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Coupling of Particle-Hole and Surface Excitations in Closed-Shell Nuclei*

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A simple model calculation is presented where particle-hole and surface excitations are coupled. An application to the case of ^{32}S is presented in detail.

The intermediate-coupling unified model,[1] where a number of nucleons are coupled to the vibrational excitations of an even-even core, has already been extensively used. This simple model, which has been successful in describing the collective properties of odd A,[2] as well as even-even nuclei[3] in vibrational regions, is well known to make no assumption on the microscopic nature of the vibrating core. Usually the core is assumed to present *harmonic* vibrations for mathematical simplicity. We note, however, that although many nuclei have part of their spectra close to harmonic, there are always deviations and the appearance of low-lying states outside the harmonic sequence. The buildup of the near collective states of an even-even nucleus is generally considered in terms of many particle-hole excitations.[4] In this study we have taken the opposite approach. Taking the harmonic spectrum for an even-even nucleus as a starting point, we consider phenomenologically the particle-hole deviation away from this picture to fit the properties of the observed spectrum. The ^{32}S nucleus, which seems to show many characteristics of an anharmonic vibrator[5] but has resisted until now any systematic theoretical description, will provide us with an appropriate example.

The classical intermediate-coupling model for two particles added to an even-even core is based on a decomposition of the nuclear Hamiltonian into four parts:

$$H = H_c + H_{sp} + H_{int} + H_{12} \tag{1}$$

with

$$H_c = \tfrac{1}{2}\hbar\omega \sum_\mu (b_\mu^* b_\mu + b_\mu b_\mu^*),$$

and

$$H_{int} = -\xi\hbar\omega\left(\frac{\pi}{5}\right)^{1/2} \sum [b_\mu + (-)^\mu b_{-\mu}^*] Y_{2\mu}(\theta, \phi).$$

Here H_c describes the phonon spectrum, H_{sp} the single-particle and hole energies, H_{int} the coupling of particles and holes to the vibrating core, and H_{12} the particle-hole interaction. The interaction Hamiltonian H_{int} is given in terms of a strength parameter ξ describing the core-single-particle interaction. Clearly here one seeks consistency between the coupling strength needed to describe the anharmonic effects of the even-even nucleus and that strength needed to describe the spectra of the $A\pm1$ nuclei.

In the ^{32}S nucleus which we take as our example, we isolate from the $J=2^+$ one-phonon state the $(s_{1/2})^{-1}(d_{3/2})$ and $(s_{1/2})^{-2}(d_{3/2})^2$ particle-hole elements and consider them as perturbations added to the harmonic vibration. In our model, H_c and

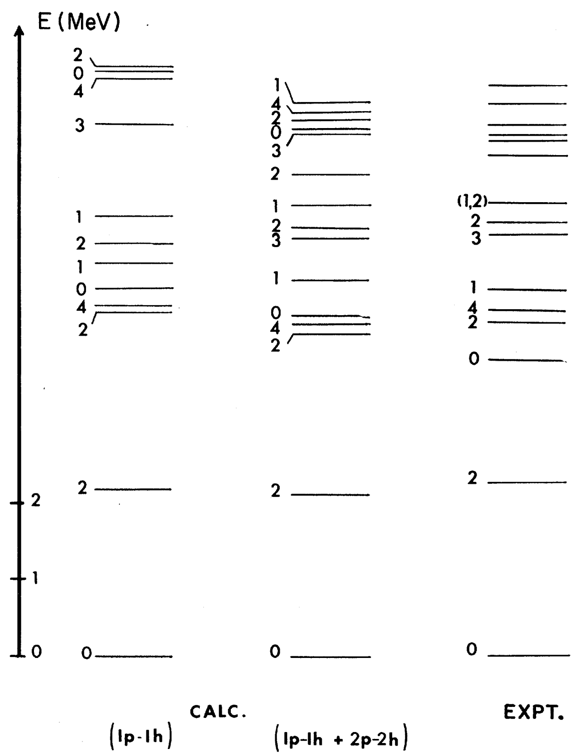

FIG. 1. Calculated and experimental low-lying levels of ^{32}S.

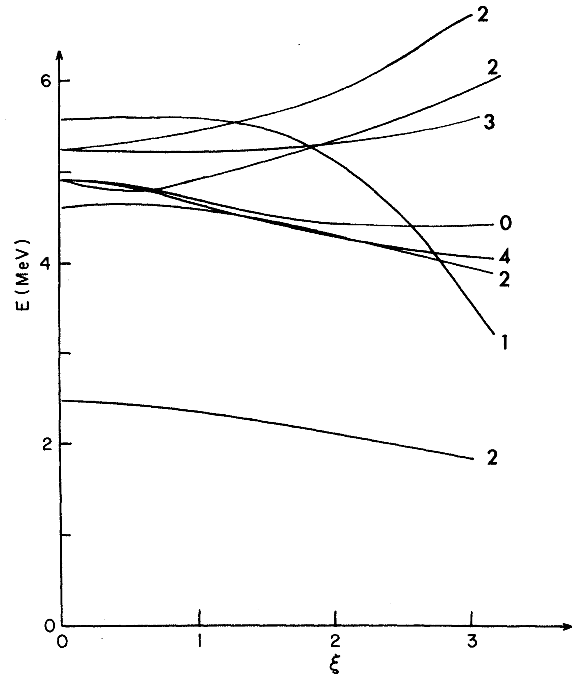

FIG. 2. Energy levels of ^{32}S as a function of the coupling strength parameter ξ.

TABLE I. Experimental and calculated $B(E2)$ in Weisskopf units in ^{32}S.

E_i (MeV)	E_f (MeV)	J_i	J_f	$B(E2)$ (a) Calc.	$B(E2)$ (b) Calc.	Expt. (Ref. 5)
2.24	0	2	0	9.2	9.8	7.0 ± 0.7
3.78	2.24	0*	2	17.6	16.4	15.0 ± 4.0
4.29	0	2*	0	0.4	0.5	1.1 ± 0.1
4.29	2.24	2*	2	11.7	10.9	7.3 ± 0.6
4.46	2.24	4	2	17.6	16.9	12.0 ± 3.0

$H_{\rm int}$ remain unchanged from their form in the classical intermediate-coupling model[1]; $H_{\rm sp}$ now represents the single-particle and single-hole energies, and $H_{\rm int}$ the corresponding 1p-1h and 2p-2h matrix elements.

Our calculations for ^{32}S depend essentially on four parameters: The phonon energy ($\hbar\omega = 2.4$ MeV); the p-h separation energy (5.1 MeV); the coupling strength parameter (ξ 2.25); and to describe the two-body interaction, we use renormalized Kuo[6] matrix elements (normalization factor $y = 1.75$). The values given in brackets are chosen to give a good fit to the experimental energy spectrum. In Fig. 1, we compare with experiment both our full calculation and a calculation including 1p-1h excitation only. It is clear that a marked improvement is obtained by the inclusion of the 2p-2h excitations.

It is interesting to note that a similar intermediate-coupling model applied to odd-A nuclei was able to give a good fit to the experimental spectra of ^{31}P and ^{35}Cl *with essentially the same value of the* ξ *parameter*.[2]

In Fig. 2 it is evident that the $J = 0*$ state stays above the $J = 2$ and 4 states of the second phonon multiplet (for $\xi > 1$), in disagreement with the experimental results. This situation was previously observed by Wilets and Jean in their study of anharmonic effects.[7]

As seen in Table I, the anharmonic character of the two-phonon states is quite well reproduced in this calculation. It is clear that the collective character of the $J = 0$ and $J = 4$ states is little influenced by a change in ξ. The second $J = 2$ state however, which is mainly 1p-1h in character for small values of ξ has already a large component of collective nature for $\xi = 2.0$ as can be seen by the relatively large $E2$ decay to the first 2^+ state. In fact, in this model, the crossover $E2$ transition, as well as the "two-phonon–one-phonon" decay from the second 2^+ state, results from a mixture of wave functions having single-particle and collective character.

It is interesting to study the single-particle contributions to these $E2$ transition rates. For this, we present two columns of results in Table I: In (a) a zero effective charge is taken; in (b) an effective charge $e_{\rm eff} = 0.5e$ is chosen, although in both cases the same collective contribution is taken. One sees clearly therefore that the $E2$ transition strengths are essentially a collective feature. We note also that the $J = 1^+$ state observed at 4.70 MeV[8] is well reproduced in our calculation, together with the $J = 3^+$ state which is mainly 2p-2h in character.

Finally it is interesting to investigate the extent to which the particle-hole perturbation succeeds in polarizing the core. One can form a simple idea of this by calculating the quadrupole moment of the first 2^+ state. Our result is $Q(2^+) = -0.09$ b, which is still small compared to a recent experimental estimate $[Q(2^+) = -0.20 \pm 0.06]$[9] but has, nevertheless, the right sign.

In conclusion, the model presented here is certainly very simple. It allows for a description of anharmonic effects in nuclei with closed subshells, by coupling 1p-1h and 2p-2h perturbations to the excited states of a harmonic vibrator. The rather satisfactory picture of the $B(E2)$ rates observed in our example seems to show that such a simple picture of an anharmonic vibrator is not inconsistent with the microscopic structure of vibrational nuclei.

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Simple Neutron-Transfer Model for the Exchange Resonance in ^{3}He-^{4}He Elastic Scattering

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A simple model based on summing neutron-transfer diagrams is developed. This model gives a broad peak in the backward ^{3}He-^{4}He elastic scattering cross section having some of the observed characteristics.

Brown and co-workers[1] have observed a bump in the differential cross section at 173.3° (c.m.) for elastic scattering of ^{3}He by ^{4}He. Data taken at smaller angles[2,3] indicated that the bump is strongly backward-peaked. In Fig. 1 we reproduce Fig. 2 of Ref. 1, which shows these results along with cluster-model predictions. Previous cluster-model calculations[3,4] had predicted a backward-peaked enhancement. Tang found that the enhancement in the cluster-model differential cross section vanished when exchange terms were omitted in the calculation. This led Brown to refer to the enhancement as an exchange resonance.

Using arguments based on an analogy with the explanation of enhancements observed in charge-exchange cross sections in atomic physics,[5] Temmer[6] in 1963 predicted exchange resonances in nuclear scattering. Underlying Temmer's prediction was the assumption that the heavier of the two nuclei consists of the exchanged particle outside a core. Such a model for ^{4}He, however, is questionable. For this reason and because it is difficult to extract the underlying mechanism responsible for the backward-peaked enhancement from the cluster-model calculation, we were led to formulate a simple model based on neutron exchange.

The simplest exchange term in ^{3}He-^{4}He scattering is neutron exchange, which is shown diagrammatically in Fig. 2(a). Scattering in the backward direction corresponds to small momentum transfer by the neutron, $\vec{k} \approx \vec{k}'$; and thus to relatively large separation of the ^{3}He cores. Therefore antisymmetrization of the ^{3}He constituent nucleon coordinates should be relatively unimportant and will be ignored. For energies in the region of the observed peak and for small momentum transfer, little energy is available at the vertices. It is reasonable, therefore, to neglect the momentum dependence of the vertex functions and the breakup thresholds in the ^{3}He and ^{4}He propagators. Since this exchange term is diagonal in spin variables, reference to spin will be suppressed.

For the exchange driving force, the scattering amplitude is the sum of two amplitudes which satisfy the coupled equations shown diagrammatically in Fig. 2(b). With the approximations mentioned and further noting that the relative n-^{3}He angular momentum is zero, the exchange term is given by

$$B(E,\vec{k},\vec{k}')=\gamma^2\left[E-\left(\frac{\vec{k}-[m_3/(m_3+m_n)]\vec{k}'}{2\mu_{3n}}\right)^2 - k'^2\left(\frac{2(m_n+m_3)m_3}{m_n+2m_3}\right)^{-1}\right]^{-1}, \quad (1)$$

where γ is the vertex "function" whose momentum dependence has been neglected. The ^{3}He and ^{4}He masses are denoted by the subscripted m's, and