Study of Low-Lying States in ^{53}Fe by the Reaction ^{50}Cr($\alpha,n\gamma$)^{53}Fe†

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Low-lying states in ^{53}Fe have been studied via the reaction ^{50}Cr($\alpha,n\gamma$)^{53}Fe. A Ge(Li) γ spectrum was measured in coincidence with neutrons to identify low-lying states and their γ transitions. A threshold study with a Ge(Li) detector was made at bombarding energies between 5.6 to 10.0 MeV. The n-γ and the threshold studies together with γ-γ coincidences suggest the existence of states at 741±1-, 1328±1-, 1423±1-, and 1697±1-keV excitation energies. An n-γ delayed-coincidence measurement has been done for the 741-keV γ ray. The resulting mean life is $\tau = 92 \pm 2$ nsec.

I. INTRODUCTION

Relatively little experimental information exists regarding the excited states of the ^{53}Fe nucleus. The only available information on the levels of ^{53}Fe is the neutron-pickup studies ^{54}Fe(p,d)^{53}Fe [1] and ^{54}Fe(^{3}He, α)^{53}Fe,[2,3] and the γ decays of the 2.5-min isomer in ^{53}Fe.[4,5] Sherr *et al.*[1] identified four excited states in ^{53}Fe below 3.2 MeV and made spin and parity assignments to two of them. Trier *et al.*[2] reported three additional levels in ^{53}Fe and suggested tentative spin-parity $\frac{3}{2}^-$ for the 757-keV state. The estimated uncertainty for the energy levels in the neutron-pickup studies is ±15 keV. Eskola[4] reported additional energy levels obtained through the decay of the 2.5-min isomer at 3.04 MeV.

Properties of the ^{53}Fe nucleus are interesting with respect to shell-model theory, since it has two proton holes and one neutron hole in the doubly closed shell of ^{56}Ni. Several theoretical calculations[6-8] have been made for the energy levels of ^{53}Fe arising from the $(\pi f_{7/2})^{-2}(\nu f_{7/2})^{-1}$ configuration. Calculations of energy levels in ^{53}Fe arising from configurations other than $1f_{7/2}$ have not been reported.

The purpose of the present experiment is to study low-lying states in ^{53}Fe and their electromagnetic transitions via the reaction ^{50}Cr($\alpha,n\gamma$)^{53}Fe. This work presents: (1) a γ-ray spectrum in coincidence with neutrons which isolates γ transitions in ^{53}Fe; (2) a study of the reaction ^{50}Cr($\alpha,n\gamma$)^{53}Fe thresholds in singles and a γ-γ coincidence measurements; and (3) an n-γ delayed-coincidence measurement.

II. EXPERIMENTAL METHOD AND RESULTS

A. ^{50}Cr($\alpha,n\gamma$); Singles and Neutron-Coincidence γ Spectra

The low-lying states of ^{53}Fe as found by various studies are shown in Fig. 1. In order to determine which states are significantly populated in the reaction ^{50}Cr($\alpha,n\gamma$), a preliminary investigation was made of singles spectra with a Ge(Li) γ-ray detector. The γ rays were produced by bombarding a thick layer of $^{50}Cr_2O_3$ (which stopped the beam) on Bi or Ta backing, with 10.0-MeV α particles. γ rays were detected with a 15-cm^3 Ge(Li) detector placed at 90° to the beam direction. The detector energy resolution was approximately 2.6 keV for the 1.332-MeV ^{60}Co γ ray. Figure 2 shows the resulting γ-ray spectrum. For the γ-energy measurement a run was taken with the beam on together with standard sources.

Figure 2 shows γ-rays singles spectrum taken at $E_\alpha = 10.0$ MeV. The energies of the prominent γ rays are listed in Table I together with the corresponding nuclei and the levels involved in the transitions. Most of the prominent γ rays correspond to γ transitions in ^{53}Mn and they have been observed previously by means of the ^{52}Cr(p,γ)[9] and ^{53}Cr($p,n\gamma$)[10] reactions with the use of Ge(Li) detectors. The 783-keV γ ray corresponds to the γ decay of the first excited state in ^{50}Cr; it is populated strongly via the ^{50}Cr(α,α') reaction. The 1011- and 1328-keV γ rays were observed previously in the decay of the 3.04-MeV isomer in ^{53}Fe.

In order to isolate the electromagnetic transitions in ^{53}Fe, the γ spectrum in coincidence with neutrons was also measured. The neutron detector was a 2×2-in NE 213 liquid scintillator coupled to an RCA 8575 photomultiplier tube. Its axis was placed at 0° to the beam direction. An appropriate absorber was placed in front of the neutron detector to attenuate low-energy γ rays. Neutron-γ pulse-shape discrimination was used to select the neutrons.[11] The neutron-induced proton recoils produce a larger amount of long-lived components than do γ-induced Compton electrons. The time difference between the initial fast component as observed at the anode and the bipolar crossover of the time-integrated linear pulse from a dynode

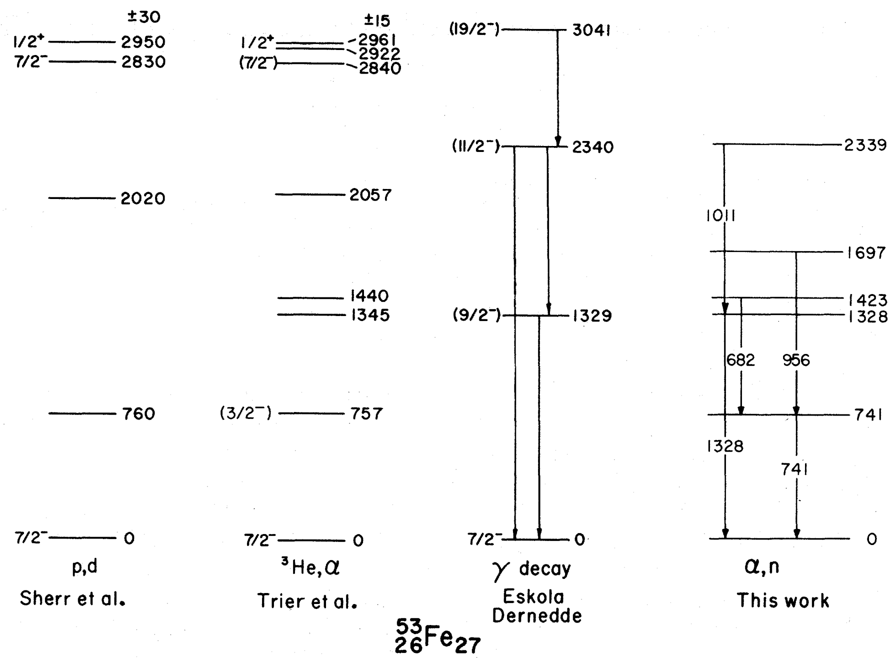

FIG. 1. States in ^{53}Fe excited by the reaction ^{54}Fe$(p,d)^{53}$Fe (Ref. 1), the reaction ^{54}Fe$(^{3}$He$,\alpha)^{53}$Fe (Ref.2), and the γ decay from the 2.5-min–3.04-MeV isomeric state (Refs. 4 and 5) compared with the present ^{50}Cr$(\alpha,n\gamma)^{53}$Fe data.

is thus different for neutrons and γ rays. A measurement of this time difference for each pulse with a time-to-amplitude converter (TAC) allows the selection of only neutrons. The neutron-γ pulse-shape discriminator is shown in the right part of Fig. 3. The time-difference spectrum, taken for ^{50}Cr + α at $E_\alpha = 10$ MeV, is displayed in Fig. 4.

The Ge(Li) γ spectrum taken in coincidence with neutrons for the reaction ^{50}Cr$(\alpha, n\gamma)^{53}$Fe is shown in the lower part of Fig. 5. A singles γ-ray spectrum for ^{50}Cr + α is shown for comparison purposes

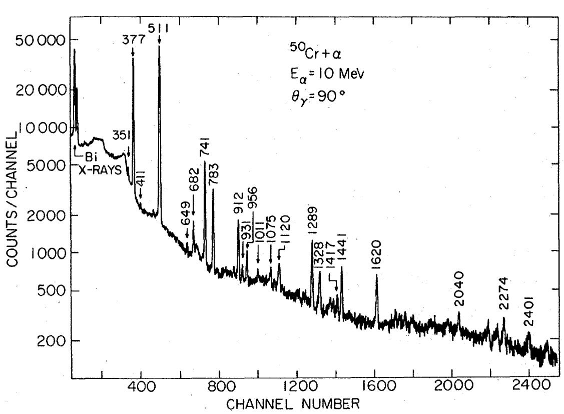

FIG. 2. γ-ray singles spectrum obtained at bombarding energy $E_\alpha = 10.0$ MeV. The Ge(Li) detector was placed at 90° to the beam direction. The γ energies in keV units are written above the peaks.

in the upper portion of Fig. 5 with identical gain and dispersion. For this neutron coincidence measurement, the random events were measured and subtracted from the spectrum. A coincidence resolution time of ~70 nsec was used in this measurement. The lower part of Fig. 5 shows five prominent peaks in the energy region up to 2000 keV; their energies are 351, 682, 741, 956, and 1328 keV.

B. Threshold Study of ^{50}Cr$(\alpha, n\gamma)$ and γ Spectrum in Coincidence with the 741-keV γ Ray

Five prominent γ rays have been observed in coincidence with neutrons. In order to check if these γ rays correspond to levels in ^{53}Fe a threshold study has been carried out. The 351-keV γ-ray peak appears also at a bombarding energy of $E_\alpha = 5.60$ MeV which is below the threshold to a 351-keV excited state in ^{53}Fe; therefore, the 351-keV γ ray is not likely to correspond to a γ transition in ^{53}Fe. It is populated with a relatively large yield at the low bombarding energy of $E_\alpha = 5.6$ MeV; thus, because of the Coulomb barrier, it probably corresponds to a γ transition of a medium or light contaminant ($A < 70$).

Figure 6 shows nine γ spectra of ^{50}Cr + α at bombarding energies of 5.84 to 10.0 MeV. The 741-keV γ ray appears in the γ spectrum taken at 6.90 MeV, while none of the 682-, 956-, and 1328-keV γ rays are seen. At higher bombarding energies the 682-, 956-, and 1328-keV γ rays are also observed as shown in Fig. 6. The 741-keV γ ray has a large yield at $E_\alpha = 9.5$ MeV as shown in Fig. 6; a γ spectrum gated by the 741-keV γ ray was measured at this energy using Ge(Li) detectors. The contribution of Compton events below the 741-keV peak has been measured by gating with a slightly higher γ window and has been subtracted from the spectrum. Figure 7 shows the resulting γ spectrum which shows that the 682- and 956-keV γ rays are in coincidence with the 741-keV γ ray. A coincidence resolution time of ~80 nsec was used in this measurement.

C. Neutron-γ Timing Measurement with the 741-keV γ Ray

The lifetime associated with the 741-keV γ ray was measured by an n-γ delayed-coincidence technique. The measurement was carried out at $E_\alpha = 10.0$ MeV using a 2×2-in. NaI scintillator placed at 90° for detection of the γ rays. Neutrons were detected by the NE 213 scintillator placed at 0°. Fast-timing signals from the NE 213 and NaI detectors were obtained from the anodes. These timing signals start and stop the TAC 2 as indicated in Fig. 3. Time calibration was made with air-dielectric trombone delay lines.

Slow-coincidence requirements were used in conjunction with the fast timing to isolate the excited state of interest and to minimize the time resolution. For the neutron detector single-channel analyzer (SCA) 1 was adjusted to accept pulses in the neutron energy region 1–3 MeV, and SCA 2 of the neutron discriminator was set to accept from TAC 1 a narrow pulse-height region appropriate to neutrons, as shown in Fig. 4. For the γ detector, SCA 3 and 4 were both fed by its amplifier (AMP 2); SCA 3 accepted only the photopeak of the 741-keV γ ray in its pulse-height window, and SCA 4 accepted only a region above the 741-keV photopeak and includes the 956-keV photopeak. The outputs of SCA 3 and 4 were also used to route the corresponding time-delay spectra into different halves of a multichannel analyzer. The resulting time spectrum for the 741-keV region is shown in Fig. 8. From several least-squares fits to different portions of the logarithmic slope the extracted mean life is $\tau = 92 \pm 2$ nsec. The time spectrum which was associated with the higher window did not show a long-lived component. Therefore, the 92-nsec mean lifetime is associated with the 741-keV γ ray. The prompt-resolution function is indicated by the narrow peak at zero time; it was produced by Compton events of higher-energy γ rays that enter the pulse-height window. The time

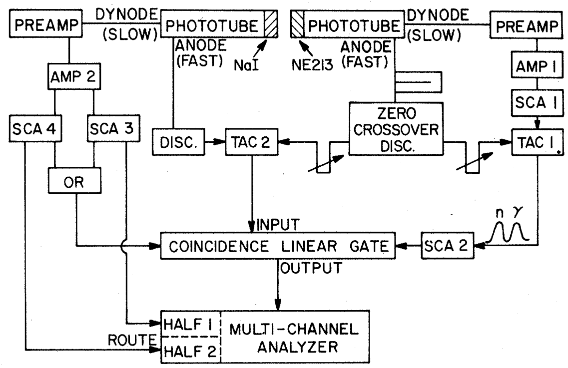

FIG. 3. A schematic diagram of the experimental electronics for the n-γ delayed-coincidence measurement.

TABLE I. Prominent γ rays from ^{50}Cr+α at $E_\alpha = 10.0$ MeV.

E_γ [a] (keV)	Nucleus	$J^\pi{}_i \to J^\pi{}_f$	E_i (keV)	E_f (keV)
377	^{53}Mn	$\frac{5}{2}^- \to \frac{7}{2}^-$	377	0
682	^{53}Fe		1423	741
741	^{53}Fe	$\frac{3}{2}(\frac{5}{2}) \to \frac{7}{2}^-$	741	0
783	^{50}Cr	$2^+ \to 0^+$	783	0
912	^{53}Mn	$\frac{3}{2}^- \to \frac{5}{2}^-$	1289	377
956	^{53}Fe		1697	741
1011	^{53}Fe	$(\frac{11}{2}^-) \to (\frac{9}{2}^-)$	2339	1328
1120	^{53}Mn	$\frac{3}{2}^- \to \frac{3}{2}^-$	2409	1289
1289	^{53}Mn	$\frac{3}{2}^- \to \frac{7}{2}^-$	1289	0
1328	^{53}Fe	$(\frac{9}{2}^-) \to \frac{7}{2}^-$	1328	0
1441	^{53}Mn	$\frac{11}{2} \to \frac{7}{2}^-$	1441	0
1620	^{53}Mn	$\frac{9}{2} \to \frac{7}{2}^-$	1620	0
2274	^{53}Mn	$\frac{5}{2} \to \frac{7}{2}^-$	2274	0

[a] The γ-ray energies are accurate to within 1 keV.

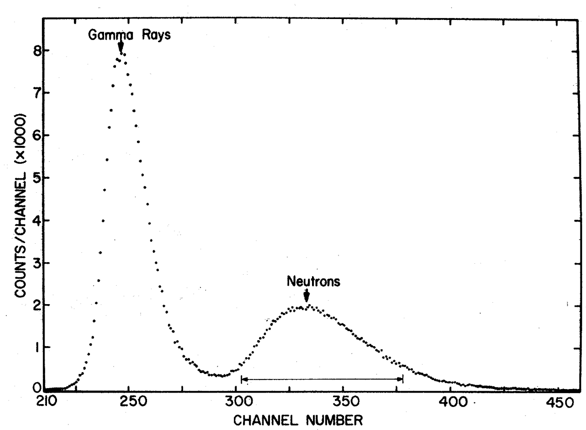

FIG. 4. Pulse-height spectrum from TAC 1 showing discrimination spectrum of neutron and γ-ray pulses from the NE 213 liquid scintillator.

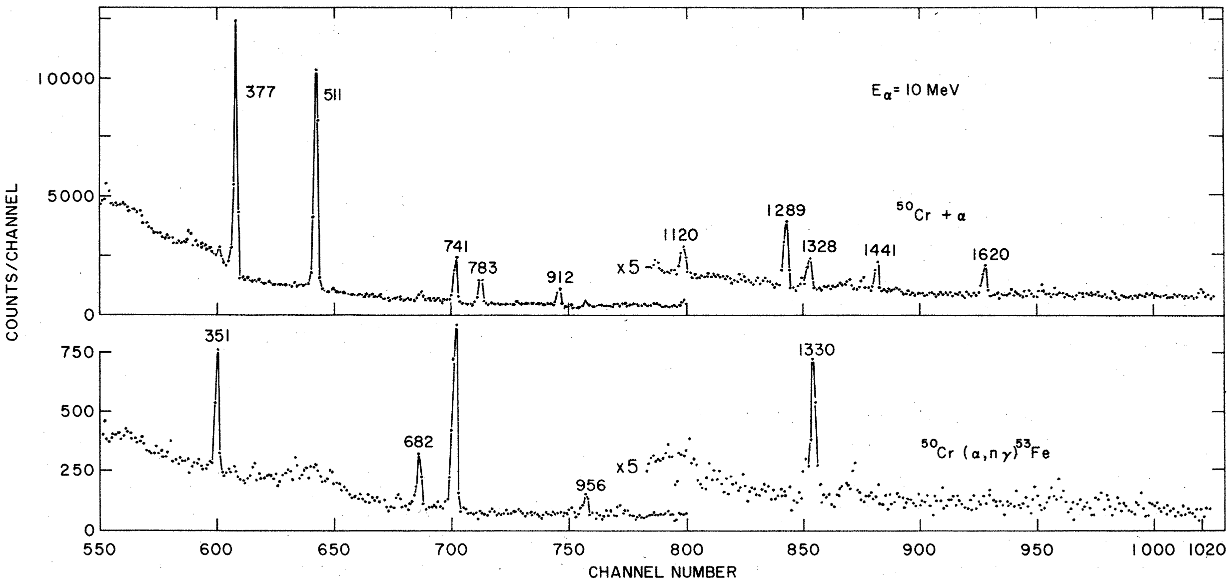

FIG. 5. The upper part is γ-ray singles spectrum from 120 to 2000 keV for E_α=10.0 MeV and θ_γ =90°. The lower part shows γ rays observed in coincidence with neutrons for a resolving time of ~70 nsec.

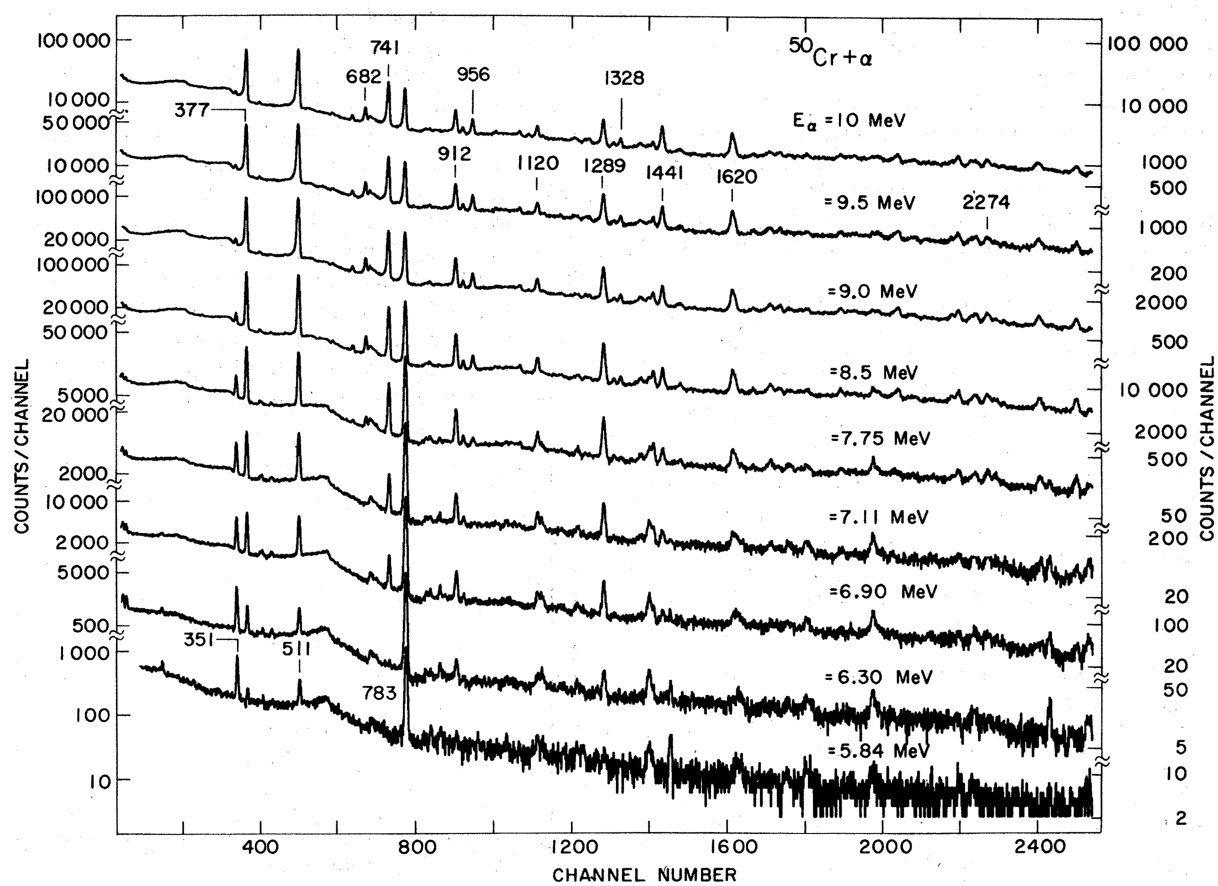

FIG. 6. γ-ray singles spectra from ^{50}Cr+α obtained at bombarding energies of 5.84, 6.30, 6.90, 7.11, 7.75, 8.50, 9.00, 9.50, and 10.00 MeV. The Ge(Li) detector was placed at 0° to the beam direction. The γ-ray energies in keV units are written above some of the peaks.

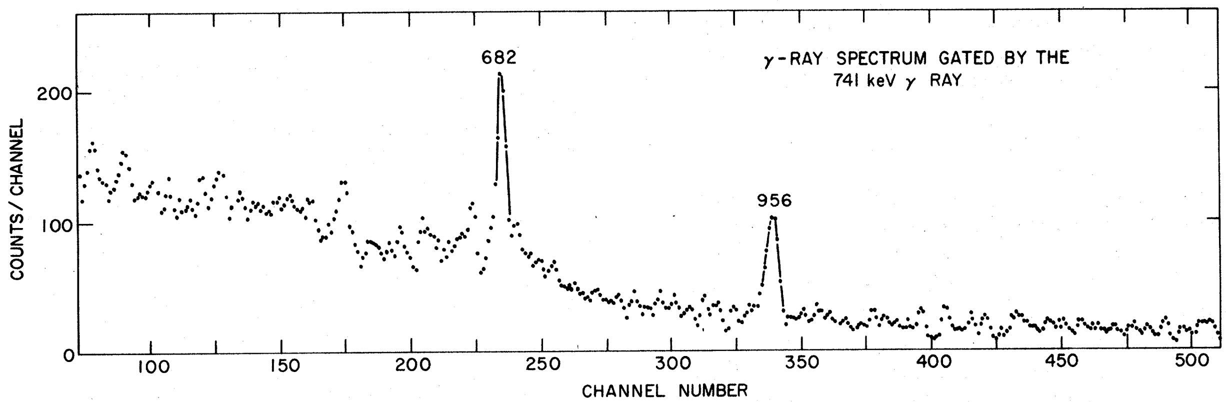

FIG. 7. A Ge(Li) γ spectrum gated by the 741-keV γ ray. Coincidence events due to Compton γ rays which fell under the 741-keV peak have been subtracted. The coincidence resolving time was ~80 nsec.

spectrum for the higher γ window gives an upper limit of $\tau < 2$ nsec for the mean life associated with the 956-keV γ ray.

III. DISCUSSION AND CONCLUSIONS

In this experiment, the mean life associated with the 741-keV γ ray of ^{53}Fe has been measured to be $\tau = 92 \pm 2$ nsec. The n-γ and γ-γ coincidence study together with the threshold measurement suggests the existence of states at 741 ± 1-, 1328 ± 1-, 1423 ± 1-, and 1697 ± 1-keV excitation energies. The levels at 1328, 1423, and 1697 keV have mean lives of $\tau < 0.4$ μsec as determined from the resolving time for the neutron-γ coincidence measurement. The detailed discussion of the 741-, 1328-, 1423-, and 1697-keV states will be given below.

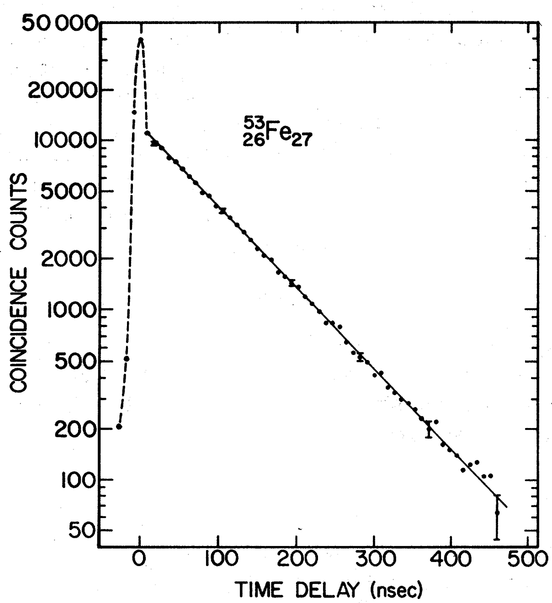

FIG. 8. The experimental decay curve for the 741-keV γ in ^{53}Fe is shown by the dots and the solid line. The n-γ time-delay spectrum was obtained relative to neutrons from the reaction ^{50}Cr$(\alpha,n)^{53}$Fe. The slope on the right side of the decay curve corresponds to a mean life $\tau = 92 \pm 2$ nsec. The sharp peak which represents the time-resolution function is a result of prompt events that satisfy the energy requirements.

A. 741-keV Level

This state was found to decay to the ground state via a 741-keV γ ray. A mean life of $\tau = 92 \pm 2$ nsec was associated with this γ ray. Black, McHarris, and Kelly[12] did not observe any γ transition to this state from the $(\frac{19}{2}^-)$ 3041-, $(\frac{11}{2}^-)$ 2340-, and $(\frac{9}{2}^-)$ 1328-keV states; therefore, it is likely to have a spin $j \le \frac{5}{2}$. A spin $\frac{1}{2}$ is excluded by the lifetime result, since the expected lifetime for an $L = 3$ transition is $\tau \gtrsim 50$ μsec. According to the calculation of Scholz and Malik[6] one would expect in this nucleus a $\frac{3}{2}^-$ state in the energy region 700–800 keV which is the right energy for this state. The 741 ± 1-keV level can be associated with the $(\frac{3}{2}^-)$ 757 ± 15-keV state observed in the neutron-pickup reactions by Sherr *et al.*[1] with the ^{54}Fe(p, d) reaction and by Trier *et al.*[2] with the ^{54}Fe$(^3$He$, \alpha)$ reaction. However, for a $\frac{3}{2}^-$ 741-keV state one would expect a mean life of $\tau < 0.1$ nsec. In this case, the lifetime is hindered by a factor of >900; a large hindrance for the $E2$ transition was also observed in the isotone ^{51}Cr in the $\frac{3}{2}^-$, 748-keV $\rightarrow \frac{7}{2}^-$, ground-state γ transition.[13] It is thus feasible to assign the present level as a $\frac{3}{2}^-$ state.

Another possible explanation is that the 741-keV state has a spin $\frac{3}{2}^+$ which requires an $M2$ transition to the ground state and is consistent with the mean life $\tau = 92 \pm 2$ nsec for the 741-keV state. A $\frac{3}{2}^+$ hole state is not expected to be populated directly with large yield at low bombarding α energies; the yield could be explained by a cascade from the 757-keV $(\frac{3}{2}^-)$ state. However, the transition $\frac{3}{2}^- \rightarrow \frac{7}{2}^-$ in ^{41}Ca and ^{47}Sc is preferred over the $\frac{3}{2}^\pm \rightarrow \frac{3}{2}^\mp$ $E1$

transitions, while here a 757-keV $\frac{3}{2}^- \rightarrow \frac{7}{2}^-$ γ transition is not observed. In any case, a spin assignment of $\frac{3}{2}$ $(\frac{5}{2})$ is suggested for the 741-keV level.

B. 1328-keV Level

As shown in Fig. 1, 1328- and 1011-keV γ rays were observed[4,5] in the decay of the 2.5-min isomer of ^{53}Fe. The order of these γ rays was not established by these studies.[4,5] The yield of the 1328 ± 1-keV γ ray is larger than that of the 1011-keV γ ray; therefore it is likely from the present study to correspond to a transition from a 1328-keV state, and thus it establishes the order of the γ rays in the cascade. This is in agreement with a recent study[12] which has shown a weak γ transition from the 3041-keV isomer to this level. The tentative spin assignment of this state made by previous studies[4,5] is $\frac{9}{2}^-$, this is consistent with the relatively weak yield observed at low bombarding α energies. The yield of the 1011-keV γ ray is smaller; this is in accord with the higher threshold energy and the high spin, $\frac{11}{2}^-$, of the 2339-keV state.

C. 1423-keV Level

The γ-γ coincidence study and the threshold measurement suggest that the 682-keV γ ray corresponds to a 1423 → 741-keV γ transition. Thus, the n-γ timing measurement with the 741-keV γ ray gives some information on the lifetime of the 1423-keV state, since the neutron window includes neutrons populating this state. No long-lived component besides $\tau = 92$ nsec is observed in the decay curve shown in Fig. 6, which suggests $\tau(1423) < 10$ nsec. Black, McHarris, and Kelly[12] did not observe γ transitions from the $(\frac{19}{2}^-)$ 3041- and $(\frac{11}{2}^-)$ 2340-keV states to the 1423-keV state. In the present study no γ decay from this state to the ground state has been observed [see Fig. 5(b)]. Therefore, it is likely to have a spin $J \leq \frac{3}{2}$, which is consistent with the fact that it decays to the $\frac{3}{2}$ $(\frac{5}{2})$ 741-keV state and has a lifetime limit of $\tau < 10$ nsec.

D. 1697-keV Level

The γ-γ coincidence study together with the threshold measurement suggests that the 956-keV γ ray corresponds to a 1697 → 741-keV γ transition. The n-γ timing measurement with the 956-keV γ ray gives an upper lifetime limit of $\tau < 2$ nsec. No γ transition from this state to the ground state has been observed in the γ spectrum in coincidence with neutrons. This together with the fact that this state decays mainly to the $\frac{3}{2}$ $(\frac{5}{2})$ 741-keV state and the lifetime limit $\tau < 2$ nsec suggest a spin $J \leq \frac{3}{2}$ for the 1697-keV state.

It should be noted that of the four excited states at 741, 1328, 1423, and 1697 keV, only the $(\frac{9}{2}^-)$ 1328-keV state is associated with the $(\pi f_{7/2})^{-2}(\nu f_{7/2})^{-1}$ configuration according to shell-model calculations of McCullen, Bayman, and Zamick,[7] and Vervier.[8] The $\frac{3}{2}$ $(\frac{5}{2})$ 741-keV state can be associated with the low $\frac{3}{2}^-$ state predicted by the Coriolis-coupling-model calculation of Scholz and Malik.[6] However, the 1423- and 1697-keV states with spins $J \leq \frac{3}{2}$ are not predicted with this model, and probably correspond to configurations including orbits other than $1f_{7/2}$.

In summary, the present study suggests the existence of states at 741, 1328, 1423, and 1697 keV. A spin assignment of $\frac{3}{2}$ $(\frac{5}{2})$ was suggested for the 741-keV level. Spins of $J \leq \frac{3}{2}$ are likely for the 1423- and 1697-keV states. A lifetime of $\tau = 92 \pm 2$ nsec was associated with the 741-keV γ ray. However, more information is needed, such as: (a) a measurement of the neutron energy spectrum by the time-of-flight technique to check if there is another state slightly above the 741-keV level; and (b) a conversion-electron measurement to obtain more definite spin-parity parameters for the 741-, 1423-, and 1697-keV states.

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[1] R. Sherr, B. F. Bayman, E. Rost, M. E. Rickey, and C. G. Hoot, Phys. Rev. 139, B1272 (1965).

[2] A. Trier, L. Gonzalez, J. Rapaport, T. A. Belote, and W. E. Dorenbusch, Nucl. Phys. A111, 241 (1968).

[3] A. G. Blair and H. E. Wegner, Phys. Rev. 127, 1233 (1962).

[4] K. Eskola, Phys. Letters 23, 471 (1966).

[5] I. Dernedde, Z. Physik 216, 103 (1968).

[6] W. Scholz and F. B. Malik, Phys. Rev. 147, 836 (1966); 150, 919 (1966).

[7] J. D. McCullen, B. F. Bayman, and L. Zamick, Phys. Rev. 134, B515 (1964).

[8] J. Vervier, Nucl. Phys. A103, 222 (1967).

[9] S. Sterner, L. Jonsson, and S. E. Arnell, Arkiv Fysik 31, 567 (1966); R. L. Auble and M. N. Rao, Nucl. Data B3(Nos. 5, 6), 127 (1970); S. Maripuu, Nucl. Phys. A149, 593 (1970).

[10]M. T. McEllistrem, K. W. Jones, and D. M. Sheppard, Phys. Rev. C 1, 1409 (1970).
[11]M. L. Roush, M. A. Wilson, and W. F. Hornyac, Nucl. Instr. Methods 31, 112 (1964).
[12]J. N. Black, W. C. McHarris, and W. H. Kelly, Phys. Rev. Letters 26, 451 (1971).
[13]W. Haar and F. W. Richter, Z. Physik 231, 1 (1970).

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Magnetic Transition Strength in ^{32}S and ^{40}Ca from 180° Electron Scattering

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The magnetic excitation structures of ^{32}S and ^{40}Ca have been studied by 180° inelastic electron scattering from natural targets of CaS and Ca at energies of 39 and 56 MeV. The significant feature of the ^{32}S spectrum is the fragmentation of the $\Delta T = 1$ magnetic dipole strength into transitions to several levels in contrast to the strong concentration into a very few that is observed in ^{20}Ne, ^{24}Mg, and ^{28}Si. In the ^{32}S spectra excitation peaks are observed at 8.13, 10.82, 11.14, 11.62, 13.37, and 13.85 MeV. Analyses of the peaks at 8.13, 11.14, and 11.62 MeV yield $M1$ ground-state transition widths Γ_0 of 2.8, 18.9, and 9.7 eV, respectively. Peaks in the ^{40}Ca spectra are observed at 5.94, 6.94, 8.43, and 10.34 MeV. The peaks at 8.43 and 10.34 MeV are found to be due to an $M2$ transition, $\Gamma_0 = 2.6\times10^{-2}$ eV, and a possible $M1$ transition, $\Gamma_0 = 7.0$ eV, respectively. The analyses of the remaining peaks in the ^{32}S and ^{40}Ca spectra are discussed. The ^{32}S results are examined in terms of the theory of Kurath.

I. INTRODUCTION

At this laboratory a systematic study has been made of the strong $\Delta T = 1$ magnetic multipole transitions in the self-conjugate nuclei of the s-d shell. Our studies of those nuclei thus far reported,[1-3] ^{20}Ne, ^{24}Mg, and ^{28}Si, have revealed in each case very strong magnetic dipole transitions to $T = 1$, 1^+ excited states in the 10-MeV region. These results have been generally consistent with the theory of Kurath[4] who indicated that such nuclei should exhibit a strong concentration of the magnetic dipole strength into transitions to the lowest few of these states. However, in marked contrast with these three nuclei, the ^{32}S spectra show considerable fragmentation of this strength. The measurements of multipolarity L, radiation width Γ_0, and transition radius R are discussed for those transitions in ^{32}S and ^{40}Ca corresponding to the peaks that were observable. These results are interpreted in terms of the theory of Kurath and also from the point of view of analog resonances.

II. THEORETICAL CONSIDERATIONS

A discussion of the relevant theory has been given in several earlier reports,[1-3,5,6] but will be again outlined here as an aid in understanding a variation in the application of the theory used for the first time in this paper and explained below. The experimental results are analyzed using the model-independent plane-wave Born-approximation (PWBA) results of Rosen, Raphael, and Überall.[7] A correction is made in the analysis for distorted-wave Born-approximation (DWBA) effects using the ratios of DWBA to PWBA cross sections for $M1$ and $M2$ transitions reported by Chertok, Johnson, and Sarkar.[8] The differential cross section for 180° magnetic scattering is given by [Ref. 7, Eq. (5)]

$$\left(\frac{d\sigma}{d\Omega}\right)_{180°} = \frac{\pi\alpha}{[(2L+1)!!]^2}\;\frac{L+1}{L}\;\frac{q^{2L}}{k_1^{\,2}}\;B(ML,q)\,, \tag{1}$$

where L is the multipolarity of the induced transition, q the momentum transfer, k_1 the incident electron momentum, and $B(ML,q)$ the reduced transition matrix element. The latter can be expressed as

$$B(ML,q) = B(ML,O)G^2(L,q,R)\,, \tag{2}$$

where the q dependence is in G, and R is the transition radius. Rosen, Raphael, and Überall[7] evalu-