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## <sup>11</sup>B States Observed in the Scattering of Neutrons from <sup>10</sup>B and in the <sup>10</sup>B(*n*, α)<sup>7</sup>Li Reaction \*

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The polarization and the differential scattering cross section for neutrons scattered from <sup>10</sup>B have been measured for  $0.075 \leq E_n \leq 2.2$  MeV. These results, together with those from the <sup>10</sup>B(*n*, α)<sup>7</sup>Li, <sup>10</sup>B(*n*, α)<sup>7</sup>Li\* (0.48 MeV), <sup>7</sup>Li(α, α)<sup>7</sup>Li, <sup>7</sup>Li(α, α')<sup>7</sup>Li\* (0.48 MeV), and other reactions leading to states in <sup>11</sup>B, have been simultaneously interpreted in one consistent *R*-matrix calculation. The calculated results are in good agreement with most of the data and give new information about states in <sup>11</sup>B. The level parameters obtained for these states and the calculated reaction cross sections are consistent with the corresponding quantities in the mirror nucleus <sup>11</sup>C. Quantitative explanations are given both for the well-known  $1/v$  behavior of the cross section for the <sup>10</sup>B(*n*, α)<sup>7</sup>Li reaction and for the α<sub>0</sub>/α<sub>1</sub> branching ratio.

### I. INTRODUCTION

The energy level structure of <sup>11</sup>B above the α-particle threshold ( $E_x = 8.664$  MeV) is quite complex. Experiments<sup>1-6</sup> with <sup>7</sup>Li(α, α)<sup>7</sup>Li, <sup>7</sup>Li(α, α')-<sup>7</sup>Li\*, <sup>7</sup>Li(α, γ)<sup>11</sup>B\*, <sup>10</sup>B(*d*, *p*)<sup>11</sup>B, and <sup>9</sup>Be(<sup>3</sup>He, *p*)<sup>11</sup>B have yielded information on spins, parities, energies, and widths for several of the states below the neutron threshold ( $E_x = 11.456$  MeV), but a number of anomalies in these observed reactions

still remain unexplained. Above the neutron threshold, virtually no definitive information exists regarding  $J^\pi$  assignments, particle widths, and the like for the  $T = \frac{1}{2}$  states in <sup>11</sup>B, although a number of broad resonances<sup>1,3,7-16</sup> in the neutron total cross section for <sup>10</sup>B as well as in the reactions <sup>10</sup>B(*n*, *n'*γ)<sup>10</sup>B\*, <sup>10</sup>B(*n*, *n*)<sup>10</sup>B, <sup>10</sup>B(*n*, α<sub>0</sub>)-<sup>7</sup>Li, <sup>10</sup>B(*n*, α<sub>1</sub>)<sup>7</sup>Li\*, <sup>7</sup>Li(α, α')<sup>7</sup>Li\*, and <sup>7</sup>Li(α, *n*)-<sup>10</sup>B have been observed in this region of excitation. ( $T = \frac{3}{2}$  analogs of the two lowest states in <sup>11</sup>Be have

been observed<sup>17</sup> at  $E_x = 12.55$  and  $13.05$  MeV in  $^{11}\text{B}$ .) In particular, the  $1/v$  dependence of the  $^{10}\text{B}(n, \alpha)^7\text{Li}$  cross section has been known<sup>18</sup> for a long time; however, there has been no progress toward an understanding of this prominent feature of the cross section in terms of the level structure in  $^{11}\text{B}$ . The large  $1/v$  cross section extends to neutron energies as high as 100–200 keV and has been employed extensively for decades in neutron detection and as a standard for flux measurements. Early efforts at an explanation<sup>19</sup> in terms of an  $s$ -wave resonance near threshold were severely hampered by the lack of neutron scattering data, as well as data on the other open channels. The results were, therefore, somewhat ambiguous and did not give clearly defined values for the parameters of such a state.

For  $^{11}\text{B}$  above 11 MeV, theoretical structure calculations<sup>20</sup> based on the shell model are at present restricted to a few negative-parity states. The formation of positive-parity states by  $s$ -wave neutrons is an important process; however, shell-model calculations of highly excited non-normal-parity states are not available.

To form a consistent interpretation of the interaction of neutrons with  $^{10}\text{B}$ , the present measurements of the differential cross section and polarization for neutrons scattered from  $^{10}\text{B}$  are combined with all other known reaction data that lead to these states in  $^{11}\text{B}$  and to mirror states in  $^{11}\text{C}$ . It is hoped that these results may lead to a better understanding of the energy levels for mass-11 nuclei, as well as to a more meaningful application of  $^{10}\text{B}$  as a standard for flux measurements.

## II. EXPERIMENT AND RESULTS

The apparatus and data-reduction methods have been described in earlier publications.<sup>21,22</sup> The reaction  $^7\text{Li}(p, n)^7\text{Be}$  provided a partially polarized beam of neutrons emitted at  $51^\circ$  relative to the incident protons. A transverse magnetic field precessed the neutron spins through  $180^\circ$  so that the product of the polarizations produced by the source and by the scatterer could be obtained. The partially polarized beam of neutrons was incident at an angle of  $45^\circ$  upon slab-shaped scatterers of enriched  $^{10}\text{B}$  (95 at.-%  $^{10}\text{B}$  and 5 at.-%  $^{11}\text{B}$ ) powder packed under vacuum in steel cans with walls 0.005 in. thick. The large faces of the scatterer measured 10 in.  $\times$  20 in., the thickness was  $\frac{1}{8}$  in. (0.022 03 atoms/b), and the average neutron transmission was 90–95%. Neutrons scattered by the boron were detected by the use of shielded tanks that contained  $\text{BF}_3$  counters in an oil moderator. Measurements were made at five angles simultaneously; these data were sufficient to specify the angular distribution completely in the neutron en-

ergy range of interest, namely  $0.075 \leq E_n \leq 2.2$  MeV. Backgrounds were measured with duplicate empty cans in place of the scatterer.

Multiple-scattering corrections to the differential scattering cross section  $\sigma(\theta)$  ranged from 3 to 8% for the scatterers used. Multiple-scattering corrections to the polarization  $P(\theta)$  were much smaller than the statistical errors and were neglected. Both  $P(\theta)$  and  $\sigma(\theta)$  were corrected for the second group of neutrons from the  $^7\text{Li}(p, n_1)^7\text{Be}^*$  reaction. With the aid of a companion measurement<sup>22</sup> of neutrons scattered from  $^{11}\text{B}$ , both  $\sigma(\theta)$  and  $P(\theta)$  were also corrected for the  $^{11}\text{B}$  content in the sample. The known<sup>23</sup> energy dependence of the detector efficiencies was employed to correct the data on the assumption that all the observed counts were due to neutron elastic scattering. Inelastic scattering to the 0.717-MeV state of  $^{10}\text{B}$  is less than 1–2% of the total scattering (elastic plus inelastic) below  $E_n \approx 1.7$  MeV. The maximum inelastic scattering in this experiment is in the neighborhood of the resonance at  $E_n \approx 1.9$  MeV, where inelastic scattering accounts for  $\sim 8\%$  of the total cross section. Thus the contribution from inelastic scattering is not large even though the detectors do not sharply discriminate against inelastic events. The energy spread in the neutron beam, caused mainly by the thickness of the lithium target, varied from approximately 30 to 100 keV (measured at a proton energy of 1.9 MeV).

Figure 1 shows the experimental  $\sigma(\theta)$  and  $P(\theta)$  (solid points) in terms of the (laboratory) energy dependence of the coefficients  $B_L$  in the Legendre-polynomial expansion of  $\sigma(\theta)$ , and in terms of the coefficients  $C_L$  in the associated-Legendre-polynomial expansion of  $\sigma_p = \sigma(\theta)P(\theta)$ , both in the center-of-mass system. Preliminary results of these measurements with partial analyses have reported earlier.<sup>11,12,24</sup> Published values<sup>25</sup> of the polarization from the  $^7\text{Li}(p, n)^7\text{Be}$  source reaction were used to derive  $P(\theta)$  from the measured product of  $P(\theta)$  and the polarization in the source reaction. Counts from the detector were converted to cross section as described previously.<sup>22</sup>

Coefficients  $B_L$  for  $L > 3$  are negligible over the entire energy interval measured, and  $B_3$  is very small and nonresonant throughout. Where error bars are not shown, the errors do not exceed the size of the points. Coefficients  $C_L$  for  $L > 2$  were everywhere negligible.

No previous polarization results have been reported for  $^{10}\text{B}$ , but differential scattering cross sections have been measured at Oak Ridge National Laboratory<sup>26</sup> at three neutron energies: 0.55, 1.0, and 1.5 MeV. At  $E_n = 0.55$  MeV the previous measurements showed complete isotropy for  $\sigma(\theta)$  and were fitted with an  $s$ -wave phase shift only.

This clearly disagrees with the present results, which give a resonance in the coefficients  $B_1$  and  $C_1$  at this energy. The mere existence of nonzero polarization (i.e., nonzero  $C_1$ ) requires that higher partial waves must be present. At 1.0 MeV the present and previous results agree. At 1.5 MeV the Oak Ridge results at the smallest angle give a value nearly twice that obtained in this work. However, there is no serious disagreement at other angles. The Oak Ridge measurements were for elastic scattering only, whereas the present results include inelastic events as well. However, the inelastic scattering cross section at this energy is less than 1% of the total scattering cross section so, as discussed above, this difference in the measurement could not account for the discrepancy. A further difficulty with the Oak Ridge 1.5-MeV data is that the integrated differential elastic scattering cross sections is considerably greater than the difference between the total cross section and that for the  $^{10}\text{B}(n, \alpha)^7\text{Li}$  alone (neglecting the cross section of the remaining open channels). The present results, on the other hand, are consistent with the values of the total and reaction cross sections.

### III. R-MATRIX ANALYSIS AND DISCUSSIONS

In this section we discuss those levels in  $^{11}\text{B}$  formed in the  $^{10}\text{B} + n$  reaction that can be adequately described by an  $R$  matrix<sup>27</sup> for a single level (plus background term) of given  $J^\pi$ . The  $\frac{7}{2}^+$   $s$ -wave levels require a more complex  $R$ -matrix description which is discussed in Sec. IV. For the single-level cases, the elements of the  $R$  matrix are

$$R_{cc'}^{J^\pi} = \left( \frac{\gamma_{\lambda c} \gamma_{\lambda c'}}{E_\lambda - E} + \delta_{cc'} R_{cc}^0 \right)^{J^\pi},$$

where  $R^0$  is assumed to be diagonal. The corresponding elements of the collision matrix are

$$U_{cc'}^{J^\pi} = e^{i(\Omega_c' + \Omega_c')} \left[ \delta_{cc'} + \frac{i(\Gamma_{\lambda c} \Gamma_{\lambda c'})^{1/2}}{E_\lambda - E + \Delta_\lambda - \frac{1}{2}i \sum_{c''} \Gamma_{\lambda c''}} \right],$$

where  $\Gamma_{\lambda c} = 2P_c' \gamma_{\lambda c}^2$ ,  $\Delta_\lambda = -\sum_c \bar{S}_c' \gamma_{\lambda c}^2$ ,  $\bar{S}_c = S_c - b_c$ , and the primed quantities  $P_c'$ ,  $S_c'$ , and  $\Omega_c'$  are the usual surface functions modified<sup>27</sup> to take account of the effect of  $R^0$  in the one-level formula. The boundary conditions  $b_c$  are chosen such that  $\Delta_\lambda = 0$  at the resonance energy. Since  $^{10}\text{B}$  has spin  $3^+$ , states in  $^{11}\text{B}$  can be formed by neutrons via two channels, namely channel spins  $s = \frac{5}{2}$  and  $\frac{7}{2}$ . These states can undergo  $\alpha$  decay to the ground state (by emission of the  $\alpha_0$  group) and to the first excited state (via  $\alpha_1$ ) of  $^7\text{Li}$ , which are separated by only 0.48 MeV. Since the energy of the  $\alpha_0$  group is at least 4.38 MeV and since the widths of

states considered in the analysis based on the single-level formula were not large, the variations in  $\alpha$  penetrabilities for these states were neglected and the widths  $\Gamma_{\lambda c}$  for  $c = \alpha_0, \alpha_1$  were assumed to be constants. The boundary condition in the  $\alpha$  channel was chosen such that  $\bar{S}_\alpha = 0$  everywhere. This approximation causes at most an error of 2 keV in  $\Delta_\lambda$  at points far from resonance. Neutron penetrabilities  $P_c$ , shift functions  $S_c$ , and hard-sphere phase shifts  $\Omega_c$  were calculated for a radius of 4.14 F. For neutron scattering calculations, at least three channels must be considered.

These are the  $\alpha$ -decay channel and the neutron channels corresponding to  $s = \frac{5}{2}$  and  $\frac{7}{2}$ , where all  $\alpha$  decay is lumped into one channel. (This can be done since all the  $\alpha$  widths are assumed constant in the single-level case.) While the reactions  $^{10}\text{B}(n, p)^{10}\text{Be}$  and  $^{10}\text{B}(n, t)^2\alpha$  are energetically possible, with separation energies just slightly below that of the neutron binding energy, their cross sections are comparatively small in the region of interest and were neglected. The coefficients  $B_L$  and  $C_L$  for scattering are then calculated<sup>28</sup> from the  $U_{cc}^{J^\pi}$ . These calculations are shown as solid curves in Fig. 1, and the parameters for the various states are given in Table I. Further discussion of the results of the calculations follows.

#### A. Anomaly at $E_x = 13.1$ MeV ( $E_n = 1.8$ MeV)

The anomaly at  $E_n = 1.8$  MeV in Fig. 1, which was first seen in the  $^{10}\text{B}(n, \alpha)$  reaction,<sup>16</sup> was originally attributed to two or more overlapping levels. According to Ref. 16, if the peak is due to single isolated level, a Breit-Wigner fit demands a spin  $J \geq \frac{1}{2}$  with  $l_n = 2$  and  $l_\alpha \geq 5$ . However, the resulting  $\alpha$ -particle reduced width exceeds the Wigner limit and thus makes such an assignment improbable. Another group<sup>9</sup> also carefully investigated this peak for signs of multilevel structure in an experiment with 50-keV resolution, but none was found. A spin  $J \geq \frac{1}{2}$  was assigned by the latter group.

In the present attempt to fit the neutron data, spins  $\frac{1}{2} \leq J \leq \frac{11}{2}$  with  $l_n = 1$  or 2 were tried on the basis of a single-level assumption. None gave a simultaneous fit to the peaks in  $B_1$  and  $B_2$ , which are displaced from each other by approximately 100 keV. In order to obtain even an approximate fit to  $B_1$  and  $B_2$  simultaneously, two states were required: a  $p$ -wave and a  $d$ -wave state. The chief contribution to the coefficient  $B_2$  was fitted by attributing it to a  $d$  state with  $J^\pi$  in the range  $\frac{3}{2}^+ - \frac{7}{2}^+$  and with  $E_r = 1.86$  MeV, and that to coefficient  $B_1$  as due to a  $p$  state with  $J^\pi = \frac{3}{2}^- - \frac{7}{2}^-$  and  $E_r = 1.75$  MeV. The interpretation of this anomaly

as two overlapping levels with  $J$  values in the range  $\frac{3}{2}-\frac{7}{2}$  also gives a better explanation for the observed strong inelastic neutron yield,<sup>10</sup> because a  $\frac{7}{2}^+$  state can decay to the 0.717-MeV level of  $^{10}\text{B}(J^\pi=1^+)$  by  $l_n=2$ , and a  $\frac{7}{2}^-$  state should decay by  $l_n=3$ , whereas a value of  $J^\pi=\frac{11}{2}^+$  would require  $l_n=4$ .

In the mirror nucleus  $^{11}\text{C}$ , the probable analog of the positive-parity component of this anomaly has been observed with a correspondingly large cross section at  $E_x=12.65$  MeV in the  $^{10}\text{B}(p, \alpha)^7\text{Be}$

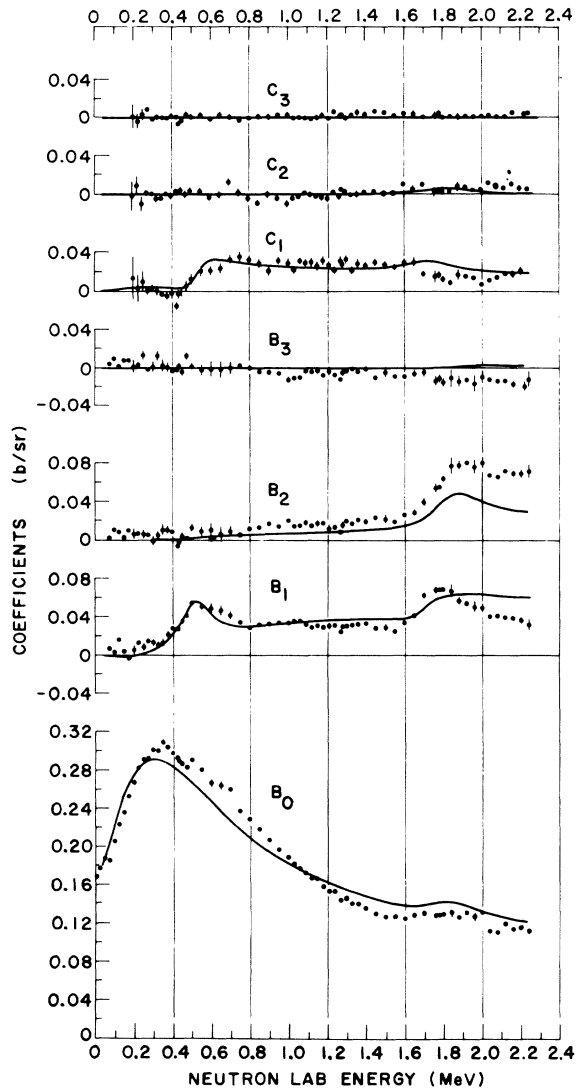


FIG. 1. Coefficients  $B_L$  in the Legendre-polynomial expansion of  $\sigma(\theta)$ , and  $C_L$  in the associated-Legendre-polynomial expansion of  $\sigma(\theta)P(\theta)$ . Data points for  $B_0$  below  $E_n=0.075$  MeV are from Ref. 39. Where not shown, errors on the data points are less than the size of the points. The integrated scattering cross section is  $4\pi B_0$ . The solid curves are the result of the  $R$ -matrix calculations with the parameters given in Table I.

reaction.<sup>29</sup> Figure 2 indicates these probable mirror levels. In Ref. 29 also, the conclusion is that although  $J=\frac{11}{2}$  fits the data on a single-level assumption, the resulting  $\alpha$ -particle reduced width (with  $l_\alpha=5$ ) exceeds the Wigner limit. A value  $J^\pi=\frac{7}{2}^+$  was tentatively assigned in that work.

The final parameters for the two closely spaced states are given in Table I. The overlapping nature of these states, together with the uncertainty in their  $J$  values makes definitive assignments of widths extremely difficult. However, the widths of Table I are more consistent with the inelastic scattering results<sup>10</sup> than with the  $(n, \alpha)$  results.<sup>9</sup>

#### B. State at $E_x=11.94$ MeV ( $E_n=0.53$ MeV)

Several reactions show evidence for a state in  $^{11}\text{B}$  near 11.9 MeV. In the reaction  $^{10}\text{B}(n, \alpha)^7\text{Li}$  to the ground state ( $\frac{3}{2}^-$ ) and the state ( $\frac{1}{2}^-$ ) at 0.48 MeV in  $^7\text{Li}$ , this state was observed<sup>1,9</sup> as a resonance at  $E_n=0.53$  MeV with a total width of 0.14 MeV, and was assigned<sup>9</sup>  $J^\pi=\frac{1}{2}^+, \frac{3}{2}^+$ , or  $\frac{5}{2}^+$  from a Breit-Wigner analysis. This state was observed<sup>1,3</sup> at  $E_x=11.88$  MeV in the  $^7\text{Li}(\alpha, \alpha')^7\text{Li}$  and  $^7\text{Li}(\alpha, \alpha')^7\text{Li}^*(0.48 \text{ MeV})$  reactions and was postulated<sup>3</sup> to have negative parity and a width of 0.150 MeV. This state is also detected at  $E_x=11.93$  MeV in the  $^{10}\text{B}(n, \alpha_0)^7\text{Li}$  reaction on the basis of reciprocity from the  $^7\text{Li}(\alpha, n)^{10}\text{B}$  reaction.<sup>14</sup> The analysis of Ref. 14 gives  $J^\pi=\frac{5}{2}^+ (l_n=0)$  or  $J^\pi=\frac{3}{2}^- (l_n=1)$  and a width of 0.3 MeV.

In the present work this state appears as a bump (Fig. 1) only in the coefficients  $B_1$  and  $C_1$  at  $E_n=0.53$  MeV. Since no coefficients  $B_L$  and  $C_L$  with  $L>1$  are resonant, this state is almost surely formed by  $p$  waves. The only assignment to fit both the  $B_L$  and  $C_L$  was  $J^\pi=\frac{5}{2}^-, l_n=1$ , and channel spin  $s=\frac{7}{2}$ . The  $\alpha$ -particle partial width used was 0.150 MeV, in agreement with the  $\alpha$  scattering work. The value  $J^\pi=\frac{5}{2}^-$  is consistent with both the  $\alpha_0$  and  $\alpha_1$  data,<sup>9</sup> since  $l_\alpha=2$  is possible for each, with the  $\alpha_0$  transition stronger because of its higher penetrability. The analog of this state in  $^{11}\text{C}$  has not yet been clearly established.

#### C. State at $E_x=11.79$ MeV ( $E_n=0.37$ MeV)

The large broad low-energy peak (Fig. 1) in the coefficient  $B_0$  was not known from previous measurements of  $\sigma_T$  or  $\sigma_{n,\alpha}$ . Since this peak does not appear in any other  $B_L$  or  $C_L$ , it probably is an  $s$ -wave state.<sup>30</sup> The conclusion is that the resonance is a broad  $s$  state with  $J^\pi=\frac{7}{2}^+$  or  $\frac{5}{2}^+$  located at  $E_n=0.37$  MeV ( $E_x=11.79$  MeV in  $^{11}\text{B}$ ) and is primarily responsible for the  $1/v$  dependence of the  $^{10}\text{B}(n, \alpha)^7\text{Li}$  cross section at low neutron energies.

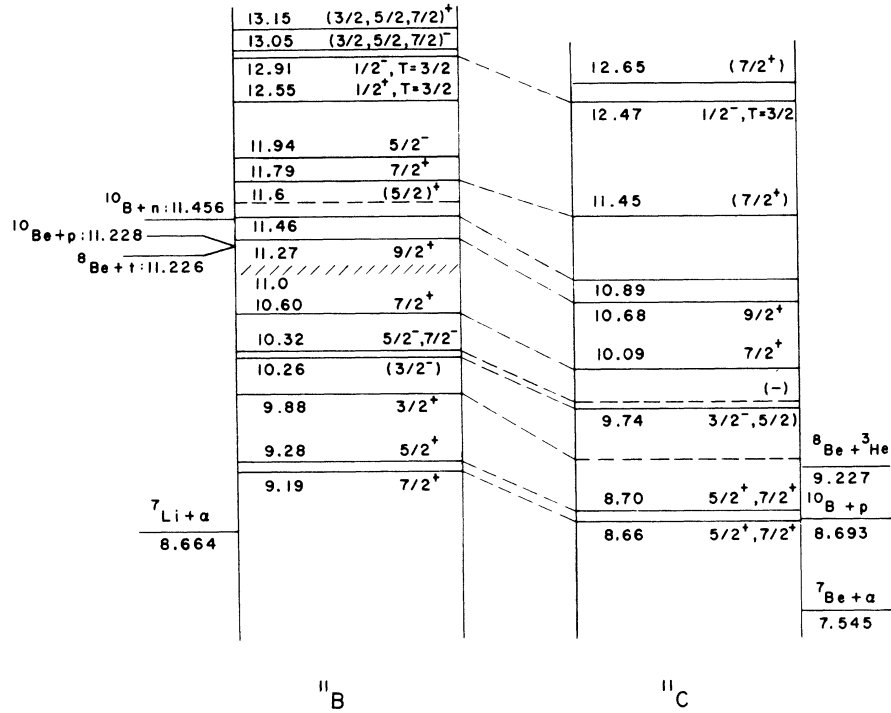


FIG. 2. Partial energy level diagram for  ${}^{11}\text{B}$  and  ${}^{11}\text{C}$ . Probable analog states are connected by dashed lines.

TABLE I.  $R$ -matrix parameters for calculations of the solid curves of Figs. 1, 3, and 4. All energies and widths are in MeV.

$J^\pi$	$\frac{7}{2}^+$ (scatt.)	$\frac{7}{2}^+$ (bound)	$\frac{5}{2}^+$	$\frac{5}{2}^+$	$\frac{3}{2}^- - \frac{1}{2}^-$	$\frac{3}{2}^+ - \frac{1}{2}^+$
$l_n$	0	0	0	1	1	2
$l_{\alpha_0}$	3	3	1	2	0, 2	1, 3
$l_{\alpha_1}$	3	3	...	...	2, 4	1, 3
$E_x$	11.79	10.61	11.61	11.94	13.05	13.15
$E_\lambda$ (c.m.)	0.335	-1.38	0.155	0.480	1.59	1.69
$E_r$ (lab)	0.37	(-0.94)	0.170	0.53	1.75	1.86
$\gamma_n^2$	0.77	0.12	0.0006	0.075	0.046	0.656
$\gamma_n^2/\gamma_{wn}^2$ <sup>a</sup>	0.19	0.03	0.0015	0.0188	0.068	0.165
$\Gamma_n$ (c.m.)	0.77	...	0.004	0.024	0.050	0.024
$\gamma_{\alpha_0}^2$	0.0004	0.054	0.063 <sup>b</sup>	0.037 <sup>b</sup>	...	...
$\gamma_{\alpha_0}^2/\gamma_{w\alpha}^2$ <sup>c</sup>	0.0006	0.078	0.091	0.055	...	...
$\Gamma_{\alpha_0}$ (c.m.)	0.001	0.030	0.296	0.150	...	...
$\gamma_{\alpha_1}$	0.076	0.402	...	...	...	...
$\gamma_{\alpha_1}^2/\gamma_{w\alpha}^2$	0.112	0.59	...	...	...	...
$\Gamma_{\alpha_1}$ (c.m.)	0.113	0.070	...	...	...	...
$\Gamma_\alpha$	0.114	0.100	0.296	0.150	0.136	0.163
$b_n$	0.0	0.0	0.0	-0.74	-1.1	-1.51
$b_{\alpha_0}$	-1.33	-1.33	-0.312	...	...	...
$b_{\alpha_1}$	-1.64	-1.64	...	...	...	...
$R_{nn}^0$	0.0	0.0	0.2	0.0	0.0	0.0

<sup>a</sup> For a neutron interaction radius of 4.14 F, the value of  $\gamma_{wn}^2 \equiv 3\hbar/2Ma^2$  is 4 MeV. Here  $M$  is the reduced mass.

<sup>b</sup> This value of  $\gamma_{\alpha_0}^2$  was deduced from  $\Gamma_{\alpha_0}/2P_{\alpha_0}$  evaluated at  $E_r$ , where  $\Gamma_{\alpha_0}$  = constant.

<sup>c</sup> For an  $\alpha$ -interaction radius of 6.0 F, the value is  $\gamma_{w\alpha}^2 = 0.68$  MeV.

The existence of an  $s$ -wave state with  $J^\pi = \frac{7}{2}^+$  near the neutron threshold in  $^{11}\text{B}$  is in agreement with other experimental evidence. From the  $^{10}\text{B}(n, \gamma)^{11}\text{B}$  reaction<sup>31</sup> at thermal energies, only 6% of the capture  $\gamma$  rays from a state near the neutron threshold in  $^{11}\text{B}$  go to the  $\frac{3}{2}^-$  ground state. Capture in a  $\frac{7}{2}^+$  state would require  $M2$  decay to the ground state and would thus be inhibited (as observed), whereas capture in a  $\frac{5}{2}^+$  state should decay rapidly by an  $E1$  transition. Further, the branching ratio of  $\alpha$  particles to the ground state and first excited state  $\frac{1}{2}^-$  at 0.48 MeV in  $^7\text{Li}$  in the reaction  $^{10}\text{B}(n, \alpha)^7\text{Li}$  induced by thermal neutrons was measured<sup>14</sup> to be only 6.75%. It would be difficult to explain this result if the reaction were assumed to proceed through a  $\frac{5}{2}^+$  state, since  $\alpha$  decay to the ground state would go via  $l_\alpha = 1$  and that to the excited state by  $l_\alpha = 3$ . For a  $\frac{7}{2}^+$  state, on the other hand, both the  $\alpha_0$  and  $\alpha_1$  transitions have  $l_\alpha = 3$ .

A further substantiation of the  $\frac{7}{2}^+$   $s$ -wave assignment is given by the reaction  $^{10}\text{B}(p, p)^{10}\text{B}$ , for which it was necessary<sup>32</sup> to assume a broad  $\frac{7}{2}^+$   $s$  state in  $^{11}\text{C}$ . The  $^{10}\text{B}(p, \alpha)^7\text{Be}$  reaction<sup>29,33</sup> also shows a broad resonance (Fig. 2) at approximately the same energy, i.e., at  $E_x = 11.45$  MeV in  $^{11}\text{C}$ . The fact that this state in  $^{11}\text{C}$  decays almost completely by  $\alpha_1$  decay to the first excited state of  $^7\text{Be}$  is further evidence for the mirror assignment, since it agrees with the branching ratio needed to fit the  $^{10}\text{B}(n, \alpha)^7\text{Li}$  data (as seen in Sec. IV). As expected, this broad state in  $^{11}\text{B}$  is not seen in the  $^7\text{Li}(\alpha, n)^{10}\text{B}$  reaction,<sup>1,14,15</sup> since the  $\alpha_0$  partial width is negligible compared to the  $\alpha_1$  partial width.

The single-level assumption was used in the first attempts to simultaneously fit both the  $B_0$  coefficient in the scattering data and the total  $^{10}\text{B}(n, \alpha)^7\text{Li}$  cross section with an  $s$ -wave state. It was possible to fit the shape of the scattering resonance in  $B_0$  at  $E_n = 0.37$  MeV and predict the  $1/v$  form of the observed total  $(n, \alpha)$  cross section, but the calculated  $(n, \alpha)$  cross section was only about  $\frac{1}{2}$  the observed value. Alternatively, when the form and magnitude of the  $(n, \alpha)$  cross section were fitted, the peak in  $B_0$  for scattering was too small. It may be possible to explain part of the missing  $(n, \alpha)$  cross section (which is predominantly the cross section  $\sigma_{n\alpha_1}$  to the first excited state) by the addition of very weak  $s$  states.<sup>34</sup> Because of the complex structure of  $^{11}\text{B}$  in this region, the possibility of states with very small neutron partial widths always exists. In an effort to explain the known data in an interpretation consistent with the well-established levels of  $^{11}\text{B}$ , a search for nearby  $s$  states indicated that the well-known<sup>3,6</sup> bound state with  $J^\pi = \frac{7}{2}^+$  at  $E_x = 10.60$  MeV

(Fig. 2) might contribute some of the missing  $(n, \alpha_1)$  cross section at low energies. The probable analog state in  $^{11}\text{C}$  at  $E_x = 10.09$  MeV is observed to be formed predominantly by  $s$ -wave protons,<sup>32,35</sup> in harmony with this interpretation. Furthermore, the experimentally determined ratio  $\Gamma_{\alpha_0}/\Gamma_{\alpha_1}$  is approximately equal to unity for the mirror states.<sup>3,35</sup> The only other nearby known  $^{11}\text{B}$  state that might contain  $l_n = 0$  strength is the 11.27-MeV state assigned<sup>3</sup>  $J^\pi = \frac{7}{2}^+$  or  $\frac{9}{2}^+$ . From the  $^{10}\text{B}(p, p)^{10}\text{B}$  reaction, the probable analog state in  $^{11}\text{C}$  at  $E_x = 10.68$  MeV is assigned<sup>32</sup>  $J^\pi = \frac{9}{2}^+$ , which excludes any possibility of formation by an  $s$ -wave neutron.

Further away at 9.19 MeV is the  $\frac{7}{2}^+$  member of an  $s$ -wave doublet of near single-particle neutron strength which might contribute to  $\sigma_{n\alpha_1}$ . The total  $\alpha$  width for this state corresponds<sup>36</sup> to approximately 4% of the Wigner limit for a radius of 5 F. Since the bombarding  $\alpha$  energy of the resonance is 0.819 MeV, approximately 0.52 MeV in the center-of-mass system, this resonance is barely above the inelastic threshold. Thus the width observed is the ground-state width  $\Gamma_{\alpha_0}$ , and the value of  $\Gamma_{\alpha_1}$  (or more precisely  $\gamma_{\alpha_1}^2$ ) is very difficult to measure for this state. Unfortunately it is just this quantity  $\gamma_{\alpha_1}^2$  that would be needed to calculate the missing  $\sigma_{n\alpha_1}$ . The 9.28-MeV  $\frac{5}{2}^+$   $s$  state is less likely to produce the missing  $\sigma_{n\alpha_1}$ , since it cannot interfere constructively with the broad  $\frac{7}{2}^+$  state at 11.79 MeV to give an enhanced cross section. Furthermore, the partial width  $\Gamma_{\alpha_1}$  for this state is also very difficult to measure because it, too, is barely above threshold. Therefore these strong doublet states (at 9.19 and 9.28 MeV) do not now appear to offer a convenient alternative explanation of the missing  $\sigma_{n\alpha_1}$ . Fortunately, it is possible to reproduce the  $(n, \alpha)$  cross sections in terms of the  $s$ -wave states at 10.60 and 11.79 MeV as is shown in the next section.

#### IV. TWO-LEVEL, THREE-CHANNEL $R$ MATRIX FOR $\frac{7}{2}^+$ $S$ -WAVE STATES

Because of interest<sup>34,37,38</sup> in the  $^{10}\text{B}(n, \alpha)^7\text{Li}$  cross sections for low neutron energies, the  $R$ -matrix calculation was modified so that the  $\alpha_0$  and  $\alpha_1$  channels were specifically included in order to compare the calculated with the observed  $\alpha$ -branching ratios. Both the 11.79- and the 10.60-MeV state were taken into account in a two-level, three-channel  $R$ -matrix calculation of  $\sigma_{n\alpha_0}$  and  $\sigma_{n\alpha_1}$  and of the coefficients  $B_L$  and  $C_L$  for neutron scattering. The scattering matrix elements are given by

$$U_{cc'}^{J^\pi} = \Omega_c W_{cc'} \Omega_{c'},$$

where

$$W_{cc'} = \delta_{cc'} + 2iP_c^{1/2}P_{c'}^{1/2}[(1 - R_{cc}^0 L_c^0)^{-1}R_{cc}^0 \delta_{cc'} + \sum_{\lambda\mu} (a_{\lambda c} a_{\mu c'}) A_{\lambda\mu}].$$

In these equations,  $P_c$  is the penetrability for channel  $c$ ,  $R^0$  is the background matrix (assumed to be diagonal), and the other quantities are defined by the expressions

$$\begin{aligned} \Omega_c &= e^{-i\phi_c}, & L_c^0 &= S_c - b_c + iP_c, \\ a_{\lambda c} &= \gamma_{\lambda c} / (1 - R_{cc}^0 L_c^0), \\ A_{11} &= \frac{\epsilon_2}{D}, & A_{12} &= A_{21} = \frac{\xi_{12}}{D}, & A_{22} &= \frac{\epsilon_1}{D}, \\ \epsilon_\lambda &= E_\lambda - E - \xi_{\lambda\lambda}, & D &= \epsilon_1 \epsilon_2 - \xi_{12}^2, \\ \xi_{\lambda\mu} &= \sum_c \frac{L_c^0 \gamma_{\lambda c} \gamma_{\mu c}}{1 - R_{cc}^0 L_c^0}, \end{aligned}$$

where  $\gamma_{\lambda c}$  is the reduced-width amplitude of level  $\lambda$  in channel  $c$ . Neutron shift factors  $S_c$ , penetrabilities  $P_c$ , and hard-sphere phases  $\phi_c$  were calculated as before, while  $\alpha$ -particle shift factors and penetrabilities were calculated from the appropriate Coulomb wave functions. The choice of the boundary condition  $b_c$  for each channel was determined by setting  $b_c$  equal to the shift factor  $S_c$  at the resonance energy of the scattering state at  $E_x = 11.79$  MeV. For these boundary conditions, the resonance condition (namely that the real part of  $D$  be a minimum at the observed resonance energies of the two  $\frac{7}{2}^+$  states) yielded the proper energies  $E_\lambda$  for the two states. The  $B_L$  and  $C_L$  for scattering were calculated from the appropriate  $U_{nn}^{J\pi}$  as before, and  $\sigma_{n\alpha_0}$  and  $\sigma_{n\alpha_1}$  were calculated from

$$\sigma_{n,x}^{J\pi} = \frac{\pi}{k_n^2} g_J |W_{nx}|^2,$$

where  $g_J$  is the statistical factor and  $x$  stands for  $\alpha_0$  or  $\alpha_1$ . Contributions to the  $(n, \alpha)$  cross section from states other than the two  $\frac{7}{2}^+$  states were calculated from a single-level formula derived from a single-level  $R$  matrix as discussed in Sec. III. The cross section is given by

$$\sigma_{n,x}^{J\pi} = \frac{\pi}{k_n^2} g_J \frac{\Gamma_n \Gamma_x}{(E_\lambda + \Delta_\lambda - E)^2 + \frac{1}{4} \Gamma_T^2},$$

where the various quantities are treated just as in the previous calculations for an  $R$  matrix for a single level plus a constant  $R^0$ .

The parameters used in the fitting for the bound 10.60-MeV state are consistent with the  ${}^7\text{Li}(\alpha, \alpha) - {}^7\text{Li}$  and  ${}^7\text{Li}(\alpha, \alpha') {}^7\text{Li}^*(0.48 \text{ MeV})$  results,<sup>3,6</sup> and also consistent<sup>32,35</sup> with the  ${}^{10}\text{B}(p, \alpha)$  reaction to the ground state and first excited state of  ${}^7\text{Be}$ ,

i.e.,  $\gamma_n^2 \approx 3\%$  of the Wigner limit and  $\Gamma_{\alpha_1} \approx \Gamma_{\alpha_0}$ . The radius in the  $\alpha$  channel was taken to be 6.0 F. The parameters for the scattering state at  $E_n = 0.37$  MeV are consistent with the results from the  ${}^{10}\text{B}(p, p) {}^{10}\text{B}$  scattering as well as with the  ${}^{10}\text{B}(p, \alpha)$  reactions to  ${}^7\text{Be}$  and  ${}^7\text{Be}^*(0.43)$  and the  ${}^{10}\text{B}(n, \alpha)$  reactions to  ${}^7\text{Li}$  and  ${}^7\text{Li}^*(0.48)$ , since almost all the  $\alpha$  width is in the  $\alpha_1$  channel, and the total widths for both reactions are large.

The small deviation from  $1/v$  in the  $(n, \alpha)$  cross section near 100–200 keV (as indicated in Fig. 3) has been interpreted<sup>34</sup> as a possible  $s$ -wave resonance. It was included in the present calculation as a  $J^\pi = \frac{5}{2}^+$  state ( $l_n = 0$ ) with a very small neutron partial width ( $\sim 4$  keV), a total width  $\Gamma_T = 300$  keV, and  $E_{\text{res}} = 0.170$  MeV as reported.<sup>34</sup> The  $\frac{5}{2}^+$  assumption is made because  $\frac{7}{2}^+$  would probably cause a noticeable interference in  $B_0$  with the broad 11.70-MeV state, and no such interference is observed. Also, the small percentage (i.e., 6%) of capture  $\gamma$  rays to the ground state [in the  ${}^{10}\text{B}(n, \gamma)$  reaction] is not seriously affected by such a weak state. The  $\alpha$  width of this state was taken to be entirely in the  $\alpha_0$  channel for two reasons. First, the  $\alpha_0$  decay from the  $\frac{5}{2}^+$  state to the  $\frac{3}{2}^-$  ground state would go much more easily by  $l_\alpha = 1$  than would the  $\alpha_1$  decay to the  $\frac{1}{2}^-$  first excited state, which would require  $l_\alpha = 3$ . And second, the  $\alpha_0$  group has been observed<sup>14</sup> to have a considerable rise

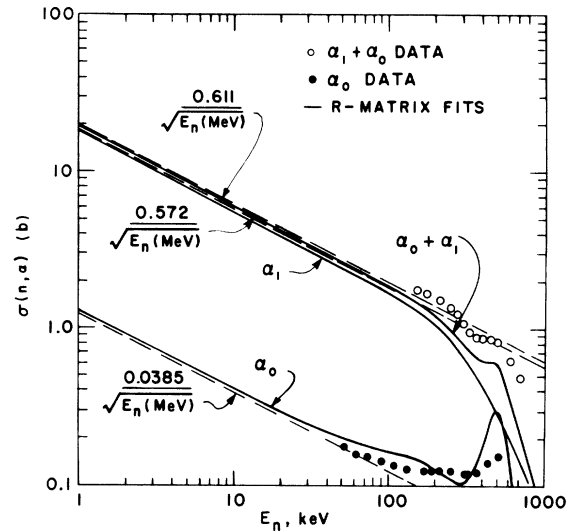


FIG. 3. Plot of  $\sigma_{n,\alpha_0}$ ,  $\sigma_{n,\alpha_1}$ , and their sum versus  $E_n$ . Data for  $(\alpha_0 + \alpha_1)$  are taken from an evaluation (Ref. 38) which corresponds for the most part to the data of Ref. 13 except for the two high points near 230 keV. Data for  $\alpha_0$  are from Ref. 14. Dashed curves are based on a  $1/v$  dependence as shown, and correspond approximately to the observed data below 100 keV in the cases of  $(\alpha_0 + \alpha_1)$  and  $\alpha_1$ , and also for  $\alpha_0$  below 50 keV. Solid curves are the results of the  $R$ -matrix calculations with the parameters given in Table I.

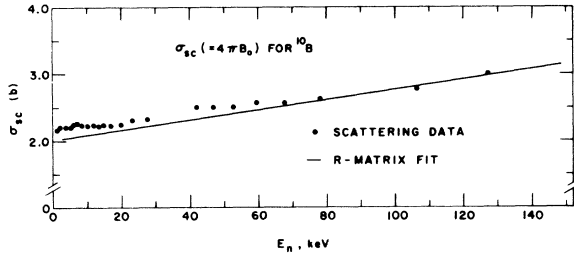


FIG. 4. The low-energy total scattering cross section  $4\pi B_0$  of neutrons, plotted as a function of  $E_n$ . The data points are from Ref. 39, and the solid curves are the result of the  $R$ -matrix calculations with the parameters given in Table I. Note the break in the vertical scale.

in the cross section above the  $1/v$  line as  $E_n$  increases above 100 keV, as seen in Fig. 3, which shows the energy dependence of the  $(n, \alpha)$  total, the  $(n, \alpha_0)$ , and the  $(n, \alpha_1)$  reaction cross sections.

When all of these states, together with the  $p$  state at  $E_n = 0.53$  MeV ( $E_x = 11.94$  MeV), are included in the  $R$ -matrix calculation for  $\sigma_{n\alpha_0}$  and  $\sigma_{n\alpha_1}$ , the solid curves in Fig. 3 result. The state near 0.5 MeV has been observed<sup>14</sup> to occur predominantly in the  $\alpha_0$  group. Thus, for the purposes of this calculation, its  $\alpha$  width was taken to be entirely in the ground-state channel. The  $1/v$  dependence of  $\sigma_{n\alpha_0}$  and  $\sigma_{n\alpha_1}$ , especially below 100 keV, is well reproduced and the resonances are

TABLE II.  $R$ -matrix parameters associated with  $\frac{7}{2}^+$  states. These values were used in calculating the solid curves of Fig. 5, and the  $\frac{5}{2}^+$  ( $l_n=0$ ) state has been omitted. All parameters for other  $J^\pi$  remain the same as in Table I. The footnotes of Table I also apply to these quantities.

$J^\pi$	$\frac{7}{2}^+$ (scatt.)	$\frac{7}{2}^+$ (bound)
$l_n$	0	0
$l_{\alpha_0}$	3	3
$l_{\alpha_1}$	3	3
$E_x$ (c.m.)	11.79	10.61
$E_\lambda$	0.335	-1.38
$E_r$ (lab)	0.37	(-0.94)
$\gamma_n^2$	0.77	0.12
$\gamma_n^2/\gamma_{un}^2$	0.19	0.03
$\Gamma_n$ (c.m.)	0.77	...
$\gamma_{\alpha_0}^2$	0.0004	0.098
$\gamma_{\alpha_0}^2/\gamma_{w\alpha}^2$	0.0006	0.14
$\Gamma_{\alpha_0}$ (c.m.)	0.001	0.055
$\gamma_{\alpha_1}^2$	0.076	0.316
$\gamma_{\alpha_1}^2/\gamma_{w\alpha}^2$	0.112	0.46
$\Gamma_{\alpha_1}$	0.113	0.055
$\Gamma_\alpha$	0.114	0.110
$b_n$	0.0	0.0
$b_{\alpha_0}$	-1.33	-1.33
$b_{\alpha_1}$	-1.64	-1.64
$R_{nn}^0$	0.0	0.0

approximately reproduced. The deviations of the curves from the data in Fig. 3 at the higher energies are caused in part by the omission of any higher-energy states, some of which are known to have large  $(n, \alpha)$  cross sections. Again, Table I shows the complete set of  $R$ -matrix parameters for all the states used in the calculations shown in Figs. 1, 3, and 4. Figure 4 compares the experimental<sup>39</sup> and  $R$ -matrix results for the total scattering cross section at low energies. The calculated results, represented by the solid curves in these figures, reproduce the data reasonably well.

The reduced widths for the levels in the different channels are shown in Table I. In order to fit these data and not be inconsistent with other data, it was necessary in the case of the two-level formula to choose the neutron reduced-width amplitudes  $\gamma_{\lambda c}$  to be of opposite sign for the two  $\frac{7}{2}^+$  levels.

Even when the very weak resonance assumed at  $E_n = 0.170$  MeV (taken to be  $J^\pi = \frac{5}{2}^+$ ) is omitted from the calculation, a reasonable fit to the low-energy data is still obtained, as observed in Fig. 5, except that  $\sigma_{n\alpha_0}$  is low above  $E_n \approx 20$  keV. Only small changes in some of the parameters result when this state is omitted and no inconsistencies with other data are produced. Table II shows the changed values of the parameters required to fit  $\sigma_{n\alpha}$  as shown in Fig. 5. All other parameters remain unchanged from those given in Table I. The

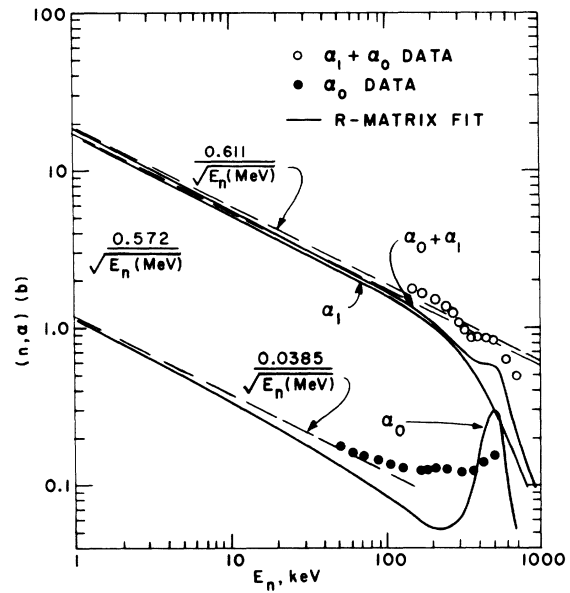


FIG. 5. Same as Fig. 3 except that the solid curves are results of  $R$ -matrix calculations without the  $\frac{5}{2}^+$  state at  $E_n = 0.17$  MeV, and the parameters used for the two  $\frac{7}{2}^+$  states are as given in Table II. All other parameters are the same as in Table I.



$B_L$ ,  $C_L$ , and the scattering cross section (Figs. 1 and 4) for these changed parameters are indistinguishable from those for the original parameters of Table I. If this weak resonance is assumed to be a  $p$  resonance,<sup>34</sup>  $\Gamma_n$  would have to be very small, since this state is not observed in the coefficients  $B_1$  or  $C_1$  in the neutron scattering data. Since the assumption of a weak  $\frac{5}{2}^+$   $s$ -wave state with  $\Gamma_\alpha = \Gamma_{\alpha_0}$  does produce a rise in the  $(n, \alpha_0)$  cross section at 100–300 keV and does not produce any other inconsistencies in the fitting, this might be considered as further evidence for the existence of such a weak  $s$ -wave state. The parameters  $E_r$ ,  $\Gamma_T$ , and  $\Gamma_n$  of this  $s$  state may still be somewhat uncertain, since very small differences between various experiments appear to result in major discrepancies in the size and location of these small bumps.<sup>8,13</sup>

Notice that if the  $\frac{5}{2}^+$   $s$ -wave resonance were moved up in energy from 170 keV to correspond more nearly with the peak<sup>13</sup> near 230 keV, the calculated curve for  $\sigma_{n\alpha_0}$  in Fig. 3 might fit the data better. The assignment of states and resonance parameters for  $^{11}\text{B}$  from small deviations among disagreeing experiments is at best somewhat speculative.

Recent measurements<sup>40</sup> of the  $^{10}\text{B}(n, \alpha_1\gamma)^7\text{Li}^*$ - (0.48 MeV) reaction point up further discrepancies in the results above 150 keV. Differences of up to 50% exist above 100 keV at some points. Again the assignment of resonance parameters from small deviations from  $1/v$  or any other energy dependence in this region should be treated with great care, at least until the independent experimental results become more consistent.

## V. COMPARISON WITH $^{11}\text{B}$ STRUCTURE PREDICTIONS

### A. Negative-Parity States

Calculations of negative-parity states on the basis of the shell model<sup>41–43</sup> and the unified model<sup>44</sup> have previously been compared with experiments.<sup>2,6,36,45</sup> The comparison is good up to  $E_x \approx 9\text{MeV}$ , the region of the  $\alpha$  threshold. Above this energy, theory and experiment do not agree so well; and above 10.60 MeV the experimental spins and parities have been somewhat uncertain, especially at higher energies.

The assignment of  $J^\pi = \frac{5}{2}^-$  ( $l_n = 1$ ) to the 11.94-MeV state is one of the more nearly certain results of the present analysis. Two shell-model calculations predict a  $\frac{3}{2}^-$   $p$  state, one<sup>20</sup> at  $E_x = 11.44$  MeV and the other<sup>41</sup> at 11.90 MeV. However, the possibility that the state observed at 11.94 MeV has spin and parity  $\frac{3}{2}^-$  was ruled out, since the sign of the resulting polarization would be opposite

that of the data. Thus, this state is not the one predicted by either calculation, even though the energies are close. It seems more probable that this state might be identified with the  $\frac{5}{2}^-$  state predicted at  $E_x = 10.69$  MeV. However, a state of  $\frac{5}{2}^-$  or  $\frac{7}{2}^-$  is already observed<sup>5,6</sup> at 10.32 MeV. A  $\frac{5}{2}^-$  state lying somewhere between 13- and 15-MeV excitation is also predicted<sup>43</sup> by a unified model, but the large discrepancy in energy would seem to reduce the likelihood of correspondence with the 11.94-MeV state. The  $p$  state at  $E_x = 13.05$  MeV ( $\frac{3}{2}^- \leq J^\pi \leq \frac{7}{2}^-$ ) is too uncertain to compare with negative-parity predictions at this time. More experimental and theoretical work is needed in order to determine the proper assignments and configurations of these negative-parity states.

### B. Positive-Parity States

Calculations of low-lying positive-parity states at 9.19 and 9.28 MeV have been made<sup>45</sup> on the basis of a  $2s_{1/2}$  nucleon coupled to a  $^{10}\text{B}$  core in the ground state. The  $^{10}\text{B}(d, p)^{11}\text{B}$  stripping results<sup>2,5</sup> show strong transitions to these states. Distorted-wave Born-approximation (DWBA) analysis<sup>46</sup> of the data<sup>5</sup> gives nearly full single-particle strength for  $l_n = 0$ . At higher excitation the probability for  $\alpha$  decay becomes large and the contributing positive-parity states are expected to be described by more complex configurations. Few calculations have been made in this region of excitation. One such<sup>47</sup> predicts a  $\frac{1}{2}^+$  state at 10.59 MeV based upon the configuration  $(1s)^4(1p)^4(2s)^3$  with the ground state of  $^8\text{Be}$  as an inert core. No corresponding state has been observed. Of particular interest is the configuration of the broad  $\frac{7}{2}^+$  state at 11.79 MeV which, according to the present interpretation, is responsible for much of the  $(n, \alpha)$  and neutron total cross section at low  $E_n$ . As seen in Table I,  $\gamma_n^2$  for this state is almost 20% of the Wigner limit for the radius used. This implies that a considerable fraction of the  $l_n = 0$  single-particle strength is included in the configuration of this state; but according to the  $(d, p)$  results discussed above, the bound states at 9.19 and 9.28 MeV used up a large part of the  $l_n = 0$  single-particle strength. However, the uncertainties in the determination of spectroscopic factors and reduced widths are such that these two results are not necessarily in conflict. The possibility that more complex configurations such as  $(1s)^4(1p)^4(2s, 1d)^3$  are responsible for some of the lower-lying positive-parity states has been suggested; however, no such calculations have been reported above the neutron binding energy. Since  $\sigma_{n\alpha_1}$  for the 11.79-MeV state is large, it would seem that the configuration of this state must include  $(^7\text{Li}^* + \alpha)$  cluster terms. Such a model has been sug-

gested<sup>6</sup> but, in its simplest form, it does not seem to agree with data on  $\gamma$  widths for states just below the neutron threshold.

In summary, this analysis represents a consistent quantitative interpretation of practically all the known reaction and scattering data that involve levels in  $^{11}\text{B}$  at high excitation energy. The present study was made in an attempt to gain a better understanding of scattering and reaction data in terms of the level structure of  $^{11}\text{B}$  and  $^{11}\text{C}$  at these excitations. Hopefully, this approach may

also be of value in using the  $^{10}\text{B}(n, \alpha)^7\text{Li}$  reaction as a standard for neutron-flux calibration.

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## Second-Class Currents in Nuclear Beta Decay\*

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Expressions for the ratio of  $ft$  values for mirror  $\beta$  decays are given in the treatment of nuclei as elementary particles with emphasis on the effects of second-class currents. The results are compared with the corresponding ones in the usual impulse-approximation treatment with off-mass-shell and meson-exchange corrections. Theoretical implications of the data of Wilkinson and Alburger are examined.

### I. INTRODUCTION

A recent analysis of all available data on mirror  $\beta$  decays by Wilkinson and Alburger<sup>1</sup> has stimulated renewed interest in the question of the presence (or absence) of second-class currents in strangeness-conserving semileptonic weak processes. Several theoretical works on the subject from various points of view have already been reported.<sup>2-7</sup> The classification of the weak currents into first- and second-class currents was first made by Weinberg<sup>8</sup> using the transformation properties of the currents under  $G$  parity. In the absence of  $CP$  (or  $T$ ) violation in semileptonic weak processes, the Weinberg classification scheme is identical to the one proposed by Cabibbo,<sup>9</sup> who used transformation properties under charge-symmetry operation.

Second-class currents, if they exist, can produce asymmetries in the matrix elements for the processes of mirror pairs. Second-class currents could, as is well known, contribute to the inequality of  $ft$  values of mirror  $\beta$  decays. In the case of nucleons, no mirror asymmetry shows up *if induced terms are neglected*, even if second-class currents exist. Therefore, the effects of the second-class-current contribution are in general suppressed owing to kinematics. Unfortunately, there are also other small effects in addition to the second-class currents (such as electromagnetic interactions, isospin mixing, and binding-energy effects for complex nuclei) which can pro-

duce mirror asymmetries. These small effects cannot safely be ignored in the analysis of experimental data, since they could probably yield terms of the same order as the second-class currents.

In this paper we present<sup>10</sup> expressions for the ratio of  $ft$  values for mirror  $\beta$  decays, including various effects which can produce mirror asymmetries. To formally include off-mass-shell and meson-exchange corrections, we have used the treatment of nuclei as elementary particles<sup>11</sup> in the derivation, with the  $B^{12}$ - $N^{12}$  system as an example. We have also derived the formula in the impulse-approximation treatment, with special emphasis on the off-mass-shell contribution. It is shown that the off-mass-shell effects, as well as the on-mass-shell-induced tensor coupling term (a second-class current) can produce mirror asymmetries proportional to the energy release. Various theoretical implications of the Wilkinson and Alburger data are examined.

### II. FIRST- AND SECOND-CLASS CURRENTS

We start with the conventional effective weak Hamiltonian for strangeness-conserving semileptonic weak processes<sup>12</sup>

$$H_w = \frac{G \cos \theta_c}{\sqrt{2}} \int d\vec{x} l_\alpha^*(\vec{x}, 0) [V_\alpha(\vec{x}, 0) + A_\alpha(\vec{x}, 0)] + \text{H.c.},$$

$$l_\alpha^*(\vec{x}, 0) = \bar{\psi}_l(\vec{x}, 0) \gamma_\alpha (1 + \gamma_5) \psi_\nu(\vec{x}, 0), \quad (1)$$