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Spectroscopy of the Even Zn Isotopes by Inelastic Alpha Scattering*

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The (α, α') reaction, at $E_\alpha = 31$ MeV, on Zn^{68} and Zn^{70} was studied with an over-all energy resolution of 70–120 keV. Angular distributions are presented. New 3^-, 4^+, and 5^- assignments are made on the basis of distorted-wave Born-approximation (DWBA) calculations. Good angular distributions were obtained for double-excitation 2^+ states at 1.88 MeV in Zn^{68} and at 1.78 MeV in Zn^{70}. Fractionation of the octupole strength has been observed. Systematics of excitation energies and strengths are discussed for Zn^{64}, Zn^{66}, Zn^{68}, and Zn^{70}.

I. INTRODUCTION

We have studied Zn^{64}, Zn^{66}, Zn^{68}, and Zn^{70} with the (α, α') reaction at 31 MeV. The results for Zn^{64} and Zn^{66} have already been published.[1] In previous inelastic scattering experiments on Zn^{68}, good angular distributions were obtained only for the strongly excited first 2^+ and 3^- states.[2-8] Spin assignments have been made to some of the other low-lying weaker levels by directional correlations[9, 10] and with the $Zn^{66}(t, p)$ reaction,[11] while assignments to levels above about 3 MeV in excitation are generally only tentative. Less is known of the Zn^{70} level structure: The first 2^+ state is known from Coulomb excitation[12]; and, more recently, some of the low-lying states, including the first 3^- state, have been identified by Calderbank *et al.*[6] and Lewis *et al.*[7] by inelastic proton scattering.

The purpose of our experiment was (i) to study the variation in excitation strength of the levels in the Zn isotopes as neutrons are added, (ii) to see if the fractionation of octupole strength which was found in several other regions of the periodic table would also be present in the Zn isotopes, and (iii) to obtain more detailed angular distributions for the low-lying levels which have generally been interpreted as double-excitation states.

In this paper we present the data obtained for states in Zn^{68} and Zn^{70}. New spin assignments are given for some of the weaker states at higher excitation energies. The measured transition strengths for levels in Zn^{64}, Zn^{66}, Zn^{68}, and Zn^{70} are compared with each other and with the results of other experiments. Finally, a discussion of the double-excitation features and systematic trends is given for all the even Zn isotopes.

II. EXPERIMENT

The Zn^{70} data were obtained with an α-particle beam of 31 MeV at the Massachusetts Institute of Technology cyclotron with a setup which was identical to the one described in our Zn^{64} and Zn^{66} paper.[1] The Zn^{68} data were obtained with an energy of 30.55 MeV at the Michigan State University cyclotron, with a very similar setup.

The Zn targets were self-supporting metal foils, about 1 mg/cm^2 thick, prepared by the Oak Ridge National Laboratory. The Zn^{68} target was en-

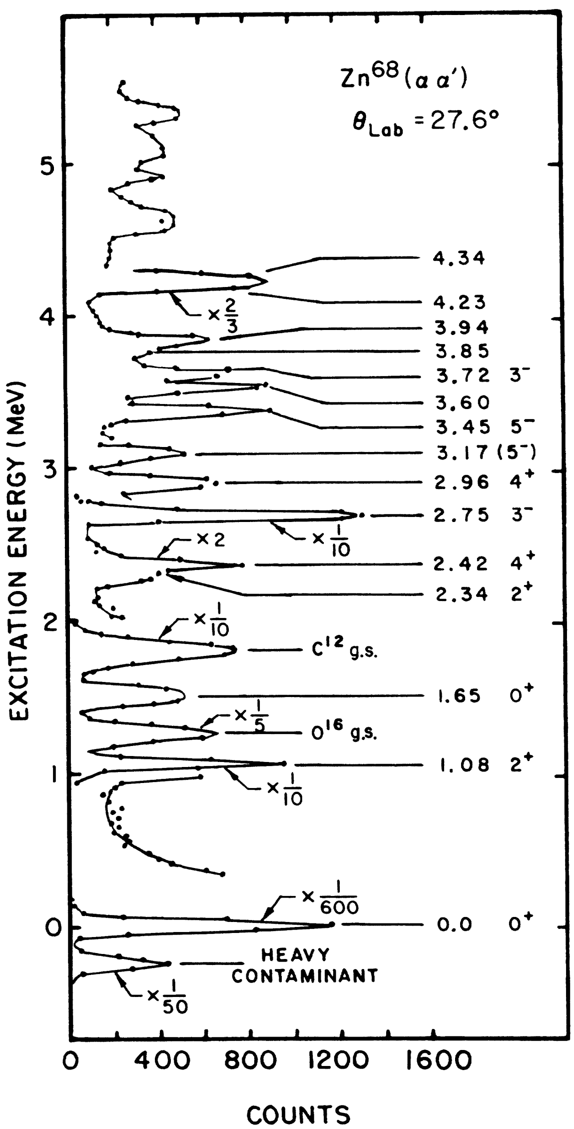

FIG. 1. Spectrum of scattered α particles from Zn^{68}. The solid line merely guides the eye.

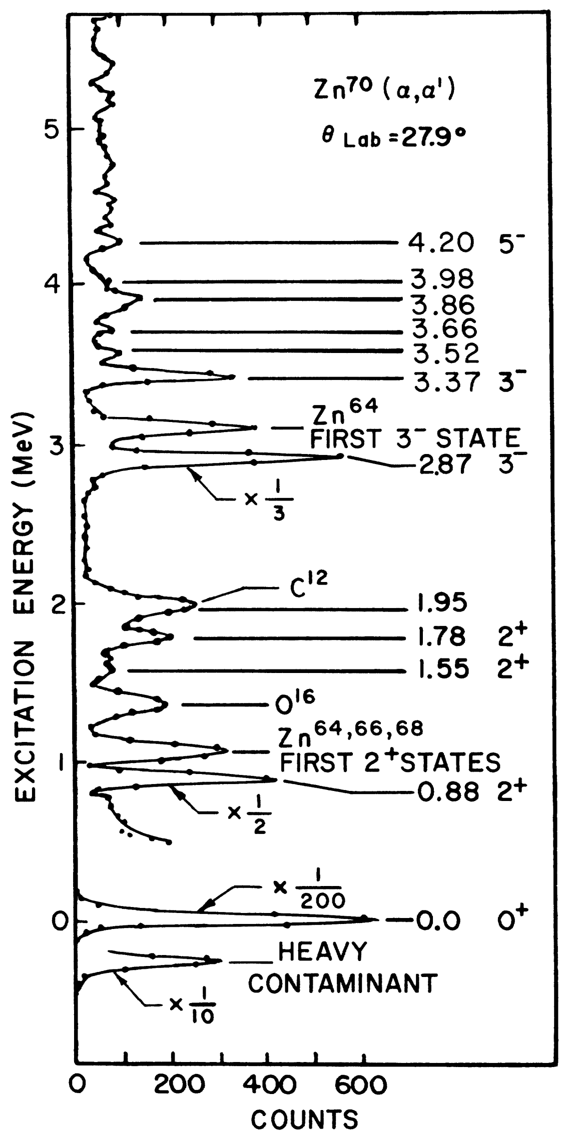

FIG. 2. Spectrum of scattered α particles from Zn^{70}. The solid line merely guides the eye.

riched to 99%, but the Zn^{70} target had impurities consisting of 7.9% Zn^{64}, 5.2% Zn^{66}, and 6.2% Zn^{68}.

The over-all energy resolution including the solid-state detector resolution, target thickness effects, beam energy spread, and electronic noise was from 70 to 100 keV. Two energy spectra are shown in Figs. 1 and 2.

The elastic and inelastic cross sections for Zn^{70} were measured with respect to fixed monitor counters. These relative scattering cross sections were normalized in a separate run as described in our $Zn^{64,66}$ paper.[1] The Zn^{68} cross sections were obtained (at Michigan State University) with respect to a beam current integrator and were checked during the normalization run for all the measured Zn isotopes. The error in the elastic cross section for one angle relative to another (the relative error) is estimated to be about 3%. The error in the inelastic scattering cross section was usually larger due to uncertainties in the background subtraction and in the peak unfolding. Data were plotted without error bars if the relative errors were less than 8%. Excitation energies were determined from the position of the elastic peaks and the well-known first excited states to an accuracy of ±30 keV.

III. THEORETICAL ANALYSIS

We calculated the elastic scattering angular distributions using the optical model with a nuclear potential of the form $V(r) = -(V+iW)/[1 + e^{(r-R)/a}]$ and a Coulomb potential of a uniformly charged sphere of radius R.

The inelastic scattering was calculated in the distorted-wave Born approximation (DWBA). We used a collective form factor to describe the nuclear interaction between the α particle and the target nucleus.[13] The Coulomb-excitation contribution to the inelastic scattering was included by adding a term b_l/r^{l+1} to the form factor.[13] This contribution is negligible for angular momentum transfers larger than $l = 3$. Since only a finite number of partial waves (48) are available in the computer program, the effect of Coulomb excitation is not properly calculated at small angles.[13] A more detailed discussion of these points has

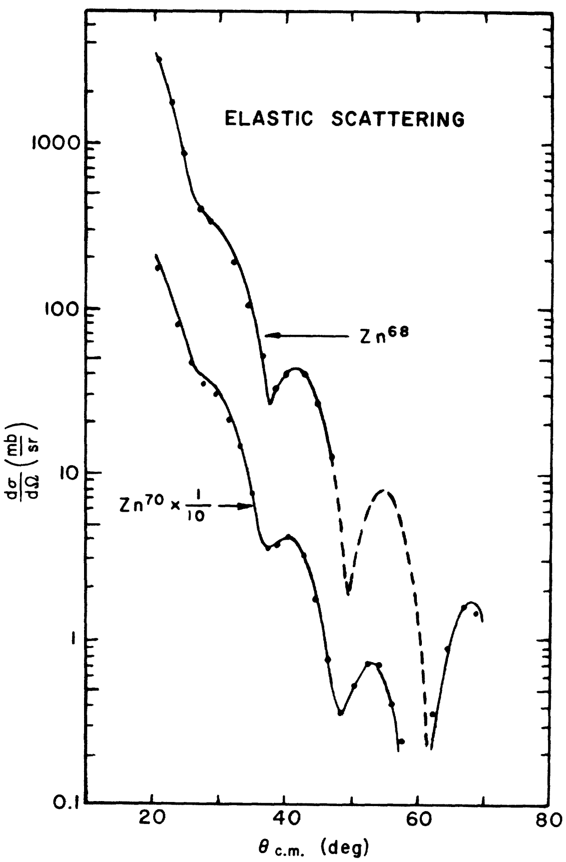

FIG. 3. Elastic scattering cross sections of 31-MeV α particles from Zn^{68} and Zn^{70}. The solid line is the optical-model fit to the data.

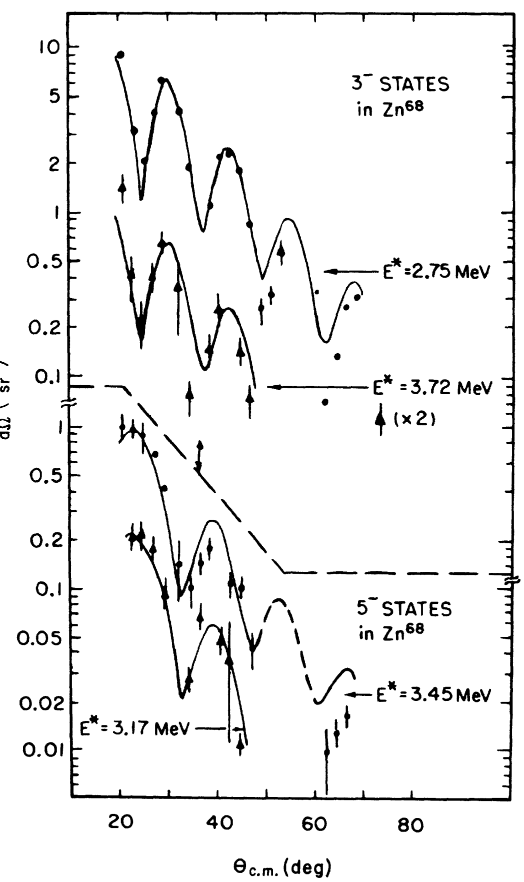

FIG. 4. Differential cross sections for the odd-parity states in Zn^{68}. The solid line is the DWBA calculation.

been given by Martens and Bernstein.[14]

Values of $(\beta_l R)^2$, which are proportional to the differential cross sections were extracted by normalizing the DWBA calculations to the experimental angular distributions at the forward diffraction peaks. The uncertainties in $(\beta_l R)^2$ depend on the quality of the fit and the error in the cross sections. The strength of one-step transitions induced by the (α, α') reaction can be related to a microscopic, one-body, isoscalar (IS) multipole transition operator analogous to the electric multipole transition operator.[15] The comparison between the magnitude of the (α, α') cross sections and the IS transition operator is possible due to the fact that the calculation of the transition matrix element is essentially model independent over the first two to three diffraction peaks in the angular distribution. In view of this model independence we computed the IS transition rates with the vibrational model assuming a Fermi mass distribution. Transition rates have usually been calculated with a uniform mass distribution; however, Owen and Satchler[16] have shown that this procedure underestimates the transition strength, with the error becoming more serious with increasing multipolarity. For the Zn isotopes we found, from the tables in Ref. 15, that the ratio F_l, of transition rates computed with the Fermi and uniform mass distributions is 1.08, 1.27, 1.62, and 2.26 for angular momentum transfers of $l = 2$, 3, 4, and 5, respectively. The IS transition rate G_l was computed in single-particle (Weisskopf) units with the formula

$$G_l = \frac{Z^2}{4\pi} F_l \frac{(l+3)^2}{2l+1}\left(\frac{\beta_l R}{1.2A^{1/3}}\right)^2,$$

where the value of the single-particle unit is given by

$$B_{\mathrm{s.p.}} = \frac{2l+1}{4\pi}\left(\frac{3}{3+l}\right)^2 (1.2A^{1/3})^{2l}\; e^2\,\mathrm{fm}^{2l}\,.$$

IV. EXPERIMENTAL RESULTS

A. Zn^{68} and Zn^{70} Angular Distributions and Spin Assignments

Energy spectra for the Zn^{68} and Zn^{70} (α, α') reactions are shown in Figs. 1 and 2. The positions of the C^{12} and O^{16} contaminants are indicated. No individual levels were resolved above an excitation energy of about 5 MeV.

The elastic scattering angular distributions for Zn^{68} and Zn^{70} are presented in Fig. 3 along with the optical-model calculations. The optical-model parameters were determined by allowing V, W, R, and a to vary until a best least-squares fit to the data was achieved. Table I shows the resulting Zn^{68} and Zn^{70} optical-model parameters.

TABLE I. Optical-model parameters.

Isotope	V (MeV)	W (MeV)	R (F)	a (F)
Zn^{68}	46.05	12.29	6.45	0.619
Zn^{70}	38.57	12.13	6.76	0.578

The spin and parity assignments are based on the DWBA predictions and, where possible, on a comparison with states of known spin and parity. The angular distributions, grouped according to parity, are shown in Figs. 4–7. The Zn^{68} and Zn^{70} odd-parity states are shown in Figs. 4 and 6, respectively; whereas, the even-parity states are shown in Figs. 5 and 7. The solid lines in these figures represent the DWBA calculations, which are in good agreement with the data. No DWBA fits could be obtained for the previously identified second 2^+ states in Zn^{68} (Fig. 5) and Zn^{70} (Fig. 7). These levels will be discussed in Sec. V E. The statistical error in $d\sigma/d\Omega$ for the 3.72-MeV level in Zn^{68} is much larger than for the low-lying levels, but due to the good DWBA fit and its similarity to the lowest 3^- state we assigned $J^\pi = 3^-$ to this state (see Fig. 4). We made only a tentative $J^\pi = 5^-$ assignment to the 3.17-MeV level in Zn^{68}, since the angular distribution extends only over a small angular region (see Fig. 4). We also obtained a partial angular distribution for a state at 1.65 MeV, which has a previous $J^\pi = 0^+$ assignment. It is about 20–50 times weaker than the first excited 2^+ state. Some other states gave very fragmentary angular distributions, which are not shown in the figures and for which no J^π assignments were made. Their excitation energies are, however, added to Table II.

Table II collects the excitation energies, spins, parities, β_l and $\beta_l R$ values, and the IS transition rates for the levels in Zn^{68} and Zn^{70}. The uncertainty in the $\beta_l R$ value depends on the quality of the DWBA fit in addition to the absolute error in the data.

B. Comparison with Other Experiments

Zn^{68} levels. The excitation energies of the Zn^{68} levels listed in Table II are in good agreement with the known and sometimes more-accurate values.[17] The excitation energies of many more levels above 3 MeV are known,[17] but most of these levels were not excited in our work due to the selective excitation mechanism of the α-nucleus interaction. The first 2^+ state at 1.08 MeV and the first 3^- state at 2.75 MeV were already measured in earlier inelastic scattering experiments.[2-8]

Our $\beta_l R$ values for those levels, listed in Table II, are in agreement with the results of the (α, α') experiments[2,4] at 22 and 43 MeV and with those of the (p, p') experiment at 50 MeV.[6,7] Among the higher-lying Zn^{68} levels we were able to identify a new 4^+ state at 2.96 MeV, a 5^- state at 3.45 MeV, a 3^- state at 3.72 MeV and make a tentative 5^- assignment to a level at 3.17 MeV. Even though our energy resolution and background was much better than the earlier (α, α') experiments we could not find clear evidence for a 3^- level at 4.33 MeV.[4] We found a broad group consisting of at least two levels in this excitation region. Neither the composite peak nor the unfolding into two peaks yields identifiable angular distributions which is an indication that more than two levels may be excited.

Zn^{70} levels. Figure 8 compares our Zn^{70} excitation energies and spin assignments with the results of the (p, p') experiment of Calderbank *et al.*[6] and Lewis *et al.*[7] There is good agreement in the excitation energies between the two experiments up to about 3 MeV. Above 3 MeV only a few levels

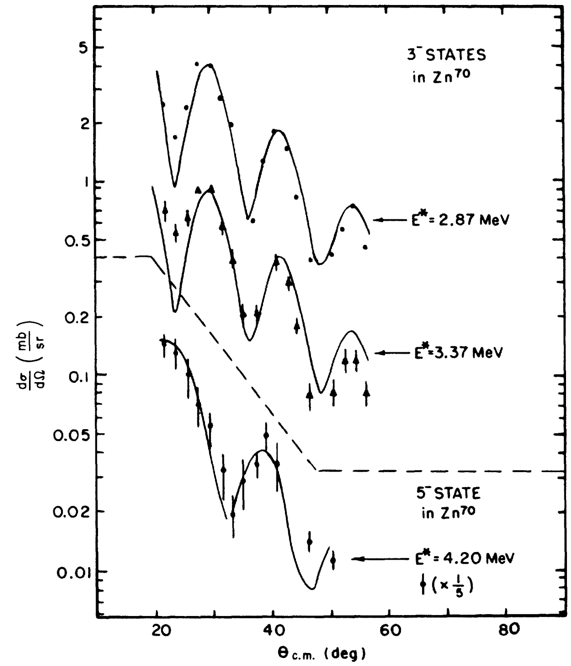

FIG. 6. Differential cross sections for the odd-parity states in Zn^{70}. The solid line is the DWBA calculation.

were resolved in the (p, p') work and it is difficult to establish the correspondence of energy levels. Our $(\beta_l R)$ values for the 0.88-MeV (2^+) and 2.87-MeV (3^-) states are in good agreement with the (p, p') results. Lewis *et al.*[7] made a coupled-

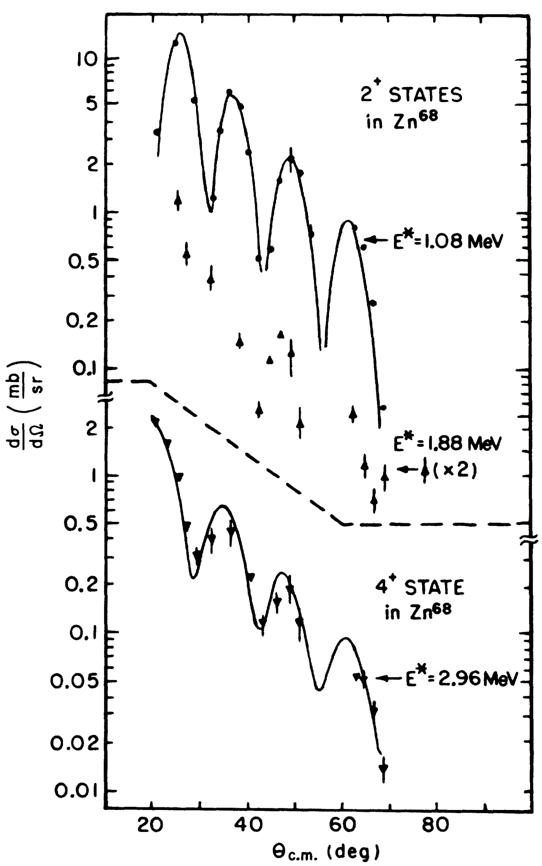

FIG. 5. Differential cross sections for the even-parity states in Zn^{68}. The solid line is the DWBA calculation. States shown without fits are discussed separately in the text (see Sec. V E).

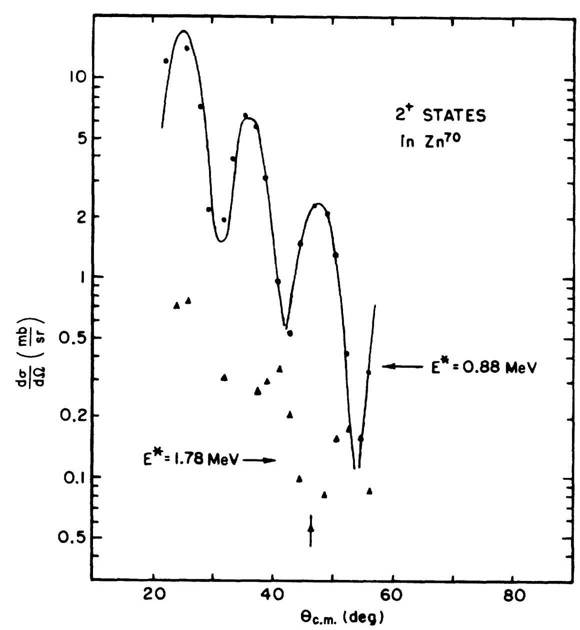

FIG. 7. Differential cross sections for the even-parity states in Zn^{70}. The solid line is the DWBA calculation. States shown without fits are discussed separately in the text (see Sec. V E).

channels calculation including the ground state and first excited state assuming a two-phonon excitation for the 1.55- and 1.78-MeV states. They found $J^{\pi}=2^{+}$ for both states. The 1.55-MeV state was very weakly excited in our experiment but the 1.78-MeV state was strong enough (to allow us) to measure its angular distribution. We plot $d\sigma/d\Omega$ for the 1.78 MeV 2^{+} in Zn^{70} in Fig. 9 together with the second 2^{+} states of Zn^{64}, Zn^{66}, and Zn^{68} as a function of momentum transfer in order to allow for the mass difference of the isotopes and the slightly lower bombarding energy of the Zn^{68} experiment. The Zn^{70} angular distribution is almost identical to those of Zn^{64} and Zn^{66} which supports Calderbank's 2^{+} assignment. The difference in shape of the Zn^{68} state will be discussed in Sec. V E.

Among the levels above an excitation energy of 3 MeV we identified a new 3^{-} state at 3.37 MeV and a new 5^{-} state at 4.20 MeV.

TABLE II. Transition rates in Zn^{68} and Zn^{70}.

E^* (MeV)	J^{π}	β_l	$(\beta_l R)$ (F)	G_l [a]
		Zn^{68}		
1.08	2^+	0.16	1.03 ± 0.05	17.1
1.65	0^+ [b]			
1.88	2^+			
2.34	2^+ [c]			
2.42	4^+			
2.75	3^-	0.16	1.03 ± 0.05	20.7
2.96	4^+	0.071	0.46 ± 0.03	5.6
3.17	(5^-)	0.026	0.17 ± 0.02	1.1
3.45	5^-	0.052	0.35 ± 0.03	4.7
3.60				
3.72	3^-	0.042	0.27 ± 0.04	1.4
3.85				
3.94				
4.23				
4.34				
		Zn^{70}		
0.88	2^+	0.16	1.09 ± 0.06	18.8
1.55	2^+ [d]			
1.78	2^+			
1.95				
2.87	3^-	0.13	0.92 ± 0.05	16.0
3.37	3^-	0.063	0.43 ± 0.03	3.5
3.52				
3.66				
3.86				
3.98				
4.20	5^-	0.047	0.32 ± 0.03	3.8

[a] G_l is computed assuming a Fermi mass distribution (see Sec. III).

[b] Reference 11.

[c] References 10 and 11.

[d] Reference 7.

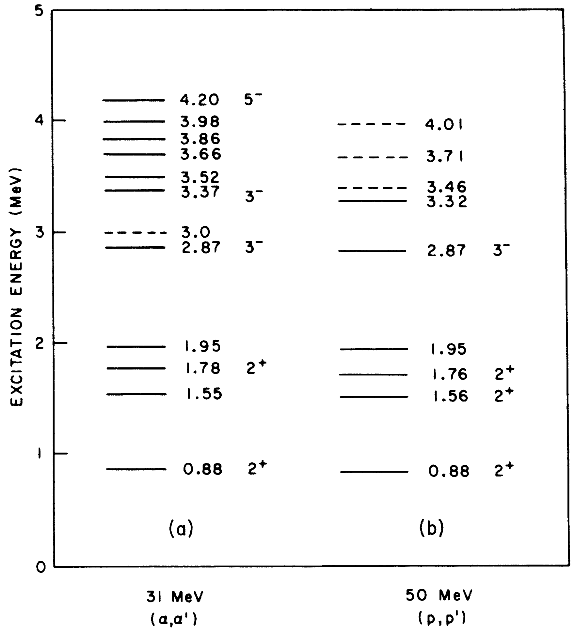

FIG. 8. Comparison of Zn^{70} level schemes. (a) This work, (b) Refs. 6 and 7.

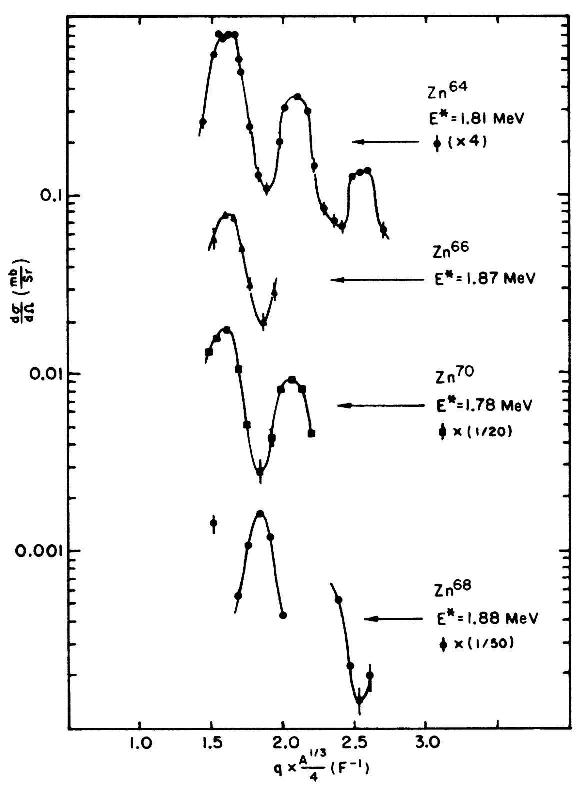

FIG. 9. Comparison of the shapes of the double-excitation 2^{+} states in Zn^{64}, Zn^{66}, Zn^{68}, and Zn^{70}. In order to allow for the slightly lower bombarding energy in the Zn^{68} experiment and the isotopic mass differences, the cross sections are plotted as a function of momentum transfer.

V. SUMMARY AND CONCLUSIONS

In Fig. 10 we include the level schemes of Zn^{64} and Zn^{66}, which were reported in Ref. 1, along with the Zn^{68} and Zn^{70} levels measured in this experiment. Other levels are included in Fig. 10 which are only known from other experiments. These are listed in the legend to Fig. 10. Figure 11 shows the results of our transition strength measurements on the Zn isotopes.

A. 2^+ States in the Zn Isotopes

The first excited state in each nucleus has a spin and parity of 2^+. Their excitation energies are shown in Fig. 10. The fact that the Zn^{68} first 2^+ state has the highest excitation energy is consistent with the weak shell closure at $N=38$ [18]; but the relatively large drop in excitation energy between Zn^{68} and Zn^{70} does not fit this picture. The transition strengths, G_2, for these levels are compared in Fig. 11(a). The strengths of the Zn^{64} and Zn^{66} first 2^+ states are each 26 single-particle units, while in Zn^{68} and Zn^{70} the strength of the lowest 2^+ state decreases by about 30%. We measured one other 2^+ state in each of the Zn isotopes at about 1.8 MeV which exhibited double-excitation characteristics. These levels are discussed separately below. Some other 2^+ states have been identified in these nuclei by other methods,[10, 11, 19-21] but in this work they were not observed or were very weakly excited.

B. 3^- States in the Zn Isotopes

At least two octupole states were measured in each of the Zn isotopes. The lowest was always

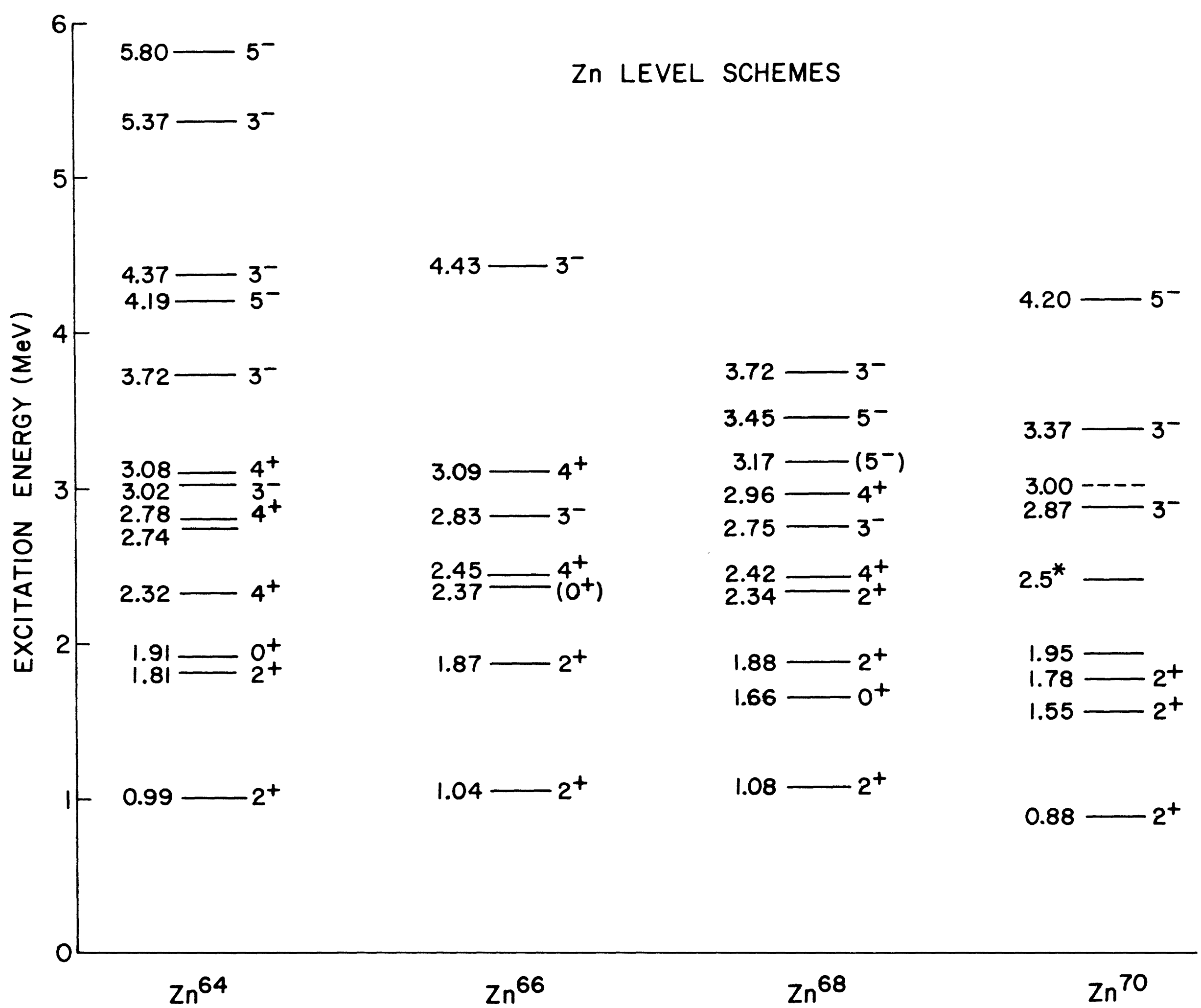

FIG. 10. Level schemes of Zn^{64}, Zn^{66}, Zn^{68}, and Zn^{70}. The $J^{\pi}=0^+$ states in Zn^{64} and Zn^{66} were not observed in this experiment (see Refs. 6 and 7). An asterisk indicates excitation of more than one level.

the most strongly excited. As shown in Fig. 11(b), the transition rate of the first 3^- state decreases almost linearly by 45% between Zn^{64} and Zn^{70}. These transition rates are higher than those found for the strongest octupole states in many other nuclei. In fact, the 3.02-MeV 3^- state in Zn^{64} has one of the highest transition rates measured (27 single-particle units) in any nucleus.

The octupole strength in Zn^{64} is distributed among four levels, while in Zn^{66}, Zn^{68}, and Zn^{70} only two 3^- states were found. The transition strength to the higher 3^- states is 22% for Zn^{64}, 15% for Zn^{66}, 7% for Zn^{68}, and 22% for Zn^{70} (the strength of the lowest octupole state was taken to be 100%). Veje[22] has calculated the excitation energies and distribution of octupole transition strengths for a variety of nuclei. Of the Zn isotopes only Zn^{70} was included in his calculations. The comparison between theory and experiment is not as good in Zn as for some other nuclei.[14]

In Fig. 12 we plot the percent of fractionation of octupole strength in spherical, even-even nuclei vs A, for $A \leq 70$. Except for Zn, the data in Fig. 12 were taken from Ref. 15. No experimental errors are shown, since the intent is only to indicate the general trend of the measurements. From Fig. 12 we see that the fractionation reaches a maximum in Ti^{48} and then falls steadily to a minimum value in Zn^{68}. While it is not clear how to relate this feature to the nuclear structure, it seems interesting that even though the strength of the lowest octupole states in the Zn isotopes is relatively high the fractionation of strength is very low.

C. 4^+ States in the Zn Isotopes

The excitation strength of the 4^+ transitions is split between single- and double-excitation 4^+ states. The double-excitation levels are discussed separately below. The single-excitation 4^+ states are all found at about the same excitation energy of 3 MeV. Such a level may also exist in Zn^{70}, but that part of the spectrum was obscured by contaminant peaks. Figure 12(d) shows that the transition strength gradually decreases from Zn^{64} to Zn^{68}, and is stronger by 20 to 30% than in most other nuclei.

D. 5^- States in the Zn Isotopes

New 5^- states were found in Zn^{64}, Zn^{68}, and Zn^{70}. The total 5^- transition rate for each nucleus is shown in Fig. 12(e). Possible 5^- transitions in Zn^{66} may have been missed due to the high level density of the Zn^{66} (α, α') spectrum. Comparison between the 5^- states measured in the Zn isotopes and those in nearby nuclei is impossible due to the lack of data. However, the fact no 5^- states at all have been found in the Ni isotopes may be significant since high-resolution experiments have been performed.[23]

E. Double-Excitation Features in the Zn Isotopes

In a previous paper[1] we compared the shapes of the angular distributions of the double-excitation 2^+ and 4^+ states in Zn^{64} with the shapes of the single-excitation 2^+ and 4^+ angular distributions.

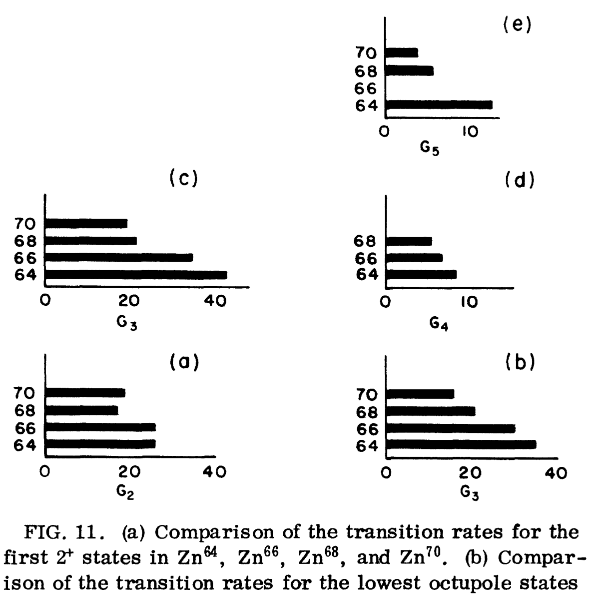

FIG. 11. (a) Comparison of the transition rates for the first 2^+ states in Zn^{64}, Zn^{66}, Zn^{68}, and Zn^{70}. (b) Comparison of the transition rates for the lowest octupole states in the Zn isotopes. (c) Comparison of the total ocutpole strength in the Zn isotopes. (d) Comparison of the transition strengths of the 3-MeV 4^+ states in Zn^{64}, Zn^{66}, and Zn^{68} (see Sec. V C of the text). (e) Comparison of the total 5^- transitions strengths in the Zn isotopes.

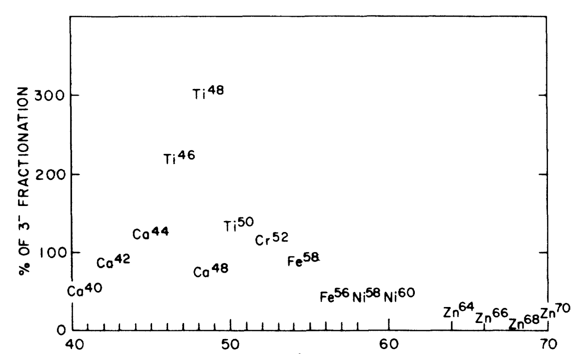

FIG. 12. Fractionation of octupole strength for spherical even-even nuclei vs A. With the exception of Zn, the data were taken from Ref. 15. No experimental errors are indicated since the intent is only to show the general trend of the measurements. The percent of octupole fractionation is defined as $(\sum_i G_{3i}/G_3)\times 100$, where $\sum_i G_{3i}$ includes all but the strongest octupole strength, G_3.

The double-excitation 4^+ states in Zn^{64} and Zn^{66} have approximately equal excitation strengths. Their angular distributions were exactly out of phase with those of the single-excitation 4^+ states, and the slope of the envelope of their angular distributions was much shallower than that of the single-excitation 4^+ states. Even though no complete angular distributions were obtained for double-excitation 4^+ states in Zn^{68} and Zn^{70}, weakly excited double-excitation levels may also exist in those nuclei since there is a known 4^+ state at 2.42 MeV in Zn^{68} and we found a weakly excited multiple peak at about 2.45 MeV in the Zn^{70} (α, α') spectra. Similar, doubly excited 4^+ states have been measured in the Ni isotopes.[24-26] Various analyses of those data[25-27] have indicated that the phasing of the angular distributions for these levels varies from one nucleus to another. This is due to competing direct- and two-step excitation modes, and also depends on the bombarding energy.[25]

In each Zn nucleus we found a second 2^+ transition (in Zn^{64} at 1.81 MeV, in Zn^{66} at 1.87 MeV, in Zn^{68} at 1.88 MeV, and in Zn^{70} at 1.78 MeV) whose phasing of the angular distribution was shifted by various amounts relative to the first 2^+ state. Their angular distributions are presented along with those of the other even-parity states. In addition they are compared with one another in Fig. 9 where they are plotted vs momentum transfer in order to allow for the different bombarding energies and masses. In contrast to the 4^+ states, the slopes of the envelopes of the single- and double-excitation 2^+ angular distributions are identical. This contradicts the predictions of the diffraction model[28] and the adiabatic perturbation series expansions of the DWBA for double-excitation states,[29] which show that the slope of the angular distributions for multiple excited states should be shallower than those of single-step excitations. It would be interesting to measure the angular distribution of the 1.81-MeV 2^+ state in Zn^{64} as a function of bombarding energy and to see if a coupled-channels calculation can reproduce both the slope and the change in phase.

It is difficult to draw detailed nuclear-structure information from our results since wave functions are not available for the Zn isotopes. It remains difficult to understand the systematics in the excitation energies of the first 2^+ states and the single-excitation 4^+ states. The particularly small variation in the excitation strengths, G_2 and G_4, as neutrons are added, indicates that the wave functions for the ground states and these excited states are affected in nearly the same manner by this addition of neutrons.

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Structure of the Random-Phase-Approximation Ground-State Wave Functions of ^{56}Ni and ^{48}Ca†

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Conventional random-phase-approximation (RPA) and corrected RPA (MRPA) calculations are carried out for ^{56}Ni and ^{48}Ca nuclei, using the Tabakin potential without and with second-order Born corrections. The obtained ground-state wave functions are analyzed. The probability for the doubly-closed-shell (zero-particle, zero-hole) configurations is found to be small for ^{56}Ni [18% (RPA), 38% (MRPA)]. Similar numbers for the ^{48}Ca nucleus are, 35% RPA, 56% (80% for protons and 69% for neutrons) MRPA. In the case of ^{56}Ni, the $T=0$ part contributes ≈95% to the shell breaking. The occupation numbers for single-particle and single-hole states are small except for the $1f_{7/2}$ hole and $1f_{5/2}$ particle states in ^{56}Ni, and for the $1d_{3/2}$ proton-hole state in ^{48}Ca. It is found that the conventional RPA using antisymmetrized matrix elements overestimates the calculated quantities of the ground state, roughly by a factor of 2. The effect of truncating the configuration space is also studied quantitatively. The results are compared with earlier investigations.

I. INTRODUCTION

Shell-model calculations in the $1f_{7/2}$ shell[1] and the $2p$-$1f$ shell[2] rely on the assumption of inert cores (^{40}Ca and ^{48}Ca or ^{56}Ni, respectively) in order to keep the dimension of the configuration space manageable. Recent experimental observations[3] and some theoretical investigations,[4-7] however, indicate the presence of appreciable core excitations in the "closed shell" ground states. While phenomenological interactions and the single-particle energies can represent core contributions to some extent, if used in the spirit of least-squares fitting, no argument along these lines can be made for the matrix elements of realistic interactions that incorporate core-polarization corrections in perturbation theory.

It was recently pointed out[8,9] that the frequently used version or conventional form of random-phase approximation (RPA) employing antisymmetric matrix elements overestimates the ground-state correlations **by a factor of 2 in intensity.** This fact is reflected in calculated quantities such as the correlation energy or the single-particle occupation probabilities, which are also off by a factor of 2. It is due to the double counting of some of the lowest-order diagrams. It was further shown[9] that the modification required to avoid this double counting is to add a term, referred to as the exchange correction term, in the exponent of the ground-state wave function. It is to be remarked that in the excited states, because of the nature of the lowest-order vertex, the problem of double counting does not arise.

We report here the results of the conventional RPA and corrected RPA (MRPA), which include