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## Effect of the 7.12-MeV Level in $^{16}\text{O}$ on the Alpha Spectrum from $^{16}\text{N}$ $\beta$ Decay

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Taking into account the final-state density of the leptons, a single resonance model for the  $^{12}\text{C} + \alpha$   $1^-$  scattering amplitude predicts a markedly asymmetric peak in the  $\alpha$  spectrum following  $^{16}\text{N}$   $\beta$  decay into continuum states of  $^{16}\text{O}$ . The observed peak is nearly symmetric and it is shown that this symmetry can arise from destructive interference from the subthreshold  $1^-$  state (7.12 MeV) and/or a background from states of  $\sim 17$ -MeV excitation in  $^{16}\text{O}$ . It is shown that the  $\alpha$  width of the 7.12-MeV state must be an order of magnitude smaller than that of the  $1^-$  state (9.58 MeV).

### I. INTRODUCTION

The  $\alpha$  spectrum from the  $\beta$  decay of  $^{16}\text{N}$  to continuum states of  $^{16}\text{O}$  has been carefully investigated<sup>1,2</sup> because of the possibility of observing parity violation in the decay of the  $2^-$  (8.88 MeV) into the  $^{12}\text{C} + \alpha$  channel. The  $\alpha$  spectrum displays a single peak due to transitions to the broad  $1^-$  state at 9.58 MeV. The peak occurs about 150 keV lower because the density of final states for the leptons rises rapidly as the excitation energy is lowered from its maximum allowed value of 10.41 MeV.

The exact shape of the spectrum is of only moderate interest to the parity-violation investigations but is of paramount interest in the present work because of the prospect of obtaining information about the  $\alpha$  width of the subthreshold  $1^-$  (7.12-MeV) state. From the standpoint of nuclear-structure theory, the  $\alpha$  widths of the low-lying states in  $^{16}\text{O}$  are quantities to be explained by cluster models of these states.<sup>3,4</sup> The  $\alpha$  width of the  $1^-$  (7.12-MeV) state is also an important parameter in fixing the rate of the reaction  $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$  that synthesizes

$^{16}\text{O}$  in stellar interiors.<sup>5</sup> The small value of the photocapture cross section ( $\sim 50$  nb at peak) has so far frustrated attempts to get the width directly from the  $\gamma$  experiment.<sup>6,7</sup>

Most of the early measurements<sup>8-10</sup> of the shape of the  $\alpha$  spectrum following  $^{16}\text{N}$  decay resulted in an asymmetric peak that fell off more slowly on the low-energy side. Such a shape can be fit quite well by a single-level expression which includes the lepton phase space. The curve marked "single level" in Fig. 1 represents a good fit to most of the data. However, even in 1961 Kaufmann and Wäffler<sup>11</sup> found that the peak was nearly symmetric and the experiments of the past two years<sup>1,2,12,13</sup> have tended in this direction. The solid lines in Fig. 1 summarize these data.

It is plain that destructive interference with the 9.58-MeV state is occurring. This interference can arise from the effects of the 7.12-MeV states, as well as from all  $1^-$  states of higher energy. ( $3^-$  states can also contribute, but the effect should be small because of the higher centripetal barrier and the absence of a  $3^-$  state in the 7-10-MeV

range.) Within the context of  $R$ -matrix theory the amplitude for the decay is given by

$$\langle \psi_E^{(-)} \| \tilde{\sigma} \tau^{(+)} \| \psi_E \rangle = \pm \sum_{\lambda} \frac{\Gamma_{\alpha\lambda}^{1/2} \langle J_{f,\lambda} \| \tilde{\sigma} \tau^{(+)} \| J_i \rangle}{E_{\lambda} - E} \frac{e^{-i\delta}}{|1 - RL^0|}. \quad (1)$$

The + or - sign is chosen to give continuity;  $\delta$  is the  $^{12}\text{C} + \alpha$  phase shift;  $R(E)$  is the  $R$  matrix, or inverse logarithmic derivative at the nuclear surface; and  $L^0 = L - B$  is the complex function of the outgoing Coulomb wave function defined by Land and Thomas.<sup>14</sup> It is clear that the effect of each state depends on the product of the square root of the  $\alpha$  width,  $\Gamma_{\alpha\lambda}^{1/2}$ , and the reduced Gamow-Teller (GT) matrix element for each level.  $M_{GT}^2$  for the 7.12-MeV state is known to be 0.029, about 3% of the estimated summed strength to  $1^-$ ,  $T=0$  states (see Sec. III) and taking a nominal energy of 9.43 MeV for the peak due to the 9.58-MeV state we get  $M_{GT}^2 \cong 0.003$  for this broad level. Thus, one could expect a significant interference from the lower state for a wide range of  $\alpha$  widths.

Symmetry arguments suggest that the background from higher levels should not swamp the effect of the lower state. The operator  $\tilde{\sigma} \tau^{(+)}$  operating on the  $^{16}\text{N}$  ground state creates states within the same super multiplet. The space symmetry of these states is different from that of  $^{12}\text{C} + \alpha$ . We have the result that there is no *a priori* reason to expect anything but random behavior of the sign of the  $\Gamma_{\alpha\lambda}^{1/2} \langle J_{f,\lambda} \| \tilde{\sigma} \tau^{(+)} \| J_i \rangle$  for the distant levels.

Weisser, Morgan, and Thompson<sup>7</sup> have attempted to fit the 9.58-MeV peak in the  $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$  cross section by a three-level  $R$ -matrix analysis. We present a similar analysis of the  $^{16}\text{N}$  decay  $\alpha$  spectrum which also uses three-level  $R$ -matrix poles representing the 7.12- and 9.58-MeV levels, as well as a distant level. The  $\alpha$  widths are constrained by the known behavior<sup>15, 16</sup> of the  $1^-$  phase shift around 9.58 MeV. Similarly, the three GT matrix elements are constrained by the absolute height of the  $\alpha$ -spectrum peak and the  $ft$  value of the 7.12-MeV state. Operationally, the reduced width of the bound state and the background  $\beta$ -decay matrix element are the independent variables that are varied to fit the  $\alpha$ -spectrum shape.

## II. $R$ -MATRIX FORMULATION

The basic formula for the differential decay rate for  $^{16}\text{N}$  into  $^{12}\text{C} + \alpha$  is<sup>17, 18</sup>

$$\frac{d\Delta}{dE_x} = (2\pi^3)^{-1} f(W_{\max}, Z) g_A^2 \hat{f}_i^{-2} \frac{1}{2\pi} \sum_J |\Gamma_{\alpha\lambda}^{J1/2} A_{\lambda\lambda'}(E_x) \times \langle J, \lambda' \| \sigma \tau^{(+)} \| J_i \rangle|^2, \quad (2)$$

$$(A^{-1})_{\lambda\lambda'} = E_{\lambda} + \Delta_{\lambda\lambda'} - E_x - i\Gamma_{\lambda\lambda'}/2.$$

In writing down this equation it has been assumed only allowed GT transitions are important so that  $J=1, 3$ .  $A_{\lambda\lambda}'$  is the inverse level matrix for the states of angular momentum  $J$ ,  $\Gamma_{\alpha\lambda}^J$  is the  $\alpha$  width of the  $\lambda$ th level. (All definitions follow Lane and Thomas.) The function  $f(W_{\max}, Z)$  takes into account the integration over the lepton final states and is tabulated in Siegbahn,<sup>19</sup> with  $W_{\max} = 10.41 - E_x$ .

While 68%<sup>20</sup> of the  $\beta$  decays of the  $^{16}\text{N}$  ground state go to the bound  $3^-$  (6.13 MeV), it can be assumed that the  $J=3$  contribution in Eq. (2) is small because of the small value of the  $l=3$  penetration factor and the absence of any  $3^-$  in the range  $7.16 \leq E_x \leq 9.90$  MeV.

The energies  $E_{\lambda}$  which enter Eq. (1) are, of course, the poles of the  $R$  matrix (here the  $R$  function),

$$R(E) = \sum_{\lambda} \gamma_{\lambda}^2 / (E_{\lambda} - E), \quad (3)$$

which should reproduce  $^{12}\text{C} + \alpha$  scattering amplitude. The connection between the collision matrix and the  $R$  matrix is

$$U \equiv e^{2i\delta} = O^{-1}(1 - RL_0^*)(1 - RL_0)^{-1}I, \quad (4)$$

$$L = S + iP.$$

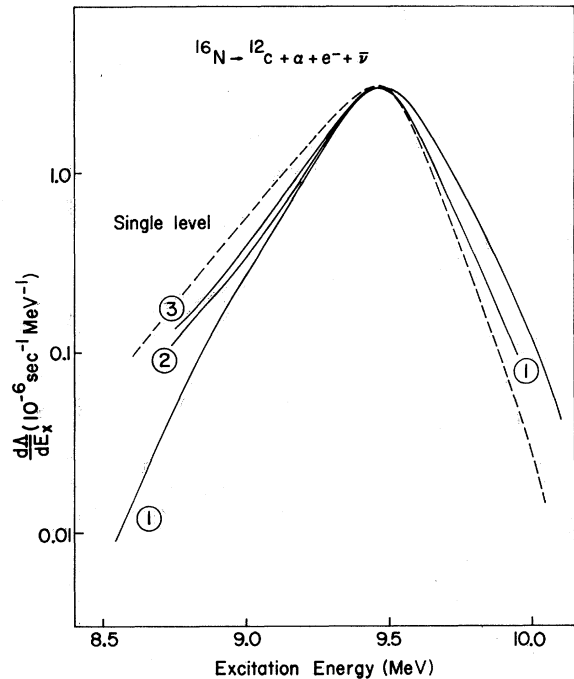


FIG. 1 Three experimental determinations of the  $\alpha$  spectrum following  $^{16}\text{N}$   $\beta$  decay. Curve 1 is from Ref. 1, curve 2 from Ref. 12, and curve 3 from Ref. 13. Curve 1 has been shifted by 60 keV so that the peak occurs at 9.48-MeV excitation energy. The dashed curve is a single-level fit to the data using the single-level parameters of Jones *et al.* (Ref. 15).

Here  $O$ ,  $I$ ,  $L_0$ , and  $L_0^*$  are functions of the asymptotic Coulomb wave functions evaluated on the nuclear surface.<sup>14</sup> Of particular importance is  $L_0 = L - B$ , since  $B$  is the boundary condition which the logarithmic derivatives of the  $|J, \lambda\rangle$  satisfy on the surface at  $E = E_\lambda$ .

A peculiarity of the  $R$ -matrix formulation is that  $B$  can be chosen so that a particular  $E_\lambda$  is the real part of the complex energy of a pole of  $U$ , but the remaining  $E_\lambda$  will, in general, not coincide with the real energies of other poles of  $U$ . For example, there is a bound state at 7.12 MeV and a resonance at 9.58 MeV in the  $1^-$  channel. We have chosen  $B = S(E_2)$ ,  $E_2 = 9.56$  MeV, so that  $R$  has a pole very near the energy at which the phase shift goes through  $90^\circ$  ( $\sim 9.58$  MeV). Then  $R$  does not have a pole at  $E_b = 7.12$  MeV, but the pole of  $U$  at this energy requires that  $R$  satisfy

$$1 - R(E_b)L_0(E_b) = 0. \quad (5)$$

For a fixed number of poles one has a family of  $R$  matrices with different values of  $B$  and, perforce, different  $\gamma_\lambda$  and  $E_\lambda$  which yield the same scattering amplitude.<sup>21</sup> Thus, an  $R$  matrix with  $B$  such that  $B = S(E_b)$  and appropriate values of the other  $E_\lambda$  will give identical results with the  $R$  matrix of our choice which we have defined in the previous paragraph. The  $\alpha$  width of the bound state is defined in terms of our parameters by

$$\gamma_1(E_b) = \pm \left[ \sum_\lambda \frac{\gamma_\lambda^2}{(E_\lambda - E_b)^2} \right]^{-1/2} \sum_\lambda \frac{\gamma_\lambda^2}{E_\lambda - E_b} \quad (6)$$

and corresponds to the reduced width of the first level of an  $R$  matrix for which  $E_1 = E_b$ . The sign must be chosen such that the right- and left-hand limits of  $\gamma(E)$  are both  $\gamma_\lambda$  as  $E \rightarrow E_\lambda$ .

In a similar way, the observed GT matrix element,  $M_{GT} = \hat{J}_i^{-1} \langle J_b \| \hat{\sigma} \vec{\tau}^{(+)} \| J_i \rangle$ , of the bound state, must be related to the matrix elements of the basis states by

$$M_{GT}(E_b) = \pm \left[ \sum_\lambda \frac{\gamma_\lambda^2}{(E_\lambda - E_b)^2} \right]^{-1/2} \sum_\lambda \frac{\gamma_\lambda \hat{J}_i^{-1} \langle J_{f,\lambda} \| \hat{\sigma} \vec{\tau}^{(+)} \| J_i \rangle}{E_\lambda - E_b}. \quad (7)$$

With regard to the sign of the interference between levels, we can choose all the  $\gamma_\lambda > 0$ , since  $R$  is independent of the sign and can choose the signs of

the  $\langle J_{f,\lambda} \| \hat{\sigma} \vec{\tau}^{(+)} \| J_i \rangle$  to generate the proper interference.

### III. DISTRIBUTION OF GAMOW-TELLER STRENGTH IN $^{16}\text{O}$

The basis set of single-particle and hole states fails in many cases to explain the negative-parity states of  $^{16}\text{O}$ , simply because there are far fewer basis states than experimentally observed levels of a given  $J^\pi$ . For example, calculations<sup>22, 23</sup> usually yield a single low-lying  $T=0$ ,  $1^-$  state that can be associated with either the  $1^-$  (7.12 MeV) or the  $1^-$  (9.58 MeV), but not both. On the other hand if one calculates the distribution of  $E1$  strength for ground-state absorption of  $\gamma$  ray, the results are remarkably close to experiment.

Since the lowest  $T=1$  state usually turns out to be of the correct spin and parity,  $2^-$ , and at about the correct energy, one can hope the  $^{16}\text{N}$  ground state is adequately represented by simple particle-hole configurations. We have, therefore, used random-phase-approximation (RPA) wave functions<sup>24</sup> to estimate the distribution of GT strength connecting the  $^{16}\text{N}$  ground state and  $^{16}\text{O}$   $1^-$  states.

An excited state of  $^{16}\text{O}$  is created by operating on the ground state with an operator that is a sum of particle-hole creation and annihilation operators,

$$\begin{aligned} |\psi_\alpha\rangle &= \Gamma^{\alpha\dagger} |\psi_0\rangle, \\ \Gamma^{\alpha\dagger} &= \sum_{m,i} x_{mi} a_m^\dagger a_i + y_{mi}^\alpha a_i^\dagger a_m. \end{aligned} \quad (8)$$

The  $\beta$ -decay interaction is written as

$$H_I = \sum_{i,k} H_{ik} a_i^\dagger a_k, \quad (9)$$

where  $H_{ik}$  is a single-body matrix element. Then the  $\beta$ -decay matrix element between physical states is written as

$$\begin{aligned} \langle \psi_\beta | H_I | \psi_\alpha \rangle &= \langle \psi_0 | \Gamma^\beta H_I \Gamma^{\alpha\dagger} | \psi_0 \rangle \\ &= \langle \psi_0 | [\Gamma^\beta, [H_I, \Gamma^{\alpha\dagger}]] | \psi_0 \rangle \\ &\quad + \langle \psi_0 | \Gamma^\beta \Gamma^{\alpha\dagger} H_I | \psi_0 \rangle. \end{aligned} \quad (10)$$

The above result follows from  $\Gamma^\beta |\psi_0\rangle = 0$ . Within the spirit<sup>25</sup> of the RPA the second term on the right-hand side is zero, since  $H_I$  acting on a closed-shell state is zero. (Only allowed transitions are being considered.) The double commutation, without further assumptions, yields

$$\langle \psi_\beta | H_I | \psi_\alpha \rangle = \sum_{m,n} [x_{ni}^{\beta*} x_{mi}^\alpha (H_{nm} \delta_{ij} - H_{ij} \delta_{nm}) - y_{nj}^{\beta*} y_{mi}^\alpha (H_{mn} \delta_{ij} - H_{ji} \delta_{nm})]. \quad (11)$$

More specifically, after putting in the angular momentum couplings and the isospin, the quantities in the

parenthesis can be written as follows:

$$\frac{1}{\sqrt{2}} \langle j'_p j'_n, J' M' T' T'_z = 0 | \sigma^{(\lambda)} \tau^{(\pm)} | j_p j_n, J M T T_z = -1 \rangle \\ = - \frac{(1 \lambda J M | J' M')}{\sqrt{2}} \left( (-1)^{j_n + j'_p + J'} \begin{Bmatrix} 1 & j_p & j_p' \\ j_n & J' & J \end{Bmatrix} \langle j'_p \| \bar{\sigma} \| j_p \rangle \delta_{hh'} - (-1)^{T + j_p + j'_n + J} \begin{Bmatrix} 1 & J_n & j'_n \\ j_p & J' & J \end{Bmatrix} \langle j_n \| \bar{\sigma} \| j'_n \rangle \delta_{pp'} \right). \quad (12)$$

The tabulated RPA wave function of Gillet and Vinh-Mau<sup>24</sup> discloses that the ground state of <sup>16</sup>N is chiefly ( $p_{1/2}$ )<sup>-1</sup> ( $d_{5/2}$ ) with a small admixture of ( $p_{3/2}$ )<sup>-1</sup> ( $d_{5/2}$ ) and extremely small admixtures of other states. We have neglected all but the principal two states in obtaining the results for  $|M_{GT}|^2$  shown in Table I. We see that the strength lies chiefly in three states at 16.6 MeV ( $T=0$ ) and at 18.1 and 22.2 MeV ( $T=1$ ). Since the lowest two  $T=0$  levels contain admixtures of the spurious c.m. state we have also evaluated  $|M_{GT}|^2$  for the lowest  $T=0$  state tabulated by Seaborn and Eisenberg,<sup>23</sup> since these authors have projected out the spurious state. The matrix element is still essentially zero because their lowest  $1^-$  state is chiefly ( $p_{1/2}$ )<sup>-1</sup> ( $2s_{1/2}$ ).

In the approximation that <sup>16</sup>O has 100% occupation of the  $1p_{1/2}$  and  $1p_{3/2}$  shells and <sup>16</sup>N g.s. is pure 1p-1h the sum of  $|M_{GT}|^2$  over all final  $J$  should be 6. This number arises from the circumstances that either the neutron-particle or the proton-hole state can make the transition. The transitions to the  $1^-$  levels account for over 50% of this strength. In the next section, the GT  $T=1$  strength for  $1^-$  states has been ignored because Suffert and Feldman<sup>26</sup> failed to find any coupling of  $\alpha$  channels to the giant-dipole state in <sup>16</sup>O between the energies of 22–25 MeV.

TABLE I. The excitation energies,  $E_x$ , are taken from Ref. 24. The Gamow-Teller matrix elements for the  $1^-$  (7.12-MeV),  $2^-$  (8.88-MeV), and  $3^-$  (6.18-MeV) states were obtained using the observed branching ratios of 5, 1.2, and 68%, respectively, and a half-life for <sup>16</sup>N of 7.35 sec.

$T$		$E_x$ (MeV)	$(M_{GT})^2$ calc	$(M_{GT})^2$ obs
0	$1^-$	4.71	0.01	0.029
		9.88	0.0	
		15.0	0.01	
		16.6	1.32	
		22.4	0.0	
1	$1^-$	13.5	0.01	
		18.1	0.30	
		19.6	0.01	
		22.2	0.84	
		25.4	0.02	
0	$2^-$	10.5	0.51	0.20
0	$3^-$	6.25	0.16	0.15

#### IV. PARAMETRIZATION OF $\alpha$ SPECTRUM

An important constraint on our reduced widths and level energies is that they reproduce the  $1^-$  phase shifts where known and also reproduce the pole of  $U$  on the real axis for  $E_x = 7.12$  MeV. Two independent phase-shift analyses<sup>15, 16</sup> of elastic scattering arrive at the result that the energy behavior of the  $1^-$  phase shift is well fitted by a single-level  $R$  matrix in the region around 9.56 MeV. Jones *et al.*<sup>15</sup> give the single-level parameters  $E_2 = 9.56$  MeV,  $\gamma_2^2 = 0.608$  MeV,  $a = 5.4$  fm. This fit is excellent for  $9.26 \leq E_x \leq 9.86$ , but the single-level phase shifts are smaller than the observed ones outside these limits. We have determined the  $R(8.96)$  and  $R(10.18)$  such that the phase shifts are matched at these energies. We have required that our parameters reproduce these values of  $R(E)$  and require, in addition,  $R(9.56) = \infty$ . In order to satisfy Eq. (5),  $R(7.12)$  is also fixed. Because of the concentration of the  $T=0$  GT strength in one state at 16.6 MeV, we have picked this to be the energy of our background level. Thus, with four values of the  $R$  matrix fixed and the background-level energy fixed,  $E_3 = 16.6$  MeV, only one parameter is free to be varied and we take this to be the width of the lowest state,  $\gamma_1$ .

The variation of  $E_1$ ,  $\gamma_2$ , and  $\gamma_3$  with  $\gamma_1$  is shown

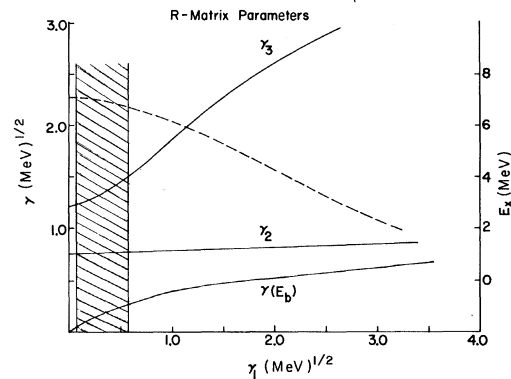


FIG. 2. Values of the reduced widths,  $\gamma_2$  and  $\gamma_3$ , and of the energy of the lowest  $R$ -matrix pole,  $E_1$ , are considered as functions of  $\gamma_1$  such that the resonant behavior of the  $1^-$  phase shift is matched for  $9.56 \pm 0.60$  MeV. The curve marked  $\gamma(E_b)$  is the width of the physical bound state at 7.12 MeV as evaluated from the  $R$  matrix. The shaded column displays the limitation of the ranges of values obtained in the present analysis.

in Fig. 2. Because the boundary condition,  $B$ , is chosen to be that appropriate to the physical resonance at 9.56 MeV,  $\gamma_2$  remains essentially unchanged for wide variations of  $\gamma_1$ . We have adopted the convention that  $\gamma_1$ ,  $\gamma_2$ , and  $\gamma_3$  are all positive. The curve marked  $\gamma(E_x)$  gives the  $R$ -matrix value for the reduced width of the physical bound state at 7.12 MeV, evaluated from Eq. (6).

In a similar way the three GT matrix elements are constrained. One sees directly from Eq. (1) that for  $E = E_\lambda$  the amplitude for  $\beta$  decay into  $^{12}\text{C} + \alpha$  depends only on the GT matrix element for that level. The peak height of the spectrum is essentially independent of  $M_1$  and  $M_3$ . The absolute peak height of  $3 \times 10^{-6} \text{ sec}^{-1} \text{ MeV}^{-1}$  was obtained from the total area under the curve and the result of Hättig, Hünchen, and Wäffler,<sup>1</sup> that the branching ratio to the 9.58-MeV state is  $(1.2 \times 10^{-3})\%$ . Then a relationship between  $M_1$  and  $M_3$  is obtained from Eq. (7) when one sets  $|M(E_b)|^2 = 0.029$ . We have chosen  $M_3$  to be the independent matrix element.

The effect of the background levels is "mocked up" through the product  $\gamma_3 M_3$  which represents the sum over a number of levels. Table I displays the

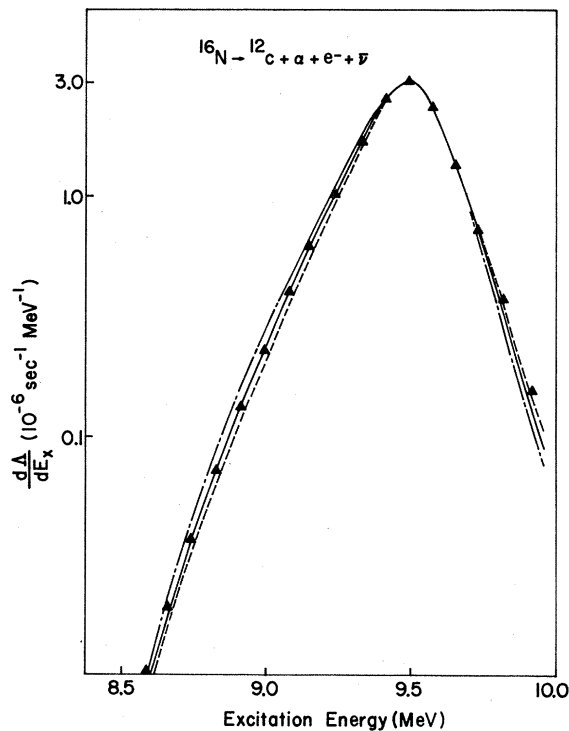


FIG. 3. Calculated curves for  $\gamma_1 = 0.37 \text{ MeV}^{1/2}$  and zero background (solid curve),  $\gamma_1 = 0.18 \text{ MeV}^{1/2}$  and destructive interference with the background (dashed curve), and for  $\gamma_1 = 0.58 \text{ MeV}^{1/2}$  and constructive interference with the background (dot-dashed curve).

fact that the  $T=0$  1p-1h state at 16.6 MeV and the  $T=1$  state at 18.1-MeV excitations carry a large portion of the GT strength. These states contribute because of the large  $(1p_{1/2})^{-1} (1d_{3/2})$  component<sup>24</sup> in their wave functions. These two states show up as broad resonances in the continuum calculations of Raynal, Melkanoff, and Sawada<sup>27</sup> and there is evidence of isospin mixing because both states have some dipole strength. However, upon comparing the peaks seen in  $^{15}\text{N}(p, \gamma)$ <sup>28</sup> and  $^{15}\text{N}(p, n)$  with those seen in  $^{12}\text{C}(\alpha, \gamma)$ <sup>26</sup> and  $^{12}\text{C}(\alpha, n)$ ,<sup>29</sup> we can find only one common  $1^-$  peak in this region, the narrow peak at  $E_x = 17.13 \text{ MeV}$ . For this peak we estimate  $\Gamma_\alpha = 1.5 \text{ keV}$ , which means the reduced width is extremely small.

In order to put an upper limit on the background we have adopted a value of  $\gamma_\alpha^2 = 20 \text{ keV}$  as the magnitude of the  $\alpha$  width of a  $T=0$  predominantly 1p-1h  $1^-$  state. This value was obtained by Hebbard<sup>30</sup> as the  $\alpha$  width of the  $1^-$  state at 12.45 MeV and agrees with the  $^{12}\text{C}(\alpha, \gamma)$  data of Larson and Spear.<sup>31</sup> The  $1^-$  states at 12.45 and 13.07 MeV have large single-particle widths because of their strong  $(p_{1/2})^{-1} (2s)$  components. Since either constructive or destructive interference from the background can occur we take

$$\gamma_3 M_3 = \pm [(0.02)(1.31)]^{1/2}. \quad (13)$$

The plus sign implies constructive interference for  $E_x < 9.56 \text{ MeV}$ .

The  $\alpha$  spectrum following the  $\beta$  decay of  $^{16}\text{N}$  was calculated, using Eq. (2), for the two values of  $\gamma_3 M_3$  given above, as well as  $\gamma_3 M_3 = 0$ . The comparison with the data of Ref. (1) is shown in Fig. 3. The data were shifted 60 keV lower in energy so that the peak occurred at about 9.48 MeV. The calculated curve for the case of zero background falls on top of the data except at the extremities

TABLE II. Values of the parameters for negative interference of the background ( $\gamma_3 M_3 > 0$ ), zero background, and positive background ( $\gamma_3 M_3 < 0$ ) with the 9.56-MeV resonant amplitude. The above signs of the interference obtain for  $E_x \leq 9.56 \text{ MeV}$  and change for  $E_x > 9.56 \text{ MeV}$ .

	Negative interference	No interference	Positive interference
$\gamma_1$	0.180 $\text{MeV}^{1/2}$	0.370 $\text{MeV}^{1/2}$	0.580 $\text{MeV}^{1/2}$
$\gamma_2$	0.764	0.771	0.784
$\gamma_3$	1.282	1.377	1.510
$E_1$	7.08 MeV	6.97 MeV	6.76 MeV
$E_2$	9.56	9.56	9.56
$E_3$	16.60	16.60	16.60
$M_1^2$	0.0295	0.0330	0.0398
$M_2^3$	0.0052	0.0053	0.0055
$M_3^2$	0.0158	0.0	0.0114

of the curve. The curve with constructive interference is shifted to lower energy and the curve with destructive interference is shifted toward higher energy. In view of the arbitrary shift of the data the small shifts of the three curves are not significant. However, the slope of the zero background curve and the negative interference curve fit the data better so that positive interference of the size of the upper limit in Eq. (13) is ruled out.

We list the  $R$ -matrix parameters in Table II along with the squares of the GT matrix elements. It should be stressed that five of the six  $R$ -matrix parameters are determined by the  $1^-$  phase shift, while the sixth,  $\gamma_1$ , is fixed by the  $\alpha$  spectrum upon the selection of a background contribution. A change of  $\pm 0.02$  in  $\gamma_1$  makes the fit markedly worse so the greatest uncertainty is in the size of the background. The parameter to be compared with  $\alpha$  widths of the bound states of  $^{16}\text{O}$  obtained from  $\alpha$  transfer reactions<sup>32, 33</sup> is  $\gamma_1^2(E_b)$ . The range represented in Eq. (13) leads to the restriction:

$$0.0088 \leq \gamma_1^2(E_b) \leq 0.073 \text{ MeV}, \quad (14)$$

$$\gamma_1^2(E_b) = 0.0339 \text{ MeV (no background)},$$

while the no-background result is listed below. In terms of the Wigner limit

$$0.013 \leq \theta_\alpha^2(7.12) \leq 0.105, \quad (15)$$

$$\theta_\alpha^2(7.12) = 0.049 \text{ (no background)}.$$

The above values overlap with the estimate of  $\theta_\alpha^2(7.12) = 0.06 - 0.14$  by Loebenstein *et al.*<sup>32</sup> and the single value  $\theta_\alpha^2(7.12) = 0.025$  by Pühlhofer *et al.*<sup>33</sup> They are also consistent with the theoretical estimate of Stephenson<sup>34</sup> who attributed the  $\alpha$  width of the  $1^-(7.12)$  to an admixture of a highly deformed state.<sup>35</sup> The deformed state is taken to be the major component of the  $1^-(9.58)$ .

In conclusion, a three-level  $R$ -matrix treatment

of the reaction  $^{16}\text{N} \rightarrow ^{12}\text{C} + \alpha + e^- + \bar{\nu}$  is capable of parametrizing the  $\alpha$  spectrum, remarkably well. It will be hard to isolate the variation of  $\gamma_1$  from that of  $\gamma_3 M_3$  better than we have done, but with more accurate determinations of the  $^{12}\text{C} + \alpha$  phase shifts it can be done. There are differences in the shapes for positive or negative interference, and the beautiful precision of the  $\alpha$  spectrum data should allow one to fix  $\theta_\alpha^2(7.12)$  to  $\pm 25\%$  once the arbitrary shifts of the curves are eliminated. We tend to accept the energy calibration of Ref. 1, since Browne and Michael<sup>36</sup> measured the energy of the second  $1^-$  state to be 9.61 MeV, considerably higher than the phase-shift result.

With regard to the problem of astrophysical interest, that of obtaining the  $^{12}\text{C}(\alpha, \gamma)$  cross section at energies of several hundred keV, our results may be of limited usefulness, since the background in the electromagnetic interaction must be determined from scratch. However, the widths for the 7.12-MeV state must come out to be the same after analyzing the data on both reactions if we are to have any confidence in the parametrization of the capture reaction.

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## New States in ${}^9\text{Li}$ from the Reaction ${}^7\text{Li}(t, p){}^9\text{Li}$

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Proton spectra from the reaction  ${}^7\text{Li}(t, p){}^9\text{Li}$  have been measured at triton energies of 15 and 19 MeV. Evidence is observed for new states in  ${}^9\text{Li}$  at excitation energies of  $4.31 \pm 0.03$ ,  $5.38 \pm 0.06$ , and  $6.41 \pm 0.02$  MeV, having widths of  $0.25 \pm 0.03$ ,  $0.6 \pm 0.1$ , and  $< 0.1$  MeV, respectively. Angular distributions were obtained at  $E_t = 15$  MeV for the ground, 4.31-, and 6.41-MeV states. The shape of the ground-state distribution is similar to an  $L=0$  distorted-wave Born-approximation calculation, in agreement with the assignment  $J^\pi = \frac{3}{2}^-$  for the ground state of  ${}^9\text{Li}$ . The relative magnitude of the cross section to the first excited state, when considered with other work, provides support for the assignment of  $J^\pi = \frac{1}{2}^-$ .

### INTRODUCTION

Although considerable experimental information is available for  $T = \frac{1}{2}$  states in  $A = 9$  nuclei, only the first two  $T = \frac{3}{2}$  states have been previously observed. Numerous studies<sup>1,2</sup> of the  $\beta$  decay (or subsequent neutron emission) of  ${}^9\text{Li}$  have given information about the ground state, and  ${}^9\text{Li}$  has been observed as a spallation product<sup>3</sup> as well as through the reactions  ${}^7\text{Li}(t, p){}^9\text{Li}$  and  ${}^{18}\text{O}({}^7\text{Li}, {}^9\text{Li})-{}^{16}\text{O}$ .<sup>4,5</sup> The only previous evidence for excited levels of  ${}^9\text{Li}$  comes from the reaction  ${}^7\text{Li}(t, p){}^9\text{Li}$  performed with 11.28-MeV tritons.<sup>4</sup> In that experiment the first excited level was observed to lie at an excitation energy of 2.691 MeV; no additional levels were found up to 4-MeV excitation. The measured angular distribution of the ground-state

proton group indicates an assignment of  $J^\pi = \frac{3}{2}^-$ , in agreement with shell-model predictions.

The formation of  ${}^9\text{C}$  from the proton bombardment of carbon and natural boron targets has been observed<sup>6</sup> by detecting delayed protons following positron decay of  ${}^9\text{C}$ . Accurate measurements of the mass of  ${}^9\text{C}$  have been made by observing  ${}^6\text{He}$  energy spectra from the reaction  ${}^{12}\text{C}-({}^3\text{He}, {}^6\text{He}){}^9\text{C}$ <sup>7</sup> and by detecting the delayed protons from  ${}^9\text{C}$  near the threshold for the reaction  ${}^7\text{Be}-({}^3\text{He}, n){}^9\text{C}$ .<sup>8</sup> No experimental observations of excited levels in  ${}^9\text{C}$  have yet been made.

The lowest  $T = \frac{3}{2}$  state has been observed in  ${}^9\text{B}$  at  $E_x = 14.67$  MeV by means of the reaction  ${}^7\text{Li}-({}^3\text{He}, n){}^9\text{B}$ ,<sup>8</sup> the reaction  ${}^9\text{Be}({}^3\text{He}, t){}^9\text{B}$ ,<sup>9</sup> the reaction  ${}^{11}\text{B}(p, t){}^9\text{B}$ ,<sup>10</sup> and the reaction  ${}^7\text{Li}({}^3\text{He}, n\gamma){}^9\text{B}$ .<sup>11,12</sup> Similarly, the lowest  $T = \frac{3}{2}$  state in  ${}^9\text{Be}$  has been