

results of Alburger and Wilkinson.<sup>1</sup> Our value of  $7978.0 \pm 1.7$  keV agrees exactly with the  $\gamma$ -ray result of  $7978.1 \pm 1.9$  keV. It is therefore 18-keV below the tabulated number<sup>3</sup> and confirms the discrepancy pointed out by Alburger and Wilkinson. Other excitation energies between 7 and 9 MeV are found to be 5 to 10 keV lower than the tabulated

values. It is to be noted that the excitations given for the 9.185- and 9.274-MeV levels with uncertainties of  $\pm 1$  keV are exactly those found in the present work. The uncertainties listed in Table I were deduced in the usual way<sup>5</sup> and are estimates of internal errors.

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## Further Comparisons and Shell-Model Analysis for Nuclei with $N = 28$ , $20 < Z \leq 28$

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Comparisons are given between newly available experimental information about electromagnetic transitions in  $^{52}\text{Cr}$ ,  $^{53}\text{Mn}$ , and  $^{54}\text{Fe}$  and the theoretical predictions of a recently presented mixed-configuration shell-model calculation which permits small  $2p_{3/2}$  and  $1f_{5/2}$  admixtures in the  $1f_{7/2}$ -shell states. The agreement obtained between theoretical and measured values supports the conclusion that this model gives a good picture of  $E2$  and  $M1$  transition strengths as well as level spacings and proton spectroscopic factors in this region.

Several recent experiments have yielded new information about electromagnetic transitions in the nuclei with  $N = 28$  and  $20 < Z \leq 28$ . This information comes primarily from nuclear-reaction studies which present  $\gamma$ -ray angular distributions from aligned nuclei, particle- $\gamma$ -ray angular correlations, and Doppler-shift attenuations of  $\gamma$ -ray energies. The measurements yield at least branching ratios for the decays of levels and, when complete, reduced transition probabilities for transitions of different multipole order.<sup>1-10</sup>

Recently a reasonably extensive shell-model analysis was presented giving level excitation energies, proton spectroscopic factors and electromagnetic transition rates.<sup>11</sup> This was a mixed-configuration analysis demonstrating for the first time the effects of adding  $1f_{5/2}$  single-particle admixtures to the predominantly  $1f_{7/2}$  shell states. In that work comparisons of  $\gamma$ -ray transition rates were on the whole limited to the lowest levels of each spin and parity, since this is where experimental information was abundant. With the advent of the new information cited above, the test of the

model can be extended to a considerably larger number of levels in several nuclei. That is the purpose of this addendum to the previous work.

The method of analysis and basis of calculation is exactly that described in Ref. 11 as calculation B. This model permits protons outside the closed shell  $^{48}\text{Ca}$  core to occupy the states of the configurations  $(1f_{7/2}^n)$ ,  $(1f_{7/2}^{n-1}2p_{3/2})$ , and  $(1f_{7/2}^{n-1}1f_{5/2})$ . The eigenstates were obtained by a modified effective-interaction calculation.<sup>11</sup> A complete effective-interaction calculation was done for a partial space permitting only states of the  $(1f_{7/2}^n)$  and  $(1f_{7/2}^{n-1}2p_{3/2})$  configurations. A modified surface- $\delta$ -function two-body residual interaction was then fitted to the effective two-particle matrix elements and the result used to calculate the additional two-particle matrices necessary for the diagonalization in the larger model space.

Proton excitation models such as the present one have the defect that they violate isospin conservation, as pointed out by Horoshko, Cline, and Lesser,<sup>12</sup> who used a similar model for  $^{51}\text{V}$ . For the nuclei of this study the  $1f_{7/2}$  neutron shell is full,

and thus only those amplitudes with a proton in the  $2p_{3/2}$  or  $1f_{5/2}$  subshell violate isospin conservation. Since the model space used here only admits one proton outside the  $1f_{7/2}$  shell, the maximum possible isospin-violating amplitudes are calculated to be 10–14%, depending upon the neutron excess. In practice the amplitudes actually obtained from the diagonalization fail to conserve isospin by a few percent or less.

The results of comparisons between some additional experimental values and theoretical values not previously reported are given in Tables I–III for levels in  $^{52}\text{Cr}$ ,  $^{53}\text{Mn}$ , and  $^{54}\text{Fe}$ , respectively. On the whole, when the comparisons given here are added to those in Ref. 11, agreement is reasonably good. The data for  $^{52}\text{Cr}$  show the only large and consistent discrepancy with the calculations. The present calculations show at least an order-of-

TABLE I. Experimental and theoretical values for  $E2$  and  $M1$  transitions in  $^{52}\text{Cr}$ . An effective charge of  $1.6e$  was used in calculating the  $E2$  transition strengths. Transition energies given are the experimental values. Lifetimes are in  $10^{-13}$  sec,  $B(M1)$  values in units of  $e^2\text{fm}^2$ , and  $B(E2)$  values in units of  $e^2\text{fm}^4$ .

Level or transition <sup>a</sup>	$E$ (MeV)	Calculated	Experimental
			$ \delta $
$2^+_2 \rightarrow 2^+_1$	1.531	0.04	$6.25 \pm 0.15^b$ $> 3.6^c$
$2^+_3 \rightarrow 2^+_1$	1.728	1.35	$0.18 \pm 0.07^b$ $0.6^{+1.9}_{-0.7}^c$
$4^+_3 \rightarrow 4^+_2$	0.647	0.27	$0.1 \pm 0.2^c$
			Branching ratio
$2^+_3 \rightarrow \begin{cases} 0^+_1 \\ 2^+_2 \end{cases}$	$\begin{cases} 3.162 \\ 1.728 \end{cases}$	$\begin{cases} 5\% \\ 95\% \end{cases}$	$\begin{cases} 12\%^c \\ 88\%^c \end{cases}$
$2^+_2 \rightarrow 2^+_1$	1.531	$\begin{cases} 8 \times 10^{-5} \\ 0.07 \end{cases}$	$\begin{cases} 0.7^{+0.9}_{-0.7} \times 10^{-5}{}^d \\ 135^{+185}_{-60}{}^d \end{cases}$
			$t_m$
$2^+_1$	1.434	16.6	$9.9^{+4.5}_{-2.6}{}^e$
$4^+_1$	2.369	152	$15^{+5}_{-3}{}^e$
$4^+_2$	2.766	318	$20^{+7}_{-5}{}^e$
$2^+_2$	2.965	22	$6.8^{+3.2}_{-1.3}{}^e$
$6^+_1$	3.114	674	$> 29^e$
$2^+_3$	3.162	4	$1.2^{+0.6}_{-0.4}{}^e$
$5^+_1$	3.617	62	$> 50^e$

<sup>a</sup> Subscript on the spin value indicates the  $n$ th level above the ground state which has that spin.

<sup>b</sup> Reference 2.

<sup>c</sup> Reference 4.

<sup>d</sup> Reference 3.

<sup>e</sup> Reference 13.

magnitude too little  $E2$  strength in the decay of the  $4^+$  levels and the second  $2^+$  level. In Ref. 11 it was noted that  $^{52}\text{Cr}$  was the only nucleus in which a known experimental level was completely missed. A second  $0^+$  state is known at 2.65-MeV excitation energy, but no calculated  $0^+$  levels occur below 4.17 MeV. If neutron excitations outside the  $1f_{7/2}$  shell are important and produce noticeable collective strengths in the levels, the difficulties of the

TABLE II. Experimental and theoretical values for lifetimes, mixing ratios, and branching ratios in  $^{53}\text{Mn}$ . An effective charge of  $1.6e$  was used in calculating the  $E2$  transition strengths. Transition energies given are the experimental values. Lifetimes are in  $10^{-12}$  sec.

Level or transition <sup>a</sup>	$E$ (MeV)	Calculated	Experimental
			$t_m$
			(Ref. 7) (Ref. 1)
$\frac{3}{2}^-_1$	1.288	4.82	2.0 0.81
$\frac{1}{2}^-_1$	1.440	2.05	1.3 0.81
$\frac{3}{2}^-_1$	1.619	1.11	0.54 0.66
$\frac{5}{2}^-_2$	2.272	0.02	0.98 0.47
$\frac{3}{2}^-_2$	2.405	0.24	0.22 0.17
$\frac{7}{2}^-_2$	2.571	0.34	0.066 0.07
			$ \delta $
			(Ref. 6)
$\frac{7}{2}^-_2 \rightarrow \frac{7}{2}^-_1$	2.571	1.00	0.13, 0.70
$\frac{7}{2}^-_2 \rightarrow \frac{5}{2}^-_1$	2.193	1.12	1.57, 0.92
$\frac{5}{2}^-_2 \rightarrow \frac{7}{2}^-_1$	2.272	0.003	0.47, 3.17
$\frac{5}{2}^-_2 \rightarrow \frac{5}{2}^-_1$	1.894	1.365	1.732
			Branching ratio
			(Ref. 6)
$\frac{5}{2}^-_2 \rightarrow \begin{cases} \frac{7}{2}^-_1 \\ \frac{5}{2}^-_1 \end{cases}$	$\begin{cases} 2.272 \\ 1.894 \end{cases}$	$\begin{cases} 99\% \\ 1\% \end{cases}$	$\begin{cases} 78\% \\ 22\% \end{cases}$
$\frac{3}{2}^-_2 \rightarrow \begin{cases} \frac{7}{2}^-_1 \\ \frac{5}{2}^-_1 \end{cases}$	$\begin{cases} 2.405 \\ 2.027 \end{cases}$	$\begin{cases} 31\% \\ 54\% \end{cases}$	$\begin{cases} 39\% \\ 15\% \end{cases}$
$\frac{3}{2}^-_2 \rightarrow \frac{3}{2}^-_1$	1.116	15%	46%
$\frac{7}{2}^-_2 \rightarrow \begin{cases} \frac{7}{2}^-_1 \\ \frac{5}{2}^-_1 \end{cases}$	$\begin{cases} 2.571 \\ 2.193 \end{cases}$	$\begin{cases} 25\% \\ 71\% \end{cases}$	$\begin{cases} 40\% \\ 60\% \end{cases}$
$\frac{7}{2}^-_2 \rightarrow \begin{cases} \frac{7}{2}^-_1 \\ \frac{5}{2}^-_1 \end{cases}$	$\begin{cases} 2.685 \\ 2.307 \end{cases}$	$\begin{cases} 77\% \\ 6\% \end{cases}$	$\begin{cases} 77\% \\ 13\% \end{cases}$
$\frac{7}{2}^-_2 \rightarrow \begin{cases} \frac{5}{2}^-_1 \\ \frac{3}{2}^-_1 \end{cases}$	$\begin{cases} 1.397 \\ \end{cases}$	$\begin{cases} 1\% \\ \end{cases}$	$\begin{cases} 10\% \\ \end{cases}$

<sup>a</sup> Subscript on spin value indicates the  $n$ th level above the ground state which has that spin.

present model are just those to be expected, i.e., the model would fail to reproduce the collective  $E2$  strength in the  $4^+$  and second  $2^+$  levels.

There is, it should be noted, some apparent inconsistency in experimental decay rates reported for the second  $2^+$  level. The  $E2$  and  $M1$  strengths listed in the first part of Table I would give an experimental lifetime shorter than the calculated value by at least a factor of 10. The second part of the table shows a comparison with very recent<sup>13</sup> lifetime measurements, and the calculated lifetime is too long by only a factor of 3. The experimental values for the  $B(E2)$  and  $B(M1)$  in Table I were obtained by Kossler<sup>3</sup> using the Monahan *et al.*<sup>4</sup> mixing ratio, which was determined in a measurement involving two unresolved  $\gamma$  rays. An earlier analysis of this transition, from  $^{52}\text{Mn}$  decay,<sup>14</sup> led to the conclusion that it was more than 90% dipole, in

TABLE III. Experimental and theoretical values for lifetimes, mixing ratios, and branching ratios in  $^{54}\text{Fe}$ . An effective charge of  $1.6e$  was used in calculating the  $E2$  transition strengths. Transition energies given are the experimental values. Lifetimes are in  $10^{-12}$  sec.

Level or transition <sup>a</sup>	$E$ (MeV)	Calculated	Experimental <sup>b</sup>
		$t_{1/2}$	
$2^+_1$	1.408	1.47	$0.76^{+0.35}_{-0.22}$
$4^+_1$	2.539	5.64	$\geq 2.1$
$0^+_2$	2.564	11.2	$\geq 1.4$
$6^+_1$	2.948	2471	$\geq 0.56$
$2^+_2$	2.961	1.18	$0.052 \pm 0.008$
$4^+_2$	3.297	0.75	$\geq 2.1$
$3^+_1$	3.345	3.75	$\geq 2.1$
		$ \delta $	
$2^+_2 \rightarrow 2^+_1$	1.550	0.58	$0.105^{+0.040}_{-0.042}$
$4^+_2 \rightarrow 4^+_1$	0.756	0.53	0.38 to 1.66
$3^+_1 \rightarrow 2^+_1$	1.936	0.92	$0.65^{+2.25}_{-0.18}$
$3^+_1 \rightarrow 4^+_1$	0.805	0.19	$0.00 \pm 0.14, \geq 3.5$
		Branching ratio	
$2^+_2 \left\{ \begin{array}{l} \rightarrow 0^+_1 \\ \rightarrow 2^+_1 \end{array} \right.$	2.961	72%	57%
	1.550	28%	43%
$4^+_2 \left\{ \begin{array}{l} \rightarrow 2^+_1 \\ \rightarrow 4^+_1 \end{array} \right.$	1.887	99%	21%
	0.756	1%	79%
$3^+_1 \left\{ \begin{array}{l} \rightarrow 2^+_1 \\ \rightarrow 4^+_1 \end{array} \right.$	1.936	38%	59%
	0.805	61%	41%

<sup>a</sup> Subscript on spin value indicates the  $n$ th level above the ground state which has that spin.

<sup>b</sup> All experimental values in this table are from Ref. 5.

agreement with the present calculations. It would seem worthwhile to measure the mixing ratio for this transition in an experiment in which it is completely resolved from any other transition.

The new measurements in  $^{53}\text{Mn}$ , given in Table II, are in reasonably good agreement with the predicted values with the exception of the lifetime and mixing ratio of the 2.272-MeV level. This level is now definitely assigned<sup>7,10</sup> a spin and parity of  $\frac{5}{2}^-$ . The model predicts too much  $M1$  strength for transitions from this level due to large calculated  $1f_{5/2}$  admixtures in the state. The levels at 2.571 and 2.685 MeV have been tentatively assigned<sup>9,10</sup> spin and parity  $\frac{7}{2}^-$ . The theoretical branching ratios given in Table II for the 2.685-MeV level do not total 100% because theoretically there are weak transitions not seen experimentally.

The experimental measurements in  $^{54}\text{Fe}$  given in Table III are all from the  $(p, p'\gamma)$  work by Moss.<sup>5</sup> Predicted values are in satisfactory agreement with experimental results with the exception of the lifetime of the second  $2^+$  state and the branching ratios of the second  $4^+$  state and the first  $3^+$  state. Spin and parity assignments of the last two states are only tentative and are assigned in order to compare them with corresponding calculated levels. Pittel has calculated  $0^+$  states of  $^{54}\text{Fe}$  including both proton and neutron excitations,<sup>15</sup> and concluded that the excited  $0^+$  state at 2.56 MeV is dominated by four-hole-two-particle amplitudes. He suggests further that strong " $\alpha$ -like" correlations are important at low excitation energies in  $^{54}\text{Fe}$ . The fact that the present proton-excitation shell model represents well the level excitation energies<sup>11</sup> and the several branching ratios and mixing ratios listed in Table III, as well as similar information in nearby nuclei, raises questions about the importance of core-excited amplitudes in  $^{54}\text{Fe}$ . That approach,<sup>15</sup> however, might be just what is needed to explain the low-lying  $0^+$  level and anomalously large  $E2$  transition rates in  $^{52}\text{Cr}$ .

The purpose of this addendum is to extend the test of the mixed-configuration calculation to cover as much experimental information as possible, particularly about electromagnetic transitions, where the theoretical predictions are highly sensitive to the configuration mixing. On the whole the comparisons presented here support our previous conclusions that this mixed-configuration model gives a good picture in this region, and that deficiencies which do occur are probably due to  $1f_{5/2}$  admixture amplitudes being somewhat too large. This gives  $M1$  transition strengths which are too large. More specifically, the small  $2p_{3/2}$  and  $1f_{5/2}$  admixtures contained in the model seem to be just what is needed to obtain good quantitative estimates of transition rates for the lowest levels of a

given spin and parity, that is, the  $B(M1)$  and  $B(E2)$  values are well reproduced. For the second levels of a given spin and parity more than half of the transition rates are well represented. While the calculated rates from a particular level may not be accurate, the relative amounts of  $M1$  and  $E2$  radiation averaged over several levels is representative of the data for second levels of a given spin and parity. The indications are that the model space becomes quite inadequate at excitation energies near 3–3.5 MeV and for  $A \geq 55$ . The model is clearly more reliable for states below 3.5 MeV

and for nuclei near the beginning of the shell.

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## Adiabatic Deuteron Model and the $^{208}\text{Pb}(p, d)$ Reaction at 22 MeV\*

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It is shown that the adiabatic treatment of deuteron breakup during stripping reactions proposed by Johnson and Soper is able to explain the results obtained previously for the  $^{208}\text{Pb}(p, d)$  reaction at 22 MeV without the use of an arbitrary radial cutoff. Some discussion is also given of the sensitivity of the predictions to the parameters of the model.

### I. INTRODUCTION

Differential cross sections for the "single-particle" neutron-hole states excited in the  $^{208}\text{Pb}(p, d)$  reaction at 22 MeV were previously measured<sup>1</sup> for scattering angles from 20 to 165°. When these data were subjected to a conventional<sup>2</sup> distorted-wave Born-approximation (DWBA) analysis, it was found to be necessary to eliminate the contribution to the transition amplitude from radii less than 8.5 F in order to obtain agreement with the measured angular distributions. Without this device, the theoretical distributions tended to peak at too small an angle and to have too much structure or

to decrease too rapidly with increasing angle. These effects were particularly marked for the pickup of  $1h_{9/2}$  and  $1i_{13/2}$  neutrons.

Since  $8.5 = 1.435A^{1/3}$  for  $A = 208$ , the radial cutoff required is appreciably outside the bulk of the matter distribution in the Pb nucleus. This must be regarded as a serious deficiency in the conventional DWBA approach. Similar problems with  $(p, d)$  reactions on medium-weight<sup>3</sup> and light<sup>4</sup> nuclei have been circumvented by the approach of Johnson and Soper<sup>5</sup> which includes approximately the contributions from diffractive breakup of the deuteron. We report here similar success for pickup from Pb. This extends the applications of the model to