Elastic Scattering of ^{16,18}O by ^{116,120}Sn at Energies near the Coulomb Barrier*

B. C. Robertson and J. T. Sample
University of Alberta, Edmonton, Alberta, Canada
and Brookhaven National Laboratory, Upton, New York 11973

and

D. R. Goosman, K. Nagatani, and K. W. Jones
Brookhaven National Laboratory, Upton, New York 11973
(Received 18 August 1971)

The elastic scattering of ^{16,18}O by ^{116,120}Sn has been studied at energies near the Coulomb barrier. Angular distributions were measured at $E_{\text{c.m.}}=57.97$ MeV and excitation curves at 75, 80, 85, and 90° from 45 to 68 MeV and from 54 to 67.5 MeV for ^{16,18}O scattered by ^{120}Sn, respectively. A single angular distribution was taken at $E_{\text{c.m.}}=48.53$ MeV for the scattering of ^{16}O by ^{120}Sn. An optical-model analysis was used to parametrize the data, and evidence is presented for a change in radius between ^{116}Sn and ^{120}Sn, and for a change of diffuseness between ^{16}O and ^{18}O.

I. INTRODUCTION

The subject of heavy-ion elastic scattering has received considerable attention for some time and has been the subject of numerous experiments.[1] Nevertheless, the need still exists for additional experiments with various targets and projectiles to establish regularities as a function of masses and energies in more detail than they are presently known.

In this work we present results for the elastic scattering of ^{16,18}O by ^{116,120}Sn in the energy range from 45 to 68 MeV. The best and most useful method of analysis for heavy-ion elastic scattering is not clear. We have chosen to use the optical model rather than the various diffraction models because the results can be conveniently used for calculating reaction cross sections in distorted-wave Born-approximation programs. The interpretation of the optical-model parameters in terms of actual physical quantities is not clear, since a justification for describing such a complex system as two colliding heavy ions in a two-body problem is not made yet. Conclusions, therefore, must be drawn with great caution. If one follows the ordinary optical-model analysis, however, it is apparently possible to derive meaningful values for differences in matter radii and diffusenesses of the nuclear potential.

II. EXPERIMENTAL TECHNIQUE AND RESULTS

Targets of ^{116,120}Sn with isotopic purity better than 95% were prepared by vacuum evaporation of metallic Sn, or Sn_2O_3 reduced with titanium powder, from a niobium boat onto 20-μg/cm^2 carbon backings. The impurities in the targets introduced negligible errors. Because, as described below, the variation observed from these two tin isotopes was small, all other tin isotopes contained in the targets must have contributed in a constant manner. Tin thicknesses of about 40 μg/cm^2 were used. Such a thickness gave adequate counting rates and reasonably small values for energy loss and multiple scattering.

Thin silicon surface-barrier detectors were used with thicknesses of about 100 μ. Energy resolution was about 190 keV which was adequate to resolve oxygen ions elastically scattered by tin from the ions scattered by target contaminants such as carbon and oxygen. Four detectors mounted at 5° intervals on a rotatable table were used during the measurements. Solid angles were defined by a slit 3.1 mm wide by 6.2 mm high which gave an angular resolution of 0.53° and a solid angle of 1.53×10^{-4} sr.

Circular collimators limited the angular divergence of the beam to a cone of half-angle less than 0.18°. Two detectors were placed at ±12.5° because the Rutherford cross section near 12.5° changes by about 10% over this cone, so that a single detector was not sufficient to serve as a monitor. The sum of the counts in the two counters provided a normalization independent to first order of the beam-angle variations during the run, and the difference of the counts provided an angular correction for the angles of the movable detectors. A single 0.79-mm-diam collimator was used on each monitor counter with a resulting angular resolution of 0.14°.

Cross sections were determined by normalizing to the yield measured in the monitor counter at 12.5° where it was assumed that the cross section was due only to Coulomb scattering. For most of the runs statistical errors were less than ±5% and the uncertainty in the normalization to the Rutherford cross section was about the same. Typical uncertainties are indicated on the curves which display our experimental results.

Data were taken at energies close to the Coulomb barrier for targets of ^{116,120}Sn and incident beams of ^{16,18}O. Figures 1 and 2 show the results of excitation curves taken at laboratory angles of 75, 80, 85, and 90° for ^{16}O by ^{116}Sn and ^{16}O by ^{120}Sn, respectively. Angular distributions for ^{16}O on ^{120}Sn at 55 and 65.75 MeV and on ^{116}Sn at 66.0 MeV are shown in Fig. 3, while in Fig. 4 results for the scattering of ^{18}O by ^{116,120}Sn at 67.0 and 66.73 MeV, respectively, are displayed. As would be expected, the results are very similar and conclusions about the influence of nuclear-structure effects must rest on the results of a detailed analysis. In the present case, we have chosen to parametrize our results in terms of an optical model, and the conclusions are discussed in the following sections.

III. ANALYSIS

The analysis of the data was carried out using the optical-model code ABACUS II.[2] The form of the potential was taken to be

$$V(r) = V_C(r) + Vf(r) + iWg(r),$$

where

$$f(r) = [1 + e^{(r-R)/a}]^{-1},$$

$$g(r) = -4a[df(r)/dr].$$

The Coulomb potential was calculated assuming a uniform charge distribution inside the nucleus, and the Coulomb radius was set equal to R at all times. The potential used has four free parameters, V, W, R, and a, where V and W are the strengths of the real and imaginary potentials and R and a are the radius and diffuseness of the Woods-Saxon form factor. While it may be physically reasonable to consider a different radius and diffuseness for the imaginary potential it was not felt that the present state of knowledge justifies such a procedure, and we have taken R and a to be the same for both imaginary and real potentials.

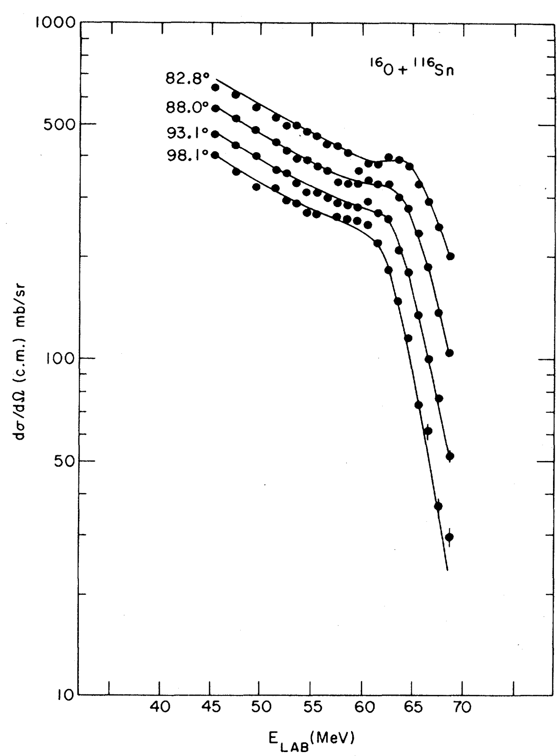

FIG. 1. Excitation function for elastic scattering of ^{16}O by ^{116}Sn at center-of-mass angles of 82.8, 88.0, 93.1, and 98.1°. The curves are optical-model fits calculated with the parameters V=40.14 MeV, W=4.503 MeV, R=9.55 fm, and a =0.50 fm. Typical uncertainties are indicated.

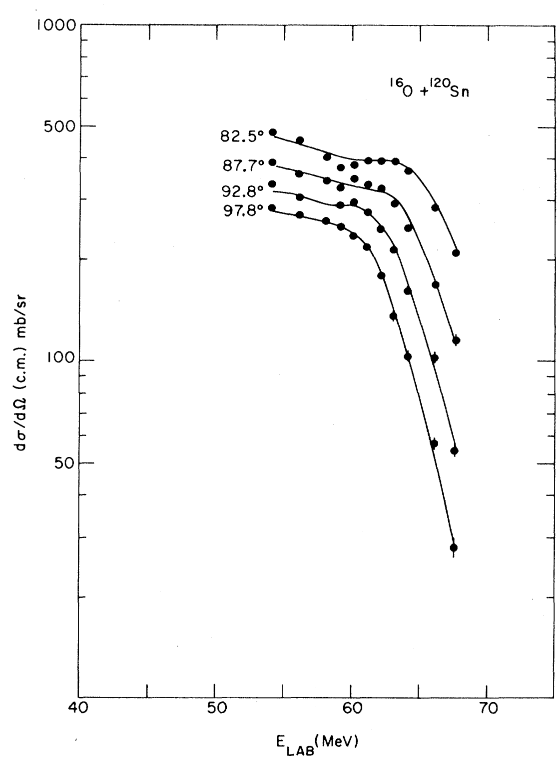

FIG. 2. Excitation function for elastic scattering of ^{16}O by ^{120}Sn at center-of-mass angles of 82.5, 87.7, 92.8, and 97.8°. The curves are optical-model fits calculated with the parameters V =39.12 MeV, W =4.417 MeV, R =9.836 fm, and a=0.453 fm. Typical uncertainties are indicated.

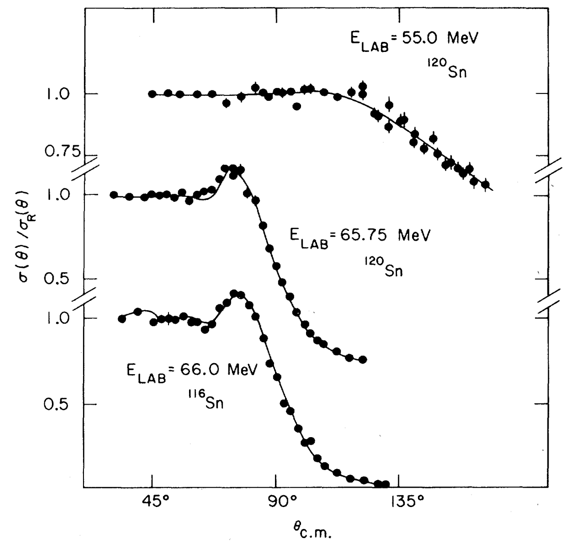

FIG. 3. Angular distributions for elastic scattering of ^{16}O by ^{120}Sn measured at bombarding energies of 55.0 and 65.75 MeV and for elastic scattering of ^{16}O by ^{116}Sn at 66.0 MeV. The latter two laboratory energies correspond to the same center-of-mass energy, 57.97 MeV. The curves are optical-model fits calculated with the following parameters for V, W, R, and a with dimensions in MeV and fm: 39.92, 4.17, 9.81, 0.43 (^{120}Sn, 55 MeV); 39.12, 4.42, 9.84, 0.45 (^{120}Sn, 65.75 MeV); and 41.23, 4.53, 9.54, 0.50 (^{116}Sn, 66.0 MeV). Typical uncertainties are indicated.

Initially, the interaction radius was taken to be about $R = 1.4(A_1^{1/3} + A_2^{1/3}) \simeq 10$ fm, and the initial values of V, W, and a were approximated from the work of Orloff and Daehnick.[3] Scans of the parameter space were then made to establish a set of approximate values, and finally a search was made over the four parameters to establish the best-fit values. The results are shown in Table I.

It is well known (see, for example, Ref. 1) that parameters such as those listed in Table I are not unique. This has been verified by Orlöff and Daehnick[3] for the scattering of ^{16}O by ^{27}Al for potentials as deep as 200 MeV. In the present case we have investigated fits for a real potential around 800 MeV deep, which would be the naive expectation, ignoring the exclusion principle,[4] based on a depth for nucleons of 50 MeV and multiplying by 16, the total number of nucleons involved. A satisfactory fit is indeed found with such a deep potential, as shown in Table I. This should not be surprising, since heavy-ion scattering at low energy is expected to be insensitive to the nuclear interior, as discussed in detail by Igo.[5]

It has been shown in Table I that good fits to the data can be obtained with the variation of three parameters only. Considering the insensitivity to the depths chosen, next we investigate the characteristics of the radius and diffuseness by fixing $V = 39.12$ MeV and $W = 4.42$ MeV. The radius and diffuseness were varied to obtain a best fit to the angular distributions. It was found that the best fits occurred for particular values of a and R and, in particular, unique curves were found for the scattering pairs (^{16}O, ^{116}Sn); (^{16}O, ^{120}Sn); (^{18}O, ^{116}Sn); and (^{18}O, ^{120}Sn). The results are shown in Fig. 5. There is a linear relationship observed between a and R for each pair. This not only demonstrates the fact that only three parameters are required but also interesting aspects of these scattering systems. If we assume that the diffusenesses for ^{116}Sn and ^{120}Sn are approximately equal we can then estimate the difference in the matter radii for ^{116}Sn and ^{120}Sn. For the ^{16}O scattering this is 0.07 fm and for the ^{18}O scattering 0.055 fm for an average of 0.06 fm. On the other hand, if we as-

TABLE I. Summary of optical-model parameters.

Projectile	Target	c.m. energy (MeV)	V (MeV)	W (MeV)	R (fm)	a (fm)	χ^2	Number of parameters varied [a]
^{16}O	^{120}Sn	48.53	799.04	50.03	9.56	0.30	1.02	4
			39.92	4.42	9.81	0.45	1.09	4
			39.96	4.50	9.61	0.50	1.17	3
		57.97	797.01	50.00	9.56	0.30	3.37	4
			39.12	4.42	9.84	0.45	2.08	4
			40.99	4.50	9.66	0.50	3.29	3
^{16}O	^{116}Sn	57.97	41.23	4.53	9.54	0.50	2.76	4
			41.19	4.50	9.54	0.50	2.65	3
^{18}O	^{120}Sn	57.97	38.86	4.52	9.78	0.51	2.82	4
			38.72	4.50	9.79	0.51	2.82	3
^{18}O	^{116}Sn	57.97	39.69	4.53	9.75	0.50	5.05	4
			39.71	4.50	9.74	0.50	5.13	3

[a] W was fixed at 4.50 MeV for the three-parameter fit.

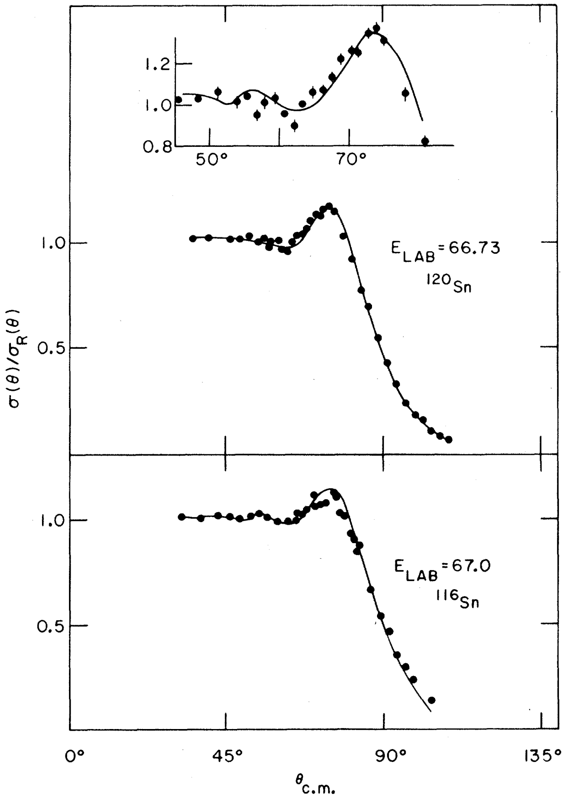

FIG. 4. Angular distributions for elastic scattering of ^{18}O by ^{120}Sn and ^{116}Sn measured at bombarding energies of 66.73 and 67.0 MeV, respectively, which correspond to a center-of-mass energy of 57.97 MeV. The curves are optical-model fits with the following parameters for V, W, R, and a with dimensions in MeV and fm: 38.86, 4.52, 9.78, 0.51 (^{120}Sn, 66.73 MeV); and 39.69, 4.53, 9.74, 0.50 (^{116}Sn, 67.0 MeV). Typical uncertainties are indicated. The upper curve shows an expanded portion of the second curve.

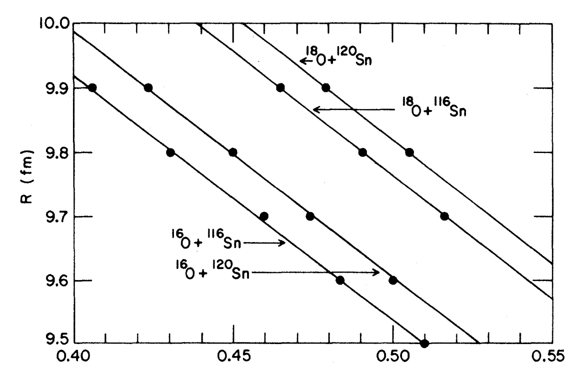

FIG. 5. Values of R and a which give a best fit to the angular-distribution data at $E_{c.m.}=57.97$ MeV when V and W are fixed at 39.12 and 4.42 MeV, respectively. The χ^2 which results are not, in general, as good as the ones listed in Table I, but are such that $3.3 \leq \chi^2 \leq 7.9$.

sume that the radii vary as $A^{1/3}$ then we estimate the change in diffuseness as 0.003 fm for ^{18}O scattering and ≤ 0.001 fm for ^{16}O scattering, which is consistent with the assumption of constant diffuseness made for the estimate of the radius above.

An alternative type of approach has been made by Bertin *et al.*[6] in analyzing the elastic scattering of ^{16}O by ^{40,44,48}Ca. They use, as a measure of the radius, the Rutherford radius R_R, the point at which the total real potential, given by the sum of the nuclear and Coulomb potentials, has a maximum value. The results of such an analysis carried out on the shallow potentials listed in Table I are shown in Fig. 6. Since the Coulomb potential varied less than 10% over the radii shown in Fig. 6, it is correct to first order to assume that the Rutherford radius should vary linearly with $A_1^{1/3}+A_2^{1/3}$. A straight line is thus fitted to the points of Fig. 6. The difference in the Rutherford radii of ^{116}Sn and ^{120}Sn is 0.09 fm, which agrees with the value obtained above.

Matter radii for the tin isotopes have also been found by Boyd *et al.*[7] in an optical-model study of proton scattering by the tin isotopes. For the case of ^{116}Sn and ^{120}Sn they find a difference of 0.1 ± 0.02 fm, which should probably be considered to be in good agreement with the results of the present experiment. If also the radius in the tin isotopes varies according to the simple rule as $1.4A^{1/3}$ fm, a difference between ^{116}Sn and ^{120}Sn of 0.07 fm is found, which is in reasonable agreement with all the values cited.

Finally, if we assume the $1.4A^{1/3}$ law for the radii of the combination of ^{16,18}O + ^{120}Sn and ^{16,18}O + ^{116}Sn we are then able to use the results shown

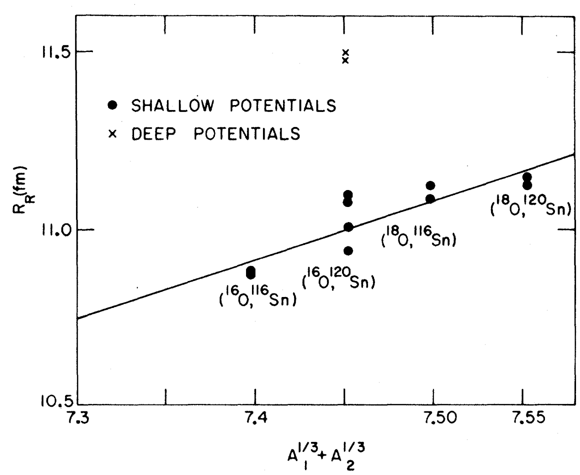

FIG. 6. Rutherford radius R_R, deduced from the potentials listed in Table I with $V \sim 40$ MeV, versus $A_1^{1/3}+A_2^{1/3}$. The line is the result of a linear least-squares fit to these points. Points obtained from the two ~800-MeV potentials are also shown for comparison.

in Fig. 5 to deduce the difference in diffuseness of ^{16}O and ^{18}O. This is 0.022 and 0.020 fm for scattering from ^{116}Sn and ^{120}Sn, respectively.

IV. CONCLUSIONS

Parameters have been presented which describe the elastic scattering of $^{16,18}O$ by $^{116,120}Sn$ in terms of an optical model. While the application of the optical model to the scattering of two heavy nuclei is not well grounded theoretically, it does appear possible to extract information about the radii and diffuseness which are in accord with results obtained by more conventional optical-model treatments.

*Work performed under the auspices of the U. S. Atomic Energy Commission and the Atomic Energy Control Board of Canada.

[1]See, for example, the review article by R. Anni and L. Taffara, Rev. Nuovo Cimento 2, 1 (1970).
[2]E. H. Auerbach, Brookhaven National Laboratory Report No. BNL 6562, 1962 (unpublished). Some modifications to the integration step size have been made in the program to adapt it to heavy-ion scattering where the wavelengths encountered can be quite small.
[3]J. Orloff and W. Daehnick, Phys. Rev. C 3, 430 (1971).
[4]L. M. Frantz, R. L. Mills, R. G. Newton, and A. M. Sessler, Phys. Rev. Letters 1, 340 (1958).
[5]G. Igo, Phys. Rev. Letters 1, 72 (1958); Phys. Rev. 117, 1665 (1959).
[6]M. C. Bertin, S. L. Tabor, B. A. Watson, Y. Eisen, and G. Goldring, Nucl. Phys. A167, 216 (1971).
[7]R. N. Boyd, J. Fenton, M. Williams, T. Kruse, and W. Savin, Nucl. Phys. A162, 497 (1971).

PHYSICAL REVIEW C VOLUME 4, NUMBER 6 DECEMBER 1971

Excitation Function and Recoil Ranges of ^{211}At Produced Through $^{40}Ar + ^{209}Bi$ Transfer Reactions

R. Bimbot, D. Gardès, and M. F. Rivet
Institut de Physique Nucléaire, Chimie Nucléaire, Orsay, France
(Received 12 July 1971)

The excitation function and projected recoil ranges for ^{211}At produced through $^{40}Ar + ^{209}Bi$ reactions have been measured for ^{40}Ar beam energies from 190 to 270 MeV. A substantial value (25 mb) was found for the cross section at high energies. The general features of the reaction (excitation function, magnitude of the cross section, variation of recoil range versus incident energy) are very similar to the results for lighter projectiles, and show unambiguously a non-compound-nucleus process. The possibility that various transfer mechanisms may lead to ^{211}At is discussed with the help of a kinematic analysis.

I. INTRODUCTION

This work was undertaken as part of a general study of transfer reactions induced in heavy targets with heavy ions.[1,2] This study consists of measuring the cross sections, angular distributions, and recoil ranges of some heavy nuclei produced through transfer reactions in ^{209}Bi. In the case of very heavy projectiles such as ^{40}Ar, very few experimental results are available,[3,4] and it is interesting to observe the evolution of the cross section and the main features of the transfer reactions when the mass of the projectile is increased. The recent possibility of using the Orsay accelerator ALICE Ar^{13+} beam made it possible to initiate this type of study by the measurement of the cross section and projected recoil range of ^{211}At produced through $^{40}Ar + ^{209}Bi$ reactions. This paper gives the results of these measurements and the conclusions which can be drawn about the mechanisms involved in ^{211}At production.

II. EXPERIMENTAL METHODS AND RESULTS

Targets of 1-mg/cm^2 Bi were prepared by evaporation onto 0.6- and 1-mg/cm^2 99.99% pure aluminium foils. A stack of aluminium catcher foils of the same thickness and purity was used with each target to measure the differential range of recoiling nuclei. The same foils were used as beam degraders. The target assembly was put into a Faraday cup and the integral beam was measured. Typical beam intensities were of the order of 5.10^{-9} A.