Odd Co Isotopes in a Unified Vibrational Model*

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The odd-A Co isotopes are studied in a version of the unified vibrational model which incorporates both anharmonic and quasiparticle effects. With the $p_{1/2}$, $p_{3/2}$, $f_{5/2}$, and $f_{7/2}$ orbitals available, quasihole states are coupled to anharmonic vibrations of the corresponding Ni cores. Very good agreement is obtained with the known properties of $^{57-59}$Co in a one-parameter fit, while the $^{61-63}$Co calculations are essentially parameter free. The systematics of the energy levels of $^{57-63}$Co can also be explained well by the model.

I. INTRODUCTION

A considerable amount of theoretical work has been devoted to studies of the low-lying structure of the Co isotopes. Calculations have ranged from shell-model investigations to more phenomenological models such as the intermediate-coupling model. The ^{57}Co nucleus represents a good example where various theoretical attempts can be compared. The early shell-model calculations of Vervier[1] and McGrory[2] were quite unsuccessful in predicting the low-lying states. Intermediate-coupling calculations[3,4] where several single-particle states and up to three-phonon states were taken into account have been more promising, but were seen to disagree in their predictions of several of the low-lying states recently observed. Both weak- and intermediate-coupling (IC) approaches assumed the core excitations to follow a pure harmonic-vibrational picture. This does not seem to be a good approximation to the vibrational structure of the Ni cores which show an obvious and strong anharmonic pattern.

The version of the IC model which we present below is essentially based on description of the phonon states using an anharmonic scheme. Also, pairing effects are not ignored and quasiparticle states have been introduced. Both these effects will be shown to have a rather drastic influence on the low-lying structure of Co isotopes, a trend which had already been established for lighter s-d-shell nuclei.[5] We devote Sec. II of this paper to a brief description of the formalism used, showing how the anharmonicity and the quasiparticle schemes are introduced. In Sec. III we examine the application of this modified IC model to negative-parity states of the Co isotopes and finally conclude in Sec. IV with a critical comparison of the various models available in that region.

II. FORMALISM

All applications of the classical weak- or intermediate-coupling models[6,7] have been based on the coupling of particles (or holes) to a harmonic-vibration core, the total Hamiltonian being written as

$$H = H_c + H_{s.p.} + H_{int}, \tag{1}$$

where $H_{s.p.}$ is the usual single-particle shell-model Hamiltonian and H_c describes the core vibrations.

Data from "vibrating" nuclei suggest, however, that it is necessary to include anharmonic terms in the phonon spectrum to describe the nondegeneracy of the two-phonon triplet. It is also known that in most cases the $E2$ rates do not follow closely the vibrational scheme [non-negligible $B(E2: 2^* \to 0)$ are frequent]. Also the quadrupole moment of the first 2^+ state is often substantially different from zero. This has led us to introduce[5] an anharmonic description of the core properties both in the core and the interaction Hamiltonian. We shall write H_c so as to take into account the fact that the two-phonon states of angular momentum J have energy $(2+\eta_J)\hbar\omega$. Here η_J is not necessarily zero and will be defined by the core spectrum. We write

$$H_c = \hbar\omega \sum_{\mu} (b_\mu^* b_\mu) + \tfrac{1}{4} \sum_{J=0,2,4} \hbar\omega \eta_J (b_\mu^* b_\mu^*)^J (b_\mu b_\mu)^J, \tag{2}$$

where $\hbar\omega$ is the phonon vibration energy. We use the basis $|j:NR:IM\rangle$ in which $H_c + H_{s.p.}$ is diagonal, where j is the particle angular momentum, R is the core angular momentum for a state of N phonons, and $\vec{I} = \vec{R} + \vec{j}$ with z component M. The

TABLE I. Systematics of parameters and input data.

	^{55}Co	^{57}Co	^{59}Co	^{61}Co	^{63}Co
k (MeV)	37.0	36.5	37.5	37.0	37.0
ξ	1.5	2.7	3.35	3.3	3.0
$\hbar\omega$ (MeV)	3.07	1.45	1.33	1.17	1.35
C (MeV)	190	100	75	75	90
$Q(2^+)$ [a]	−0.18	−0.15	−0.03	0.04	0.07
$\eta(J=0)$	−0.16	0.03	−0.28	−0.25	−0.31
$\eta(J=2)$	−0.50	−0.08	−0.38	−0.04	0.12
$\eta(J=4)$	−0.55	−0.30	−0.11	−0.01	−0.07

[a] Reference 12.

eigenfunctions of H at an energy E^α can be expanded as

$$|E^\alpha, IM\rangle = \sum_{jNR} a_\alpha(j, NR; I)|j : NR : IM\rangle . \quad (3)$$

The core-particle interaction will be described by

$$H_{\rm int} = -\xi\hbar\omega(\tfrac{1}{5}\pi)^{1/2}\sum_\mu Q_{2\mu}Y_{2\mu}(\theta,\phi), \quad (4)$$

where Q is the quadrupole operator for the core and $Y_{2\mu}(\theta,\phi)$ is the angular part of the quadrupole operator for the particle. Here ξ is a parameter which describes the strength of the coupling and, as we shall see below, is essentially the only parameter in this calculation.

In the present model we shall consider the quasiparticles to be in the $p_{1/2}$, $p_{3/2}$, $f_{5/2}$, and $f_{7/2}$ orbitals. These are introduced to the calculation through the interaction Hamiltonian whose matrix elements are

$$\langle j' : N'R' : IM|H_{\rm int}|j : NR : IM\rangle = (-1)^{I+R+1/2}\xi\hbar\omega[\tfrac{1}{4}(2j+1)(2j'+1)]^{1/2}\begin{pmatrix} j & 2 & j' \\ \frac{1}{2} & 0 & -\frac{1}{2}\end{pmatrix} \times \begin{Bmatrix} I & R & j \\ 2 & j' & R'\end{Bmatrix}(N'R'\|Q\|NR)(u_j u_{j'} - v_j v_{j'}). \quad (5)$$

Here u_j and v_j represent the quasiparticle and quasihole amplitude in state j, respectively. In the classical version of this model, the matrix elements of Q take their pure harmonic values and are nonzero only where $|N-N'|=1$. Here they are deduced from the observed transition rates and quadrupole moments of the core nucleus. The off-diagonal matrix elements of Q are chosen[8] to be

$$(N'R'\|Q\|NR) \propto (2R+1)^{1/2}[B(E2 : R \to R')]^{1/2}, \quad (6)$$

normalized so that $(12\|Q\|00)$ has its harmonic value of $\sqrt{5}$. The diagonal matrix elements are defined by

$$(NR\|Q\|NR) \propto \frac{5}{4}\left(\frac{7}{2\pi}\right)^{1/2} eQ_{NR}, \quad (7)$$

with the same normalization, which ensures that the coupling parameter ξ is identical to that in the classical model.

In this work we shall also present results of electromagnetic decays involving $M1$ and $E2$ transitions, for which formulas have appeared in a previous treatment[5] of a quasiparticle-coupling model (taking into account a correction in the $E2$ formulas recently reported by Brussaard[9]). These transitions differ essentially from their corresponding classical analogs in their use of quasiparticle amplitudes as multiplicative factors.

Finally we shall study spectroscopic factors $S(I)$ for a single-particle-transfer reaction leading from the core nucleus to a state of spin I. These will be obtained from the relation

$$S(I) = u_j^2 a^2(j, 00; I)\delta_{Ij}, \quad (8)$$

where the u_j (and v_j) are linked to the proton and neutron occupation probabilities of the core states.

TABLE II. Spectroscopic factors $[(2J+1)C^2S$ for pickup leading to Co nuclei].

	^{55}Co	^{57}Co			^{59}Co		
Level	Calc.	Calc.	Exp. [a]	Exp. [b]	Calc.	Exp. [a]	Exp. [b]
$(\frac{7}{2})_1$	6.54	5.56	5.53	5.50	5.34	5.50	6.38
$(\frac{7}{2})_2$	0.60	1.48	1.37	1.40	1.60	0.93	1.21
$(\frac{7}{2})_3$	...	0.01	0.20	0.84	0.03	(0.70)	1.08
$(\frac{3}{2})_1$	0.54	0.44	0.06	0.05	0.39	0.42	0.24
$(\frac{3}{2})_2$	0.01	0.01	0.19		0.05	0.09	
$(\frac{1}{2})_1$	0.11	0.09			0.10		
$(\frac{5}{2})_1$	0.0	0.0			0.0		
$(\frac{5}{2})_2$	0.0	0.0	(0.10)		0.0		

[a] A. G. Blair and D. D. Armstrong, Phys. Rev. 151, 930 (1966).

[b] G. Mairle *et al.*, Nucl. Phys. A134, 180 (1969).

TABLE III. Decay properties of ^{57}Co. Experimental results are given in parentheses under the calculated values in the last three columns. Weisskopf units are designated by W.u.

Transition	$B(E2)$ (W.u.) Calc.	$B(E2)$ (W.u.) Exp.[a]	$B(M1)$ (W.u.) Calc.	$B(M1)$ (W.u.) Exp.[a]	δ [a, b]	Branching ratio[b]	τ_m [a] (fsec)
$(\frac{9}{2})_1$-$(\frac{7}{2})_1$	20.7	18 ± 4	0.21	0.19 ± 0.04	−0.25	100	98.4
					(−0.27 ± 0.01)		(84 ± 18)
$(\frac{3}{2})_1$-$(\frac{7}{2})_1$	3.61	0.45 ± 0.08	···	···	···	100	3.0 psec
						(100)	(28.0 psec)[c]
$(\frac{1}{2})_1$-$(\frac{3}{2})_1$	4.64	<21.8[c]	0.67	0.02[c]	−0.01	100	17.4 psec
						(100)	(870 psec)[c]
$(\frac{11}{2})_1$-$(\frac{9}{2})_1$	11.9	34 ± 20	0.64	0.5 ± 0.2	−0.05	52	267
					(−0.09 ± 0.01)	(56)	
$(\frac{11}{2})_1$-$(\frac{7}{2})_1$	9.55	10 ± 4	···	···	···	48	(230^{+120}_{-90})
						(44)	
$(\frac{3}{2})_2$-$(\frac{1}{2})_1$	0.31	···	0.027	<0.1	−0.03	2	431
						(<2)	
$(\frac{3}{2})_2$-$(\frac{3}{2})_1$	0.63	···	0.022	···	−0.06	4	(360 ± 70)
						(0)	
$(\frac{3}{2})_2$-$(\frac{7}{2})_1$	6.70	10.5 ± 2.0	···	···	···	94	
						(98)	
$(\frac{7}{2})_2$-$(\frac{3}{2})_1$	0.35	···	···	···	···	0	73
						(0)	
$(\frac{7}{2})_2$-$(\frac{9}{2})_1$	4.58	1 ± 1	1.08	0.68 ± 0.20	−0.02	65	(120 ± 30)
					(−0.02 ± 0.01)	(60)	
$(\frac{1}{2})_2$-$(\frac{7}{2})_1$	5.38	0.1	0.056	0.02 ± 0.01	0.36	35	
					$(0.04^{+0.22}_{-0.17})$	(40)	
$(\frac{5}{2})_1$-$(\frac{3}{2})_1$	0.00	···	0.012	···	0.00	<0.1	
						(0)	
$(\frac{5}{2})_1$-$(\frac{9}{2})_1$	0.13		···	···	···	0	21
						(0)	(30 ± 12)
$(\frac{5}{2})_1$-$(\frac{7}{2})_1$	3.42		0.27	$(0.16^{+0.10}_{-0.05})$	0.14	100	
						(100)	
$(\frac{5}{2})_2$-$(\frac{1}{2})_1$	0.55	47 ± 25	···	···	···	0	
						(3)	
$(\frac{5}{2})_2$-$(\frac{3}{2})_1$	0.05	8.9 ± 5.0	0.007	0.02 ± 0.01	0.05	0.5	41
					$(0.34^{+0.20}_{-0.11})$	(14)	
$(\frac{5}{2})_2$-$(\frac{9}{2})_1$	0.05	···	···	···	···	0	(420 ± 120)
						(0)	
$(\frac{5}{2})_2$-$(\frac{7}{2})_1$	0.19	<0.03	0.075	0.007 ± 0.002	0.08	100	
					(0.06 ± 0.07)	(83)	

TABLE III *(Continued)*

Transition	$B(E2)$ (W.u.) Calc.	$B(E2)$ (W.u.) Exp.[a]	$B(M1)$ (W.u.) Calc.	$B(M1)$ (W.u.) Exp.[a]	δ[a, b]	Branching ratio[b]	τ_m[a] (fsec)
$(\frac{7}{2})_3$-$(\frac{3}{2})_2$	0.00	...	...	...	...	0	
						(0)	
$(\frac{7}{2})_3$-$(\frac{11}{2})_1$	1.71	...	...	...	...	1	457
						(0)	
$(\frac{7}{2})_3$-$(\frac{3}{2})_1$	7.36	$24.3^{+15.0}_{-8.0}$	...	...	...	16	(320 ± 90)
						(8)	
$(\frac{7}{2})_3$-$(\frac{9}{2})_1$	0.69	1.4 ± 0.6	0.008	0.05 ± 0.02	0.30	38	
					(−0.13 ± 0.02)	(68)	
$(\frac{7}{2})_3$-$(\frac{7}{2})_1$	0.02	$0.2^{+0.5}_{-0.2}$	0.0018	$0.002^{+0.002}_{-0.001}$	0.19	46	
					(0.4 ± 0.6)	(24)	
$(\frac{13}{2})_1$-$(\frac{11}{2})_1$	12.5		0.38		−0.14	88	
$(\frac{13}{2})_1$-$(\frac{9}{2})_1$	13.1		...	...	...	12	58
$(\frac{15}{2})_1$-$(\frac{13}{2})_1$	4.15		0.74		−0.02	45	
$(\frac{15}{2})_1$-$(\frac{11}{2})_1$	16.4		...	...	...	55	303

[a] Experimental results from Ref. 16.
[b] Experimental results from Ref. 19.
[c] Reference 10.

In the present study, these occupation numbers were derived from experimental results[10] for neutron-pickup and proton-stripping reactions on the even Ni cores by the methods described in the work of Castel *et al.*[11] The proton occupation numbers used were $v_{1/2}{}^2 = 0.10$, $v_{3/2}{}^2 = 0.15$, $v_{5/2}{}^2 = 0.0$, and $v_{7/2}{}^2 = 0.90$ for ^{55,57,59}Co and $v_{1/2}{}^2 = 0.0$, $v_{3/2}{}^2 = 0.20$, $v_{5/2}{}^2 = 0.0$, and $v_{7/2}{}^2 = 0.90$ for ^{61,63}Co. In deriving these occupation numbers we are neglecting the blocking effect of the extra quasiparticle, which modifies the number operator for the j_1 state to $N = \sum_j (2j+1) v_j{}^2 + 1 - 2v_{j_1}{}^2$, and implies that the v_j are a function of the particular quasiparticle level involved. However, the modifications caused in the $v_j{}^2$ are very small, and are well within the uncertainties in the input experimental information on spectroscopic factors.

III. RESULTS FOR THE ODD Co ISOTOPES

General Comments

As we mentioned earlier, the aim of these calculations is to achieve a correct description of the odd Co isotopes as proton quasihole states coupled to Ni cores using a description of the anharmonic core vibrations derived entirely from experimental data. The low-lying structure of these Ni isotopes is already well known and we display in Table I the input data used in this calculation (taken primarily from experimental studies of the Ni isotopes). The quadrupole moments of the first 2^+ states of ^{58,60,62}Ni were adopted from recent experimental measurements.[12] The $Q(2^+)$ for ^{64}Ni, which has not been measured, was deduced from a number of calculations which successfully predict those of the other core nuclei. The case of ^{56}Ni is considered in the section dealing with ^{55}Co. The strength parameter ξ is related to the core-particle coupling matrix element $k = \langle jl | k(r) | j'l' \rangle$ by $\xi = k(\frac{5}{2}\pi\hbar\omega C)^{1/2}$. Since both $E(2^+) = \hbar\omega$ and C can be derived from experiment,[12] we have tried as much as possible to limit our freedom in the choice of ξ. The calculations for ^{57}Co and ^{59}Co were essentially one-parameter in nature with ξ being the only adjustable parameter. A best fit for the ^{57,59}Co low-lying spectra was obtained for $\xi = 2.7$ and $\xi = 3.35$, respectively. These correspond to values of k of 36.5 and 37.5 MeV. The ^{61,63}Co calculations were then performed with a value of ξ deduced from C, $\hbar\omega$ (both from experiment), and $k = 37$ MeV and are therefore in principle parameter-free.

The energies of the single-quasihole states were taken to be the same for all members of the $^{57-63}$Co series. These energies, relative to the $f_{7/2}$ state, were $E_{1/2} = 2.8$ MeV, $E_{3/2} = 1.5$ MeV, and $E_{5/2} = 2.6$ MeV, and are consistent with single-particle energies derived from analysis of pickup and stripping reactions on ^{62}Ni, the core nucleus of the series considered for which the greatest experi-

mental detail is available. They are also similar to the values used by Zamick in a recent calculation,[13] the decreased separation between the $f_{7/2}$ states and the others being due to the quasiparticle nature of our states. Throughout the calculations we use effective values for the single-particle charge and gyromagnetic ratios, namely $e_p = 2e$ for the proton effective charge, $g_R = 0$, and $g_s(\text{eff}) = 0.6\, g_s(\text{free})$, values which have been used frequently in calculations in this region.

^{57}Co Results

A large amount of experimental data has accumulated recently on this nucleus. The ground-state spectroscopic factor (for proton pickup from ^{58}Ni) is correctly calculated (see Table II) and confirms that the value of ξ used is reasonable. The magnetic moment ($\mu = 4.49\mu_N$) also compares well with the observed[14] $\mu_{\text{exp}} = 4.65 \pm 0.05\mu_N$.

The $J = \frac{9}{2}$ state at 1.22 MeV is found to be 80% $(\frac{7}{2} \times 2^+)$ in character and its decay properties are correctly reproduced (see Table III). The observed[10] properties of the two low-lying $J = \frac{3}{2}$ states are somewhat conflicting. The higher of the two is the more strongly excited in the reaction ^{58}Ni$(t, \alpha)^{57}$Co, indicating that the $J = \frac{3}{2}$ hole strength is largely concentrated there. However, it has a strong $E2$ decay to the ground, whereas the decay from the lower $\frac{3}{2}$ state is very weak.

The calculated spectroscopic factor of the $J = \frac{3}{2}^-$ state at 1.38 MeV is larger than the experimental value. The ^{56}Fe$(^3$He$, d)^{57}$Co spectroscopic factor, however, indicates that this state should be mainly single particle. From various measurements, Van Esch, Rots, and Coussement[15] deduce single-particle amplitudes of $\alpha = 0.93$, 0.89, 0.87, and 0.80, which compare well with our present estimate of $\alpha = 0.84$. Also the long $E2$ lifetime indicates little collective component (see Table III). The calculated magnetic moment $\mu = 2.23\mu_N$ is close to a recent estimate[15] ($\mu = 3.0\mu_N$).

Since our calculated $E2$ rates agree well with experiment[16] and since the ^{56}Fe$(^3$He$, d)^{57}$Co spectroscopic factors indicate that the lower $\frac{3}{2}$ state is the more strongly excited, we conclude that the higher of the two is the collective one [of $(\frac{7}{2} \times 2^+)$ nature] and that the lower is largely single particle. As we shall see, this is not true of the higher Co isotopes.

In general, an excellent correspondence is established between the observed excitation energies of the low-lying levels and the corresponding calculated quantities (see Fig. 1). As is seen from Table III the general collective behavior of the low-lying states is correctly reproduced and the most

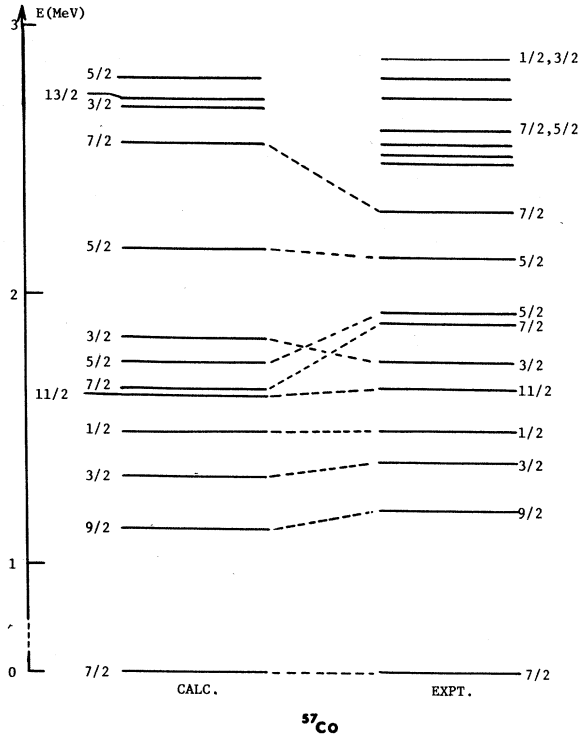

FIG. 1. Comparison of the observed and calculated spectra of ^{57}Co.

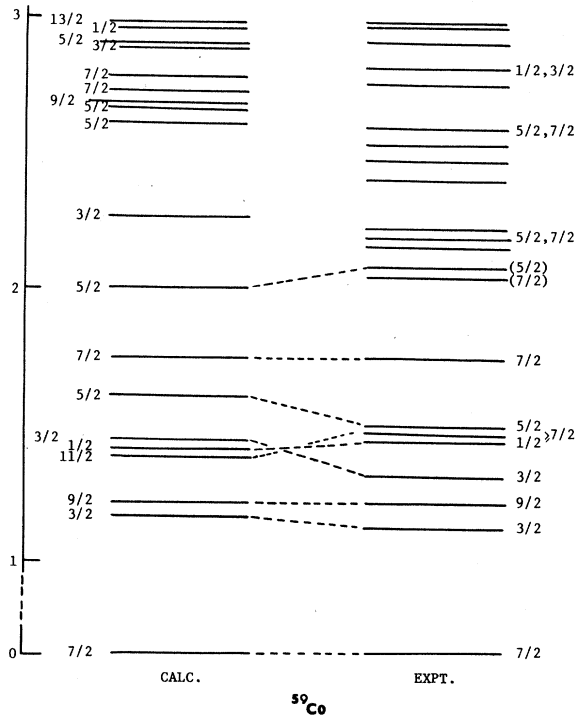

FIG. 2. Comparison of the observed and calculated spectra of ^{59}Co.

TABLE IV. Decay properties of ^{59}Co. Experimental results are given in parentheses in the last three columns.

Transition	$B(E2)$ (W.u.) Calc.	$B(E2)$ (W.u.) Exp.[a]	$B(M1)$ (W.u.) Calc.	$B(M1)$ (W.u.) Exp.[a]	δ[b]	Branching ratio[b]	τ_m[a] (fsec)
$(\frac{3}{2})_1$-$(\frac{7}{2})_1$	7.72	11.0 ± 3.0	···	···	···	100	3536
						(100)	(3100 ± 400)
$(\frac{9}{2})_1$-$(\frac{7}{2})_1$	20.7	14.7 ± 3.0	0.24	0.27 ± 0.07	−0.26	100	73
					(−0.25 ± 0.05)	(100)	(70 ± 20)
$(\frac{3}{2})_2$-$(\frac{3}{2})_1$	2.65		0.099		−0.03	6	942
						(7)	
$(\frac{3}{2})_2$-$(\frac{7}{2})_1$	9.63	17.6 ± 3.0 1.0 ± 0.4	···	···	···	94 (93)	(890 ± 150) (13 500 ± 5000) (85 000)
$(\frac{1}{2})_1$-$(\frac{3}{2})_2$	0.92		0.17		0.00	0	4050
						(76)	
$(\frac{1}{2})_1$-$(\frac{3}{2})_1$	4.90		0.57		−0.02	100	(135 psec)
						(24)	
$(\frac{11}{2})_1$-$(\frac{9}{2})_1$	7.32		0.61		−0.01	7	847
						(<10)	
$(\frac{11}{2})_1$-$(\frac{7}{2})_1$	14.2	5.3 ± 2.0	···	···	···	93	
						(>90)	
$(\frac{5}{2})_1$-$(\frac{3}{2})_2$	5.65		0.64		−0.01	0.2	
						(0)	
$(\frac{5}{2})_1$-$(\frac{9}{2})_1$	2.72		···	···	···	0	28
						(0)	
$(\frac{5}{2})_1$-$(\frac{3}{2})_1$	1.07		0.091		−0.03	0.6	
						(20)	
$(\frac{5}{2})_1$-$(\frac{7}{2})_1$	4.56		0.27		0.15	99	
						(80)	
$(\frac{7}{2})_2$-$(\frac{5}{2})_1$	9.77		0.72		−0.01	0.4	
					(−0.02 ± 0.13)	(9)	
$(\frac{7}{2})_2$-$(\frac{9}{2})_1$	2.26		1.01		−0.02	31	62
					−0.02 ± 0.05	(32)	
$(\frac{7}{2})_2$-$(\frac{3}{2})_1$	0.34		···	···	···	0	
						(0)	
$(\frac{7}{2})_2$-$(\frac{7}{2})_1$	4.77		0.059		0.36	69	
					$(0.89^{+0.25}_{-0.17})$	(59)	
$(\frac{5}{2})_2$-$(\frac{5}{2})_1$	0.93		0.10		−0.03	1	
$(\frac{5}{2})_2$-$(\frac{3}{2})_2$	1.10		0.046		−0.06	1	
$(\frac{5}{2})_2$-$(\frac{9}{2})_1$	0.00		···		···	0	35
$(\frac{5}{2})_2$-$(\frac{3}{2})_1$	0.36		0.028		0.07	2	
$(\frac{5}{2})_2$-$(\frac{7}{2})_1$	0.46		0.11		0.09	97	

[a] Experimental values from Ref. 10.

[b] Experimental values from Ref. 18.

obvious differences seem to occur for some $M1$ decay rates from higher states.

^{59}Co Results

The ground-state spectroscopic factor is well reproduced, as are the quadrupole moment ($Q = 0.412$ b compared with $Q_{exp} = 0.407 \pm 0.040$ b)[14] and the magnetic moment ($\mu_{calc} = 4.42\mu_N$ and $\mu_{exp} = 4.65\mu_N$).[14] In fact, very little change is observed between ^{57}Co and ^{59}Co for both ground-state spectroscopic factors and magnetic moments.

The low-lying $J = \frac{9}{2}$ and $\frac{11}{2}$ states show little change from ^{57}Co in their decay as is seen from Table IV. We suggest that our $\frac{11}{2}$ state should be identified with the high-spin state observed at 1.46 MeV (Fig. 2). A $J = \frac{5}{2}$ assignment for the 1.19-MeV state has apparently not been completely ruled out by experiment. Our results are quite incompatible with such an assignment on grounds of energy, $E2$ strength, and the sign of the mixing ratio δ. We note that our calculation predicts the observed inversion between ^{57}Co and ^{59}Co of the order of the $J = \frac{3}{2}$ and $\frac{9}{2}$ states (at 1.10 and 1.19 MeV, respectively), and the halving of the separation of the $J = \frac{9}{2}$ and $\frac{11}{2}$ states. Both these effects arise in this model from the change in the $Q(2^+)$ value of the core.

As in ^{57}Co, the observed properties of the two $\frac{3}{2}$ states do not form an entirely coherent picture. The lower is preferentially excited in the reaction ^{60}Ni$(t, \alpha)^{59}$Co, in agreement with our results, but the higher is the stronger in stripping reactions. The ground-state $E2$ strength from the lower $\frac{3}{2}$ state is much greater than in ^{57}Co, with which our results agree. For the higher $\frac{3}{2}$ state, three experiments,[10] using Coulomb excitation and β decay, lead to measurements of the $E2$ lifetime differing by 2 orders of magnitude. The present calculation favors a strong $E2$ decay, with the higher $\frac{3}{2}$ state containing the greater collective strength, though there is considerable mixing between these two states. The decay properties of the higher-lying states are not well documented. Available branching ratios are generally well described in our model calculation.

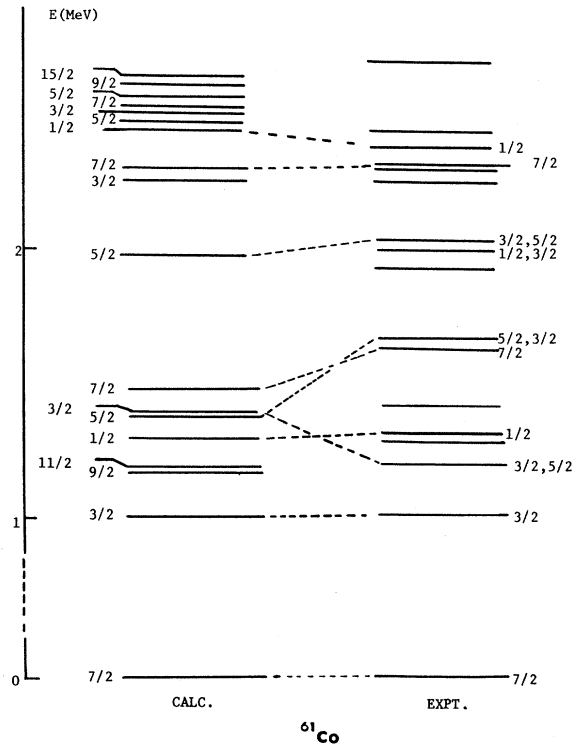

FIG. 3. Comparison of the observed and calculated spectra of ^{61}Co.

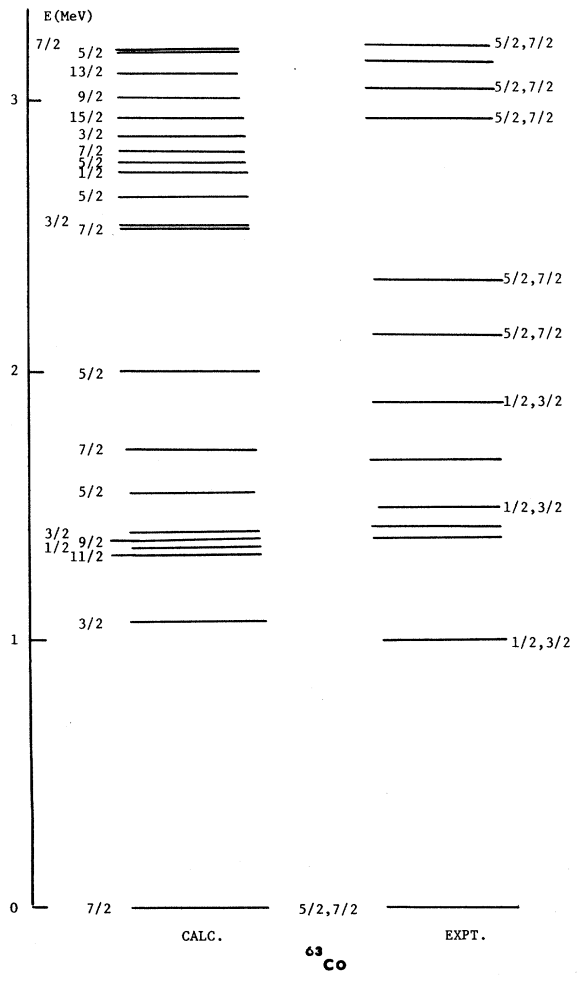

FIG. 4. Comparison of the observed and calculated spectra of ^{63}Co.

$^{61-63}$Co Results

Little is yet known about the properties of the excited states of these two isotopes. In our calculation, the $J = \frac{9}{2}$ and $\frac{11}{2}$ states form a close doublet both in ^{61}Co and ^{63}Co (see Figs. 3 and 4). Their close proximity may make their identification difficult experimentally. The 1.29-MeV level in ^{61}Co is a candidate for such a doublet, and a high-resolution study of it would be of interest. The very recent results[17] of an experiment using the reaction ^{62}Ni$(t, \alpha)^{61}$Co indeed indicate that there are two states here, at 1.272 and 1.287 MeV. If this is the case, it is possible to associate each of our calculated levels below 1.7 MeV with an observed level. This would leave only the possible experimental level[10] at 1.42 MeV unaccounted for. Its existence seems in any case doubtful, since it was not observed in either of the most recent studies[17,18] of ^{61}Co.

Coop, Graham, and Titterton[18] suggest that the single-particle character lies in the low-lying

TABLE V. Decay properties of ^{61}Co.

Transition	$B(E2)$ (W.u.) Calc.	Exp.	$B(M1)$ (W.u.) Calc.	Exp.	δ	Branching ratio[a]	τ_m (fsec)
$(\frac{3}{2})_1$-$(\frac{7}{2})_1$	16.4		...	...	...	100 (100)	2612
$(\frac{9}{2})_1$-$(\frac{7}{2})_1$	20.8		0.21		−0.27	100 (100)	90
$(\frac{11}{2})_1$-$(\frac{9}{2})_1$	4.39		0.57		−0.01	0.1	1903
$(\frac{11}{2})_1$-$(\frac{7}{2})_1$	12.6		...	...	...	100	
$(\frac{3}{2})_2$-$(\frac{9}{2})_1$	4.21		0.19		−0.03	62 (5)	4700
$(\frac{3}{2})_2$-$(\frac{7}{2})_1$	1.07		...	...	...	38 (95)	
$(\frac{1}{2})_1$-$(\frac{9}{2})_2$	2.39		0.43		0.00	0 (18)	10 750
$(\frac{1}{2})_1$-$(\frac{9}{2})_1$	6.24		0.12		−0.05	100 (82)	
$(\frac{5}{2})_1$-$(\frac{3}{2})_2$	2.49		0.32		−0.01	0	42
$(\frac{5}{2})_1$-$(\frac{1}{2})_1$	0.14		...	...	...	0	
$(\frac{5}{2})_1$-$(\frac{9}{2})_1$	3.50		...	...	...	0	
$(\frac{5}{2})_1$-$(\frac{3}{2})_1$	5.26		0.52		−0.03	4 (12)	
$(\frac{5}{2})_1$-$(\frac{7}{2})_1$	5.13		0.24		0.15	96 (88)	
$(\frac{7}{2})_2$-$(\frac{3}{2})_2$	0.02		...	...	...	0	129
$(\frac{7}{2})_2$-$(\frac{9}{2})_1$	2.35		0.88		−0.01	20 (45)	
$(\frac{7}{2})_2$-$(\frac{3}{2})_1$	0.91		...	...	...	0	
$(\frac{7}{2})_2$-$(\frac{7}{2})_1$	5.84		0.046		0.41	80 (55)	
$(\frac{5}{2})_2$-$(\frac{1}{2})_1$	0.06		...	...	...	0	45
$(\frac{5}{2})_2$-$(\frac{3}{2})_2$	0.91		0.010		−0.16	0.5	
$(\frac{5}{2})_2$-$(\frac{9}{2})_1$	0.04		...	...	...	0	
$(\frac{5}{2})_2$-$(\frac{3}{2})_1$	0.00		0.036		0.00	5	
$(\frac{5}{2})_2$-$(\frac{7}{2})_1$	0.33		0.080		0.10	95	

[a] Experimental results (in parentheses) from Ref. 18.

TABLE VI. Decay properties of ^{63}Co.

Transition	$B(E2)$ (W.u.) Calc.	Exp.	$B(M1)$ (W.u.) Calc.	Exp.	δ	Branching ratio	τ_m (fsec)
$(\frac{3}{2})_1$-$(\frac{7}{2})_1$	14.4		...	...	...	100 (100)	2588
$(\frac{11}{2})_1$-$(\frac{7}{2})_1$	12.8		...	...	...	100	1069
$(\frac{1}{2})_1$-$(\frac{3}{2})_1$	17.7		0.20		−0.06	100	9113
$(\frac{9}{2})_1$-$(\frac{11}{2})_1$	2.57		0.67		0.00	0	54
$(\frac{9}{2})_1$-$(\frac{7}{2})_1$	19.4		0.21		−0.31	100	
$(\frac{3}{2})_2$-$(\frac{1}{2})_1$	0.36		0.28		0.00	0.2	
$(\frac{3}{2})_2$-$(\frac{3}{2})_1$	4.90		0.20		−0.04	34	1850
$(\frac{3}{2})_2$-$(\frac{7}{2})_1$	3.76		...	...	...	66	
$(\frac{5}{2})_1$-$(\frac{3}{2})_1$	4.30		0.33		−0.04	4	33
$(\frac{5}{2})_1$-$(\frac{7}{2})_1$	4.84		0.23		0.17	96	
$(\frac{7}{2})_2$-$(\frac{9}{2})_1$	2.01		0.88		−0.01	11	
$(\frac{7}{2})_2$-$(\frac{3}{2})_1$	1.27		...	...	...	0	102
$(\frac{7}{2})_2$-$(\frac{7}{2})_1$	5.33		0.047		0.43	89	
$(\frac{5}{2})_2$-$(\frac{1}{2})_1$	0.19		...	...	...	0	
$(\frac{5}{2})_2$-$(\frac{3}{2})_1$	0.00		...	...	...	0	38
$(\frac{5}{2})_2$-$(\frac{3}{2})_1$	0.01		0.048		0.01	5	
$(\frac{5}{2})_2$-$(\frac{7}{2})_1$	0.47		0.098		0.10	95	

TABLE VII. Decay properties of ^{55}Co.

Transition	$B(E2)$ (W.u.) Calc.	Exp.	$B(M1)$ (W.u.) Calc.	Exp.	δ	Branching ratio	τ_m (fsec)
$(\frac{3}{2})_1$-$(\frac{7}{2})_1$	4.86		...	...	...	100 (100)	281
$(\frac{9}{2})_1$-$(\frac{7}{2})_1$	20.0		0.079		−0.89	100	14
$(\frac{1}{2})_1$-$(\frac{3}{2})_1$	0.88		0.68		−0.02	100	99
$(\frac{7}{2})_2$-$(\frac{9}{2})_1$	11.2		1.16		−0.03	5	
$(\frac{7}{2})_2$-$(\frac{3}{2})_1$	0.19		...	...	...	0	19
$(\frac{7}{2})_2$-$(\frac{7}{2})_1$	10.7		0.017		1.63	95	
$(\frac{11}{2})_1$-$(\frac{9}{2})_1$	14.5		0.60		−0.19	6	19
$(\frac{11}{2})_1$-$(\frac{7}{2})_1$	11.4		...	...	...	94	
$(\frac{5}{2})_1$-$(\frac{9}{2})_1$	0.12		...	...	...	0	
$(\frac{5}{2})_1$-$(\frac{3}{2})_1$	0.02		0.006		−0.04	0.2	9
$(\frac{5}{2})_1$-$(\frac{7}{2})_1$	6.90		0.069		1.63	99.8	
$(\frac{3}{2})_2$-$(\frac{1}{2})_1$	0.12		0.011		−0.02	0	
$(\frac{3}{2})_2$-$(\frac{3}{2})_1$	0.13		0.003		−0.17	0	22
$(\frac{3}{2})_2$-$(\frac{7}{2})_1$	7.66		...	...	...	100	

$J = (\frac{3}{2})_1$ state in ^{57}Co and in the higher $(\frac{3}{2})_2$ state in $^{59-63}$Co. The present calculations give a predominant single-particle character to the $(\frac{3}{2})_1$ state in $^{57, 59, 63}$Co and to the $(\frac{3}{2})_2$ state in ^{61}Co (where $\hbar\omega$ is smallest) (see Tables V and VI). There is, however, very strong mixing of the single-particle and collective components. We do not tabulate the spectroscopic factors for these nuclei – they are very similar to those for $^{57, 59}$Co, with the exception that the two $J = \frac{3}{2}$ states have nearly equal strengths.

^{55}Co Results

We have reserved discussion of this nucleus until last, since it does not appear to submit so well to treatment by this present model. The nucleus ^{56}Ni differs greatly from the other core nuclei in its spectrum and in being doubly magic and unstable, and it seems possible that its excited states may also be somewhat different in structure. In a previous calculation using the anharmonic model,[5] it was found that the quadrupole moment of the 2^+ state plays an important role in determining the ordering and spacing of the levels. Unfortunately no $Q(2^+)$ value is known for the ^{56}Ni core. By examining the systematics of the $Q(2^+)$ values in the Ni isotopes (see Table I) we can expect a rather large negative value for this moment. A value of $Q(2^+) = -0.18$ b as chosen seems to allow a general correspondence to be established between the few observed levels and the calculated ones (see Fig. 5). The states observed at 2.66 and 2.92 MeV seem likely to have spins $\frac{9}{2}$ and $\frac{7}{2}$, respectively. The $J = (\frac{3}{2})$ state at 2.56 MeV, however, is not accounted for, and may be supposed to be a state of higher seniority.

We should point out that the single-particle energies used for the ^{55}Co case represent 1.5 times the values used for the other Co isotopes, and that $\hbar\omega$ is taken to be larger than $E(2^+)$. In Table VII, we present results of calculated decay properties. It is perhaps noteworthy that none of the levels considered decays strongly to more than one lower state. However, too little is yet known about transitions in ^{55}Co to allow for any possible comparison.

General Systematics

It has been shown by Coop, Graham, and Titterton[18] that there are interesting similarities and systematic differences between the spectra of the odd-A Co isotopes. One of the main objectives of this study has been to understand the systematics of the variation of energy levels and electromagnetic decays from isotope to isotope in terms of the properties of the core nuclei. The most important of these properties, as discussed in the

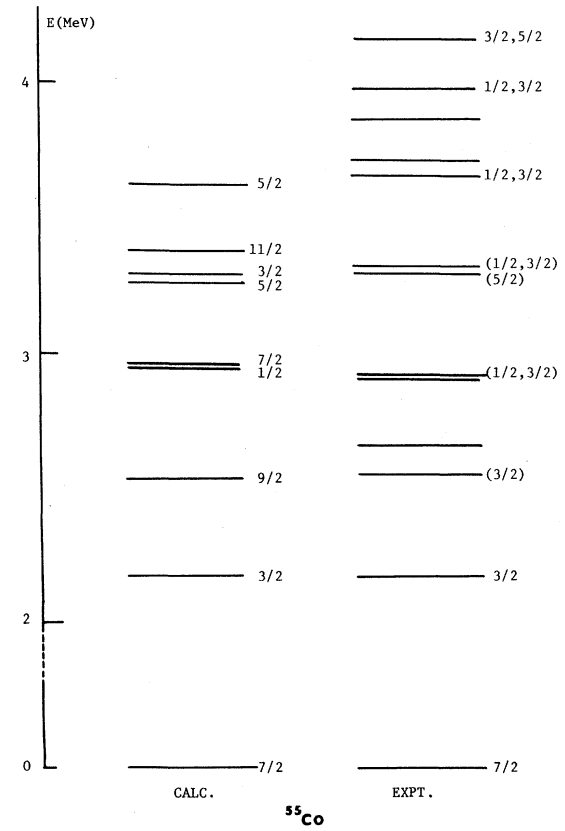

FIG. 5. Comparison of the observed and calculated spectra of ^{55}Co.

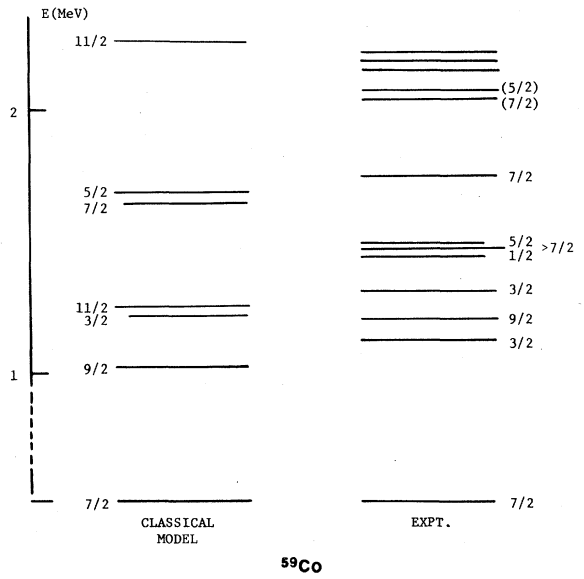

FIG. 6. Comparison of the observed spectra of ^{59}Co with a "classical" IC model calculation.

next section, is the quadrupole moment of the lowest 2^+ state. We now discuss in more detail the behavior of certain groups of levels.

(i) $\frac{9}{2}$ *and* $\frac{11}{2}$ *levels.* These are widely separated (0.67 MeV) in ^{57}Co, but are much closer in ^{59}Co, and as suggested above may be almost degenerate in ^{61}Co and ^{63}Co, the 1.29- and 1.37- or 1.43-MeV states, respectively, being candidates. This behavior is almost exactly reproduced by the calculation, and is due almost entirely to the systematic variation of $Q(2^+)$ for the core. The wave functions for these states change little from isotope to isotope, however, and accordingly the reduced transition rates from the states do likewise.

(ii) $\frac{3}{2}$ *levels.* Two low-lying $J=\frac{3}{2}$ levels appear in the spectra of all Co isotopes. The structure of these states has already been discussed. We simply remark here that the systematic variation of the energies of these states is well explained by this calculation.

(iii) $\frac{1}{2}$ *level.* The systematic slow decrease in energy of this state from ^{57}Co to ^{61}Co is well accounted for. The calculated lifetime, however, is always much shorter than the observed one, although the closeness of the $\frac{1}{2}$ and two $\frac{3}{2}$ levels complicates the situation.

(iv) Levels $(\frac{5}{2})_1$, $(\frac{5}{2})_2$, $(\frac{7}{2})_2$. These levels appear both in theory and experiment between 1.4 and 2.1 MeV in the spectra of ^{57}Co and ^{59}Co, and probably in similar positions in ^{61}Co and ^{63}Co. The interchange in the order of $(\frac{5}{2})_1$ and $(\frac{7}{2})_2$ between ^{57}Co and ^{59}Co is reproduced, but not that between ^{59}Co and ^{61}Co. The $(\frac{5}{2})_2$ state is remarkably stable in energy, at about 2.0 MeV. The wave functions for the $(\frac{7}{2})_2$ state appear to be satisfactory in that they contain the correct single-particle component and reproduce the observed branching ratios in their decay. It is not possible, however, to understand the observed[18,19] mixing ratios for the $(\frac{7}{2})_2$-$(\frac{7}{2})_1$ decay, these being $\delta = 0.04^{+0.22}_{-0.17}$ and $\delta = 0.89^{+0.25}_{-0.17}$ in ^{57}Co and ^{59}Co, respectively.

The calculated decays of the $J=\frac{5}{2}$ states are in general less satisfactory. As with the $J=\frac{1}{2}$ states, this may arise from the fact that we are dealing with quasiparticle states of these spins which are of very small amplitude.

(v) $\frac{13}{2}$ *and* $\frac{15}{2}$ *levels.* The calculations predict levels of these spins to occur below 3 MeV in $^{57-63}$Co, the $\frac{13}{2}$ being lower in ^{57}Co and ^{59}Co, and the $\frac{15}{2}$ in ^{61}Co and ^{63}Co. We have predicted the decay of

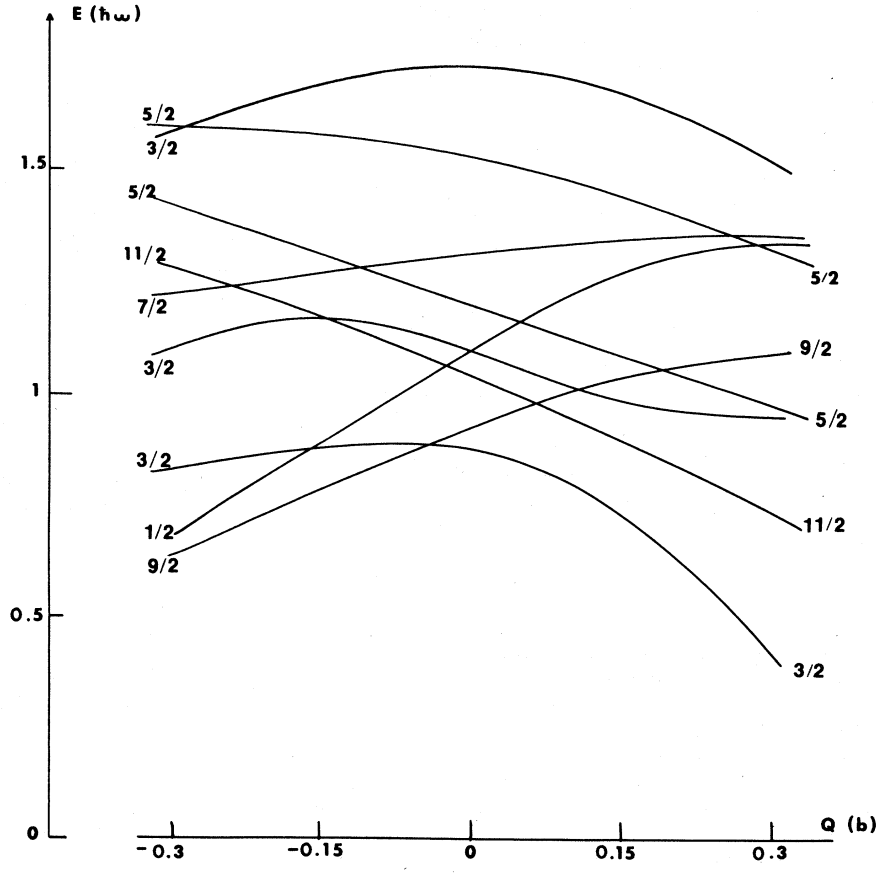

FIG. 7. Dependence of the calculated energy levels of ^{59}Co on the quadrupole moment of the 2^+ state of the core nucleus.

such states for ^{57}Co, and suggest that their possible detection would be of interest.

(vi) Density of levels above 2 MeV. It is clear from Figs. 3 and 4 that our calculated density of levels is too low near 2.5 MeV, and then becomes much too high around 3 MeV, whereas the observed density increases in a more uniform manner. This arises from the fact that we are unable to include any core states above the "two-phonon" states, as insufficient experimental detail is available on such levels. Such core states would be important in the wave functions of levels above about 2.5 MeV, and would have the effect of breaking the near degeneracy of levels in the region near 3 MeV. The outcome would be a redistribution of the levels between 2.5 and 3.5 MeV, though not a significant change in their number.

IV. GENERAL COMMENTS AND CONCLUSION

As mentioned earlier, many model calculations are available for the odd Co isotopes. The phenomenological approaches[3,4] based on the vibrational excitations of the Ni cores have been unsuccessful in their predictions of the ordering of the low-lying states. In Fig. 6 we present a comparison between a recent calculation of Satpathy and Gujrathi[3] using the IC model and experiment; it is clear that a much improved picture of the low-lying level scheme is obtained by introducing anharmonic features. To show how important their effects are, we have calculated the variation of the energy levels with $Q(2^+)$ for a specific nucleus. Although the $Q(2^+)$ value is taken from experiment and therefore not a variable parameter, it introduces a large amount of anharmonicity in the core structure. Our calculations for ^{59}Co are shown on Fig. 7, where it is clear that the right ordering and spacing of the levels is obtained for a value of $Q(2^+)$ close to the observed one ($Q_{exp} = -0.03$ b). The influence of quasiparticle effects has also proven to be important. However, there are no significant changes from one odd-A Co isotope to the other so that the general systematics are not affected by our use of quasihole states.

Another point worth emphasizing is that for almost all transitions the mixing ratio δ is predicted with the correct sign. Little attention has been paid to this point in earlier studies with the IC model, although this positive result throughout a whole series of isotopes is an indication of the goodness of fit of transition rates.

The most recent shell-model calculations on Co nuclei are those of Gatrousis *et al.*[20] for ^{57}Co, and do not include transition rates. The level scheme obtained for ^{57}Co can explain all the observed low-lying levels, although the ordering of levels is not exact and an unobserved $(\frac{9}{2})_2$ level is predicted below 2 MeV. We note that the structure of our wave functions is quite consistent with theirs, in the sense that their dominant configurations have the same "last-particle" j states as ours. The differences between the calculations made a more detailed comparison impossible.

A complete theoretical description for Co isotopes on this model would require detailed calculations for the Ni core. These are becoming quite successful.[21] The quadrupole moments of the 2^+ states are well predicted, but some of the $E2$ rates are not so satisfactory. These calculations would have the advantage of showing the exact nature of the particle-hole excitations responsible for the collective vibrations and would therefore show to what extent the coupling of an extra core nucleon could violate the Pauli principle. A recent attempt[22] at isolating the main 1p-1h and 2p-2h components from an "anharmonic" one-phonon wave function has shown that small p-h components could be responsible for relatively large anharmonic effects. More generally, if such extended calculations became possible, they could form the basis for a more complete calculation of the properties of the higher (>2.5 MeV) levels in the Co isotopes. This could also be expected to be successful, since it appears from the present results that these nuclei can in fact be represented as a proton hole coupled to a Ni core, with the systematics of the energy levels being dominated by the energy and quadrupole moment of the 2^+ states of the cores.

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