given spin and parity, that is, the $B(M1)$ and $B(E2)$ values are well reproduced. For the second levels of a given spin and parity more than half of the transition rates are well represented. While the calculated rates from a particular level may not be accurate, the relative amounts of $M1$ and $E2$ radiation averaged over several levels is representative of the data for second levels of a given spin and parity. The indications are that the model space becomes quite inadequate at excitation energies near 3–3.5 MeV and for $A \geq 55$. The model is clearly more reliable for states below 3.5 MeV and for nuclei near the beginning of the shell.

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Adiabatic Deuteron Model and the ^{208}Pb(p, d) Reaction at 22 MeV*

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It is shown that the adiabatic treatment of deuteron breakup during stripping reactions proposed by Johnson and Soper is able to explain the results obtained previously for the ^{208}Pb(p, d) reaction at 22 MeV without the use of an arbitrary radial cutoff. Some discussion is also given of the sensitivity of the predictions to the parameters of the model.

I. INTRODUCTION

Differential cross sections for the "single-particle" neutron-hole states excited in the ^{208}Pb(p, d) reaction at 22 MeV were previously measured[1] for scattering angles from 20 to 165°. When these data were subjected to a conventional[2] distorted-wave Born-approximation (DWBA) analysis, it was found to be necessary to eliminate the contribution to the transition amplitude from radii less than 8.5 F in order to obtain agreement with the measured angular distributions. Without this device, the theoretical distributions tended to peak at too small an angle and to have too much structure or to decrease too rapidly with increasing angle. These effects were particularly marked for the pickup of $1h_{9/2}$ and $1i_{13/2}$ neutrons.

Since $8.5 = 1.435A^{1/3}$ for $A = 208$, the radial cutoff required is appreciably outside the bulk of the matter distribution in the Pb nucleus. This must be regarded as a serious deficiency in the conventional DWBA approach. Similar problems with (p, d) reactions on medium-weight[3] and light[4] nuclei have been circumvented by the approach of Johnson and Soper[5] which includes approximately the contributions from diffractional breakup of the deuteron. We report here similar success for pickup from Pb. This extends the applications of the model to

TABLE I. Optical-potential parameters.

	V (MeV)	r_0 (F)	a (F)	W_D (MeV)	r_0' (F)	a' (F)	V_{so} (MeV)	r_{so} (F)	a_{so} (F)
Proton	$58.4-0.3E_p$	1.25	0.65	10	1.25	0.76	6	1.12	0.47
Deuteron	$\overline{V}$	1.25	0.682	$\overline{W}_D$	1.25	0.783	6	1.12	0.47
Neutron	V_n [a]	1.25	0.65	0	⋯	⋯	$(\lambda=25)$ [b]	1.25	0.65

[a] Adjusted to give binding equal to separation energy; $V_n \approx 46$ MeV.
[b] Spin-orbit coupling of λ times the Thomas term.

a selection of targets with A from 12 to 208. It is hoped that these results will stimulate more extensive and detailed studies of the model.

II. THE MODEL

According to the prescription of Johnson and Soper, instead of using in the DWBA an optical potential which fits the observed deuteron elastic scattering, one should use the effective deuteron potential

$$\overline{U}(r)=\frac{1}{D_0}\int [U_n(\vec{r}+\tfrac{1}{2}\vec{s})+U_p(\vec{r}-\tfrac{1}{2}\vec{s})]\,V_{np}(s)\phi_d(s)d\vec{s}\,, \tag{1}$$

where

$$D_0=\int V_{np}(s)\phi_d(s)d\vec{s}\,.$$

Here V_{np} is the neutron-proton interaction, ϕ_d is the wave function for the deuteron ground state, and U_n, U_p are the optical potentials for a neutron, proton, respectively, with $\frac{1}{2}$ the bombarding energy of the deuteron. The distorted wave generated by this potential includes "deuterons" which have been broken up in the average field of the target but whose constituent neutron and proton continue to move together in a ^3S state with little relative momentum.

The main effect[3] of the averaging Eq. (1) of the U_i over the shortranged function $D(s)=V_{np}(s)\phi_0(s)$ is to give $\overline{U}$ approximately the same radius as U_i but with a larger surface thickness. (We assume here that U_n and U_p differ only in depth.) Hence we chose the real and imaginary parts of $\overline{U}$ to have the same radius parameters r_0 and r_0' as the corresponding parts of the U_i, but increased the surface-diffuseness parameters a and a' so that the mean square radius (MSR) of $\overline{U}$ equaled the sum

TABLE II. Integrated cross sections in mb for $S=1$.

Case	$\overline{V}$	$\overline{W}_D$	$3p_{1/2}$	$3p_{3/2}$	$2f_{5/2}$	$2f_{7/2}$	$1h_{9/2}$	$1i_{13/2}$
1	102	13.5	5.96	4.93	1.61	1.17	0.095	0.263
2	102	19.4	5.36	4.37	1.45	1.03	0.085	0.209
3	112	19.4	5.34	4.43	1.48	1.09	0.092	0.188
4	92	13.5	6.97	6.08	2.16	1.66	0.145	0.331

of the MSR for U_i and $\frac{1}{4}$ the MSR for the function D (the MSR of D is about 4×0.6 F^2 if Hulthén functions are used). At the same time, the depths were adjusted so that the volume integral of $\overline{U}$ equaled that of (U_n+U_p). If a Woods-Saxon shape is used for the real parts, to order $(a/R)^2$ the increase in diffuseness is given by

$$\begin{aligned}\delta a&=0.0362\delta\langle r^2\rangle/a\\&=0.0217/a\quad\text{if } \delta\langle r^2\rangle=0.6\ \text{F}^2,\end{aligned} \tag{2}$$

and the corresponding reduction in depth from $\overline{V}=V_n+V_p$ is

$$\delta\overline{V}/\overline{V}=-(\delta a/a)(\pi a/R)^2,$$

where $R=r_0A^{1/3}$. With the derivative of a Woods-Saxon shape for the imaginary parts these relations become

$$\begin{aligned}\delta a'&=0.0304\delta\langle r^2\rangle/a'\\&=0.01824/a'\quad\text{if } \delta\langle r^2\rangle=0.6\ \text{F}^2,\end{aligned} \tag{3}$$

and

$$\frac{\delta\overline{W}}{\overline{W}}=-\frac{\delta a'}{a'}\left[1+\frac{2}{3}\left(\frac{\pi a'}{R'}\right)^2\right].$$

The spin-orbit potential has little effect and was taken to be simply the sum of the neutron and proton terms.

The proton optical potential used was based on one proposed[6] for 17-MeV protons on Pb and also found to give good fits to cross sections and polarizations for protons of 19, 20, 25, and 30 MeV scattered from Pb. The parameters are given in Table I, using standard notation. For the effective deuteron potential, the diffuseness parameters are increased by Eqs. (2) and (3) to the values given in Table I. The well depths are needed for a neutron and proton energy of about 8 MeV. The proton contribution was taken directly from Table I. If the neutron and proton potentials are assumed equal, we get the values of $\overline{V}$ and $\overline{W}_D$ given as case 3 in Table II. However, there are strong indications that the neutron potential is weaker than that for a proton; at 8 MeV the real depth is[7] $V\approx46$ MeV and the imaginary depth is $W_D\approx6.5$ MeV when $a'=0.47$ F. If we assume we can scale W_D for a'

=0.76 F by keeping the product $W_D a'$ constant, we get $W_D \approx 4$ MeV. These choices result in case 1 in Table II.

The picked-up neutron was assumed to be bound

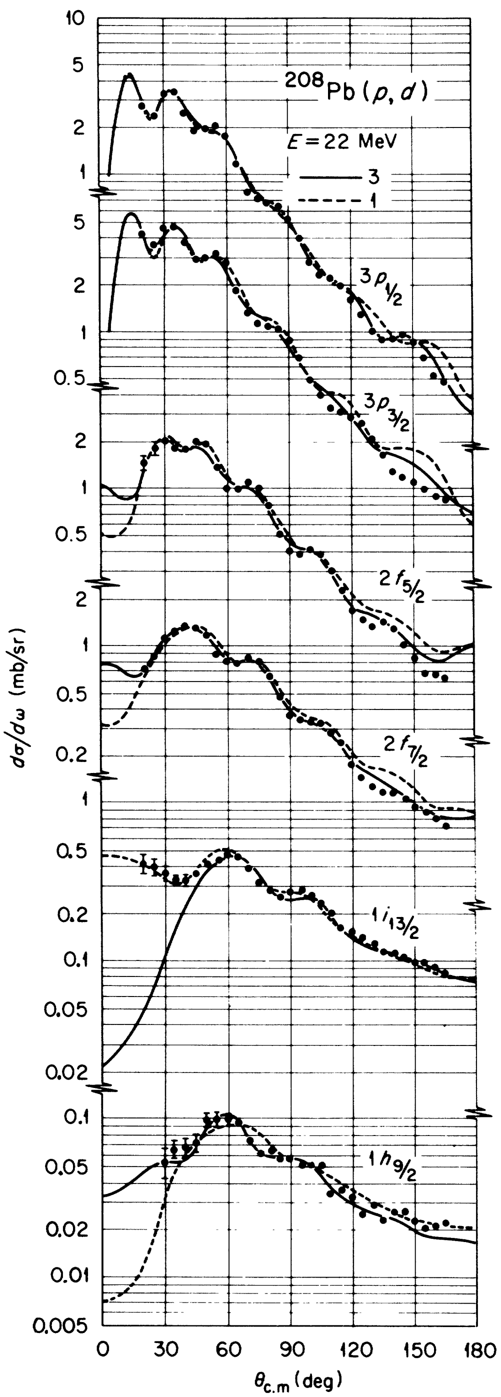

FIG. 1. Comparison of theoretical distributions obtained with the Johnson-Soper model with the measurements. The two curves correspond to cases 1 and 3 of Table II.

TABLE III. Spectroscopic factors for Fig. 1.

Orbit	$3p_{1/2}$	$3p_{3/2}$	$2f_{5/2}$	$2f_{7/2}$	$1g_{9/2}$	$1i_{13/2}$
E_x (MeV)	0	0.894	0.570	2.334	3.430	1.634
S (case 3)	2.1	3.9	6.0	6.0	7.0	15.0
S (case 1)	2.0	3.7	6.0	6.0	7.0	13.0
S (Ref. 1) [a]	2.1	4.0	6.1	6.0	6.1	13.4

[a] With radial cutoff at 8.5 F and fitted to peak cross section.

by the corresponding separation energy in a potential whose parameters are included in Table I and which is the same as that used for the conventional DWBA calculations of Ref. 1.

Except for the new prescription for the effective deuteron optical potential, the calculations are the same as for the usual DWBA and were made using the code JULIE. Spin-orbit effects on the distorted waves were omitted for the $h_{9/2}$ and $i_{13/2}$ transitions. For strict consistency with the folding integral (1) the same finite-range function $D(s)$ should be used in the stripping calculation.[2] However, the effects here were found to be at most a few percent, so most of the DWBA calculations were made in the zero-range approximation.[2]

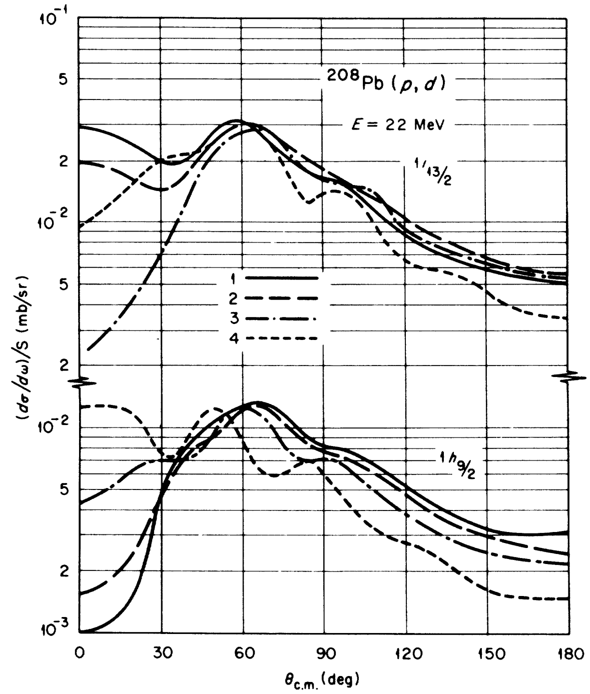

FIG. 2. The angular distributions for $l=5$ and $l=6$ pickup for the various choices of effective deuteron potential listed in Table II. For ease of comparison, the curves for case 4 have been reduced by factors of 0.5 ($l=5$) and 0.6 ($l=6$). Also the $l=5$ curve for case 3 was reduced by 10%, the $l=6$ curve for case 1 by 20%.

III. RESULTS

Figure 1 compares the calculated angular distributions for cases 1 and 3 with the data.[1] No radial cutoff was used and the agreement is good. The spectroscopic factors used in drawing the curves are listed in Table III; they are very close to those obtained[1] from the usual DWBA. Table II also lists the integrated theoretical cross sections for unit spectroscopic factor and for various choices of the effective deuteron well depths.

One result is the relative insensitivity to the strength $\overline{W}_D$ of the imaginary part of the effective deuteron potential; a similar situation exists for $1p$-shell nuclei.[4] Comparison of cases 1 and 2 in Table II shows that a 44% increase in $\overline{W}_D$ reduces the integrated cross sections by only 10% (except for the $h_{9/2}$, which is reduced 20%). The changes in angular-distribution shape are also small, being confined to the most forward angles. This is illustrated in Fig. 2 for the $h_{9/2}$ and $i_{13/2}$ transitions which are the most sensitive to any changes in parameters.

The calculations are most sensitive to variations in the real potential depth $\overline{V}$. Comparison of cases 1 and 4 (Table II) shows that a 10% reduction in $\overline{V}$ leads to increases in the integrated cross section ranging from 17 to 53% and produces large changes in the angular distributions (see Fig. 2). Comparison of cases 2 and 3 shows that these effects are nonlinear; there a 10% change in $\overline{V}$ produces changes of only a few percent in the cross sections, although the angular distributions for $h_{9/2}$ and $i_{13/2}$ (see Fig. 2) still show appreciable effects.

Corrections for nonlocality of the distorting potentials[7] were introduced using the local-energy approximation.[2] Except for small changes in the $h_{9/2}$ and $i_{13/2}$ distributions at the most forward angles, the effects were entirely negligible.

Calculations were also made with nucleon potentials of the Bechetti-Greenlees type[8] with very similar results. The spectroscopic factors obtained were close to those given in Table III. The angular distributions give rather poorer fits to the data than those shown in Fig. 1, since they tend to decrease more slowly with increasing angle.

IV. CONCLUSIONS

Previous work[3,4] has shown the Johnson-Soper model to be successful for targets of light- and medium-weight nuclei. The present results show similar success for the ^{208}Pb nucleus. In each case the conventional DWBA has required the use of a radial cutoff at a large radius in order to fit the measured angular distributions; such a cutoff is not needed when the adiabatic model is used. The spectroscopic factors obtained from the two methods are essentially the same.

The results of the adiabatic model are sensitive to the depth of the real part of the effective deuteron potential. Figure 1 shows that (except for the forward rise in the $i_{13/2}$ distribution) the deeper real potential of case 3 gives a better fit to the measured angular distributions although case 1 is most likely to be closer to the effective potential prescribed by the model. It will be interesting to see whether there are corrections to the adiabatic model which will account for this discrepancy.

It is hoped that these results may help to stimulate more extensive studies of this model.

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