(d, p) and (d, t) Studies of the Actinide Elements. II. ²⁴³Cm, ²⁴⁵Cm, ²⁴⁷Cm, and ²⁴⁹Cm

T. H. Braid, R. R. Chasman, J. R. Erskine, and A. M. Friedman

Argonne National Laboratory, Argonne, Illinois 60439

(Received 11 March 1971)

Properties of states in 243 Cm, 245 Cm, 247 Cm, and 249 Cm were determined by (d,p) and (d,t) reactions on targets of 244 Cm, 246 Cm, and 248 Cm. Assignments of one-quasiparticle states are made. Single-particle level schemes are extracted from the data by use of a pairing calculation. The extracted single-particle level schemes are compared with single-particle calculations, which include β_4 and β_6 deformations. Equilibrium values of β_2 , β_4 , and β_6 are estimated for the odd-mass Cm isotopes.

I. INTRODUCTION

This paper is the second in a series of studies of the single-particle states of actinide nuclei populated in single-neutron transfer reactions. Information from stripping and pickup reactions is a valuable adjunct to the data obtained in investigations of nuclear decays. A major purpose of this work is to identify the one-quasiparticle states in the actinides and to compare the measured properties of these states with theoretical predictions. In the first paper' of this series, we presented a detailed study of ^{235}U . In the present paper, we consider the odd-mass Cm isotopes. Preliminary results for some thorium, uranium, and plutonium isotopes have also been published.²

One of the noteworthy features of our study of the curium isotopes is the unusual nature of the four targets used. The target nuclei are rare isotopes, some of which are highly radioactive. The ²⁴⁴Cm target had an activity of 10^{10} α disintegrations/min.

More complete reaction data would be desirable, particularly complete angular distributions that could provide better identification of the l value transferred. However, the low yield of the reactions, the intense radioactivity, and the fragility of the targets prevented us from acquiring more extensive data. Nevertheless, the data we do have furnish extensive new information on the excitation energies and the yields of reactions to many levels of four curium isotopes. The interpretation of many of the levels is unambiguous, but other level assignments are of necessity rather tenuous.

II. THEORETICAL ANALYSIS

In the first paper of this series,¹ we presented the theoretical framework used to analyze our single-neutron-transfer data. To make the present work self-contained, we sketch the major features of the analysis.

The basic approximation is that the differential cross section can be factored into two parts: Θ_{J}^{DW} depending on the reaction mechanism and S_I^K depending on the nuclear eigenstates. In the case of an even-even target nucleus, this approximation is written as

$$
\frac{d\sigma_J^K}{d\Omega} = (2J+1)\Theta_J^{\text{DW}} S_J^K,
$$
\n(1)

where J is the total spin of the state populated and K denotes the specific state being populated. We have computed the factor Θ_J^{DW} by using the distorted-wave Born-approximation code JULIE.³ The spectroscopic factor S_t^K is calculated for the nuclear Hamiltonian

$$
H = H_{\text{single-particle}} + H_{\text{pairing}} \tag{2}
$$

Each of the two terms in the nuclear Hamiltonian affects the magnitude of the spectroscopic factor S_J^K . The single-particle wave functions in the deformed field are generated⁴⁻⁶ from $H_{\text{single-p}}$ The wave functions can be written in terms of a spherical basis as

$$
\psi_K^{\Omega} = \sum_{\mathbf{J}} C_{\mathbf{J}}^K \phi_{\mathbf{J}}^{\Omega} (r, \theta, \phi) , \qquad (3)
$$

where $\phi_{\iota}^{\Omega}(r, \theta, \phi)$ is a normalized function. The spectroscopic factor for the rotational level having spin J, based on the single-particle state $\psi_{\mathbf{k}}^{\Omega}$, is proportional to $(C_J^K)^2$. Pairing forces influence the spectroscopic factor in that they determine the occupation probability of the various singleparticle states in the initial and final systems. The effect is most apparent when we consider the ratios of (d, p) and (d, t) spectroscopic factors. This ratio is

$$
R_K = \frac{S^K(d, p)}{S^K(d, t)} \approx \frac{1 - \langle N_K(A - 1) \rangle}{\langle N_K(A + 1) \rangle}, \qquad (4)
$$

where $\langle N_{\kappa}(A \pm 1) \rangle$ is the occupation probability of the state ψ_{κ}^{Ω} in the ground state of the nucleus having mass $A \pm 1$. R_K goes to infinity for particle

 $\overline{4}$

247

FIG. 1. Triton spectrum for the reaction 244 Cm(d, t) 243 Cm at $\theta = 140^{\circ}$.

states at high excitation and goes to zero for hole states at high excitation.

Combining these effects, we find that the differential cross sections are

$$
\frac{d\sigma^{\scriptscriptstyle{K}}_J(d,\,p)}{d\Omega}=2(C^{\scriptscriptstyle{K}}_J)^2[~1-\langle\,N_{\scriptscriptstyle{K}}(A\,-\,1)\rangle\,]\Theta^{\rm DW}_J(d,\,p)~,~~(5)
$$

$$
\frac{d\sigma_J^K(d,t)}{d\Omega}=2(C_J^K)^2\langle N_K(A+1)\rangle\Theta_J^{\rm DW}(d,t)\;.\qquad\qquad(6)
$$

For a more detailed discussion of the analysis, we refer the reader to Ref. 1.

III. EXPERIMENTAL PROCEDURE

The experimental techniques and methods of analysis of the data are the same as were described previously for the case of ^{235}U ,¹ except that special care had to be taken because the targets (especially ²⁴⁴Cm) were highly radioactive. The targets were prepared by simultaneously collecting the 244 Cm, 246 Cm, and 248 Cm fractions in a mass separation of 200 mg of highly irradiated Cm. The target backings used in the isotope separator were $40-\mu$ g/cm² carbon films supported by 10-20 μ g/ cm² of Formvar, and the method of separation and collection were as described by Lerner.⁷

FIG. 2. Proton spectrum for the reaction 244 Cm(d, p)²⁴⁵Cm at $\theta = 140^{\circ}$.

FIG. 3. Triton spectrum for the reaction 246 Cm(d, t) 245 Cm at $\theta = 140^\circ$.

The protons and tritons from the (d, p) and (d, t) reactions were recorded with a Browne-Buechner magnetic spectrograph.⁸ Most experiments were done at a bombarding energy of 12 MeV and at angles of 90 and 140°. In addition, the 248 Cm(d, t)-⁴⁷Cm reaction at E_d = 16 MeV was studied at 10, 20, 30, 40, and 50'.

IV. EXPERIMENTAL RESULTS

Figures 1-7 show some of the spectra we obtained. The energies and cross sections of peaks observed in the $^{244}Cm(d, t)^{243}Cm$ reaction are listed in Table I. Those seen in the 244 Cm(d, p) 245 Cm and the 246 Cm(d, t)²⁴⁵Cm reaction are listed in Table II, those from the 246 Cm(d, p)²⁴⁷Cm and the 248 Cm(d, t)²⁴⁷Cm in Table III, and those from the ²⁴⁸Cm(d, p)²⁴⁹Cm reaction in Table IV. The groundstate Q value for each reaction is given in the table headings. These have been previously reported.' Only the differential cross sections obtained at 140' with 12-MeV deuterons are given in the tables. The cross sections obtained at 90° were used to test l-value assignments whenever it was possible to do so in the manner explained in Ref. 1. The assignments of Nilsson orbitals have also been listed in Tables I-IV. The labels A, B, and C indicating confidence levels denote well-established, probable, and plausible assignments, respectively. More detailed definitions of the confi-

FIG. 4. Proton spectrum for the reaction 246 Cm(d, p)²⁴⁷Cm at $\theta = 140^\circ$.

FIG. 5. Triton spectrum for the reaction ${}^{248}Cm(d,t) {}^{247}Cm$ at $\theta = 140^\circ$.

dence levels appear in Ref. 1.

Figures 8-11 are the level schemes deduced for 243 Cm, 245 Cm, 247 Cm, and 249 Cm, respectively, on the basis of these experiments.

V. DISCUSSION

Level Assignments A.

1. ^{243}Cm

Since only the ²⁴⁴Cm(d, t)²⁴³Cm reaction was available to study the levels of this nucleus, it is difficult to make positive assignments for many of the observed peaks. Table I lists the excitation energies of the levels and our assignments.

The ground state of 243 Cm is assigned as the $\frac{5}{2}$ state of the $\frac{5}{2}$ + [622] band on the basis of its signature and the fact that the unhindered α -decay transitions from this state populate the $\frac{5}{2}$ state of the $\frac{5}{2}$ +[622] band in ²³⁹Pu.¹⁰ The signatures of the $\frac{1}{2}$ +[631] band and $\frac{1}{2}$ -[501] band are well characterized, and are similar to the signatures of these states as seen in other actinide nuclei. The signature of the $\frac{7}{2} + [624]$ band is consistent with the calculated signature when band mixing is taken into account. However, the effects of mixing are so large that this assignment is given a C confidence level. The signature of the peaks assigned to the $\frac{3}{2}$ + [631] band agrees with the signature of this

FIG. 6. Proton spectrum for the reaction 248 Cm(d, p) 249 Cm at $\theta = 140^{\circ}$.

band as seen in other actinide nuclei.

In addition to the $\frac{1}{2}$ -[501], the other hole states expected to have very large (d, t) cross sections are the states $\frac{5}{2} - [503]$ and $\frac{3}{2} - [501]$. On the basis of calculations, we estimate that the cross section to the $I=\frac{3}{2}$ level of the $\frac{3}{2}$ -[501] band is ~30% larger than the cross section to the $I = \frac{1}{2}$ level of the $\frac{1}{2}$ -[501] band. We also estimate that the cross section to the $I = \frac{5}{2}$ level of the $\frac{5}{2} - [503]$ band is ~50% smaller than the cross section to the $I=\frac{1}{2}$ level of the $\frac{1}{2}$ - [501] band. On this basis, we have assigned peak 23 as the $I=\frac{5}{2}$ level of the $\frac{5}{2}$ – [503] band. We assign this band also in 245 Cm and 247 Cm on the same basis.

Another hole state that we have assigned in ²⁴³Cm, ²⁴⁵Cm, and ²⁴⁷Cm is the state $\frac{3}{2}$ + [631]. The evidence for this assignment is also quite scant. We do see two peaks separated by ~ 95 keV in all of these isotopes and at about the same energy relative to the state $\frac{1}{2} - [501]$. The singleparticle state that corresponds best to this signature is $\frac{3}{2} + [631]$.

$$
2. \quad {}^{245}Cm
$$

In Table II we list the energies of the levels observed in 245 Cm produced in both the 246 Cm(d, t) and 244 Cm(d, p) reactions. Our assignments and the confidence levels of the assignments are also given. Figure 7 shows both the (d, p) and (d, t) spectra placed back to back with aligned excitation energies. The various rotational bands assigned are marked in this figure. The levels of the $\frac{7}{2}$ + [624], $\frac{5}{2}$ + [622], and $\frac{9}{2}$ – [734] bands have been
observed in the α decay of ²⁴⁹Cf.^{11, 12} Our spectr observed in the α decay of 249 Cf.^{11, 12} Our spectral show no evidence of the 644- and 703-keV levels observed in these α -decay studies. However, if these two levels are the $\frac{7}{2}$ and $\frac{9}{2}$ members of the these two levels are the $\frac{7}{2}$ and $\frac{9}{2}$ members of the $\frac{7}{2}$ – [743] rotational band, as postulated by Ahmad,¹¹

FIG. 7. A comparison of (d, p) and (d, t) spectra leading to levels in ²⁴⁵Cm, plotted on an energy scale in which the excitation energies are in alignment.

then we should not have populated those states. Ahmad¹¹ also observed a state at 775 keV, probably the $I = \frac{11}{2}$ member of the $\frac{7}{2} - [743]$ band. We have assigned a peak at 771 keV, which is barely distinguishable from background, to be this level. The signatures of the $\frac{1}{2}$ + [620] and $\frac{1}{2}$ + [631] bands consist of several moderately large peaks. We

feel that our identifications are quite reliable. The $\frac{5}{2}$ + [622] signature is not quite so distinctive and we have assigned it only with confidence level C.

In 245 Cm, we observe some large peaks in the (d, p) spectrum above the $\frac{3}{2} + [622]$ band. On the basis of a single-particle-model calculation, the

^a Assumed doublet.

TABLE II. Energy levels in ²⁴⁵Cm, their excitation energies, differential cross sections in (d, p) and (d, t) reactions, orbital assignments, and confidence level. The ground-state Q values were measured (Ref. 9) to be 3.297 ± 0.007 MeV for the (d, t) and (d, p) reactions, respectively. A, B, and C denote well-established, probable, and plausible, respectively.

Excitation			Assignment			
Level No.	energy (keV)	$d\sigma(d, p)$ at 140° $(\mu b/sr)$	$d\sigma(d, t)$ at 140° $(\mu b/sr)$	Nilsson orbital		Confidence level
31	1083 ± 3	20	$60 + 11$	$\frac{3}{2} + [631]$		$\mathbf C$
32	1103 ± 3	20	102 ± 15			
33	1259 ± 5	50 ± 25	20 ± 5			
34	1271 ± 2	74 ± 18	293 ± 60	$\frac{5}{2}$ – [503]		C
35	1473 ± 3	20	45 ± 15			

TABLE II (Continued)

^aUnresolved doublet.

Combined cross section since levels could not be resolved.

 c Observed only on the 90° (*d*, *t*) exposure.

likeliest assignment for these peaks is the band $\frac{1}{2}$ -[750]. This band¹³ should be characterized by large cross sections to the $\frac{3}{2}$ and $\frac{7}{2}$ members of the rotational band and a large negative decoupling parameter. %e have assigned levels 25 and 27 as the $I = \frac{3}{2}$ and $\frac{7}{2}$ members of the $\frac{1}{2} - [750]$ band, both with confidence level C. If we assume a rotational constant of $\neg 6$ keV for this band, which is a typical value for neutron rotational bands in the Cm isotopes, we find that the decoupling parameter is $a \approx -3.0$. This value is in agreement with theoreti-

cal expectations. This $\frac{1}{2} - [750]$ assignment is strengthened considerably by our clear-cut identification of this band in 249 Cm.

3. ^{247}Cm

The levels found in 247 Cm are shown in Table III. Three rotational bands in 247 Cm – namely, the $\frac{9}{2} - [734], \frac{5}{2} + [622], \text{ and } \frac{1}{2} + [620] \text{ bands} - \text{have been}$ identified in studies of the α decay¹⁴ of ²⁵¹Cf and the β ⁻ decay^{15, 16} of ²⁴⁷Am. The $\frac{9}{2}$, $\frac{11}{2}$, and bers of the $\frac{9}{2}$ -[734] band have been seen in decay-

FIG. 8. Level scheme for ²⁴³Cm. Dashed lines indicate weakly populated levels which may be due to contaminants in the target. The labels A, B, and ^C denote well-established, probable, and plausible confidence levels, respectively.

TABLE III. Energy levels in ²⁴⁷Cm, their excitation energies, differential cross sections observed with the (d, p) and (d, t) reactions, orbital assignments, and confidence levels. The ground-state Q values were deduced be +0.053 ± 0.008 and 2.935 ± 0.008 MeV for the (d, t) and (d, p) reactions, respectively. A, B, and C denote well-established, probable, and plausible, respectively.

Excitation				Assignment			
Level No.	energy (keV)	$d\sigma(d, p)$ at 140° $(\mu b/sr)$	$d\sigma(d, t)$ at 140° $(\mu b/sr)$	Nilsson orbital	$\mathbf I$	Confidence level	
$\pmb{0}$	$\boldsymbol{0}$	$\!<\!10$	$\mathord{<}3$	$\frac{9}{2}$ – [734]	$\frac{9}{2}$	A	
1	59 ± 3	\bf{a}	7 ± 3	$\frac{9}{2}$ – [734]	$\frac{11}{2}$	A	
$\,2$	217 ± 2		38 ± 10	$\frac{9}{2} - [734]$	$\frac{15}{2}$	A	
3	227 ± 1	43 ± 13^{b}	66 ± 15	$\frac{5}{2} + [622]$	$\frac{5}{2}$	A	
$\overline{\mathbf{4}}$	287 ± 4	$\!<\!20$	5 ± 3	$\frac{7}{2} + [624]$	$\frac{7}{2}$	$\, {\bf B}$	
5	309 ± 3	$\!<\!20$	36 ± 10				
$\,6$	318 ± 3	87 ± 26	123 ± 17	$\frac{5}{2} + [622]$	$\frac{9}{2}$	A	
7	336 ± 5	$\!<\!20$	24 ± 10				
8	344 ± 2	${<}20$	$8\,7\,\pm\,13$	$\frac{7}{2}$ + [624]	$\frac{9}{2}$	В	
9	(370 ± 3)	${<}\,20$	9 ± 4				
10	381 ± 5	${<}\,20$	24 ± 7	$\frac{5}{2} + [622]$	$\frac{11}{2}$	$\mathbf C$	
11	398 ± 4	$\!<\!20$	20 ± 8	$\frac{7}{2}$ + [624]	$\frac{11}{2}$	$\mathbf C$	
12	404 $^{\rm c}$	164 ± 40	66 ± 11	$\frac{1}{2} + [620]$	$\frac{1}{2}$	A	
13	(417 ± 6)	\mathbf{a}	16 ± 6	$\frac{1}{2} + [620]$	$\frac{3}{2}$	$\mathbf C$	
14	439 ± 3	168 ± 40	40 ± 20	$\frac{7}{2} + [613]$	$\frac{9}{2}$	$\mathbf C$	
15	452 ± 3	124 ± 25	18 ± 6	$\frac{1}{2}$ + [620]	$\frac{5}{2}$	A	
16	506 ± 3	$\!<\!20$	83 ± 25	$\frac{1}{2} + [631]$	$\frac{1}{2}$	$\, {\bf B}$	
17	520 ± 2 $^{\rm d}$	77 ± 15	293 ± 40	$\frac{1}{2}$ +[631] and	$\frac{3}{2}$	$\, {\bf B}$	
				$\frac{1}{2} + [620]$	$\frac{7}{2}$	$\mathbf C$	
18	550 ± 2	77 ± 15	22 ± 10	$\frac{1}{2} + [620]$	$\frac{9}{2}$	$\mathbf C$	
19	584 ± 3	${}_{<20}$	23 ± 8				
20	592 ± 2	70 ± 20	54 ± 13				
21	604 ± 3	$\!<\!20$	20 ± 8	$\frac{1}{2} + [631]$	$\frac{7}{2}$	$\mathbf C$	
$\bf{^{22}}$	668 ± 5	114 ± 27	7 ± 3	$\frac{3}{2} + [622]$	$\frac{3}{2}$	$\mathbf C$	
23	687 ± 5	89 ± 25	16 ± 7	$\frac{9}{2} + [615]$	$\frac{11}{2}$	$\mathbf C$	
$\bf{^{24}}$	699 ± 2 $^{\rm d}$	$\bf a$	75 ± 15	$\frac{3}{2}$ + [622] and	$\frac{5}{2}$	$\mathbf C$	
				$\frac{1}{2} + [631]$	$\frac{9}{2}$	$\, {\bf B}$	
25	749 ± 5	$2\,50\,\pm\,100$	$\!<\!4$	$\frac{3}{2} + [622]$	$\frac{7}{2}$	$\mathbf C$	
26	784 ± 4	217 ± 49	$\!<\!4$	$\frac{1}{2}$ – [750]	$\frac{3}{2}$	$\mathbf C$	
27	803 ± 5	114 ± 45	$\!<\!4$				
$\bf{28}$	819 ± 4 ^d	346 ± 65	$\!<\!4$	$\frac{1}{2}$ -[750] and	$\frac{7}{2}$	$\mathbf C$	
				$\frac{3}{2} + [622]$	$\frac{9}{2}$	$\mathbf C$	
$\bf 29$	$8\,56\pm5$	115 ± 40	$\!<\!4$				

	Excitation		Assignment			
Level	energy	$d\sigma(d, p)$ at 140°	$d\sigma(d\,,\,t)$ at $\,140^{\circ}$	Nilsson		Confidence
No.	(keV)	$(\mu b/sr)$	$(\mu b/sr)$	orbital	$\mathbf I$	level
30	897 ± 5	104 ± 40	$\mathord{\text{<}} 4$	$\frac{1}{2}$ – [750]		$\mathbf C$
31	947 ± 2	< 40	74 ± 18			
32	957 ± 2	<40	308 ± 40	$\frac{1}{2}$ – [501]	$\frac{1}{2}$	$\, {\bf B}$
33	988 ± 5	<40	35 ± 10			
34	1001 ± 2	<40	157 ± 42	$\frac{1}{2}$ - [501]	$\frac{3}{2}$, $\frac{5}{2}$	$\mathbf C$
35	1044 ± 3	< 40	41 ± 10			
36	1064 ± 3	<40	48 ± 14			
37	1079 ± 3	< 40	109 ± 20	$\frac{3}{2} + [631]$	$\frac{5}{2}$	$\mathbf C$
38	1091 ± 3	<40	51 ± 15			
39	1159 ± 3	$\bf a$	43 ± 10			
40	1182 ± 3	\mathbf{a}	58 ± 15	$\frac{3}{2} + [631]$	$\frac{9}{2}$	$\mathbf C$
41	(1199 ± 4)	\mathbf{a}	27 ± 10			
42	1239 ± 5	<40	30 ± 10			
43	1247 ± 5	<40	30 ± 10			
44	1271 ± 3	<40	48 ± 15			
45	1283 ± 3	<40	187 ± 26	$\frac{5}{2}$ – [503]	$\frac{5}{2}$	$\mathbf C$
46	1317 ± 3	217 ± 50	37 ± 10			
47	(1356 ± 3)	$\bf a$	26 ± 13			
48	1364 ± 4	$\mathbf a$	37 ± 10			
49	1372 ± 4	\bf{a}	43 ± 15			
50	1512 ± 6	< 50	30 ± 10			

TABLE III (Continued)

'Obscured by another peak.

Combined cross section for states ² and 3.

 c Reference state whose excitation energy is assumed to be 404 keV (Ref. 18).

Assumed doublet.

scheme studies. We have assigned confidence level A to these levels although we do not see them in the reaction studies because of low values of C_j². We expect to see only the $I = \frac{15}{2}$ member of this band. We have assigned the level at 217 keV as the $I = \frac{15}{2}$ member of this band. The level at 227 keV has been observed in α decay and found to be a 25 - μ sec isomeric state. A similar state was also found¹⁷ in the isotone $249Cf$. Both the lifetime and conversion coefficient of the γ decay lead to an assignment of $\frac{5}{2}^+$ for this level. Peaks 3, 6, and 10 at 227, 318, and 381 keV give the wellknown signature of the $\frac{5}{2}$ +[622] band. This is consistent with previous assignments given by Chetham-Strode et al.¹⁴ Peak 4 at 287 ± 4 keV is close in energy to the level at 285 keV observed' in the β decay of ²⁴⁷Am. This peak, together with peaks 8 (344 keV) and 11 (398 keV), fit the signature of the $\frac{7}{2}+[624]$ state. These peaks are assigned as $\frac{7}{2}$ + [624] with a B confidence level.

The unhindered α transitions from the decay of ²⁵¹Cf should populate the levels of the $\frac{1}{2}$ + [620] band in ²⁴⁷Cm. These transitions were observe
by Chetham-Strode et al.¹⁴ Their energies and by Chetham-Strode $et~al.^{14}$ Their energies and assignments are 407 keV $(I = \frac{1}{2})$, 434 keV $(\frac{3}{2})$, and 450 ($\frac{5}{2}$). The (d, p) signature of the $\frac{1}{2}$ + [620] band is such that we should observe the $\frac{1}{2}$, $\frac{5}{2}$, $\frac{7}{2}$, and $\frac{9}{2}$ states, and the $\frac{3}{2}$ weakly. We observe these states at 404 keV $(I=\frac{1}{2})$, 452 keV $(\frac{5}{2})$, 520 keV $(\frac{7}{2})$, and 550 keV $(\frac{9}{2})$.

Since we do not observe the ground state in '⁴⁷Cm, all our excitation energies are based on an assumed value of the excitation energy of the $I=\frac{1}{2}$ level of the $\frac{1}{2}+$ [620] band. We have used a value of 404±1 keV taken from the work of Brown<mark>e</mark>
and Asaro.¹⁸ and Asaro.

FIG. 10. Level scheme for 247 Cm. The conventions are as in Fig. 8.

The peaks in the (d, t) spectrum at 506, 520, 604, and 699 keV constitute the characteristic signature of the $\frac{1}{2}$ + [631] band. The fact that a level at 516 keV (which we equate with the peak at 520 keV) is populated¹⁴ by the α decay of a nucleus

with $I = \frac{1}{2}$ is consistent with this assignment.

The (d, p) spectrum of 247 Cm between 700 and 900 keV is difficult to interpret. There are more peaks than we can account for in terms of the single-particle states expected in this energy range.

TABLE IV. Energy levels in 249 Cm, their excitation energies, differential cross sections in (d, p) reactions, orbital assignments, and confidence levels. The ground-state Q value was measured (Ref. 9) to be 2.488 ± 0.006 MeV. A, B, and C denote well-established, probable, and plausible, respectively.

	Excitation $d\sigma(d\,,\,p)$			Assignment		
Level No.	energy (keV)	at 140° $(\mu b/sr)$	Nilsson orbital	I	Confidence level	
$\boldsymbol{0}$	$\pmb{0}$	171 ± 33	$\frac{1}{2} + [620]$	$\frac{1}{2}$	\boldsymbol{A}	
$\mathbf 1$	25 ± 2	13 ± 5	$\frac{1}{2} + [620]$		A	
$\,2$	48 ± 1	110 ± 19	$\frac{1}{2} + [620]$	$rac{3}{2}$ $rac{5}{2}$	A	
3 ^a	110 ± 1	202 ± 25	$\frac{1}{2}$ +[620] and	$\frac{7}{2}$	A	
			$\frac{7}{2} + [613]$	$\frac{9}{2}$	Α	
$\overline{\mathbf{4}}$	146 ± 3	45 ± 11	$\frac{1}{2} + [620]$	$\frac{9}{2}$	A	
5	208 ± 1	80 ± 30	$\frac{3}{2} + [622]$	$\frac{3}{2}$	в	
6	220 ± 5	20 ± 10	$\frac{9}{2} + [615]$	$\frac{9}{2}$	$\mathbf C$	
7	242 ± 1	67 ± 15	$\frac{3}{2} + [622]$	$\frac{5}{2}$	$\, {\bf B}$	
$\,8\,$	288 ± 5		$\frac{3}{2} + [622]$	$\frac{7}{2}$	$\, {\bf B}$	
9	300 ± 5	$175 \pm 35^{\text{ b}}$	$\frac{9}{2} + [615]$	$\frac{11}{2}$	$\mathbf C$	
10	350 ± 1	79 ± 18	$\frac{3}{2} + [622]$	$rac{9}{2}$	$\, {\bf B}$	
$11\,$	469 ± 2	138 ± 28	$\frac{1}{2}$ – [750]	$\frac{3}{2}$	$\, {\bf B}$	
$12\,$	498 ± 3	240 ± 48	$\frac{1}{2}$ – [750]	$\frac{7}{2}$	B	
13	(518 ± 4)	23 ± 8				
14	528 ± 3	60 ± 20				
15	550 ± 3	70 ± 23				
${\bf 16}$	575 ± 3	80 ± 25	$\frac{1}{2} - [750]$	$\frac{11}{2}$	${\bf C}$	
17	634 ± 2	23 ± 5				
18	668 ± 2	81 ± 20				
19	870 ± 4	60 ± 20				
20	915 ± 2	128 ± 28				
$\bf 21$	1030 ± 7	160 ± 40				
$\bf{22}$	1208 ± 7	80 ± 20				
$\bf 23$	1353 ± 7	190 ± 40				
24	1382 ± 7	60 ± 15				
25	1528 ± 7	120 ± 30				
26	1550 ± 7	180 ± 45				
$2\sqrt{7}$	1570 ± 7	120 ± 30				
28	1650 ± 7	120 ± 30				

^a Assumed doublet. Combined cross section since levels are not resolved.

There is the real possibility that some of the peaks are heavy-element contaminants that we have not identified. We interpret this spectrum by assuming that the various single-particle states should have about the same cross sections as in 245 Cm. This is indeed the case for the levels we attribute to the $\frac{1}{2}$ + [620] and $\frac{7}{2}$ + [613] bands. In addition, we expect to see six peaks —four having a cross section of ~100 μ b/sr for the band $\frac{3}{7}$ + [622] and two with cross sections of $\sim 200 \mu b/sr$ for the band $\frac{1}{2}$ -[750]. We assign levels 22, 24, 25, and 28 as the $\frac{3}{2}$, $\frac{5}{2}$, $\frac{7}{2}$, and $\frac{9}{2}$ members of the $\frac{3}{2}$ +[622] band. This leaves \sim 150 ± 100 μ b/sr yet to be accounted for in level 25 and \sim 250 ± 55 μ b/sr to be accounted for in level 28. Next we assign peaks 26 and 28 as the $\frac{3}{2}$ and $\frac{7}{2}$ members of the $\frac{1}{2}$ -[750] band. This leaves us with peaks 27 and 29 still unassigned.

The peaks at 309 keV (5}, 336 keV (7), and 370 keV (9) are seen only in (d, t) reactions on the ⁴⁸Cm target and have small cross sections. They cannot be fitted into our level scheme and are probably too low in energy to be collective states. Again, they may be due to heavy-element impurities in the target, but they have been left in Table III as unassigned states since there is a possibility that they are real.

4. ^{249}Cm

In Table IV we list the levels observed in the 48 Cm(d, p)²⁴⁹Cm reaction. Since no ²⁵⁰Cm targe was available, we have no information on the hole

states in $^{249}\mathrm{Cm}$. The ground-state band of $^{249}\mathrm{Cm}$ is expected to be $\frac{1}{2}$ + [620] and the cross sections of the peaks we observe at 0, 25, and 48 keV agree with the signature of this band as seen in 47 Cm. The peak we observe at 110 keV is much too large to be only the $I = \frac{7}{2}$ member of the $\frac{1}{2}$ +[620] band. Therefore, we assume that this peak is a doublet. We have assigned the major portion of this peak as the $I = \frac{9}{2}$ member of the $\frac{7}{2}$ +[613] band, which is close in energy to the $\frac{1}{2}$ +[620] band in the other Cm isotopes. The $\frac{7}{2}$ + [613] band is calculated to have virtually no j $=\frac{7}{2}$ component – a result which explains why we do not observe the $\frac{7}{2}$ band head. The state $\frac{3}{2}$ +[622] can be assigned rather unambiguously here. The assignment of the state $\frac{9}{2}$ +[615] is somewhat more questionable.

As mentioned in Sec. VA2, we believe that we have made the first clear-cut identification of the single-particle state¹⁹ $\frac{1}{2}$ -[750] in ²⁴⁹Cm. We have assigned levels 11, 12, and 16 as the $\frac{3}{2}$, $\frac{7}{2}$, and $\frac{11}{2}$ members of the rotational band. The few keV uncertainty in the energies of these levels prevents us from making a precise determination of the decoupling parameter. We find that either a rotational-energy parameter of 5.5 keV and a decoupling parameter of -3.2 or a rotational-energy parameter of 6 keV and a decoupling parameter of -3.5 give good fits to these levels. A rotational-energy parameter in the range 5.5-6 keV is expected on the basis of the values determined for other rota-

I/2+ 1620] FIG. 11. Level scheme for 249 Cm. The conventions are as in Fig. 8.

tional bands in the Cm isotopes.

B. Single-Particle Energies

From the assignments made in this series of experiments, we have constructed single-particle energy level diagrams. This was done by assuming that the pairing-interaction constant in MeV is given by $G = 22/A$ and varying the assumed singleparticle spectrum until energy level spacings in agreement with experiment were obtained as the expreement with experiment were obtained as the
output of a pairing-force calculation.²⁰ In additio to the experimental data of this work, we have included recent experimental data²¹ on hole states in ²⁵¹Cf, which is isotonic to ²⁴⁹Cm. In ²⁵¹Cf, the state $\frac{9}{2}$ -[734] has been seen at 434 keV above the $\frac{1}{2}$ + [620] ground state; also the state $\frac{5}{2}$ + [622] is seen at 544-keV excitation energy. We have assumed that these two hole states are at the same energy relative to the $\frac{1}{2}+$ [620] state in ²⁴⁹Cm as they are in 251 Cf.

In Fig. 12, we present the experimentally observed single-particle spacings in the Cm isotopes, and in Fig. 13, we show the extracted single-particle energies. The dashed lines in Fig. 13 indicate the Cf data we have used. In Figs. 12 and 13, we have plotted all energies relative to the state $\frac{1}{2}$ + [631] for the states in ²⁴³Cm, ²⁴⁵Cm, and 247 Cm. Because we have not identified the $\frac{1}{2}$ +[631] state in ²⁴⁹Cm, the energy of the state $\frac{1}{2}$ +[620] is held constant in going from ²⁴⁷Cm to 249 Cm.

The major difficulty in understanding the extracted single-particle spectrum is the crossing of the states $\frac{5}{2} + [622]$ and $\frac{7}{2} + [624]$ in going fron ⁴⁵Cm to ²⁴⁷Cm. This level crossing cannot be

FIG. 12. Measured excitation energies of singleparticle states in the odd-A curium isotopes.

plausibly ascribed to changes in the equipotential shapes⁴ in going from 245 Cm to 247 Cm. We conclude, therefore, that the shift in the state $\frac{5}{7}$ +[622] in ²⁴⁷Cm is due to residual interactions; i.e., a $\frac{5}{2}$ state which consists of the $\frac{9}{2}$ -[734] state and a $2⁻$ phonon is interacting strongly with the $\frac{5}{2}+[622]$ state. A 2⁻ state has been identified¹¹ in 246 Cm at 843-keV excitation energy.

In view of the fact that the $\frac{5}{7}$ + [622] state is very near the Fermi level in 247 Cm, we have examine how a shift in its single-particle energy affects the extracted single-particle spectrum. We have Increase of the pairing calculation for ²⁴⁷Cm under the assumption that the extracted $\frac{5}{2} + [622]$ energy is the same as the $\frac{1}{2}$ + [631] extracted energy. We found that the $\frac{9}{7}-$ [734] ground state and all of the hole states were unshifted in energy (except, of course, the $\frac{5}{2}+$ [622]). In addition, the energies of all of the particle states were lowered by only \sim 50 keV in the extracted spectrum. From this calculation, we conclude that the extracted spectrum in 247 Cm is not sensitive to the single-particle energy of the state $\frac{5}{2}$ + [622].

Perhaps the most interesting feature in the extracted single-particle spectrum is the change in the 152-neutron gap as a function of neutron number. The state $\frac{9}{2}$ -[734] is dropping substantially in energy as a function of increasing neutron number. Pairing effects are particularly strong for the hole states in nuclei having 153 neutrons, so this large shift does not show up in the experimental data.

FIG. 13. Single-particle level scheme extracted from the data by means of a pairing-force calculation.

have extracted from the neutron-transfer studies of the Cm isotopes are in good agreement with the spectrum we have obtained for $235U$. It is tempting to ascribe the shifts in single-particle spacing in going from one Cm isotope to another to slight changes in the deformation parameters describing the central field. To investigate this possibility, we have carried out calculations¹⁹ of single-particle energies in a momentum-dependent Woods-Saxon potential, with provisions for deformations of the form $\beta_4 Y_4^0(\theta, \phi)$ and $\beta_6 Y_6^0(\theta, \phi)$. We introduce the shape deformations into the potential via the substitution

$$
r^{2} - r^{2} \left[\sin^{2} \theta e^{2 \beta_{2}/3} + \cos^{2} \theta e^{-4 \beta_{2}/3} + \beta_{4} Y_{4}^{0} (\theta, \phi) + \beta_{6} Y_{6}^{0} (\theta, \phi) \right].
$$

We have carried out calculations for $\beta_2 = 0.24$, which gives reasonable agreement with the data. In Fig. 14, we show the variation of single-particle energies as a function of β_4 ; and in Fig. 15, they are shown as a function of β_{6} . In Fig. 14, we have done the calculations for the three points $\beta_4 = -0.04$, 0.0, +0.04. The same three values were used for β_6 in Fig. 15. Straight lines are drawn to connect the appropriate points. The no-crossing rule has been ignored, and the lines are drawn to connect states with maximum overlap. We note that

FIG. 14. Variation of calculated single-particle energies as a function of β_4 ($\beta_2 = 0.24$, $\beta_6 = 0.0$).

several of the single-particle energy shifts that we see between 247 Cm and 249 Cm (or 251 Cf) in Fig. 13 also occur with increasing β_6 in Fig. 15 - specifically, (a) the crossing of $\frac{7}{2} + [613]$ and $\frac{1}{2} + [620]$, and (b) the increased separation of both $\frac{1}{2}$ -[750] and $\frac{9}{2} - [734]$ from $\frac{1}{2} + [620]$. These features indicate that β_6 increases by 0.01-0.02 as we go from ⁴⁷Cm to ²⁴⁹Cm. A second feature to emphasize is that $\frac{1}{7} - [750]$ is ~300 keV above $\frac{3}{7} + [622]$. From Figs. 14 and 15, we see that this feature rules out the possibility of either β_4 or β_6 being more negative than -0.⁰² if the other of the two coefficients is zero or negative; and it excludes positive values greater than +0.02 for either if the other is zero or positive. A reasonable characterization of the equilibrium deformation based on the singleparticle spectra is $\beta_2 = 0.24 \pm 0.01$, $\beta_4 = 0.00 \pm 0.01$, and $\beta_6 = 0.01 \pm 0.01$. As noted above, β_6 is larger in 249 Cm than in 247 Cm. The calculations of Nilsson *et al.*²² also indicate that β_4 is small for the Cm isotopes.

VI. CONCLUSION

The major result of this work is the experimental observation of many heretofore unobserved single-particle states in the odd-mass Cm isotopes. By removing pairing-force effects from the experimental data, we have been able to compare exper-

FIG. 15. Variation of calculated single-particle energies as a function of β_6 ($\beta_2=0.24$, $\beta_4=0.0$).

imental single-particle energies with the predictions of single-particle models. To make these comparisons, we have also carried out single-particle calculations with $Y_2^0(\theta, \phi)$, $Y_4^0(\theta, \phi)$, and $Y_6^0(\theta, \phi)$ deformations of a Woods-Saxon potential. We found that the generalized deformed single-particle model with residual pairing forces gives a

)Work performed under the auspices of the U. S. Atomic Energy Commission.

¹T. H. Braid, R. R. Chasman, J. R. Erskine, and A. M. Friedman, Phys. Rev. C 1, 275 (1970).

²T. H. Braid, R. R. Chasman, J. R. Erskine, and A. M. Friedman, Phys. Letters 18, 149 (1965).

3R. H. Bassel R. M. Drisko, and G. R. Satchler, Oak Ridge National Laboratory Report No. ORNL-3240 (unpublished) .

 4 R. R. Chasman, Phys. Rev. C 1 , 2144 (1970).

⁵S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. 29, No. 16 (1955).

 6 E. Rost, Phys. Rev. 154, 994 (1967).

7J. Lerner, in North American Symposium on Electromagnetic Isotope Separation, Oak Ridge National Laboratory, September 1966 (unpublished).

⁸J. R. Erskine, Phys. Rev. 135, B110 (1964).

⁹J. R. Erskine, A. M. Friedman, T. H. Braid, and R. R. Chasman, in Proceedings of the Third International Conference on Atomic Masses, Winnipeg, Canada, 1968 (University of Manitoba Press, Winnipeg, Canada, 1968).

 10 E. K. Hyde, I. Perlman, and G. T. Seaborg, The Nuclear Properties of the Heavy Elements (Prentice-Hall, Englewood Cliffs, New Jersey, 1964), Vol. II.

¹¹I. Ahmad, University of California Lawrence Radiation Laboratory Report No. UCRL-16888, 1966 (unpublished).

¹²S. A. Baranov, V. M. Shatinskii, and V. M. Kulakov, Yadern. Fiz. 10, 889 (1969 [transl.: Soviet J. Nucl. Phys. satisfactory over-all description of our observations.

ACKNOWLEDGMENTS

The authors would like to thank J. Lerner for performing the mass separation of the curium isotopes and Irshad Ahmad for helpful discussions.

 $10, 513 (1970)$.

 $\overline{^{13}}$ There is a problem of nomenclature associated with this band; it is often labeled $\frac{1}{2}$ -[761], but in view of the fact that the single-particle wave function is calculated to be ~80% Λ = 0 and only ~20% Λ = 1 we use the more descriptive asymptotic label $\frac{1}{2} - [750]$.

~4A. Chetham-Strode, Jr., R. J. Silva, J. R. Tarrant, and I. R. Williams, Nucl. Phys. A107, 645 (1968).

 $¹⁵C.$ J. Orth, W. R. Daniels, B. H. Erkkila, F. O. Law-</sup> rence, and D. C. Hoffman, Phys. Rev. Letters 19, 128 (1967).

¹⁶P. R. Fields, I. Ahmad, R. K. Sjoblom, R. F. Barnes, and E. P. Horowitz, J. Inorg. Nucl. Chem. 30, ¹³⁴⁵ (1968).

 17 I. Ahmad, A. M. Friedman, R. F. Barnes, R. K. Sjoblom, J. Milsted, and P. R. Fields, Phys. Rev. 164, 1537 (1967).

 ^{18}E . Browne and F. Asaro, University of California, Lawrence Radiation Laboratory Report No. UCRL-19530, 1969 (unpublished) .

 ^{19}R . R. Chasman, Phys. Rev. C 3, 1803 (1971). Singleparticle signatures are given in Table III.

²⁰R. R. Chasman, Phys. Rev. 134, B279 (1964).

 21 I. Ahmad, F. T. Porter, M. S. Freedman, R. F.

Barnes, R. K. Sjoblom, F. Wagner, Jr., J. Milsted, and P. R. Fields, Phys. Rev. C 3, 390 (1971).

 22 S. G. Nilsson, in Nuclear Structure and Nuclear Reactions, Proceedings of the International School of Physics $\emph{'Enrico Fermi,''}$ Course XL, 1967, edited by M. Jean and R. A. Ricci (Academic Press Inc., New York, 1969).