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PHYSICAL REVIEW C VOLUME 4, NUMBER 5 NOVEMBER 1971

Nuclear Deexcitation γ Rays in ^{14}N, ^{14}C, and ^{15}N Following π^- Capture on ^{16}O†

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(Received 30 June 1971)

Negative pions produced by the National Aeronautics and Space Administration Space Radiation Effects Laboratory cyclotron were stopped in water and the nuclear deexcitation γ rays observed. γ rays from ^{14}N, ^{15}N, and ^{14}C were identified, and yields to particular states in these nuclei obtained. A prominent γ-ray peak in our spectra was from the 3.945-MeV 1^+ state in ^{14}N which was formed with a rate of 1.8% per stopped pion. The Doppler-broadened line shape of the decay γ ray for this state was analyzed to obtain the momentum distribution of the recoiling ^{14}N nucleus. This momentum distribution is compared with momentum distributions obtained by related experiments, and to theoretical predictions. Yields to various states are presented and discussed.

I. INTRODUCTION

Several years ago it was pointed out by Ericson[1] that π^- capture is a useful probe of nuclear structure. The experiment considered here is of a new type using π^- absorption. We have observed nuclear γ rays from states in nuclei left after π^- absorption. We obtain from the relative yields of particular γ rays information on the relative rates for producing the nuclear states which precede these γ rays. The Doppler-broadened line shapes of some transitions contain implicitly the momentum spectrum of recoiling nuclei and hence the sum-momentum distribution of ejected particles. Since the π^- absorption process proceeds with high probability by ejecting two nucleons,[2] this momentum distribution is the sum momentum of nucleon pairs. Further, since the level from which the γ ray comes is known, comparison between theoretical predictions and the experimental distributions can be made.

It is clear from, e.g., $\pi^+, 2p$ work[3,4] that there are nuclear-structure effects in pion absorption. One finds very different excitation-energy distributions for different residual nuclei. These reactions which were at 80-MeV π^+ energy indicate that about 20% of the $\pi^+, 2p$ reaction on ^{16}O leads to bound states in ^{14}N. Thus one might expect reasonable yields of nuclear γ rays from the related reaction $(\pi^-, 2n)$.

There have been several experiments probing the momentum distributions of absorbing nucleon pairs.[4,5] These have looked at the proton pairs from $\pi^+, 2p$ reactions for slow but not stopped π^+, and the neutron pairs from stopping $\pi^-, 2n$ reactions. In both cases the energy resolution for the excitation energy of the residual nucleus has been limited to about 5 MeV. The present experiment in a favorable case has energy resolution of 3 keV.

II. EXPERIMENTAL PROCEDURE

Figure 1 shows schematically the experimental setup for most of the data taking. The water target bulged to a thickness of 4 in. perpendicular to its face, and was 6 in. thick to the beam. This target allowed us to have 5×10^5 (123) events/sec, 1.5×10^5 (1234)/sec, 3.5×10^5 pion stops/sec ($123\bar{4}$), 1.6×10^3 γ-ray discriminator pulses/sec, and about 250 stop γ coincidences/sec. All of the plastic scintillators were $\frac{1}{4}$ in. thick. The pions were from the meson channel of the Space Radiation Effects Laboratory (SREL) 600-MeV synchrocyclotron. They entered the absorbers with a nominal 200 MeV/c momentum. Variation of the absorber thickness was done to maximize the π^- stop rate. The experimental arrangement is very similar to that used for measurement of pionic x rays[6] at SREL.

The coincidence γ-ray energy signals were sent through a preamplifier and base-line restorer, and then gated into an 8192 Kicksort analog-to-digital converter (ADC) with a digital stabilizer. The gain was arranged so that we had approximately 1 keV/channel, and hence could look to about 8 MeV for photopeaks and 9 MeV for double-escape peaks.

The γ-ray detector was a 40-cm^3 Ge(Li) detector. It had 3.5-keV resolution on ^{60}Co at the beginning of the run and worsened to 4.5 keV at the end because of radiation damage.

The γ-discriminator pulses were used to start a time-to-amplitude converter (TAC) for which the suitably delayed π stops (timing determined by the 3 detector) preceded the stop signal. The signal for the TAC was digitized and simultaneously with the energy signal fed into the interface of an on-line computer (Yale system IBM 360-44). Two software energy "analyzers" were set up. One corresponded to γ rays coincident [full width at half maximum (FWHM) $\approx$ 60 nsec)] with a stopped pion. The other corresponded to γ rays with times separated both preceding and delayed from the prompt region by about 100 nsec and extending for nearly a μsec in both directions. The second region enabled us to be sure that our energy signals were not associated with random background or from μ^- absorption.

Extensive checks were made to verify that the γ rays of interest were indeed produced by π^- absorption. These included variation of the target thickness, stopping the pions in a carbon block immediately before the water target, comparison of the prompt γ spectra with spectra associated with times delayed by a few μsec, and comparison to lines produced by neutrons bombarding ^{16}O. All

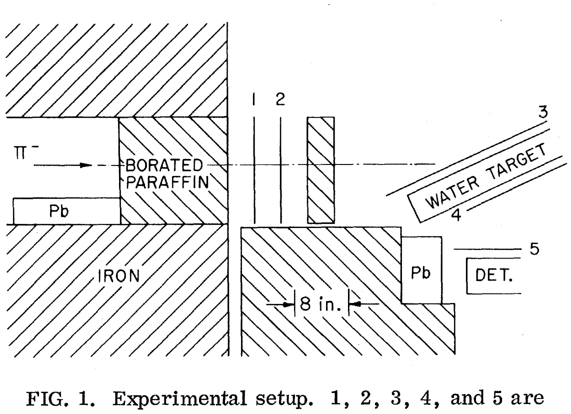

FIG. 1. Experimental setup. 1, 2, 3, 4, and 5 are $\frac{1}{4}$-in.-thick, square plastic scintillators.

TABLE I. γ-ray line intensities.

Nucleus	Transition J_i^π-J_f^π	E_γ (MeV)	$N_\gamma\times10^{-6}$	Error %	(N_γ/N_π) $\times100$	N_γ/N_i[b]
^{14}N	$1_2^+\rightarrow0_1$	1.634	1230	2	1.79	1
^{14}N	$0_1^+\rightarrow1_1^+$	2.311	1538	2	2.24	1.25
^{13}C	$\frac{3}{2}{}_1^-\rightarrow\frac{1}{2}{}_1^-$	3.682	465	22	0.69	0.38
^{13}C	$\frac{5}{2}{}_1^+\rightarrow\frac{1}{2}{}_1^-$	3.856	170	11	0.25	0.14
^{14}N	$2_1^-\rightarrow1_1^+$	5.104	102	40	0.15	0.08
^{15}N	$\frac{5}{2}{}_1^+\rightarrow\frac{1}{2}{}_1^-$	5.269	103	15	0.15	0.08
^{14}C	$3_1^-\rightarrow0_1^+$	6.727	22	40	0.032	0.018
^{12}C	$2_1^+\rightarrow0_1^+$	4.433	575	7	0.84	0.47
^{16}O	$3_1^-\rightarrow0_1^+$	6.129	219	3	0.32	0.18
^{208}Pb	$3_1^-\rightarrow0_1^+$	2.614	605	4	0.88	0.49

[a] These are the statistical errors only. Systematic errors might be expected to be of the order of 20%.

[b] N_1 is the number of γ rays produced in the ^{14}N $1_2^+\rightarrow0_1^+$ transitions, i.e., 1.23×10^9.

of these checks unambiguously pointed to pion absorption as the origin of these γ rays.

Calibration of the absolute photopeak efficiency of the detector was done using calibrated ^{60}Co, ^{56}Co, ^{228}Th, ^{22}Na, and ^{137}Cs sources. These sources were placed at three distances from the detector, with the target in place, yielding an over-all efficiency curve after geometry and absorption were factored in. Interpolation between the energies of the calibration γ rays, and extrapolation to high-energy γ rays was accomplished using a power law for the efficiency vs energy. Double-escape-peak efficiency was obtained by using the stronger lines observed during the run, and beyond 6-MeV γ-ray energy, by extrapolating using the power law for the photopeak efficiency together with the energy dependence of the pair-production cross section. To be conservative we assigned 40% errors to the intensity of those γ rays obtained using only the double-escape peaks.

III. DATA ANALYSIS

Analysis of the γ-ray spectra was accomplished as follows. A region of the spectrum was chosen surrounding the peak or peaks of interest. Intervals were chosen on each side of the peak to yield a background parametrization for the whole region. This was expressed in the form B_0+B_1X, where X is channel number referred to the starting channel of the region. Once background was subtracted from the data, we obtained the peak areas by fitting with simple Gaussians, one per peak. The parameters associated with a Gaussian were the height, the width, and the centroid position. The areas obtained from these parameters are presented in Table I. The statistical errors include the error introduced by subtracting the background.

An intrinisic ambiguity in this method of obtaining pion-capture rates to particular states is the extent to which the initial state for the γ transition was produced by direct pion absorption to it or whether pion absorption produced states which γ cascade to this initial state. In some cases this ambiguity can be at least partially resolved. For the 3.945-MeV ^{14}N 1^+ state we could make the following arguments. First, those states above the proton-decay threshold for ^{14}N (7.6 MeV) will decay by nucleon emission, since γ widths are at least 2 orders of magnitude less than the proton widths for such states. For the states below 7.6 MeV one should note that since the ground state has the same quantum numbers, $J=1^+, T=0$ as the 3.945-MeV state, one has a severe energy-dependent inhibition against decay to the 3.945-MeV state. The most energetic state which is known below the proton-decay threshold is at 7.03 MeV. For such a state we have $(7.03/3.09)^3$ which is a factor of 12 favoring ground-state decay. Now when we consider the observed branching ratios[7] we see that only the 6.44-MeV 3^+ has any appreciable branch to the 3.945-MeV state. (It has an approximate 20% branch). This 3^+ state should, however, be composed mostly of $s_{1/2}d_{5/2}$ and not for instance $(p_{3/2}{}^{-2})$ for this doubly breaks the $p_{3/2}$ closed shell and is hence very high in excitation. Cohen and Kurath[8] give no 3^+ states below 10 MeV for p shell only. This state is not strongly excited[9] in (d,α) or $(p,{}^3\text{He})$ reactions on ^{16}O nor by (α,d) reactions on ^{12}C, nor (α,α') on ^{14}N; and this state is populated by decays from the 9.6- and 10.43-MeV, 2^+ $T=1$ states[10] all of which gives strong support for the $(s_{1/2}d_{5/2})$ assignment. A $s_{1/2}d_{5/2}$ configuration cannot be easily formed by two-hole formation as in π^- absorption. Further we see no indication of a 2.49-MeV transition between the 6.44- and the 3.945-MeV states. In fact by the absence of a $6.44 \rightarrow 3.945$-MeV γ ray a limit of <5% can be placed on the intensity of the 1.634-MeV transition which is preceded by excitation of the 6.44-MeV state. Thus we can see that it is more than reasonably likely that the 3.945-MeV 1^+ state is predominantly formed directly by π^- absorption.

We observe ^{14}N 3.945-MeV state to decay with a Doppler-broadened γ ray of 1.634-MeV energy. To find the detector resolution unfolded Doppler-broadened γ-ray line shape for this transition the following procedure was employed. First, an unbroadened line was found near this peak (the double-escape peak of the ^{208}Pb 2.614-MeV transition). This unbroadened, background-subtracted line was fitted with Gaussian-plus-tail parametrization, and separately also with a third-order polynomial times a Gaussian. Both gave satisfactory and essentially equivalent fits. It was decided to use the latter, since this made the subsequent fit of the Doppler-broadened line computationally easier. The normalized χ^2 for this parametrization was $\chi^2_n = 1.28$. The form of the line shape for the ^{208}Pb transition was $G(x) = (1770 + 0.608x + 0.447x^2 - 0.386x^3)e^{-0.043x^2}$, where x refers to channel number.

We assumed that the underlying line shape, i.e., the one which is folded with the detector resolution to yield the observed line shape, is symmetric. One would expect a polynomial times a Gaussian for the underlying γ-ray line shape to the extent that harmonic-oscillator wave functions are appropriate when one considers the momentum of nucleon pairs. This follows since integrals of polynomial times Gaussians are again polynomials times Gaussians. Thus we expect that an appropriate parametrization of the line shape is

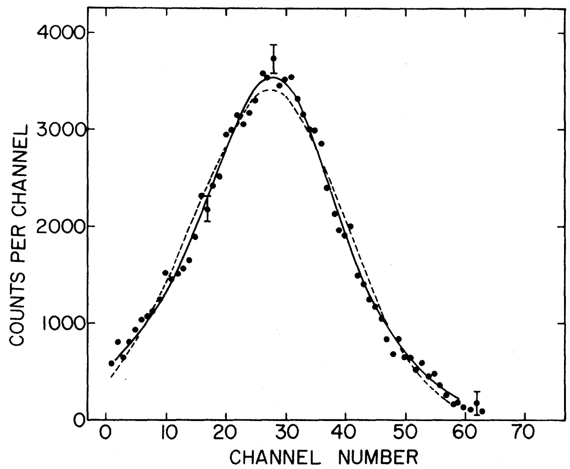

FIG. 2. Doppler-broadened line shape. The curves are obtained by folding an unbroadened line shape with parametrized broadened line shapes. Fit A, the solid line was obtained using a polynomial with up to cubic terms, multiplied by a Gaussian. The dashed curve was obtained using a Gaussian alone for the broadened line shape, Fit B.

$$F(x)=\{A_0+A_1|x-x_0|+A_2(x-x_0)^2 + A_3|x-x_0|^3+\cdots\}e^{-(x-x_0)^2/2\Sigma^2},$$

where the absolute values on the odd powers assure symmetry. It turns out that if the A_1 term is nonzero one will have an infinite momentum density at zero momentum. To disallow this we forced A_1 to be zero. We felt we needed to keep terms to at least x^3 to allow for possible structure in the momentum distribution.

Thus we fit the data $D(x)$ with the function $F(x)$ folded with the resolution function $G(x)$:

$$D(x)=\int F(x')G(x-x')dx'.$$

The data and two fits are presented in Fig. 2. The normalized χ^2 for fit A is $\chi^2_n=0.614$, while that for B is $\chi^2_n=1.35$. The parameters for fit A are $A_0=0.122$, $A_2=0.120\times10^{-4}$, $A_3=0.124\times10^{-6}$, and $\Sigma^2=153$. For fit B we held A_1, A_2, and A_3 at zero, and hence had a simple Gaussian. The momentum distribution obtained from these to fits are not much different and, further, since the fit A has a χ^2_n which is indicative of a very good fit we felt higher-order-polynomial terms would not be obtained sensitively. Below we use fit A.

Doppler Broadening and Momentum Distribution

The Doppler shift of an emitted γ ray, ΔE, is equal to $E_\gamma\, v/c\cos\theta$, where E_γ is the energy of the transition, θ is the angle between the recoil direction and the detector, and v is the speed of the recoiling ion, i.e., its momentum divided by its mass. Let us for the moment fix attention on nuclei recoiling with momentum, of magnitude between K and $K+dK$, randomly oriented with respect to decay γ rays; arbitrarily good γ-ray energy resolution; and with a very short nuclear lifetime.[11] The Doppler-broadened γ-ray line shape so produced is rectangular with width $W=2E_\gamma K/mc$; and height $h=N/W$. N is the number of γ rays of this transition observed. Calling dN/dK the number of γ rays observed associated with recoil momentum magnitude between K and $K+dK$, i.e., in a spherical shell in momentum space, the line shape now is

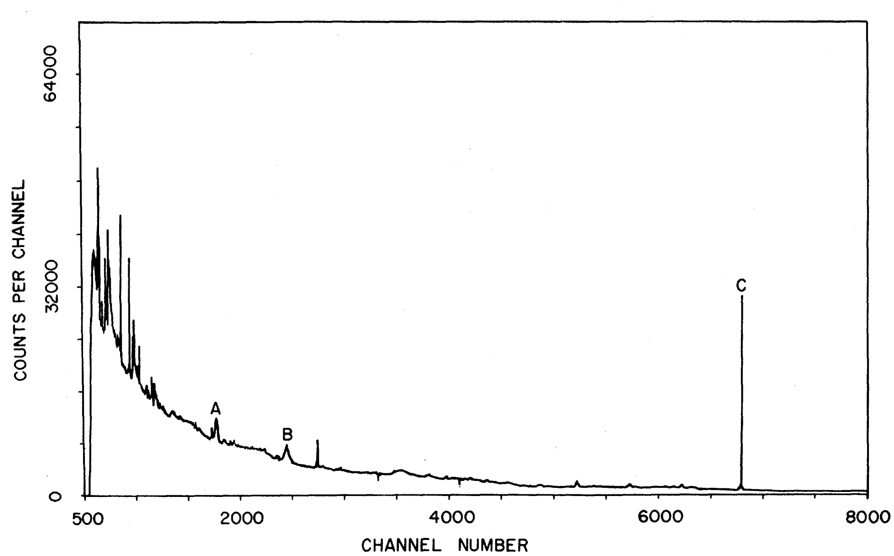

FIG. 3. γ-ray spectrum. Peak A is from the 1.634-MeV transition, peak B is from the 2.311-MeV transition, and peak C is due to the pulser.

$$N(\Delta E') = -\int_{\infty}^{K'} \frac{dN}{dK} \frac{dK}{W}.$$

If we now differentiate this integral with respect to its limit we obtain

$$\frac{dN}{dK} \sim \Delta E \frac{dN(\Delta E)}{d\Delta E}. \quad (1)$$

Note that we assume above that there is no correlation between the recoil momentum and the γ ray. This is open to some question but can be dealt with for particular γ rays. For the 1.634-MeV transition there exist arguments favoring this assumption. This point will be considered further in the discussion section.

IV. RESULTS AND DISCUSSION

Figure 3 shows the γ-ray spectrum we obtained. It is considerably compressed on this scale. We used 8192 channels for the data and had approximately 1 keV per channel. As one can see there are many γ-ray lines in evidence. We present energy-level diagrams for nuclei of interest in Fig. 4. Table I lists the areas and number of γ rays associated with the more prominent peaks. Weaker- and low-energy lines associated with activation were not analyzed. Examples of these latter are the ^{206}Pb transition at 0.803 MeV, the ^{56}Fe transition at 0.845 MeV, and the several transitions associated with neutron interactions in the Ge(Li) detector.[12] The states between 5.69 and 6.21 MeV in ^{14}N do not appear to be strongly excited; the more energetic γ rays are more difficult to observe in general, since their widths are greater due to the enhanced Doppler broadening, and the efficiency of the detector is less. The strong γ ray at 1.634 MeV is of special interest and will be discussed in detail below.

The observation of the ^{15}N transition with E_γ = 5.269 is to be noted. This transition involves single-nucleon emission in the absorption process. The initial γ decaying state is a $\frac{5}{2}^+$ state whose configuration should be $(p^{-2})^0 d_{5/2}$ as illustrated in Fig. 5. Thus the transition might be thought to involve absorption on p-shell particles leading to a final state with one nucleon free and the other in the shell-model $d_{5/2}$ state. It seems reasonable that the momentum transfer associated with single-nucleon emission might thus be obtained by reversing the motion of the nucleon which remains in the nucleus.

We see only the 6.729 3^- ground-state transition in ^{14}C. This may well be due to the broadness of other lines. This line is narrow because of the slow $E3$ transition rate. This state is weakly excited relative to the 3.945-MeV state of ^{14}N. This weak excitation seems reasonable, since the 3^- state should be mostly made from particle-hole states relative to the ground state of ^{14}C, and hence the pion absorption must either go through admixtures in wave functions or through two-step processes.

The strong excitation of the 3.945-MeV 1^+ level and the weak excitation of the 2.311-MeV 0^+ level is in agreement with theoretical results of Kopaleishvili *et al.*[13] They also predict strong excitation of a spin-2 state (we presume 2^+, but we know of no 2^+ states in ^{14}N in the energy region indicated by the figure in that paper). Eisenberg and LeTourneux[14] predict a strong excitation of $(p_{3/2}p_{1/2})^{-1}$ configurations and weak excitation of $p_{1/2}^{-2}$ configuration which is also in agreement with our result where one observes that the dominant configuration as assigned, e.g., by Pehl *et al.*[9] to the second excited state, is the former config-

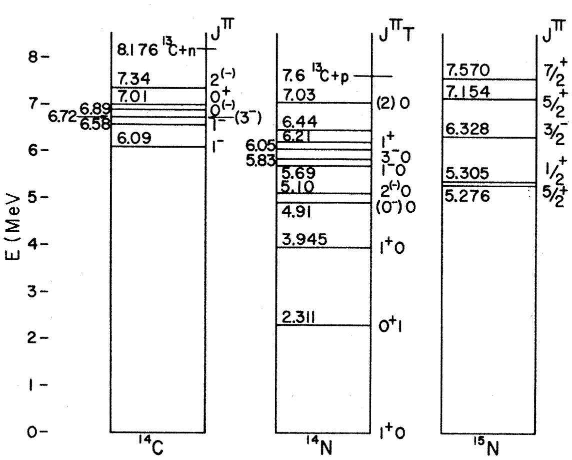

FIG. 4. Energy-level diagrams for ^{14}C, ^{14}N, and ^{15}N. The energies, spins, and parities have been taken from the National Research Council Nuclear Data Sheets. For ^{14}C and ^{14}N the lowest particle-emission thresholds are indicated.

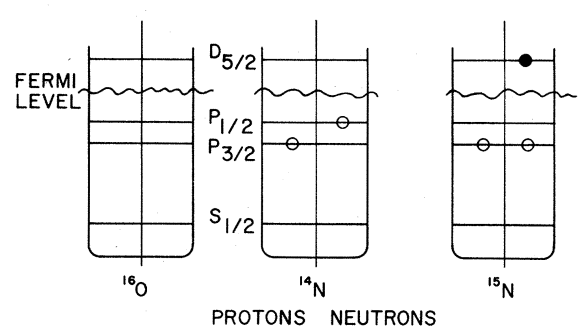

FIG. 5. Schematic description of particle-hole configurations of states of interest. In the case of ^{15}N both holes are coupled to zero angular momentum and may arise from either $p_{3/2}^{-2}$ or $p_{1/2}^{-2}$ configurations. $p_{1/2}^{-2}$ is lower in energy.

uration and the first and ground states are the latter.

The direct pion-absorption reaction rate to the 2.311-MeV 0^+ $T=1$ state is of the order of 14% or less of that to the 3.945-MeV 1^+ $T=0$ state. This relative rate is arrived at by first adding to the capture rate of the 3.945-MeV state the γ-decay rate associated with the ground-state branch. This is a 3% effect. Then we reduce the observed yield of the 2.311-MeV γ ray by those parts due to feeding from other states. 80% is due to the 3.945-MeV state and 6% is from the 5.104-MeV state. This only accounts for the observed feeding. The relatively weak excitation of this 0^+ $T=1$ state can be interpreted as follows[15]:

The interaction between the pion and the nucleons is considered to be[16]:

$$H_{\text{int}} \sim \sum_{i=1}^{A} \tau_{i}^{-} \vec{\sigma}_i \cdot (\vec{\nabla}_\pi - \vec{\nabla}_i) . \tag{2}$$

Using calculated values of atomic-state populations and experimentally measured yields,[17,18] pion absorption on ^{16}O takes place over 97% from $l=1$ mesonic orbits, and hence the gradient implied by Eq. (2) on the pion wave function is finite at the origin, while the gradient of the nucleon wave function is multiplied by the pion wave function itself which is zero at the origin and small over the nucleus.[19] Hence we assume that the interaction is now

$$H_{\text{int}} \sim \sum_{i=1}^{A} \tau_{i}^{-} \vec{\sigma}_i \cdot \vec{\nabla} \varphi_{\text{IP}}$$

and only operates on the nucleon coordinates. For a particular m projection of the pion wave function this becomes

$$H_{\text{int}} \sim \sum_{i=1}^{A} \tau_{i}^{-} \sigma_{im} . \tag{3}$$

At this point let us consider the nuclear states involved. If we leave the nucleus in a $T=1, J=0^+$ state and the reaction takes place in one step, then these must be the quantum numbers associated with the absorbing nucleons. These nucleons are in the nuclear p shell. If the relative angular momentum $l=0$, the angular momentum $\mathcal{L}$ of center of mass of the nucleon pair must be 0 or 2 to have positive parity. If $\mathcal{L}=2$ then the spins and the orbital angular momentum cannot be coupled to the required $J=0$. Similarly if the relative angular momentum $l=2$, then the required $\mathcal{L}=0$ and the spins cannot couple to $J=0$ either. So we are left with an $l=0$ $\mathcal{L}=0$, $^1S_0$ state; or an $l=1$ $\mathcal{L}=1$, $^3P_0$ state. Now we require that the pion absorption only takes place on nucleons which in their initial states are close together, i.e., relative S states. A nearly equivalent requirement would be that the relative linear momentum be high. This favors the high principal quantum-number relative S state [see Eq. (6b) below]. The interaction [shown in Eq. (3)] does not contain nucleon spatial coordinates, and hence the final state, relative, and center-of-mass angular momenta will remain the same. Thus the final state contains two neutrons in a $^1S_0$ state which is required for antisymmetry. Now we see that when we evaluate the matrix element of σ_m from Eq. (3) between these two states it vanishes, and hence we would expect to only excite this state to the extent that the assumptions we have made are invalid. It should be noted that the 1^+ $T=0$ state does not suffer this inhibition, for there the initial state of the two nucleons contains ^3S components.

The pion-absorption process is in some ways analogous to a (d, α), $(p, {}^3\text{He})$, or $(\alpha, {}^6\text{Li})$ reaction. These reactions have been carried out on ^{16}O.[7] For the (d, α) and $(\alpha, {}^6\text{Li})$ reactions both incoming and outgoing particles have zero isospin. In these cases the 0^+ $T=1$ state is not excited. For the $(p, {}^3\text{He})$ reaction the projectiles each have isospin $\frac{1}{2}$ and therefore can and, in fact, do excite this 0^+ state. The excitation of this 0^+ state is comparable to the excitation of the 1^+ at 3.945 MeV in contrast to the small excitation of the 0^+ state by π^- absorption as discussed above. It should be noted that there is no selection rule comparable to the pion selection rule which was discussed for the pickup reaction $(p, {}^3\text{He})$.

Momentum Distributions

In Fig. 6 we have the momentum distribution which one obtains using Eq. (1) and our best-fit Doppler-broadened line shape. This distribution is the relative number of occurrences of a recoil momentum magnitude falling between K and $K+dK$, i.e., (dN/dK). A related momentum distribution is

$$\frac{d^3N}{dK^3} = \frac{dN}{dK}\,\frac{1}{K^2} .$$

(d^3N/dK^3) is shown in Fig. 7.

An interesting comparison we can make is to the (d^3N/dK^3) measured by Grashin and Shalamov[20] for the $\pi^+, 2p$ reaction on freon (see Fig. 7). The general agreement is striking. This can be interpreted to mean that the average np absorbing pair has the same sum-momentum distribution as the particular np pair which is studied in our experiment.

A calculation of pion absorption on nucleons to essentially the same state as has been carried out here on ^{6}Li by Koltun and Reitan.[21] The np pair in ^{6}Li nucleons is coupled, also 1^+ $T=0$.

They come from the nuclear p shell with essentially the same LS configuration, i.e., $^3S_1$ and a slightly different radius. There is a substantial difference in the pion's state, for ^{6}Li it was assumed to be in the $1s$ atomic orbit,[22] while for ^{16}O it is, as mentioned before, almost surely in an $l=1$ orbit. The comparison between our experiment and the predictions on ^{6}Li appropriately scaled for nuclear well radius change is shown in Fig. 6.

Reactions of the sort p, pd can be thought to be quite similar to this $(\pi^-, 2n)$ reaction. Such reactions have been carried out on ^{16}O.[23] The average kinetic energy for an assumed deuteron cluster with Gaussian momentum distribution $(d^3N/dK^3 \sim e^{-K^2/Q^2}$, an $n=0, \mathcal{L}=0$ cluster) was 14 MeV in ^{16}O. This corresponds to a parameter $Q=190$ MeV/c which is larger than our case of $(\pi^-, 2n)$ excitation for the 1^+ state in ^{14}N. This is similar to the case of ^{6}Li, where the $(\pi^-, 2n)$ reaction leads to ^{4}He which has been studied by Davies, Muirhead, and Woulds.[24] Davies, Muirhead, and Woulds obtain a value of Q for the pair of approximately 70 MeV/c, while the p, pd reaction has an average kinetic energy of 8 MeV and hence an rms momentum near 170 MeV/c.

Simple Model

Ericson[1] has pointed out that the momentum distribution one measures is related to the Fourier transform of the wave function of the center-of-mass coordinate of the absorbing pair. If we use the interaction given in Eq. (1) and again make the assumption that velocity of the pion term is most important in the interaction, we can write the reaction rate for ^{14}N recoil momentum between K and $K+dK$ as:

$$\frac{dN}{dK} \sim \int d\Omega_q \, d\Omega_K \left| \int d\vec{r}\, d\vec{R}\, e^{-i(\vec{q}\cdot\vec{r}+\vec{K}\cdot\vec{R})} \Psi^{*1^+ T=0}_{^{14}N} \times \sum_{i=1}^{A} \tau_{\bar{i}} \sigma_{im} F(\vec{r}) \Psi^{0^+}_{^{16}O} \right|^2 qK^2 . \quad (4)$$

The $F(r)$ introduced is a correlation function, qK^2 is proportional to the phase space, and q is the relative momentum of the two nucleons. We can write the ^{16}O ground state as

$$\Psi^{0^+}_{^{16}O} = \sum_m (-1)^m \Psi^{1^+}_{^{14}N^{-m}} (A_s\,{}^3S_1 + A_p\,{}^1P_1 + A_D\,{}^3D_1) , \quad (5)$$

where the constants $A_S = 0.945$, $A_P = 0.284$, and $A_D = 0.165$ are derived from the wave function of Cohen and Kurath.[25] (If one uses the Moshinsky brackets[26] to obtain each of the three spatial wave functions written using relative r and center-of-mass coordinates R, one finds that the P wave function has no relative S part and hence should not enter.) Upon averaging over-all direction of K, cross terms between ^3S and ^3D drop out, also for that matter, the cross terms between ^3S and ^1P, and ^3P and ^3D. This means that the small coefficients A_D and A_P only enter dN/dK in the square and hence may be ignored. If we carry out the Moshinsky transformation, integrate over the relative coordinate r, center-of-mass coordinate R, and the directions q and K and ignore the slow dependence of the phase space on q, we obtain

$$\frac{dN}{dK} \sim K^2 \left[F_{00}(\sqrt{2}\, q) R_{10}\left(\frac{K}{\sqrt{2}}\right) - F_{10}(\sqrt{2}\, q) R_{00}\left(\frac{K}{\sqrt{2}}\right) \right]^2 , \quad (6a)$$

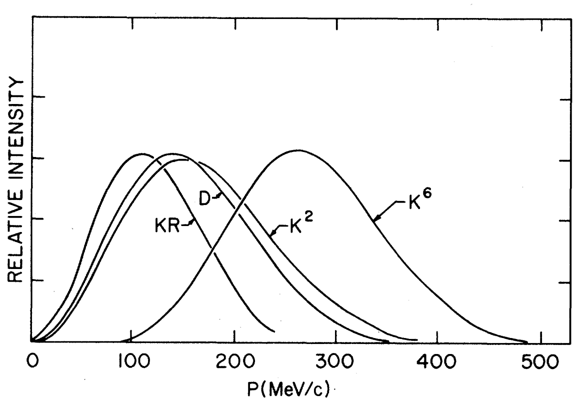

FIG. 6. Momentum distributions: dN/dK. D is the momentum distribution derived from our data; KR is the distribution from Koltun and Reitan (Ref. 21) for ^{6}Li scaled to ^{16}O by varying the harmonic-oscillator-potential-radius parameter; K^2 represents $K^2 e^{-K^2/2}$; and K^6 represents $K^6 e^{-K^2/2}$, where K is measured in units of 107 MeV/c (see text).

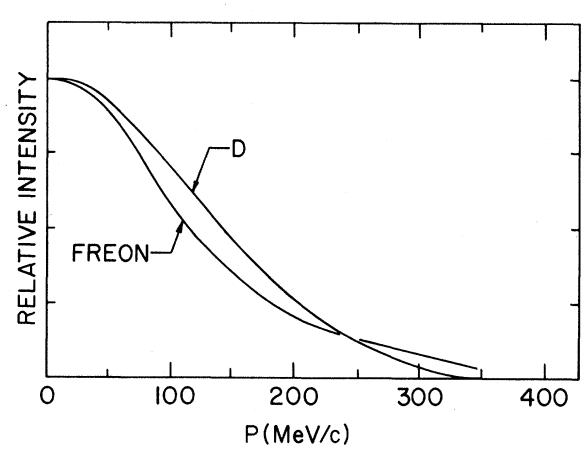

FIG. 7. Momentum distributions: d^3N/dK^3. D is the momentum distribution derived from our data and freon is the smooth curve based on a cluster model drawn by Grashin and Shalamov (Ref. 20) through their π^+, $2p$ data.

where

$$F_{no}(\sqrt{2}\,q) = \int e^{i\vec{q}\cdot\vec{r}} R_{n0}\left(\frac{r}{\sqrt{2}}\right) d\vec{r}\,. \tag{6b}$$

We have dropped all proportionality constants and used the fact that the Fourier transforms of the harmonic-oscillator wave functions are again the same harmonic-oscillator wave functions.[27] R_{nl} is the radial part of the harmonic-oscillator wave function. The momentum distribution depends upon $F(r)$. If we let $F(r)$ be a δ function, this produces upon integration a result proportional to the relative S wave function at $r=0$;

$$\frac{dN}{dK} \sim K^2\left[R_{10}\left(\frac{K}{\sqrt{2}}\right) - \sqrt{\tfrac{3}{2}}\, R_{00}\left(\frac{K}{\sqrt{2}}\right)\right]^2. \tag{7}$$

We have used the harmonic-oscillator radius parameter obtained from electron scattering[28] ($\sqrt{\hbar/m\omega} = 1.3 m_\pi^{-1} = 1.82$ fm). It turns out then that K is measured in units of approximately 107 MeV/c. Our result for this model is then

$$\frac{dN}{dK} \sim K^6 e^{-K^2/2}\,, \tag{8}$$

with lower powers of K canceling out. A comparison between this prediction and our data is presented in Fig. 6. If the results of our calculation were to agree, the harmonic-oscillator parameter would have to be $\sqrt{\hbar/m\omega} = 2.5 m_\pi^{-1}$. It may be of some interest to note that if the reaction strongly favored the relative 1S as opposed to the 0S wave function then the momentum distribution would be of the form

$$\frac{dN}{dK} \sim K^2 e^{-K^2/2}\,. \tag{9}$$

This is presented in Fig. 6 and indicated by K^2. This form is also what one would obtain if one assumes a cluster model for the two nucleons upon which absorption takes place and further assumes an $n=0$ $\mathcal{L}=0$ state function for its center-of-mass wave function. Another viewpoint which also predicts this latter form for dN/dK is to use Eq. (3) without correlations, i.e., $F(r)$ is a constant, and ignoring ^3D and ^1P wave function components as before, then the integrations which led to Eq. (7) lead now to

$$\frac{dN}{dK} \sim q^{K^2}\left| R_{00}(\sqrt{2}q)R_{10}\left(\frac{K}{\sqrt{2}}\right) - R_{10}(\sqrt{2}\,q)R_{00}\left(\frac{K}{\sqrt{2}}\right)\right|^2.$$

The relative momentum magnitude, q is determined by energy conservation. It is slowly varying with K in the region of interest, and has a large magnitude. Thus we are sampling the relative momentum components at high momentum. This strongly favors the $R_{10}(\sqrt{2}\,q)$ term and hence the $R_{00}(K/\sqrt{2})$ center-of-mass wave function.

There are several features to the data interpretation and theoretical comparisons which should be pointed out. There has been assumed to be no angular correlation between the recoiling nucleus direction and the emitted γ-ray direction. This will be true if the absorption takes place on $^3S_1$ nucleons and one uses the interaction described. For ^3D admixtures there may well be some angular correlation.[29] No attempt has been made to correct the recoil momentum for the effect of the nuclear potential on the outgoing nucleons. This could be done, e.g., by using an optical model for the residual nucleus. This effect should cause the momentum distribution of the recoil nucleus to be different from that of the initial absorbing pair, but the details of this alteration are not clear to the authors. Our simple model calculation is more naive than many that appear in the literature.[18] It was felt that it does carry the basic physical phenomena. A more complete approach to this problem might indicate those features of our model which are inaccurate. In particular we have left out any detailed description of initial- or final-state correlations. These would affect the contributions of the relative coordinate wave functions and through them the center-of-mass coordinate parts, and hence the observed momentum distributions. This is illustrated by the dramatic change produced with our model going from a δ-function correlation, which led to Eq. (7), to a smoother correlation, Eq. (9). While there exist calculations of similar quantities to the dN/dK for this $^{16}O \to {}^{14}N$ 1^+ transition we did not find any which dealt with the particular transition.

ACKNOWLEDGMENTS

We would like to express appreciation for helpful discussions with M. K. Banerjee, G. E. Brown, D. H. Wilkinson, and J. M. Eisenberg. We would further like to thank R. Winter, R. Welsh, and C. Perdrisat for careful reading of the manuscript and useful comments. Lastly, we express our gratitude to the staff of SREL for their hospitality and support.

†Work supported in part by the National Aeronautics and Space Administration.

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$$[(m_\pi/M)k_n(R_{\rm nuc}/2)]^2$$

relative to that of the pion gradient term. We have used $\nabla\phi_\pi R_{\rm nuc}/2$ as an average value for the pion wave function, the pion gradient being present in both terms factors out. Cross terms should not be present, since we average over quantization directions for the pion orbit with respect to nucleon momentum direction. Using $k_n = 340$ MeV/c (corresponding to sharing of the available energy by each of the ejected nucleons) and $R_{\rm nuc} = 3$ fm, we find the nucleon gradient term contribution to be of the order of 10%. The assumption that we may neglect nucleon gradient terms leaves us with an easily calculable theory against which to compare our experiments.

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