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Exchange Effects in $He^3 + He^4$ Scattering*

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The elastic scattering of He^3 by He^4 at 44.5 MeV (c.m.) is considered within the resonating-group formalism. The existence of open reaction channels is taken into account approximately by the introduction of a phenomenological local imaginary potential into the usual resonating-group formulation of the scattering problem. By choosing the shape and strength of this potential in a reasonable manner, it is found that a fairly good fit to the experimental differential cross section can be obtained in both the forward and the backward directions. From this study it is also found that the contribution to the differential cross section at backward angles comes mainly from exchange processes which are contained in the resonating-group formalism through the use of a totally antisymmetrized wave function. The real part of the phase shift is found to have a distinct odd-even feature, and from this it is shown that if one wishes to describe the experimental scattering data by using a complex local potential, then the real part of this potential must have a significant amount of Majorana space-exchange component.

I. INTRODUCTION

The resonating-group method in the one-channel approximation has been used to describe the elastic scattering of He^3 by He^4.[1-8] In the low-energy region close to the reaction threshold, it was found that the phase shifts calculated with this method are in good agreement with those determined from a phenomenological analysis[9] of the experimental data. At higher energies, where a large number of reaction channels are open, one expects that the one-channel calculation will become less valid. Indeed, at an energy of 18 MeV[10] a comparison between the calculated and experimental differential cross sections showed that the calculated positions of the diffraction maxima and minima agree well with the experimental positions, but that the calculated magnitude is too large.[3] This indicates that at these energies the influence of reaction channels on the elastic scattering cross section becomes quite important, and an approximate way of taking these channels into account must be incorporated into the resonating-group formalism.

In a recent investigation of the $\alpha+\alpha$ system[11] a local phenomenological imaginary potential was introduced into the resonating-group formalism. By choosing the energy-independent geometry parameters of this potential in a reasonable manner and by varying the strength with energy, it was found that the resultant complex phase shifts agreed quite well with the empirical shifts of Darriulat *et al.*[12] at energies from 24 to 60 MeV. This suggests that in the energy region where a number of reaction channels exist, such a simple description may in fact be quite adequate. In the present calculation we shall adopt this procedure to exam-

ine the He^3+He^4 scattering problem at 44.5 MeV, where the differential cross section in the angular region from 20 to 160° has recently been measured by Fetscher *et al.*[7]

We have chosen to analyze this particular data for two reasons. First, the energy is high enough so that the probability that narrow resonance levels might exist in this energy region is small. This gives us reason to believe that our calculation, which employs a macroscopic description of the reaction channels and omits spin-orbit effects by the adoption of a central nucleon-nucleon potential, could yield a reasonable description of the experimental data. Second, the exchange effects, which are present in our formulation through the use of a totally antisymmetrized wave function, will be particularly evident. At such an energy the direct potential, which is the only nuclear interaction between the two clusters if antisymmetrization is not carried out and which is of the form commonly employed for the real central part of the usual optical potential,[13] will yield a scattering amplitude which is very small in backward directions. Thus, in these directions, the main contribution to the scattering amplitude will come from exchange processes, and any omission of these processes would cause a serious discrepancy between theory and experiment.[14,15] In fact it has been found by Fetscher *et al.*[7] that the use of the usual optical model at this energy is entirely inadequate to describe the differential cross section at backward angles.[16] In this paper we shall attempt to demonstrate the importance of the exchange processes in a straightforward and transparent manner.

It should be mentioned that Fetscher *et al.*[7] have also analyzed their data by transforming the real phase shifts obtained from a one-channel resonating-group calculation into complex quantities through the introduction of an arbitrary absorption factor and obtained qualitatively successful results. As will be seen below, our results agree better with experiment simply because the complex phase shifts in our calculation arise from the introduction of an imaginary potential which, though phenomenological, does have some logical foundation.

In Sec. II, we shall present a brief formulation of this problem. Section III is devoted to a presentation of the results and a discussion of the exchange contributions. Finally, in Sec. IV, a conclusion from this investigation is given.

II. FORMULATION

The formulation of the He^3+He^4 problem with no imaginary potential was discussed in Refs. 1 and 3, and the full details of the formulation will not be repeated here. With the introduction of a local imaginary potential $iW(r)$, the function $F(\vec{r})$, which describes the relative motion of the two clusters, satisfies the integrodifferential equation[17]

$$\left[\frac{\hbar^2}{2\mu}\nabla^2+E-V_D(r)-V_C(r)-iW(r)\right]F(\vec{r}) = \int K(\vec{r},\vec{r}')F(\vec{r}')d\vec{r}', \tag{1}$$

where μ is the reduced mass, E is the total kinetic energy of the two clusters at large separation in the c.m. system, and $V_D(r)$, $V_C(r)$, and $K(\vec{r},\vec{r}')$ are the direct nuclear potential, the direct Coulomb potential, and the kernel function for the nonlocal interaction, respectively.

The quantities $V_D(r)$, $V_C(r)$, and $K(\vec{r},\vec{r}')$ are derived using a nucleon-nucleon potential of the form

$$V_{ij}=-V_0e^{-\kappa r_{ij}^2}(w-mP_{ij}^{\sigma}P_{ij}^{\tau}+bP_{ij}^{\sigma}-hP_{ij}^{\tau})+\epsilon_{ij}\frac{e^2}{r_{ij}}, \tag{2}$$

where the values of the various parameters are given in Ref. 3. When this nucleon-nucleon potential is used and if the wave function describing the system is not antisymmetrized with respect to the exchange of nucleons between the He^3 and He^4 clusters, then the nonlocal interaction represented by the right side of Eq. (1) will vanish, and the only nuclear interaction between the two clusters will be the direct potential $V_D(r)$ given by

$$V_D(r)=-3V_0(4w-m+2b-2h)\left[\frac{12\alpha\bar{\alpha}}{12\alpha\bar{\alpha}+\kappa(8\alpha+9\bar{\alpha})}\right]^{3/2} \times\exp\left[-\frac{12\alpha\bar{\alpha}\kappa}{12\alpha\bar{\alpha}+\kappa(8\alpha+9\bar{\alpha})}r^2\right], \tag{3}$$

with α and $\bar{\alpha}$ being the cluster width parameters specified by Eq. (9) in Ref. 3.

For the choice of the absorptive potential $W(r)$ we use the information learned recently from a study of the $\alpha+\alpha$ scattering problem.[11] There we found that the complex phase shifts determined empirically by Darriulat *et al.*[12] can be fitted rather well by using

$$W(r)=-W_0\left\{\frac{1}{1+e^{(r-R)/a}}+\frac{4e^{(r-R)/a}}{[1+e^{(r-R)/a}]^2}\right\}, \tag{4}$$

which contains both a volume and a surface term. The geometry parameters used were $R=3.0$ F and $a=0.5$ F. These parameters are such that the rms radius of the volume part of $W(r)$ is approximately equal to the rms radius of the direct nuclear potential V_D. Since it seems reasonable to expect that the absorption mechanism in the He^3+He^4 case

should be similar to that in the $\alpha+\alpha$ case, we shall also use Eq. (4) for $W(r)$, and since the He^3 nucleus is somewhat larger than the He^4 nucleus, we choose[18]

$$R = 3.2 \text{ F}, \quad a = 0.5 \text{ F}. \tag{5}$$

With $V_D(r)$, $V_C(r)$, and $K(\vec{r},\vec{r}')$ determined from the resonating-group calculation, and the geometry parameters in $W(r)$ chosen according to Eq. (5), there is only one parameter, W_0, left, which will then be chosen to yield a best fit to the experimental data for He^3+He^4 elastic scattering at 44.5 MeV.

III. RESULTS

Differential cross sections calculated with $W_0 = 0$ and $W_0 = 4$ MeV are compared with the experimental data in Fig. 1. From this figure we see that the addition of an imaginary potential affects the magnitude of the differential cross section, but not the general features. The agreement with the experimental data is quite satisfactory in the forward and backward directions, but is rather poor in the intermediate angular region around 90° where the calculated diffraction minimum is much too deep. At first, we thought that this discrepancy is caused by the omission of spin-orbit effects in our calculation, but this is unlikely, because the recent calculation of Fetscher *et al.*[8] showed that spin-orbit effects are unimportant at the energy

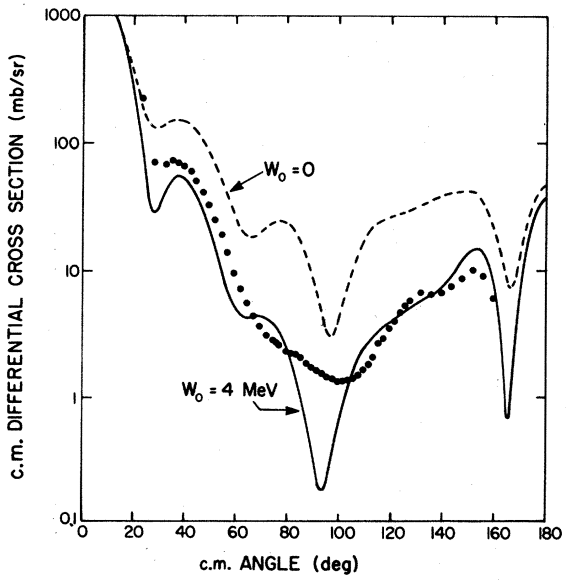

FIG. 1. Comparison of the differential cross section for He^3+He^4 scattering calculated from Eq. (1) with $W_0 = 0$ (dashed line) and $W_0 = 4$ MeV (solid line) with experimental data at 44.5 MeV. The experimental data are those of Ref. 7.

under consideration. Instead, it is our present opinion that, in this particular angular region, both direct and exchange processes are contributing significantly, and therefore, because of the interference between these processes, even minor deficiencies in the model could cause the observed discrepancy. This point should be examined in the future with the hope that such an examination would help reveal features missing in our calculation.

The calculated values of the real and imaginary parts of the lth-wave phase shift, δ_l^R and δ_l^I, are tabulated in Table I for $W_0 = 0$ and $W_0 = 4$ MeV. Here it is seen that even with W_0 as large as 4 MeV, resulting in a total reaction cross section of 477 mb, the values of δ_l^R are not markedly different from those for $W_0 = 0$. The largest difference is only about 10°, which occurs at $l = 4$ in this particular case. This indicates that even at relatively high energies where the reaction cross sections are not small, the values of δ_l^R obtained in a one-channel resonating-group calculation could still be used as starting values in a phase-shift analysis of the experimental data.

To bring out an important feature of the phase-shift behavior, we show in Fig. 2 the values of δ_l^R as a function of l with $W_0 = 4$ MeV. These values are shown as ×'s with a connecting dashed line as a visual aid. In this figure it is apparent that the phases posses an odd-even feature wherein the odd-l and even-l phases show distinctly different decreasing trends with l.[19]

To obtain a connection between the odd-even behavior in δ_l^R and the effective local interaction[20] between the He^3 and He^4 clusters, we are guided by the information learned from a recent investigation in which the properties of the local and nonlocal interaction in the $n+\alpha$ problem were studied in detail.[15] In that investigation it was

TABLE I. Real and imaginary parts of the phase shift, in degrees, calculated with $W_0 = 0$ and $W_0 = 4$ MeV.

l	$\delta_l^R + i\delta_l^I$ ($W_0 = 0$)	$\delta_l^R + i\delta_l^I$ ($W_0 = 4$ MeV)
0	174.4	$175.6 + i\,28.9$
1	186.5	$186.7 + i\,16.4$
2	137.9	$140.2 + i\,31.6$
3	144.0	$144.5 + i\,18.3$
4	68.5	$78.0 + i\,41.4$
5	77.6	$79.4 + i\,23.2$
6	15.8	$11.8 + i\,22.5$
7	20.0	$18.6 + i\,11.0$
8	1.9	$1.5 + i\,4.2$
9	4.2	$4.1 + i\,1.8$
10	−0.2	$-0.2 + i\,0.7$
11	0.9	$0.9 + i\,0.3$

found that one can construct an effective local interaction between the clusters which, in the Born approximation, yields the same scattering amplitude as that calculated with the resonating-group method. This effective local interaction has the form

$$V_{\text{eff}} = (V_D + V_a) + V_b P^r, \tag{6}$$

where P^r is a Majorana operator exchanging the position coordinates of the two clusters, and V_a and V_b are potentials representing exchange processes[21] which are automatically included in the resonating-group calculation through the use of a totally antisymmetrized wave function. Thus, in our present case of He^3+He^4 scattering, we are led to see whether or not the odd-even feature in δ_l^R can be reproduced by the use of such an effective potential; that is, instead of Eq. (1) we shall solve the following equation:

$$\left[\frac{\hbar^2}{2\mu}\nabla^2 + E - V_{\text{eff}}(r) - V_C(r) - iW(r)\right]F(\vec{r}) = 0, \tag{7}$$

where $W(r)$ is again given by Eqs. (4) and (5) with $W_0 = 4$ MeV. The results with $V_{\text{eff}} = 0.95V_D$, $1.15V_D$ and $1.35V_D$ are shown by the solid and dot-dashed lines in Fig. 2. Here one sees that the real parts of the odd-l and even-l resonating-group phase shifts are fairly well reproduced by using effective potentials of $1.35V_D$ and $0.95V_D$, respectively, while the curve representing the result with $V_{\text{eff}} = 1.15V_D$ cuts right across the dashed line. From this we conclude that, to a fair approximation, it is possible to represent the interaction between the He^3 and He^4 clusters by an effective potential of the form given by Eq. (6) with

$$V_a = 0.15V_D, \tag{8}$$

and

$$V_b = -0.2V_D. \tag{9}$$

The fact that V_a and V_b are quite different from zero indicates that at this relatively high energy of 44.5 MeV, the effects of antisymmetrization are very important and cannot at all be neglected.

As a comparison we note that in the $n+\alpha$ problem, V_a and V_b are about equal to $0.06V_D$ and $-0.08V_D$, respectively,[22] when the energy is chosen such that the wave number k is the same as that in the He^3+He^4 case at 44.5 MeV. This indicates that at relatively high energies, the effects of antisymmetrization are more important for He^3+He^4 scattering than for $n+\alpha$ scattering, a result which is more or less to be expected.

In a local-potential approach to solving scatter-

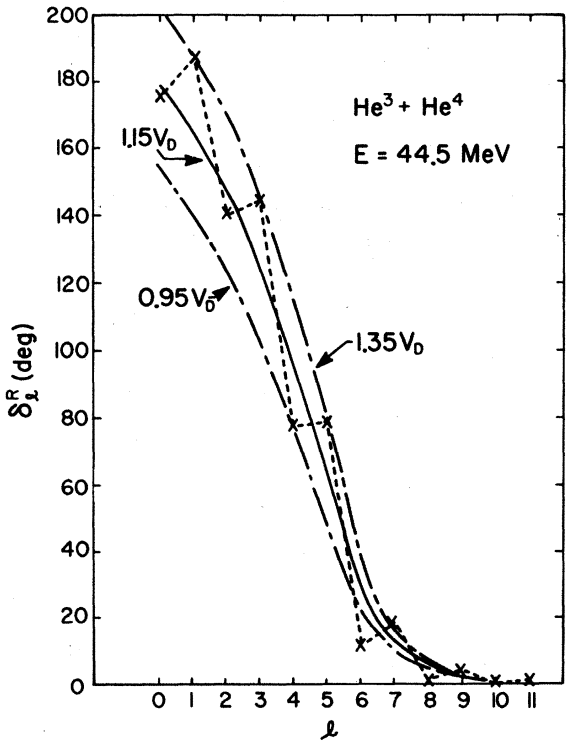

FIG. 2. Real part of the phase shift as a function of the orbital angular momentum. The ×'s represent the values obtained from Eq. (1) with $W_0 = 4$ MeV, while the solid and dot-dashed lines represent the values obtained from Eq. (7) with $V_{\text{eff}} = 0.95V_D$, $1.15V_D$, and $1.35V_D$.

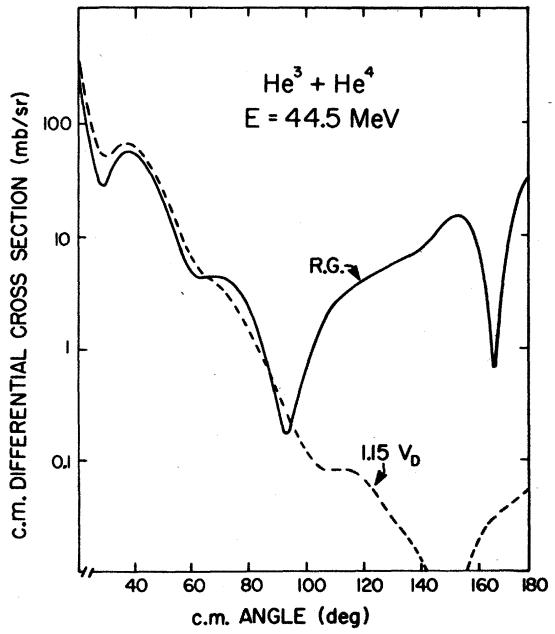

FIG. 3. Comparison of the differential cross section calculated with Eq. (1) and $W_0 = 4$ MeV (solid line) with the differential cross section calculated with Eq. (7) and $V_{\text{eff}} = 1.15V_D$ (dashed line).

ing problems, such as the usual optical-model consideration of nucleon-nucleus scattering, it has been common practice to employ an ordinary local potential without a Majorana space-exchange component. To see the consequence of such a simplification, we calculated the He^3+He^4 differential scattering cross section using Eq. (7) with V_{eff} set equal to $1.15V_D$ only. The result is shown by the dashed line in Fig. 3, in which a comparison is made with the differential cross section calculated with Eq. (1), shown by the solid line. As is clearly seen, the agreement between the results of these two calculations is fairly good at forward angles, but becomes very poor at backward angles. This shows convincingly that to obtain a satisfactory description of the differential scattering cross section at backward angles, proper consideration of the exchange processes is very important, and one simple way to accomplish this is by adding a Majorana component to the real part of the ordinary local potential.[14]

Finally, we have made a brief study of the behavior of the imaginary part of the phase shift. In Fig. 4 the values of δ_l^I, calculated with Eq. (1) and $W_0 = 4$ MeV, are shown as a function of l by the dashed line. Here it is seen that for small values of l less than about 6, δ_l^I also has a distinct odd-even feature. As a comparison, we have shown by a solid line in this same figure the values of δ_l^I calculated using Eq. (7) and with V_{eff} given by Eqs. (6), (8), and (9). Quite surprisingly, one finds that even though the reaction cross sections obtained in these two calculations are about the same (≅480 mb), the solid line demonstrates almost no odd-even structure. This indicates that if one wishes to perform a relatively simple analysis of the scattering data with a complex local

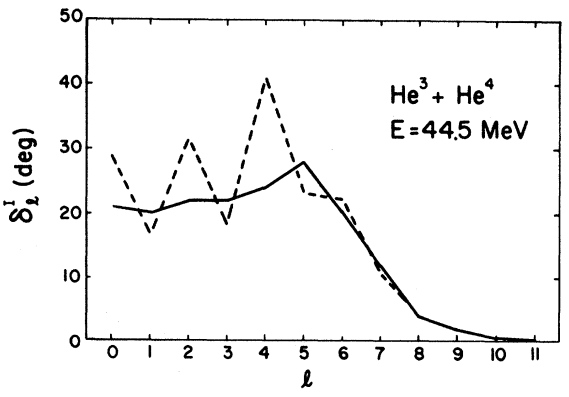

FIG. 4. Imaginary part of the phase shift as a function of the orbital angular momentum. The dashed line represents the values obtained from Eq. (1) with $W_0 = 4$ MeV, while the solid line represents the values obtained from Eq. (7) with V_{eff} given by Eqs. (6), (8), and (9).

potential, then it may even be necessary to introduce a Majorana component into the imaginary potential.[23] At present, we are examining a number of scattering problems involving light- and medium-weight nuclei to ascertain the necessity of this extra precaution. Meanwhile, however, we will consider this merely as an interesting possibility but not yet proven, since, unlike the real quantities $V_D(r)$, $V_C(r)$, and $K(\vec{r}, \vec{r}\,')$, the imaginary potential $W(r)$ used in this calculation is not derived from microscopic considerations, but is constructed from a phenomenological viewpoint.

IV. CONCLUSIONS

The elastic scattering of He^3 by He^4 at 44.5 MeV has been considered with a modification of the usual one-channel resonating-group formalism. This modification consists in taking approximate account of the existence of many open reaction channels at this energy by including in the theory a phenomenological local imaginary potential. For this potential, the form and geometry parameters are chosen according to the information gained from a recent study of the $\alpha+\alpha$ problem using a similar method.[11] The strength parameter is then chosen to yield best agreement with the experimental data. In this way it is found that a rather good fit can be obtained in both the forward and the backward directions.

From this study we have also found that, at this energy, the contribution to the differential cross section at backward angles comes mainly from exchange processes which are automatically included in the resonating-group formalism through the use of a totally antisymmetrized wave function. If these exchange processes are not taken into account, then the calculated differential cross section in this particular angular region will be much smaller than the experimental cross section.

The real parts of the phase shifts have been shown to exhibit a marked degree of odd-even structure, wherein the odd-l and even-l phases follow distinctly different decreasing trends with the orbital angular momentum. From this finding we conclude that if one wishes to describe the scattering data by a complex local potential, then the real part of this potential must have a significant amount of Majorana space-exchange component.

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[17]The notation used here is the same as that used in Ref. 3.

[18]We have also made various calculations in which the form of $W(r)$ is chosen to be slightly different from that of Eq. (4) and in which the values of R and a are chosen to be slightly different from those of Eq. (5). In all these calculations, the results obtained are quite similar to the result obtained with the choices given in Eqs. (4) and (5).

[19]It should be mentioned that Giamati *et al.* [C. C. Giamati, V. A. Madsen, and R. M. Thaler, Phys. Rev. Letters 11, 163 (1963)] have noticed an odd-even feature in their analysis of proton-α scattering.

[20]Note that the effective local interaction V_{eff}, as defined in the following Eq. (7), is a real quantity.

[21]For example, in the case of $He^3 + He^4$ scattering, the term in the kernel function with $i = 13$ [Eq. (A1) in Ref. 1] represents a process in which the incoming He^3 particle picks up a neutron and leaves as an α particle – this is commonly referred to as a neutron-transfer process (see also Ref. 6).

[22]This information is contained in Table I of Ref. 15.

[23]As was suggested by Benöhr and Wildermuth [H. C. Benöhr and K. Wildermuth, Nucl. Phys. A128, 1 (1969)], an alternate description is to relinquish the local-potential approach, but to introduce an additional phenomenological non-Hermitian integral kernel into the one-channel resonating-group formalism.