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Magnetic Transition Strength in ^{32}S and ^{40}Ca from 180° Electron Scattering

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The magnetic excitation structures of ^{32}S and ^{40}Ca have been studied by 180° inelastic electron scattering from natural targets of CaS and Ca at energies of 39 and 56 MeV. The significant feature of the ^{32}S spectrum is the fragmentation of the $\Delta T=1$ magnetic dipole strength into transitions to several levels in contrast to the strong concentration into a very few that is observed in ^{20}Ne , ^{24}Mg , and ^{28}Si . In the ^{32}S spectra excitation peaks are observed at 8.13, 10.82, 11.14, 11.62, 13.37, and 13.85 MeV. Analyses of the peaks at 8.13, 11.14, and 11.62 MeV yield $M1$ ground-state transition widths Γ_0 of 2.8, 18.9, and 9.7 eV, respectively. Peaks in the ^{40}Ca spectra are observed at 5.94, 6.94, 8.43, and 10.34 MeV. The peaks at 8.43 and 10.34 MeV are found to be due to an $M2$ transition, $\Gamma_0=2.6\times 10^{-2}$ eV, and a possible $M1$ transition, $\Gamma_0=7.0$ eV, respectively. The analyses of the remaining peaks in the ^{32}S and ^{40}Ca spectra are discussed. The ^{32}S results are examined in terms of the theory of Kurath.

I. INTRODUCTION

At this laboratory a systematic study has been made of the strong $\Delta T=1$ magnetic multipole transitions in the self-conjugate nuclei of the s - d shell. Our studies of those nuclei thus far reported,¹⁻³ ^{20}Ne , ^{24}Mg , and ^{28}Si , have revealed in each case very strong magnetic dipole transitions to $T=1$, 1^+ excited states in the 10-MeV region. These results have been generally consistent with the theory of Kurath⁴ who indicated that such nuclei should exhibit a strong concentration of the magnetic dipole strength into transitions to the lowest few of these states. However, in marked contrast with these three nuclei, the ^{32}S spectra show considerable fragmentation of this strength. The measurements of multipolarity L , radiation width Γ_0 , and transition radius R are discussed for those transitions in ^{32}S and ^{40}Ca corresponding to the peaks that were observable. These results are interpreted in terms of the theory of Kurath and also from the point of view of analog resonances.

II. THEORETICAL CONSIDERATIONS

A discussion of the relevant theory has been given in several earlier reports,^{1-3, 5, 6} but will

be again outlined here as an aid in understanding a variation in the application of the theory used for the first time in this paper and explained below. The experimental results are analyzed using the model-independent plane-wave Born-approximation (PWBA) results of Rosen, Raphael, and Überall.⁷ A correction is made in the analysis for distorted-wave Born-approximation (DWBA) effects using the ratios of DWBA to PWBA cross sections for $M1$ and $M2$ transitions reported by Chertok, Johnson, and Sarkar.⁸ The differential cross section for 180° magnetic scattering is given by [Ref. 7, Eq. (5)]

$$\left(\frac{d\sigma}{d\Omega}\right)_{180^\circ} = \frac{\pi\alpha}{[(2L+1)!!]^2} \frac{L+1}{L} \frac{q^{2L}}{k_1^2} B(ML, q), \quad (1)$$

where L is the multipolarity of the induced transition, q the momentum transfer, k_1 the incident electron momentum, and $B(ML, q)$ the reduced transition matrix element. The latter can be expressed as

$$B(ML, q) = B(ML, 0)G^2(L, q, R), \quad (2)$$

where the q dependence is in G , and R is the transition radius. Rosen, Raphael, and Überall⁷ evalu-

ate G by expansion of the spherical Bessel function involved and obtain [Ref. 7, Eq. (13a)]

$$G(L, q, R) = 1 - \frac{L+3}{L+1} \frac{(qR_2)^2}{2(2L+3)} + \frac{L+5}{L+1} \frac{(qR_4)^4}{8(2L+3)(2L+5)} - \dots, \quad (3)$$

where in each term the transition radius R_{2n} is defined differently as in Ref. 7 [R_2 and R_4 are the same as R_M and R_M^* , respectively, in expressions (13b) and (13c) of Ref. 7]. Previously we have used for G the expression (3) truncated to three terms (as shown) and have assumed $R = R_2 = R_4$.

However, for larger radii (heavier nuclei), the effect of truncation of the series becomes important. Therefore we employ the entire series, but with the same transition radius R in every term. We then have

$$G(L, q, R) = \frac{(2L+1)!!}{(L+1)(qR)^L} \frac{\partial}{\partial (qr)} [qr j_L(qr)]_{r=R}, \quad (4)$$

where $j_L(qr)$ is the spherical Bessel function. This treatment is equivalent to adoption of the model of Isabelle and Bishop⁹ in which the interacting nucleons are concentrated on a nuclear shell of radius R ; their Eq. (39) is equivalent to Eq. (4) here. In practice we employ the empirical form

$$G(L, q, R) = e^{-y/a}(1 - y + by^2) \quad (5)$$

with parameters such that Eq. (5) is quite accurate to the first zero of the equation. For $M1$ transitions, one finds that $y = 0.163106(qR)^2$, $a = 4.42$, and $b = 0.15124$. For $M2$ transitions, the parameters are $y = 0.088255(qR)^2$, $a = 2.866$, and $b = 0.18492$.

For $qR \lesssim 2$, the model dependence introduced in our results by assuming R is the same in every term of G is not very different from that introduced by assuming $R = R_2 = R_4$ and $R_6 = \dots = R_{2n} = 0$. It is of interest to mention that models which render $R_2 = R_4$ to within 10% are quite reasonable. However, we do not intend to attach physical significance¹⁰ to R , and regard it at this time as a convenient parameter.

By equating the ratio of experimental cross sections (reduced from DWBA) obtained at the two bombarding energies to the corresponding ratio of expressions (1), $B(ML, 0)$ cancels out and one obtains the ratio of G^2 at the two energies, which ratio depends on the unknowns L and R . Trial values of L are assumed; the one yielding a value of R closest to that of the nuclear-matter radius is chosen. The values of L and R are then used to determine $B(ML, 0)$ by means of Eqs. (1) and (2). The radiation width to the ground state can

then be determined and is [Ref. 7, Eq. (15a)]

$$\Gamma_0 = \frac{8\pi}{[(2L+1)!!]^2} \frac{L+1}{L} \frac{2J_0+1}{2J+1} \omega^{2L+1} B(ML, \omega), \quad (6)$$

where ω is the excitation energy, and J_0 and J are the ground- and excited-state spins, respectively.

III. EXPERIMENTAL CONSIDERATIONS

In this experiment natural targets of CaS (95.0% ³²S) and Ca (96.97% ⁴⁰Ca), 74.0 and 64.6 mg/cm² thick, respectively, were bombarded with 39- and 56-MeV electrons from the Naval Research Laboratory 60-MeV linac. A CaS target was used to obtain the ³²S data because targets of elemental sulphur were found to be quickly eroded by the electron beam. Although cooling and moving-target techniques with sulphur do exist,¹¹ it was judged simpler at the time to use the present method. Since little magnetic dipole structure was anticipated in the ⁴⁰Ca spectrum, it was felt that the latter spectrum could be easily subtracted from the CaS spectrum to obtain the ³²S spectrum. Furthermore, from earlier results with ²⁰Ne, ²⁴Mg, and ²⁸Si, it was expected that one or two dominant peaks resulting from $M1$ transitions would appear in the ³²S spectrum. Such large peaks would make the inaccuracy attendant with subtraction relatively less important. However, as discussed below, considerable fragmentation of the magnetic dipole strength was found; thus, on hindsight, it would have been better to use a more sophisticated technique with a sulphur target.

Since CaS reacts readily with oxygen and Ca reacts with both oxygen and nitrogen, care was taken to minimize the period of contact of the targets with the air. A spectrographic as well as chemical analysis was made of both the target materials before target fabrication and of the targets themselves after the experiment was run. No significant differences appeared in the analyses before and after the experiment. The largest impurities were those of about 3.1% CaSO₄ and 0.5% AlSiO₄ in the CaS. This amount of oxygen was not judged sufficient to make a significant contribution to the spectrum. The 180° scattering apparatus including the magnetic spectrometer, associated detector system, and the beam-measuring system has been described in detail in Ref. 5. The methods of energy calibration⁵ and of the treatment of the data^{1,3} are also given in earlier reports. The raw data are reduced to spectra in terms of $d^2\sigma/dE d\Omega$ by means of a computer program.

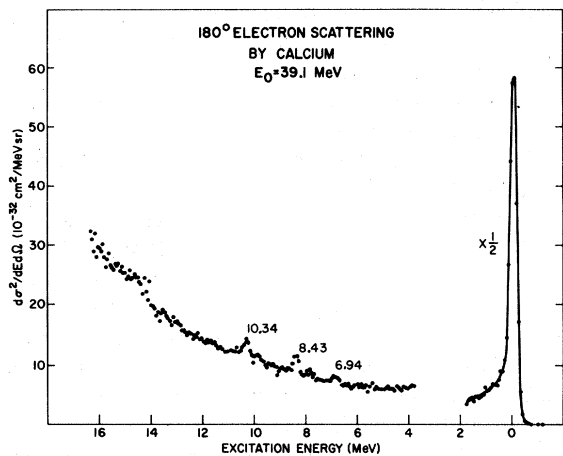


FIG. 1. Differential cross section for 180° scattering of 39.1-MeV electrons from calcium. The gap in the spectrum is present because a preliminary survey, as well as the 56-MeV data, showed no structure of interest in this region.

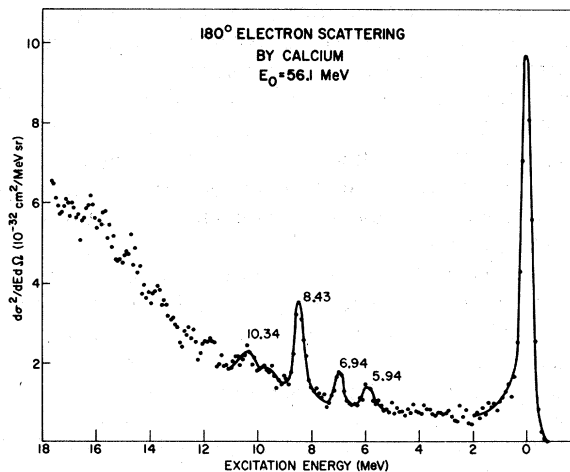


FIG. 2. Differential cross section for 180° scattering of 56.1-MeV electrons from calcium.

IV. RESULTS AND DISCUSSION

A. ^{40}Ca

Except for the early work of Barber *et al.*¹² who found no inelastic peaks in the ^{40}Ca spectrum, there have been no 180° electron scattering experiments performed on this nucleus. Although a number of electron scattering studies¹³⁻²¹ have been made of this nucleus at more forward scattering angles, little information has been given relevant to some of the transitions discussed in this paper.

The spectra resulting from 39- and 56-MeV bombardments of Ca, covering an excitation en-

ergy range from 0 to about 17 MeV, are presented in Figs. 1 and 2, respectively. The pronounced increase in intensities of the peaks at 5.9, 6.9, and 8.4 MeV in the 56-MeV spectrum relative to the corresponding intensities at 39 MeV is usually characteristic of $M2$ transitions with 180° electron scattering in this bombarding energy range. This behavior is to be contrasted with that of the 10.34-MeV peak which responds qualitatively as an $M1$ transition. The gap in the 39-MeV spectrum exists because the 56-MeV spectrum showed no significant peaks in this region.

As indicated in Table I, of the peaks observed at 5.94, 6.94, 8.43, and 10.34 MeV, quantitative analysis was undertaken only for the last two, for which values of the multipolarity L , the transition radius R , and the transition width Γ_0 are

TABLE I. Differential cross sections, spin and parity, transition radius, and radiation width for energy levels in ^{40}Ca and ^{32}S , including DWBA corrections.

Nucleus	Level energy (MeV)	$(d\sigma/d\Omega)_{56}$ (10^{-34} cm ² /sr)	$(d\sigma/d\Omega)_{39}$ (10^{-34} cm ² /sr)	J^π	R (fm)	Γ_0 (eV)
^{40}Ca	5.94 ± 0.07	34 ± 5	23 ± 12	$(1^-, 2^-)$		
	6.94 ± 0.07	38 ± 7	60 ± 19	(1^-)		
	8.43 ± 0.07	102 ± 12	119 ± 21	2^-	4.3 ± 0.5	$2.6^{+1.0}_{-0.8} \times 10^{-2}$
	10.34 ± 0.06	40 ± 13	129 ± 22	(1^+)	$3.5^{+0.4}_{-0.6}$	$7.0^{+2.9}_{-2.2}$
^{32}S	8.13 ± 0.07	38 ± 13	104 ± 31	1^+	$3.4^{+0.5}_{-0.9}$	$2.8^{+1.8}_{-1.4}$
	10.82 ± 0.07	48 ± 12	67 ± 37	1^+	$2.0^{+1.2}_{-2.0}$ ^a	$2.9^{+3.4}_{-1.4}$
				2^-	$5.0^{+0.7}_{-2.2}$	$7^{+8}_{-5} \times 10^{-2}$
	11.14 ± 0.07	54 ± 12	219 ± 46	1^+	3.9 ± 0.3	$18.9^{+7.4}_{-6.3}$
	11.62 ± 0.07	48 ± 13	123 ± 39	1^+	$3.4^{+0.5}_{-0.9}$	$9.7^{+6.1}_{-4.8}$

^a If the data and equations yield $R^2 < 0$, the lower limit of R is taken to be zero.

given. The uncertainties given for the measured cross sections are based solely on baseline uncertainties and counting statistics. Since R results from the ratio of two cross sections, the uncertainties in R are based on those for the cross sections. The radiation widths are given as absolute values; an additional uncertainty is included to allow for the absolute cross-section calibration and other factors.

The intensity of the peak at 5.94 MeV varies with bombarding energy qualitatively as if it corresponds to an $M2$ transition. This peak may arise from some combination of unresolved transitions to a 1^- level at 5.902 MeV and a possible 2^- level at 6.026 MeV observed with the reaction $^{39}\text{K}(^3\text{He}, d)^{40}\text{Ca}$.²² Under the circumstances discussed below for the 6.94-MeV transition, an $E1$ transition can simulate an $M2$ transition. Because of these ambiguities, values of Γ_0 and R for this transition are not given.

Several investigators²³⁻²⁶ find an electric dipole transition to a level at 6.94 MeV. Although transverse $E1$ transitions are not often observed in 180° electron scattering, in this case we may be exciting one. However, the observation of a transverse $E1$ transition can only be consistent with our results if it is either a T -forbidden or a spin-flip transition. This conclusion arises from the fact that the approximate measured q dependence of the cross section for this transition is q^4 . Such a dependence would appear to us as an $M2$ transition. Itoh, Oyamada, and Torizuka²¹ encountered the same problem with this transition which, to them at more forward angles, behaves as an $E3$ transition. On the other hand, since there seem to exist at least three closely spaced levels in this region,^{25, 27} there may be a chance that a 2^- state is among them. Gerace and Green,²⁸ using random-phase-approximation (RPA) techniques, suggest the presence of a 2^- level at about this energy. In any event, in view of the ambiguity associated with the observation of this peak, we have chosen not to give a value for Γ_0 or R .

That the peak at 8.43 MeV corresponds to an $M2$ transition is given some support by the work of Erskine²² who finds a possible 2^- level at 8.465 MeV. This level may be the $T=1$ analog of a 2^- level^{26, 29} at 800 keV in ^{40}K .

Leenhouts²⁷ observes a 2^- state at 9.65 MeV. Although we do see an indication of intensity in this region, our analysis is not sensitive enough to confirm this observation. It should also be mentioned that the calculations of Hill³⁰ indicate the presence of a 2^- , $T=1$, spin-isospin resonance at 9.27 MeV in ^{40}Ca .

Table I indicates an $M1$ transition to a level at 10.34 MeV. Although very little $M1$ strength is

expected in ^{40}Ca , occasionally such a transition does occur in doubly-closed-shell nuclei. For example, Stroetzel and Goldmann³¹ report a 1^+ level in ^{16}O at 16.21 MeV. Furthermore, Twin, Olsen, and Wang²⁹ report a 1^+ level in ^{40}K at 2.290 MeV which may be the analog of the level at 10.34 MeV in ^{40}Ca .

Actually, the existence of an $M1$ transition is not in itself as alarming as the strength ($\Gamma_0 = 7$ eV) of the transition. However, all of our analysis is based on the assumption that the observed transitions are magnetic. This has usually been a valid assumption, but there is always a chance that we could be observing a transverse electric quadrupole transition which simulates an $M1$ transition in behavior. If this is the case, we obtain an approximate value of $\Gamma_0 = 5$ eV. This value can only

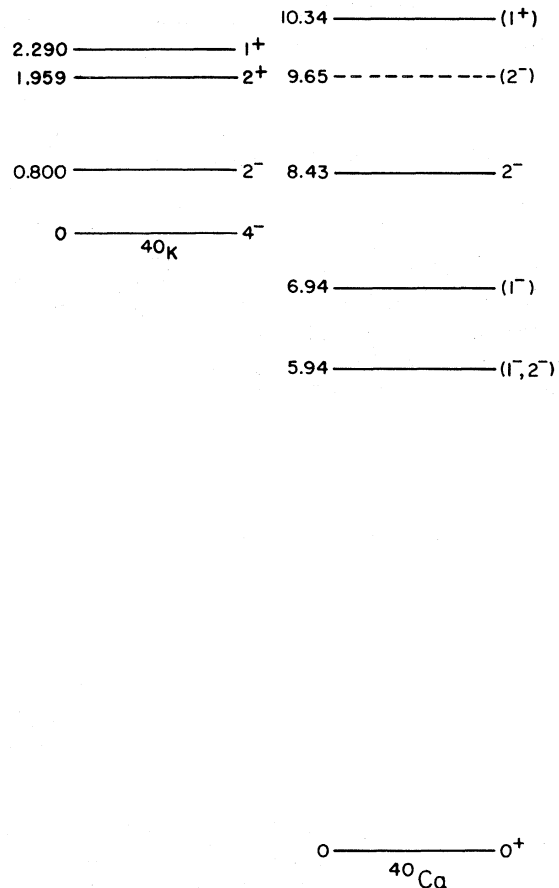


FIG. 3. Energy-level diagram of only those states in ^{40}Ca involved in transitions observed in this work and their analog states in ^{40}K (plus the ^{40}K ground state). The energy scales are adjusted so that the ^{40}K ground state is aligned with its analog state at 7.661 MeV in ^{40}Ca . The energies of the ^{40}K levels are obtained from Ref. 29. The level at 9.65 MeV is dashed to indicate our inconclusive observation of this transition.

be regarded as a rough estimate, since it was obtained using the DWBA corrections for a longitudinal $E2$ transition. The corrections for transverse $E2$ transitions are not yet available.

Some support for the 10.34-MeV transition being $E2$ may be derived from the existence of a possible 2^+ analog state in ^{40}K at 1.959 MeV (not as close energetically, however, as the 1^+ state at 2.290 MeV) which is shown in the simplified energy-level diagram in Fig. 3. This diagram also generally summarizes all of the foregoing remarks.

B. ^{32}S

Considerable study of this nucleus has been conducted by means of electron scattering at more forward angles.³²⁻³⁷ Earlier experiments^{12, 38} at 180° have also been reported, but were done with poorer resolution than with the work presented here. However, these two experiments did not suffer from the disadvantage of subtraction of another spectrum.

The spectrum resulting from a 56-MeV bombardment of CaS covering an excitation energy range from 0 to 14 MeV is presented in Fig. 4. We give this "molecular" spectrum to illustrate clearly that it is the Ca peaks that dominate the spectrum and not the anticipated peaks due to sulphur. The S spectrum at 56 MeV was obtained by subtracting the Ca spectrum from the above spectrum.

In Fig. 5 we show the net S spectrum from a 39-MeV bombardment resulting from a subtraction of a Ca from a CaS spectrum and covering an excitation energy range from 4 to 16 MeV. The gap from 0- to 4-MeV excitation exists, since preliminary data revealed no peaks in this region, as

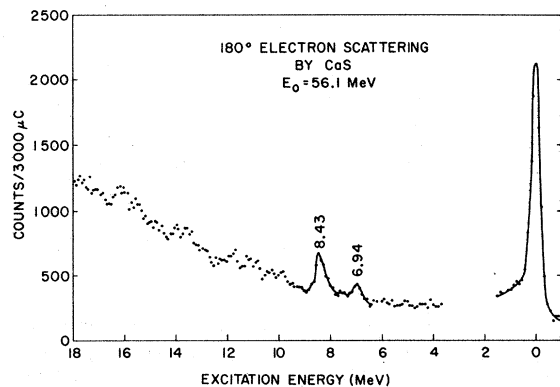


FIG. 4. Spectrum obtained by 180° scattering of 56.1-MeV electrons from CaS. The gap in the spectrum is present because a preliminary survey showed no structure of interest in this region.

is confirmed by the 56-MeV spectrum. Because of the subtraction, the statistical uncertainties are obviously greater than in the calcium spectra. However, peaks are observable at 8.13, 11.14, 11.62, 13.37, and 13.85 MeV. On analysis an additional peak (which appears more clearly in the S spectrum at 56 MeV) emerges at 10.82 MeV as the main contribution to the low-energy side of the peak at 11.14 MeV.

The four lowest-energy peaks were subjected to quantitative analysis, the results of which are included in Table I. The analysis for the 10.82-MeV transition yields ambiguous results which indicate a value of $L=1$ or 2 is possible. Results of analysis on the two peaks at about 13 MeV are not presented, since our estimate of the location of the baseline (elastic radiation tail, plus low-level inelastic intensity, plus instrumental effects) was considered too inaccurate for a meaningful determination of the peak area. These peaks could represent some additional $M1$ strength. However, some $M2$ transition strength could also be present, since such transitions have been observed^{2, 39} in this energy region in ^{24}Mg and ^{28}Si . Furthermore, Hill and Überall⁴⁰ predict the existence of a 2^- , $T=1$ spin-isospin resonance in ^{32}S at 13.5 MeV.

The fragmentation of the magnetic dipole strength is evident in Fig. 5. It is interesting to note that fragmentation also prevails in the ^{36}Ar spectra recently observed⁴¹ at this laboratory. As mentioned earlier, in the light of the concentration of $M1$ strength characteristic of our past work¹⁻³ with ^{20}Ne , ^{24}Mg , and ^{28}Si , this fragmentation has come as a surprise. This past work has been in general agreement with the theory of Kurath⁴ dealing with the self-conjugate nuclei in the p and $s-d$ shells. In particular, the theory predicts the strong concentration of $M1$ strength into the low-

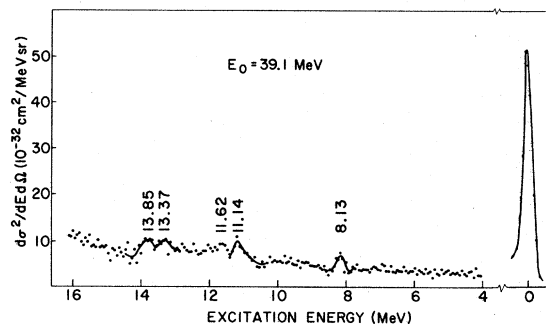


FIG. 5. Differential cross section for 180° scattering of 39.1-MeV electrons from sulphur. This spectrum is the result of a subtraction of a Ca spectrum from a CaS spectrum. The gap in the spectrum exists because a preliminary survey showed no structure of interest in this region.

est few $T=1$ states available for such transitions. This prediction for the s - d shell was based on a reasonable extrapolation of behavior in the p shell. However, a convincing theoretical study now seems needed to explain why the first three self-conjugate nuclei within the s - d shell manifest a strong concentration of $M1$ strength, while the last two show a marked fragmentation of this strength.

Kurath⁴ also gives a sum rule which is an approximate expression applicable to self-conjugate nuclei in the s - d shell. In the form most convenient for our use, it is given by:

$$\frac{0.614 \text{ MeV}}{-a} \sum_j \frac{\Gamma_{0j}}{\text{eV}} \left(\frac{10 \text{ MeV}}{\omega_{0j}} \right)^2 \cong \left\langle 0 \left| \sum_i \tilde{I}_i \cdot \tilde{s}_i \right| 0 \right\rangle, \quad (7)$$

where Γ_{0j} is the radiative width of the $M1$ transition from the excited state of energy ω_{0j} to the ground state. We take the coupling parameter to be $a = -2.03$ MeV, based on the 5.08-MeV state⁴² and the ground state of ^{17}O .

Using the values of Γ_0 given in Table I for the three $M1$ transitions and inserting them in Eq. (7), we obtain a value of 8.1 for the ls -coupling matrix element. It should be noted that we have not included in the sum the transition at 10.82 MeV, nor the ones in the 13-MeV region. We feel it is doubtful that the value given above represents all of the strength present. However, a value of about 6.0 can be given as a lower limit for this matrix element. Castel *et al.*⁴³ give values of 10.0 and 9.5 deduced from measured occupation numbers and projected Hartree-Fock calculations, respectively.

With the assumption that the ground state of the ^{32}S nucleus is axially symmetric and using the Nilsson model, values of $\langle \tilde{I} \cdot \tilde{s} \rangle$ can be calculated as a function of the deformation parameter η . The

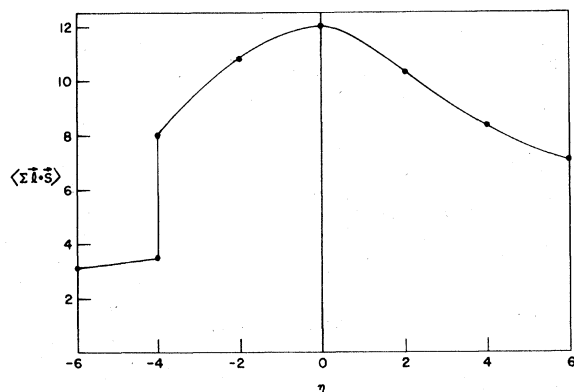


FIG. 6. The value of the ground-state ls -coupling matrix element of ^{32}S as a function of the deformation parameter η according to the Nilsson model. The discontinuity at $\eta = -4$ is due to crossing of Nilsson levels 8 and 9.

result of these calculations is presented in Fig. 6. It is clear from this curve, even assuming that the value for $\langle \tilde{I} \cdot \tilde{s} \rangle$ is probably somewhat larger than 8.1, that our results would be consistent with either a prolate or oblate deformation. Nakai *et al.*⁴⁴ and Hausser,⁴⁵ using Coulomb-excitation reorientation techniques, find the ^{32}S ground state prolate.

In this connection, it must be mentioned that the preliminary conclusions in an earlier report,⁴⁶ which arose from a qualitative inspection of the sulphur spectra are in error. Because of the poorer statistics attendant with the subtraction process coupled with a larger elastic peak than experienced in any of our previous work with lower- Z nuclei, the spectra deceptively appeared to be lacking in $M1$ strength. This qualitative result coupled with the use of Kurath's sum rule enabled us to suggest that, if the ^{32}S nucleus were sufficiently oblate, agreement with the Kurath theory would obtain. This result can be seen from Fig. 6

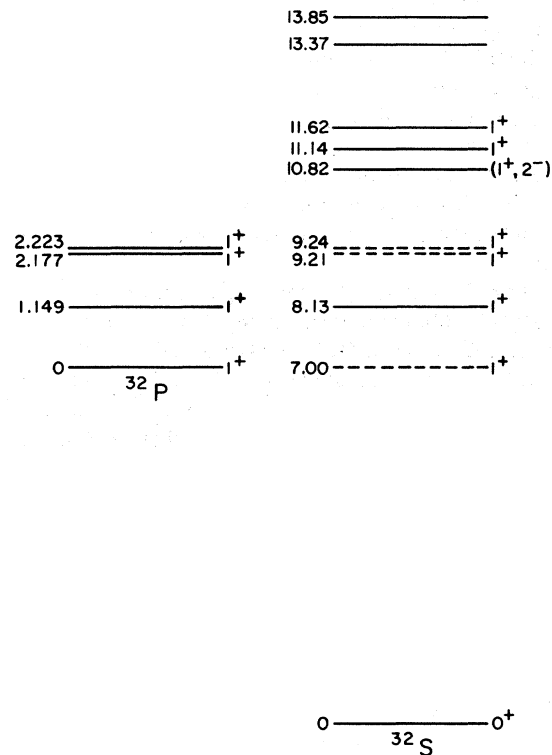


FIG. 7. Energy-level diagram principally of states in ^{32}S involved in transitions observed in this work and their analog states in ^{32}P . The energy scales are adjusted so that the ^{32}P ground state is aligned with its analog state at 7.00 MeV in ^{32}S . The energies of the ^{32}P levels are obtained from Ref. 47. The energies of the dashed ^{32}S levels (not observed by us) are also obtained from Ref. 47 but are given with only two decimal places.

where, because of the crossing of Nilsson levels 8 and 9, there is a discontinuity at $\eta = -4$ which renders the value of $\langle \vec{I} \cdot \vec{S} \rangle$ for $\eta < -4$ relatively low. However, our present analysis has invalidated such a suggestion.

Graue *et al.*⁴⁷ using the reaction $^{31}\text{P}(^3\text{He}, d)^{32}\text{S}$ have observed a 1^+ level in ^{32}S at 6.997 MeV and possibly others at 9.207 and 9.240 MeV. Gales *et al.*⁴⁸ using the reaction $^{31}\text{P}(p, \gamma)^{32}\text{S}$ confirm the presence of 1^+ states at 7.001 and 9.207 MeV. Both groups of investigators observe the 1^+ state at 8.13 MeV. Although there is some small indication of intensity in the 7.0- and 9.2-MeV regions of our spectra, it is insufficient to justify quantitative analysis. Their strength would probably add a small amount to the value of $\langle \vec{I} \cdot \vec{S} \rangle$ we have given above. It should be mentioned that Gales *et al.*⁴⁸ have observed decays from the 0^+ , $T=2$ state at 12.05 MeV to the 1^+ levels at 7.001, 8.126, and 9.207 MeV.

The state at 8.13 MeV is probably the $T=1$ analog^{47, 49} of the 1^+ level at 1.149 MeV in ^{32}P . The analogs of the ^{32}P ground, 2.177-, and 2.223-MeV states are probably the 1^+ states in ^{32}S at 6.997, 9.207, and 9.240 MeV, which as mentioned above we do not observe. This discussion is summarized in the energy-level diagram presented in Fig. 7.

It is of particular interest that we were not able

to observe the transition to the 7-MeV level. The analog β^- transition from the ^{32}P ground state is known to have an anomalously low transition probability ($\log ft = 7.9$). According to Primakoff⁵⁰ this situation could be of significance in terms of possible evidence for the existence of meson-exchange currents. In principle, if the ^{32}S γ -decay and ^{32}P β^- -decay matrix elements were known with sufficient accuracy, then, since the strictly nuclear parts of these matrix elements are small, the meson-exchange parts might be observable.

It is unfortunate that our results on ^{32}S are burdened with the handicap of the subtraction technique. It is our intention both from the standpoint of the 7-MeV transition as well as others in this nucleus to repeat this investigation when higher-resolution equipment now under test is in operation.

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Non-Normal Parity States in $A = 40$ and 41 Nuclei*

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The energy levels of the positive-parity states in ^{41}Sc (^{41}Ca) and ^{41}K are calculated by an effective particle-hole potential determined by a least-squares fit of the low-lying levels of the negative-parity states in ^{40}Ca . Some definite predictions are made on the spin-parity assignment for a few low-lying levels of ambiguous experimental spin-parity determination in ^{41}Sc .

1. INTRODUCTION

A least-squares fit to the low-lying energy levels of the non-normal-parity states in the $1f2p$ -shell nuclei by the standard Talmi procedure has been performed by Dieperink and Brussaard¹ with in the description of the $1f_{7/2}^n 1d_{3/2}^{-1}$ configurations ($n \geq 1$). For $A = 40$ and 41 nuclei, however, many more excited levels have been observed.²⁻⁵ For example, in ^{40}Ca two 1^- and two 3^- higher excited levels are well identified besides the lowest 2^- , 3^- , 4^- , and 5^- levels generally described by the $1f_{7/2} 1d_{3/2}^{-1}$ configuration.⁵ To account for these higher excited levels and, at the same time, to give a better fit for the lower-lying levels we need to include in the calculation more active orbits in the $1f2p$ and $1d2s$ shells. In this paper we discuss a calculation of the non-normal-parity states per-

formed in the configuration space including the single-particle states $1f_{7/2}$, $2p_{3/2}$, and $2p_{1/2}$, and single-hole states $1d_{3/2}$ and $2s_{1/2}$. We discuss only $A = 40$ and 41 nuclei, since the corresponding experimental data for heavier $1f2p$ -shell nuclei are scarce.

Since the spin-parity assignments for many low-lying levels in ^{41}Sc (or ^{41}Ca) have not been uniquely determined, our calculation is performed by the following procedure. An effective particle-hole potential is assumed and determined through a least-squares fit to the low-lying levels of $T = 0$ and $T = 1$ states in ^{40}Ca (Sec. 2). The resulting potential is then applied to calculate the energy levels of the positive-parity states of ^{41}Sc (or ^{41}Ca) and ^{41}K (Sec. 3). The most interesting result of our calculation is that a comparison of the calculated levels with the observed spectrum of ^{41}Sc (the spectrum of ^{41}Ca is less clear-cut) gives a