications are not only that the $v = 5$ states must be included in the calculation [assumption (b) above], but that the surface δ interaction may not approximate the physical interaction between the valence protons to a sufficient accuracy to give satisfactory agreement with the observed fragmentation of single-particle transition strengths (since these are a sensitive function of the positions of the $v = 3$ states).

2. *Results for Even-N Nuclei*

Calculations of states with predominant $v = 0$ and $v = 2$ components for the even $N = 82$ isotones Ba^{138}, Ce^{140}, and Nd^{142} were made in the three shell-model basis spaces described earlier. The results of these calculations are shown in Figs. 17–19. These figures also show the results of Wildenthal's detailed shell-model calculation and the experimental energies, spins, and parities of the states as observed in the present single-nucleon-transfer reactions.

Calculations of type 1 are not in very good agreement with the experimental results and show that excitation of protons out of $g_{7/2}, d_{5/2}$ levels are important. Calculations of type 2 which approximate most closely the truncation of the shell-model basis used by Wildenthal are in good agreement both with the detailed calculations of Wildenthal and with the experimental results. The improvement in the calculation achieved by allowing one proton to be excited into the $3s_{1/2}, 2d_{3/2}$ doublet (type-2 results vs type-1 results) is an indication of the importance of excitations of this type. The most satisfactory results are obtained for

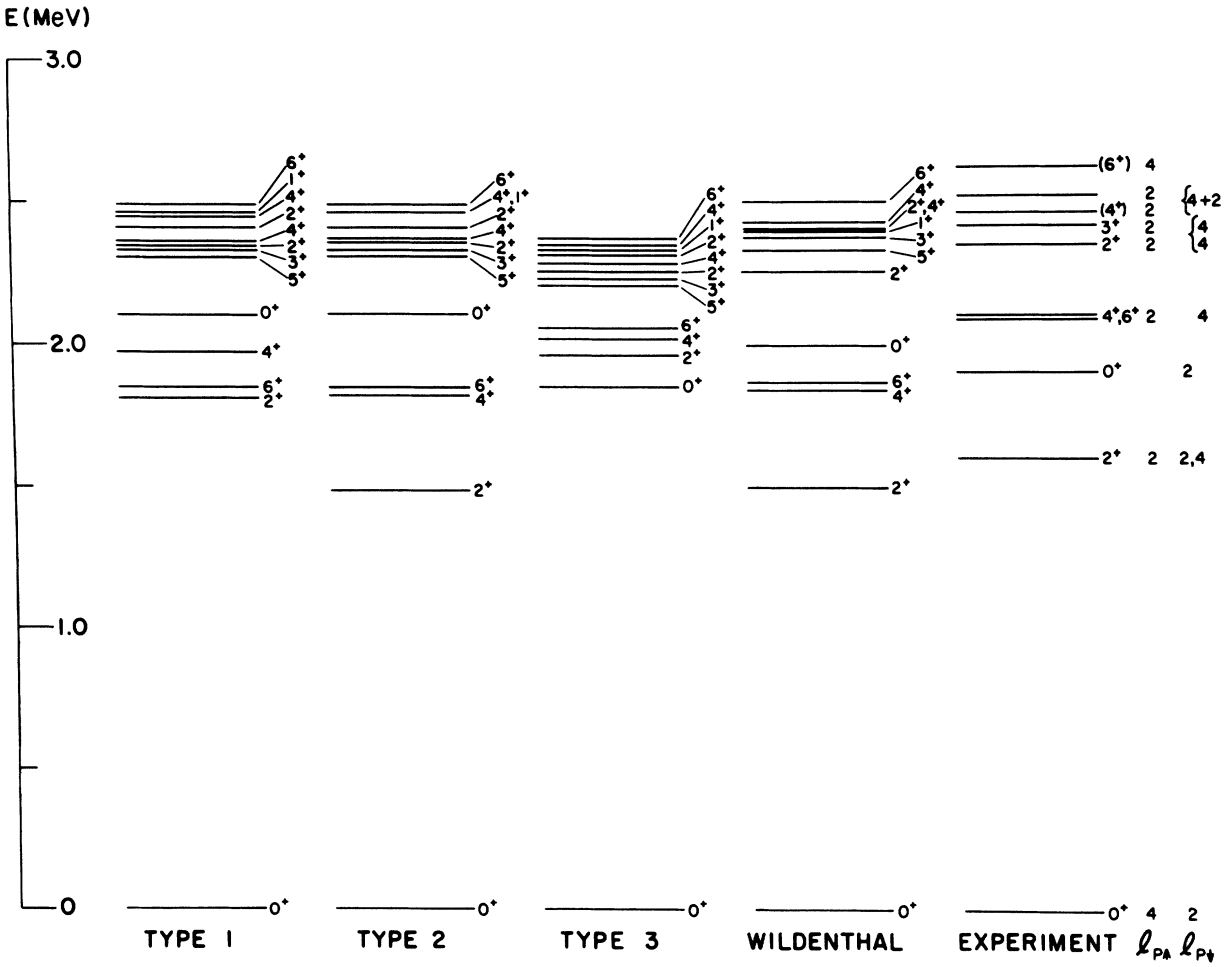

FIG. 18. Calculated and observed states in Ce^{140}.

Ce^{140}. Each of the states calculated lies within 100 keV of the corresponding state in Wildenthal's detailed calculation. The results for Ba^{138} (Fig. 17) are almost as successful. Energy levels calculated in the basis space of type 2 agree with the corresponding levels in Wildenthal's calculation to within a difference of at most 130 keV and more commonly 75 keV or less.

The results for Nd^{142} (Fig. 19) are the least successful of the three calculations. There may be two reasons for this. First, the $(g_{7/2}, d_{5/2})$ space is nearly full and the effects of more complicated configurations (configurations which have been neglected) are more important. Secondly, Nd^{142} is three protons away from the half-full $(g_{7/2}, d_{5/2})$ shell. This fact taken together with the relatively large single-particle splitting (1 MeV) may mean that the basic pseudo spin-orbit coupling scheme does not give as accurate a description of the low-lying states as in the case of Ba^{138} and Ce^{140}, which lie one proton on either side of the half-full pseudo f shell.

The calculations in the basis spaces, where excitations of a pair of protons into the $h_{11/2}$, $s_{1/2}$, and $d_{3/2}$ levels were allowed (type 3), were not as successful as the calculations without such pair excitations. The major problem seems to be the following: These pair excitations have a much larger effect on the 0^+ ground states than the other states. In the calculations of type 3 a partial compensation for this effect has been achieved by an adjustment in the strength of the surface δ interaction (a decrease in strength from $G = 0.38$ to 0.20 MeV). However, it was not possible to choose a value for G which gave both a reasonable fit for the energy of the first excited 0^+ state and a realistic separation between the lowest 2^+, 4^+, and 6^+ states. Since the results of the pickup reactions on these nuclei indicate that some excitations of pairs into the $h_{11/2}$, $s_{1/2}$, and $d_{3/2}$ levels should be considered, results for the calculations of type 3 are quite disappointing. It

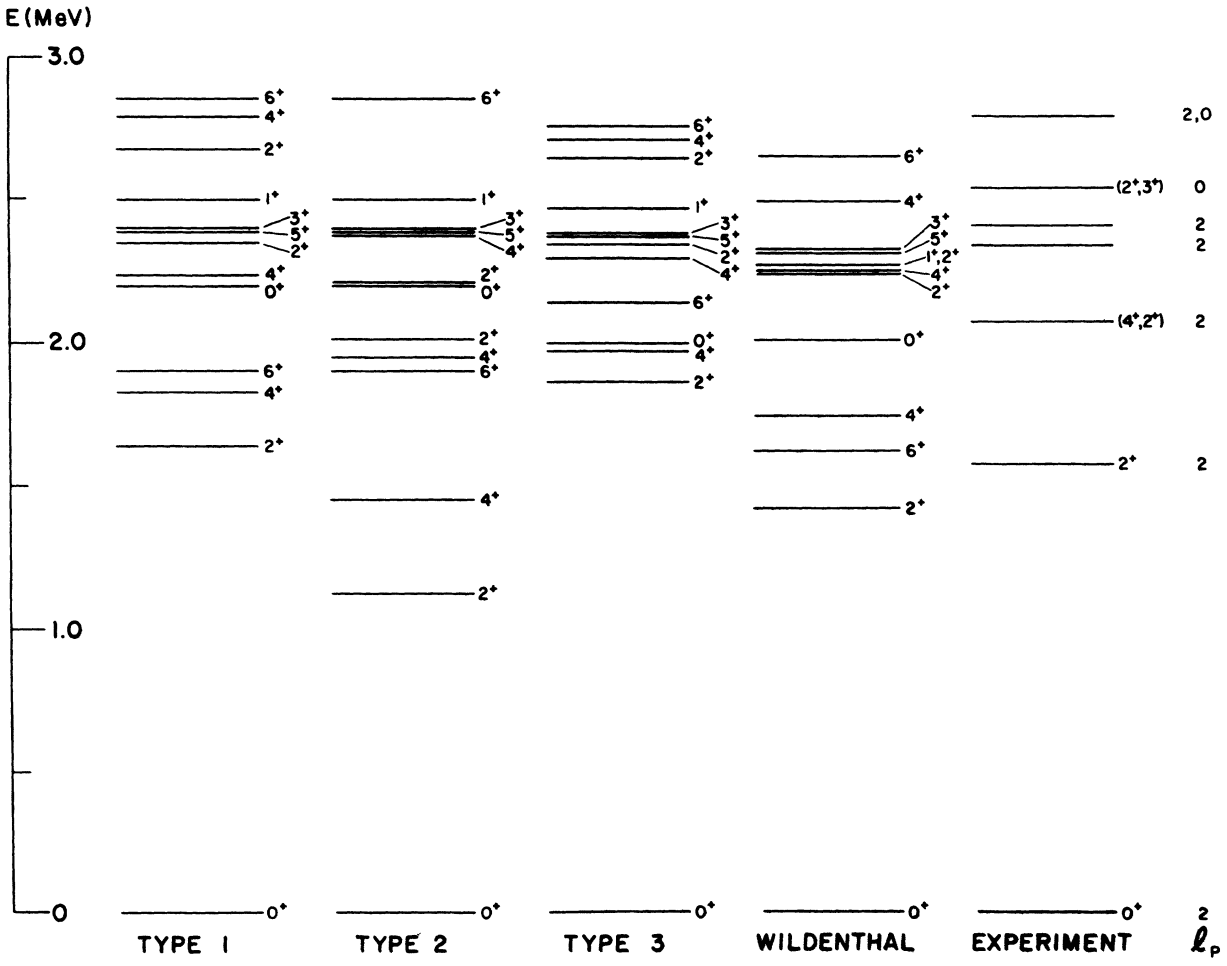

FIG. 19. Calculated and observed states in Nd^{142}.

is believed that such pair excitations should not be completely negligible. The failure of calculations of type 3 may therefore be an indication of the inadequacy of the surface δ interaction. (It is believed that it gives too much strength to the $J = 0$ pairs, particularly for the $h_{11/2}$ orbit, compared with the strength for favored pairs with $J \neq 0$.)

The good agreement between the shell-model calculations of Wildenthal and the experimental results, however, shows that the surface δ interaction can serve as a good, effective interaction for this particular truncation of the shell-model basis. Insofar as the present calculations of type 2 most closely approximate this truncation, the good agreement between these calculations and the more detailed calculations of Wildenthal shows the feasibility of a highly simplified shell-model calculation, based on the pseudo spin-orbit coupling scheme and the GPI for high-seniority states. Such simplified shell-model calculations may be particularly useful for nuclei requiring even richer configurations for which a conventional detailed shell-model approach is no longer possible.

V. DISCUSSION OF SPECTROSCOPIC FACTORS

A comparison between the experimental and theoretical transition strengths gives a measure of the "goodness" of the shell-model calculations. In addition, the predicted transition strengths may aid in making assignments of spins and parities more definitive than those possible from a knowledge of the observed l transfers. For transitions to nuclei of even Z, the present calculations of type 2 give results very similar to those of Wildenthal. Since Wildenthal's more detailed calculations are more complete for the case of odd nuclei, the spectroscopic strengths measured in the proton-transfer reactions of this investigation are compared in this section with the strengths calculated by Wildenthal.

A. Odd-Z Nuclei

The spectroscopic strengths calculated by Wildenthal[2] for the $1g_{7/2}$ and $2d_{5/2}$ transitions in the reactions $Ce^{140}(d, He^3)La^{139}$ (Fig. 20) and $Ce^{140}(He^3, d)Pr^{141}$ (Fig. 21) agree with those extracted in the present experiments to within 20%. This agreement is quite satisfactory in view of the uncertainties involved in extracting spectroscopic strengths from the measured cross sections. The agreement for the $3s_{1/2}$ and $2d_{3/2}$ transitions is less satisfactory. The calculation predicts too much splitting for the $2d_{3/2}$ strength and not enough for the $3s_{1/2}$ strength. This is not too surprising in light of the difficulties, discussed earlier, to be expected for the $J = \frac{1}{2}$ and $J = \frac{3}{2}$ states. The problems arising from the incompleteness of

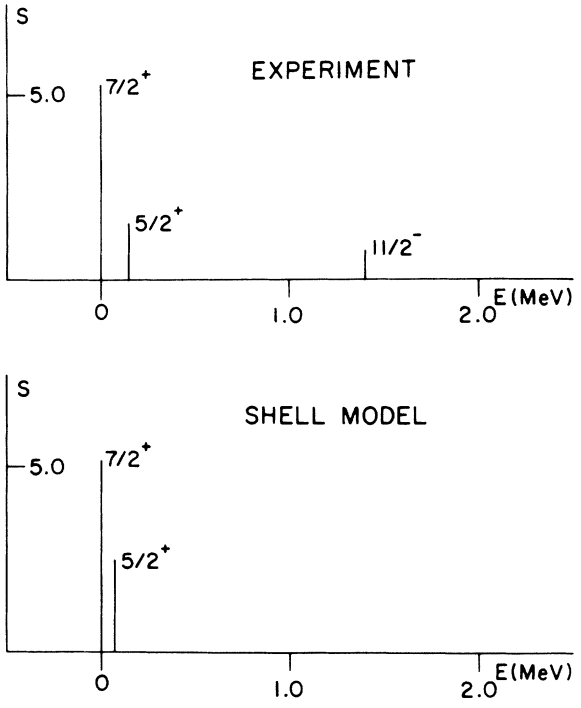

FIG. 20. Spectroscopic strengths for the $Ce^{140}(d, He^3)La^{139}$ reaction.

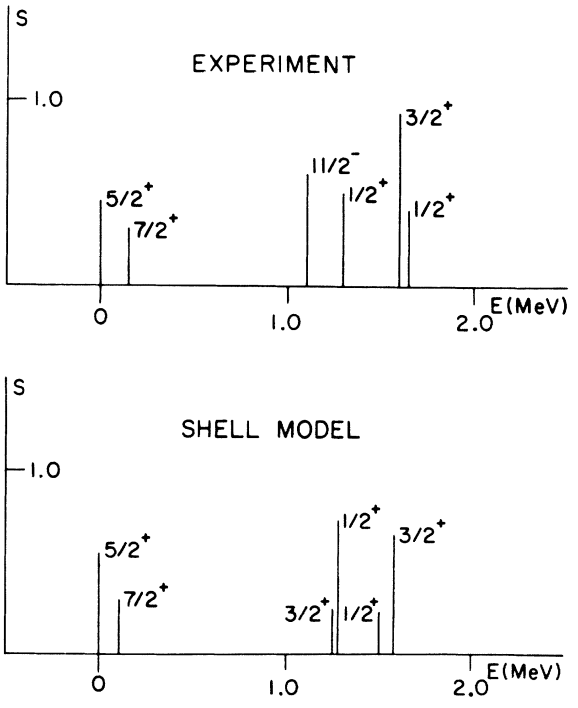

FIG. 21. Spectroscopic strengths for the $Ce^{140}(He^3, d)Pr^{141}$ reaction.

the basis space and the inadequacies of the surface δ interaction apply to Wildenthal's results in much the same way as they do to the simplified calculations based on the pseudo spin-orbit coupling scheme.

B. Ce^{140}

The spectroscopic strengths measured in the stripping reaction $La^{139}(He^3, d)Ce^{140}$ (Fig. 22) agree well with those calculated by Wildenthal[2] for those levels for which spins and parities are known with certainty so that a direct comparison can be made. The strengths measured for the ground-state transition and the summed $2d_{5/2}$ transition strengths for the 4^+ and 6^+ states at 2.08 and 2.11 MeV of excitation in Ce^{140} agree with the calculated strengths to within 30% (in both cases the measured strengths are larger). The $g_{7/2}$ transition strengths predicted for the 2^+ state at 1.60 MeV and the 0^+ state at 1.90 MeV are smaller than 0.8. A weak $l=2$ transition to the 2^+ state at 1.60 MeV is observed (Table VI). The strongest $1g_{7/2}$ transition is to a state (or cluster of states) at 2.63 MeV of excitation in Ce^{140}. It seems reasonable to associate much of this spectroscopic strength to a transition leading to the 6^+ state predicted at a slightly lower energy. It may well be that either or both a 2^+ and 4^+ state lie very close to 2.63 MeV (within 30 keV) and are also being populated. Several states between 2.35 and 2.54 MeV are populated by the stripping of a $2d_{5/2}$ proton. The sum of the four observed $l=2$ transition strengths agree quite well with the sum of the strengths predicted in that region of excitation; but without complete knowledge of the spins of the individual states being populated, it is not possible to make a detailed comparison of the strengths with the calculated strengths.

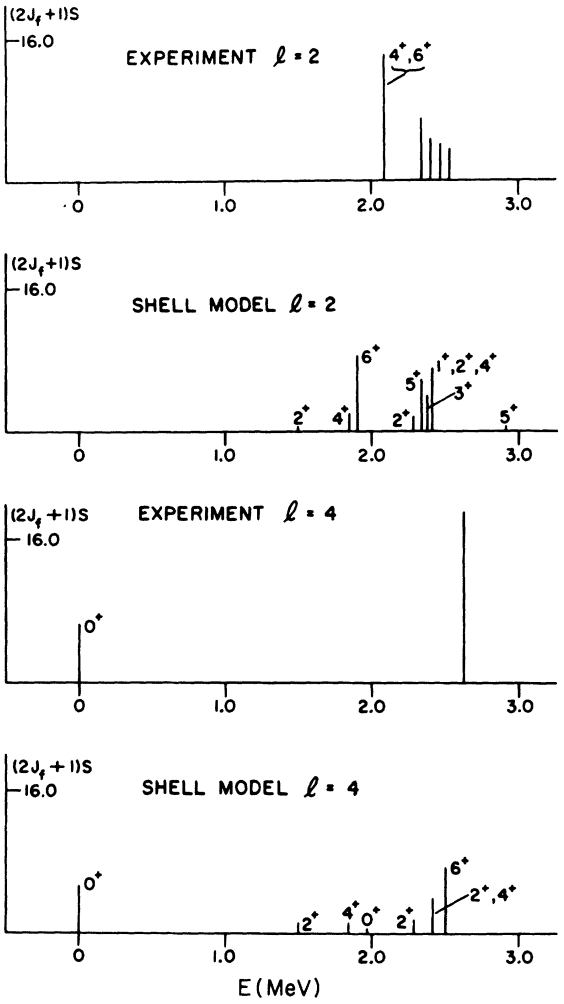

FIG. 22. Spectroscopic strengths for the $La^{139}(He^3, d)Ce^{140}$ reaction.

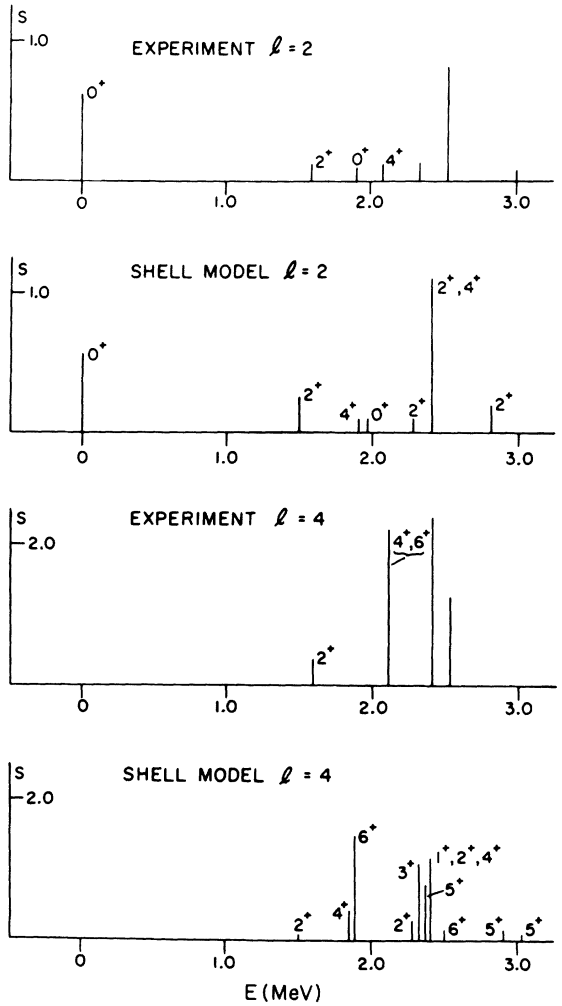

FIG. 23. Spectroscopic strengths for the $Pr^{141}(d, He^3)Ce^{140}$ reaction.

In the pickup reaction $Pr^{141}(d, He^3)Ce^{140}$ (Fig. 23) good agreement between the predicted and measured transition strengths is again obtained for the transitions leading to the ground state and the 4^+ and 6^+ states at 2.08 and 2.11 MeV, respectively. Good agreement is also achieved for the transition to the 2^+ state at 1.60 MeV. It is likely that the states at 2.52 and 2.54 MeV populated by the mixture of $l = 2$ and $l = 4$ transition strengths correspond to the 2^+ and 4^+ states predicted at nearly that energy. Some of the $l = 4$ strength could also be feeding the states with odd final spin. The sum of the measured $l = 4$ strengths for the states at 2.35 and 2.53 MeV agree well with the sum of the predicted transition strengths in that energy interval.

C. Ba^{138}

The spectroscopic strengths measured in the $La^{139}(d, He^3)Be^{138}$ reaction (Fig. 24) for the transitions to the ground state and 2^+ state at 1.43 MeV agree quite well with those calculated by Wildenthal.[2] Since the spins of the higher-lying states are unknown, it is difficult to make a detailed comparison of the measured and predicted strengths.

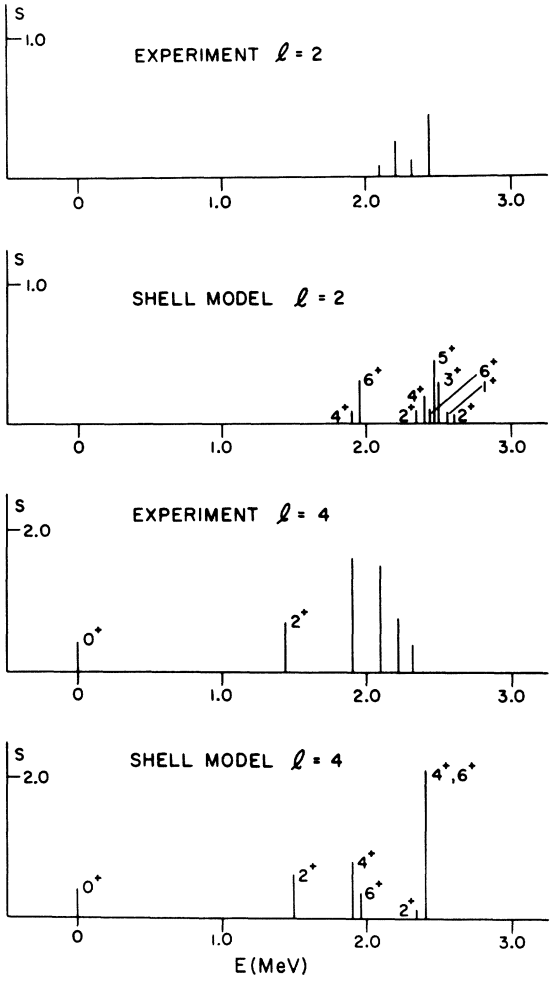

FIG. 24. Spectroscopic strengths for the $La^{139}(d, He^3)Ba^{138}$ reaction.

D. Nd^{142}

Figure 25 shows a comparison of the $l = 2$ $(2d_{5/2})$ transition strengths measured in the stripping reaction $Pr^{141}(He^3, d)Nd^{142}$ with those calculated by Wildenthal.[2] Satisfactory agreement is obtained for the strengths of the transitions leading to the ground state and 2^+ state at 1.57 MeV. On the basis of its transition strength the spin of the state observed at 2.09 MeV is likely to be 4^+. The spins of the states observed at 2.34 and 2.40 MeV are probably 2^+ and 4^+, respectively. The strength of the transition leading to the 2.40-MeV state was about twice that of the transition leading to the 2.34-MeV state. This is comparable to the ratio of strengths predicted for transitions going to 4^+ and 2^+ states at a slightly lower energy. Experimentally this ratio was determined by comparing peak intensities at angles where the resolution was most favorable. Only a single angular distribution was measured for these states.

No $l = 4$ transition strength was observed in this reaction. It would have been difficult to observe states populated by $l = 4$ transitions, which had spectroscopic strengths comparable to those predicted by the shell-model calculation.

VI. CONCLUSIONS

In summary, it may be concluded that the experimentally observed spectroscopic factors for

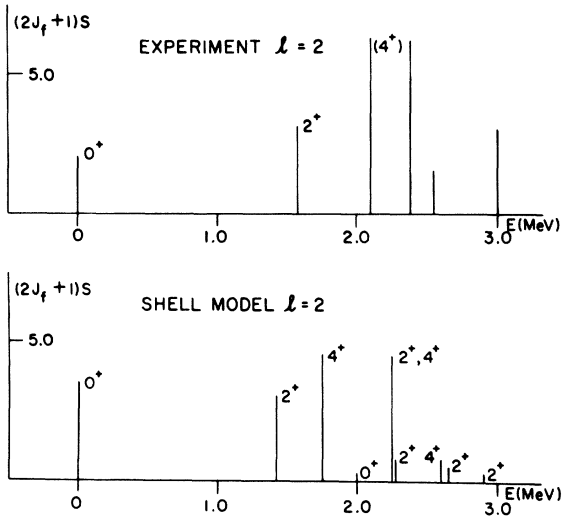

FIG. 25. Spectroscopic strengths for the $Pr^{141}(He^3, d)Nd^{142}$ reaction.

the transfer of $g_{7/2}$ and $d_{5/2}$ protons are in overall good agreement with the shell-model calculations. However, in order to give a good account of the observed fragmentation of the $2s_{1/2}$, $2d_{3/2}$, and $1h_{11/2}$ spectroscopic strengths, a considerable refinement of the shell-model calculations may be required. In the case of the stripping reactions to levels above ~2.8 MeV, in particular, the shell-model space may have to be enlarged by an order of magnitude and may be beyond the realm of possible detailed calculations, even with some of the simplifying techniques introduced in this investigation.

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