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# PHYSICAL REVIEW C VOLUME 4, NUMBER 1 VOLUME 1971

# Level Structure of <sup>42</sup>Ca by <sup>41</sup>K(<sup>3</sup>He, d)<sup>42</sup>Ca and <sup>39</sup>K( $\alpha$ , p)<sup>42</sup>Ca Reactions at 10 MeV

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The level structure of <sup>42</sup>Ca, below 6 MeV in excitation, has been studied by <sup>41</sup>K( ${}^{3}$ He,  $d$ )<sup>42</sup>Ca and  ${}^{38}K(\alpha,p){}^{42}Ca$  reactions at 10-MeV bombarding energy. Particles detection was achieved with a surface-barrier detector telescope. Comparison of 12 measured  $({}^{3}He, d)$  angular distributions with zero-range distorted-wave Born-approximation calculations indicated a predominance of  $l = 3$  and  $l = 1$  transfers, and gave information about states arising from the  $(1f_{7/2}1d_{3/2}^{-1})$  and  $(2p_{3/2}1d_{3/2}^{-1})$  configurations. Shell-model calculations have been performe to investigate the level scheme in <sup>42</sup>Ca arising from the  $(1f_{7/2}$ <sup>3</sup> $1d_{3/2}$ <sup>-1</sup>)<sub>JT</sub> configuration using four different sets of effective interaction parameters between the  $1f_{7/2}$  particle and  $1d_{3/2}$ hole. The resulting wave functions were used to calculate spectroscopic factors, which have been compared with the experimental ones. The measured  $(\alpha, p)$  angular distributions analyzed with Hauser-Feshbach calculations indicated a predominance of compound-nucleus mechanism for this reaction. Good agreement has been found between the integrated experimental and theoretical cross section.

#### I. INTRODUCTION

Low-lying negative-parity states in  $^{42}$ Ca may arise from excitation of an odd number of nucleons from the sd shell to the  $fp$  shell or from the excitation of a valence neutron, outside an inert  ${}^{40}Ca$ core, to the  $g$  shell. However, since the  $g$  shell is well above the  $fp$  shell, the dominant and simplest configuration expected for the negative-parity states of <sup>42</sup>Ca is that of a  $1d_{3/2}$  hole coupled to particles in the  $fp$  shell. In addition, there is experimental and theoretical evidence that core excited states play an important role also in the lowlying positive-parity states. In fact the  ${}^{40}Ca(t, b)$ -Ca reaction of Bjerregaard et  $al$ <sup>1</sup> has exhibite six  $J^{\pi} = 0^+$  and seven  $J^{\pi} = 2^+$  states below 7-MeV excitation energy, and Federman and Pittel' have

shown that some of these  $0<sup>+</sup>$  states can be understood in terms of 2p-Oh and 4p-2h configurations. The complexity of  $42$ Ca spectrum suggested a further investigation of the nuclear structure of  $^{42}Ca$ , paying attention to the negative-parity states described mainly by excited-core components. One way to determine experimentally such negativeparity states is to study the <sup>42</sup>Ca spectrum through the  $^{41}K(^{3}He, d)^{42}Ca$  reaction. The  $^{41}K$  nucleus in the simple shell-model picture is described in its ground state by the  $(1f_{7/2}^2 1d_{3/2}^{-1})$  configuration. Thus, in  $({}^{3}He, d)$  reaction we should expect to excite the  $0^+$  ground state of <sup>42</sup>Ca by a  $1d_{3/2}$  proton transfer and then several negative-parity states by f-p proton transfer. The  ${}^{39}K(\alpha, p)^{42}Ca$  reaction in the direct-interaction region should provide complementary information about such negativeparity states. In a stripping model, assuming that the interaction involves only the four nucleons in the  $\alpha$  particle, we have the seniority selection rule  $\Delta v_n = 0$ , 2 for the neutron pair and  $\Delta v_p = 1$  for the proton transfers.<sup>3</sup> So long as the two neutrons are transferred with a seniority zero, the  ${}^{39}K(\alpha, \rho)$ -<sup>42</sup>Ca and <sup>41</sup>K(<sup>3</sup>He, d)<sup>42</sup>Ca reactions should be equivalent. Thus, it seemed interesting to investigate the nuclear structure of  $^{42}$ Ca through the  $^{41}K(^{3}He, d)$ -<sup>42</sup>Ca and <sup>39</sup>K( $\alpha$ ,  $p$ )<sup>42</sup>Ca reactions.

During the performance of this experiment Forster  $et$   $al.^4$  have published experimental results obtained by the  $^{41}K(^{3}He, d)^{42}Ca$  reaction at 11 MeV. Their spectroscopic factors can be compared with those obtained in the present experiment. For the  $^{39}K(\alpha, p)^{42}$ Ca reaction Schiffer<sup>5</sup> reported the proton energy spectrum at an  $\alpha$  bombarding energy of 8.22 MeV.

#### II. EXPERIMENTAL PROCEDURE

The experiment was performed using the 10- MeV doubly ionized  ${}^{3}$ He and  ${}^{4}$ He beam from the CN Van de Graaff accelerator of Legnaro, Padova. The beam was analyzed by a  $90^\circ$  bending magnet with 70-cm radius of curvature and monitored by an ELCOR current integrator model No. A309C. The scattered particles were detected with a  $\Delta E$ -E telescope of surface-barrier detectors. The thickness of detectors was 85 and 2000  $\mu$ m and the detection solid angle was  $6.2 \times 10^{-4}$  sr. Reaction

products were selected by an ORTEC particle identifier model No. 423. The targets were prepared by evaporation onto a thin carbon backing of potassium iodides of natural potassium and isotopically enriched  $^{41}K$  (99.18% in  $^{41}K$ ). Absolute cross sections were evaluated by measuring the <sup>3</sup>He and <sup>4</sup>He elastic scattering yield at 4 MeV from potassium. Assuming that at this lower energy the elastic scattering on potassium is given by the Rutherford formula, a comparison of the elastic and reaction yields gave the absolute cross section of the reaction. The error in the absolute cross section has been estimated to be of the order of  $\pm 15\%$  for both reactions. Evaluation of absolute cross section from the Rutherford scattering of iodine at 10 MeV has been judged not suitable because the target composition was gradually changing during the measurements owing to decomposition of iodides followed by evaporation of the iodine. Reaction and elastic scattering yields were measured in 5 steps. The angular ranges were from <sup>25</sup> to 90' for the <sup>41</sup>K(<sup>3</sup>He, d)<sup>42</sup>Ca reaction and from 25 to 160<sup>°</sup> for the <sup>39</sup>K( $\alpha$ ,  $p$ )<sup>42</sup>Ca reaction. The angular ranges for the elastic scattering yields were from 25 to  $140^{\circ}$  for <sup>3</sup>He on <sup>41</sup>K and from 22 to 160 $^{\circ}$  for <sup>4</sup>He on  $^{39}K$ .

# III. EXPERIMENTAL RESULTS.  $^{11}$ K( $^{3}$ He, $d$ )<sup>42</sup>Ca REACTION

A spectrum of the  ${}^{41}K({}^{3}He, d){}^{42}Ca$  reaction is shown in Fig. 1. The peaks are labeled by their



FIG. 1. Deuteron spectrum for the <sup>41</sup>K $(^{3}$ He,  $d)^{42}$ Ca reaction at  $\theta_{1ab} = 30^{\circ}$ .



FIG. 2. Deuteron spectrum for the  $^{41}K(^{3}He, d)^{42}Ca$ reaction taken with a thick target and with a large detection solid angle.

excitation energies and by their spins and parities, where known. The error associated with the  $^{42}Ca$ excitation energy is  $\pm 40$  keV. An inspection of this spectrum shows an evident excitation of the 0' ground state and strong peaks above <sup>3</sup> MeV of neg.' 've-parity states excited by the transfer of an  $f$ - or  $p$ -wave proton. The excitation of the known positive-parity states below 3 MeV has been investigated by means of measurements with a heavier target (about four times thicker) and with a larger detection solid angle (about four times larger). One of these spectra is shown in Fig. 2, where it is possible to observe with the ground state some excitation of the  $2^*$ ,  $0^*$ , and  $2^*$  states



FIG. 3. Optical-model fits to the elastic scattering data with potentials described in Table I. The curves show the effect of volume (continuous line) and surface (dashed line) absorption in the  ${}^{3}$ He optical potential.

at  $1.52$ ,  $1.84$ , and  $2.42$  MeV. The measured  ${}^{3}$ He elastic scattering angular distribution plotted aa  $(d\sigma/d\sigma_{\text{Ruth}})$  versus  $\theta_{\text{c.m.}}$  along with the optical-model curves is shown in Fig. 3. The curves are derived by the optical-potential parameters' listed in Table I. The figure shows that the curve with volume absorption (full line) fits the experimental data better than the curve (dashed line) with surface absorption. Thus, the potentials corresponding to the volume absorption were used to generate the incoming 'He wave functions in the distortedwave Born-approximation (DWBA) analysis. The deuteron optical potentials shown in Table I are those obtained' by optical-model fits to the elastic scattering of deuterons at 12 MeV from a  $^{42}$ Ca target. In Fig. 4 are shown the angular distributions with the DWBA curves of deuterons leading to the several excited states of  $^{42}$ Ca observed in this experiment. In our analysis we chose the approximaperment. In our analysis we chose the approximation used by Forster  $et$  al.<sup>4</sup> in order to have a meaningful comparison between their and our experimental results. Hence the theoretical calculations were carried out using the DWBA code  $DWUCK<sup>6</sup>$  and included the use of a zero-range approximation with nonlocal corrections applied for the incident, exit, and bound-state channels. The spectroscopic factors have been extracted from the experimental data in the usual fashion by comparing the experimental cross section to the prediction of DWBA as expressed by

$$
\left(\frac{d\sigma}{d\Omega}\right)_{\text{expt}} = 4.42 \left(\frac{2J_f+1}{2J_i+1}\right) C^2(T_f)
$$
  
\$\times S\_{1j}(J\_fT\_f)[\frac{1}{2}(2s+1)\sigma\_{\text{DMUCK}}(\theta)/(2j+1)]\$,

TABLE I. Optical-model parameters. The optical potential is

$$
V(r) = -V(e^x + 1)^{-1} - i(W - 4W_D d/dx') (e^{x'} + 1)^{-1} + 4V_{so} \frac{1}{r} \frac{d}{dr} (e^x + 1)^{-1} \vec{L} \cdot \vec{S} + i4W_{so} \frac{1}{r} \frac{d}{dr} (e^{x'} + 1)^{-1} \vec{L} \cdot \vec{S} + V_C(r, r_C),
$$





<sup>a</sup> Adjusted to give the transferred proton a binding energy of  $Q(^{3}He, d) + 5.49$  MeV.

where  $J_f$  and  $J_i$  are the spins of the residual and target nuclei, and  $j$  is the total angular momentum of the transferred particle;  $C^2(T_f)$  is an isospin Clebsh-Gordan coefficient given by  $C^2(T_i^{-1}\frac{1}{2}T_i-\frac{1}{2}|T_fT_i-\frac{1}{2}),$  which couples the strippe proton and target nucleus to final states with  $T_f$  $=T_i - \frac{1}{2}$  or  $T_f = T_i + \frac{1}{2}$ . For stripping to states with  $T_f = T_i - \frac{1}{2}$  we have  $C^2 = \frac{3}{4}$ , and for stripping to states with  $T_f = T_i + \frac{1}{2}$  we have  $C^2 = \frac{1}{4}$ ;  $S_{1f}(J_fT_f)$  is the spectroscopic factor (usually written as S), and the remaining term in parentheses represents the calculated cross section. The spectroscopic information for the  $l=2$  and  $l=3$  curves has been derived on the region of the primary maximum of the experimental angular distribution, while for the  $l = 1$  curves the spectroscopic information has been derived from the region of the secondary maximum. Table II lists the excitation energies,  $l$  values,  $J^{\pi}$  limits and, spectroscopic strengths  $(2J<sub>f</sub>+1)S$  excited by (<sup>3</sup>He, *d*) reaction. The uncertainty associated with the extraction of spectro-

scopic factors has been estimated to be of the order of 10% with exception of the ground state, which uncertainty is of the order of  $20\%$ . This larger uncertainty is mainly due to a poor fit of the theoretical curve to the experimental one for  $\theta_{\rm c.m.}$  > 40°. Table II includes the spectroscopic  $v_{\text{c.m.}}$ ,  $v_{\text{c.m.}}$  is a ratio in incrudes the spectroscopic factors determined by Forster *et al.*<sup>4</sup> at 11 MeV. The agreement with the present work is satisfactory within the experimental uncertainty and differences in the distorted-wave analysis.

With the aim to see how the spectroscopic factors depend on the choice of the optical-model parameters, we have repeated DWBA calculations using the different sets of optical potentials reported in Table III. The first set is that used by Seth et al.<sup>7</sup> in the analysis of  $39K(^3He, d)^{40}Ca$  reaction at 14 MeV, and the other two are taken again from Ref. 4 and include surface absorption for both 'He and deuterons. In these calculations we have considered the  $l = 2$ ,  $0^+$  ground-state transition, the  $l=3$  leading to the 5<sup>-</sup>, 4.10-MeV state and the  $l=1$ 

TABLE II. Experimental results for levels in <sup>42</sup>Ca excited by <sup>41</sup>K(<sup>3</sup>He, d)<sup>42</sup>Ca reaction.

Excitation energy				Excitation energy <sup>a</sup>	
(MeV $\pm 0.04$ MeV)		$J^{\pi}$ limits	$(2J_f + 1)S$	(MeV $\pm 0.02$ MeV)	$(2J_f + 1)S^a$
$\theta$	$\overline{2}$	$0^+$	2.50	$\bf{0}$	2.90
1.52	2	$2^+$	< 0.20		
1.84	2	$0^+$	< 0.39		
2.42	2	$2^+$	< 0.24		
3.43	3	$3-$	2.31	3.44	2.11
3.95	3	(47)	4.59	3.95	4.17
4.09	3	$5 -$	6.05	4.10	5.91
4.41	3	$(2-)$	1.40	4.39	1.78
4.69		$3^-$	0.42	4.68	0.43
4.91		$(1^-)$	0.48	4.90	0.61
5.08		$(1^-, 3^-)^b$	0.48	5.07	0.60
5.37		$(1^-, 3^-)^b$	0.35	5.41	0.33
5.49		$3-$	0.78	5.52	0.65
5.63		$(1^-, 3^-)$ <sup>b</sup>	0.44	5.61	0.50
5.81	1	$(0^-, 1^-, 2^-, 3^-)$	0.36	5.79	0.36

<sup>a</sup> From Ref. 4.

<sup>b</sup> These states should have natural parity because they are excited in the <sup>40</sup>Ca(t, p)<sup>42</sup>Ca reaction (Ref. 1).



FIG. 4. Theoretical fits to the deuteron angular distributions for the  ${}^{41}K({}^{3}He,d){}^{42}Ca$  reaction using the distortedwave calculations described in the text.

corresponding to the 3<sup>-</sup>, 5.52-MeV state. The DWBA calculations with the Seth parameters were

carried out with the zero-range approximation and local option  $(ZR/L)$ . The resulting spectroscopic factors are larger than those reported here, the ratios being 1.12, 1.18, 1.14 for  $l = 2$ ,  $l = 3$ ,  $l=1$ , respectively. For the second set of parameters the calculations were carried out with the zero-range nonlocal  $(ZR/NL)$  approximation. The resulting spectroscopic factors were still slightly larger then ours with the following ratios: 1.14, 1.16, and 1.13 for  $l=2$ ,  $l=3$ , and  $l=1$ . The last set of parameters  $(ZR/NL)$  gives the largest spectroscopic factors, the corresponding ratios being 1.39, 1.34, and 1.26. In conclusion we can say that the optical-model parameters used in the present analysis tend to lower the spectroscopi . factors.

# IV. THEORY

# A. Shell-Model Calculation for the  $(1f_{7/2}^3 1d_{3/2}^{-1})$ Configuration in <sup>4</sup>Ca

Calculations for <sup>42</sup>Ca have been performed mainly for positive-parity states. Among these we can mention the recent work of McGrory, Wildenthal, and Halbert,<sup>8</sup> who assumed for the calcium isotopes an inert <sup>40</sup>Ca core and active neutrons in the  $1f_{7/2}$ ,  $2p_{3/2}$ ,  $1f_{5/2}$ ,  $2p_{1/2}$ , and  $1g_{9/2}$  orbits. They calculated a density of  $^{42}$ Ca states above 3 MeV much smaller than the observed one, and interpreted the extra observed states as a manifestation of the importance, above 3 MeV, of the core

excited components. Federman and Pittel' included core excited components for the study of the  $0^+$ states of  $^{42}$ Ca considering 2p-0h and 4p-2h configurations. Goode' described the negative-parity states of  $42$ Ca in terms of a core-particle-coupling model using the random-phase-approximation core states of Gerace and Green.<sup>10</sup> In particular, the calculations give rise to configuration in which the two  $f_{7/2}$  neutrons, coupled in a state with seniority 2, are themselves coupled to the  $3<sup>-</sup>$  and  $5<sup>-</sup>$  core states of <sup>40</sup>Ca. The calculations reproduce the high density of negative-parity states but fail to account for the spectroscopic factors in the  ${}^{41}$ K- $(^{3}He, d)^{42}$ Ca reaction. Finally, Dieperink and Brussaard" have calculated the level scheme of the  $A = 40 - 48$  nuclei in terms of  $(1f_{7/2})^n$  and  $(1f_{7/2})^{n+1}(1d_{3/2})^{-1}$  configurations. For <sup>42</sup>Ca they calculated  $3^-$  states at 3.38, 4.36, and 4.90 MeV and a  $5$ <sup> $-$ </sup> level at 4.41 which are in good agreement with the experimental spectrum. In order to get more information about  $42$ Ca spectrum and to calculate spectroscopic factors, it seemed worthwhile to repeat the shell-model calculations for the  $(1f_{7/2}^{3}1d_{3/2}^{-1})_{JT}$  configuration, using different sets of effective-interaction parameters between the  $1f_{7/2}$  particle and  $1d_{3/2}$  hole.

For states of <sup>42</sup>Ca coming from the  $(1f_{7/2}^{3}1d_{3/2}^{21})$ configuration the wave function may be denoted by

$$
\Psi(JT)\!=\!\sum\alpha(J'T')\,\big|\,1f_{7/2}{}^3(J'T')1d_{3/2}{}^{-1};JT\rangle\;.
$$

The general matrix element  $a_{ik}$  for the JT state between the  $[1f_{7/2}{}^3 (J'T') 1d_{3/2}{}^{-1}]_{JT}$  and  $[1f_{7/2}$ <sup>3</sup>(J"T") $1d_{3/2}$ <sup>-1</sup>]<sub>JT</sub> configuration is given by

$$
\begin{aligned} \big[ 3E\big(1f_{7/2}\big) + E\big(1d_{3/2}{}^{-1}\big) + E_C\big]\delta_{i\,\kappa} + \big\langle 1f_{7/2}{}^3(J'T')\big|\sum_{\text{pairs}}V_{ff}\big|\, 1f_{7/2}{}^3(J''T'')\big\rangle\delta_{J'J''}\delta_{T'T''} \\ + \big\langle 1f_{7/2}{}^3(J'T')1d_{3/2}{}^{-1}, JT\big|\, \sum_{\text{pairs}}V_{fd}{}^{-1}\big|\, 1f_{7/2}{}^3(J''T'')1d_{3/2}{}^{-1}, JT\big\rangle \, . \end{aligned}
$$

 $E(1f_{7/2})$  is the single nucleon energy of a  $1f_{7/2}$  neutron with respect to the  $^{40}$ Ca core and is taken equal to  $-8.36$  MeV; similarly, the energy of a  $1d_{3/2}$  hole is taken from <sup>39</sup>Ca equal to 15.63 MeV.  $E_C$  is the Coulomb energy and involves the interaction of a  $1f_{7/2}$  proton with the  $1d_{3/2}$  proton hole action of a  $1f_{7/2}$  proton with the  $1d_{3/2}$  proton hole<br>and is taken equal to  $-0.32$  MeV.<sup>11</sup> In the isospin Clebsch-Gordan coupling of the energy matrix, the hole is partially a proton hole and partially a neutron hole. Thus for the  $(J, T=1)$  states coming from the  $[1f_{7/2}^3(J'T')]d_{3/2}^{-1}]_{JT}$  configuration with  $T' = \frac{1}{2}$  and  $T' = \frac{3}{2}$ , the Coulomb energies are  $-0.32$  and  $-0.08$  MeV, while for the  $(J, T=2)$ states coming only from  $T' = \frac{3}{2}$  the Coulomb energy is -0.24. The last two terms in the matrix element contain the interaction of the  $f$  particles and that between the  $f$  particles and  $d$  hole. The reduction of the many-particle matrix elements in terms of two-body matrix elements has been performed with coefficients of fractional parentage<sup>12</sup> and with standard Racah techniques<sup>13</sup> which are similar to those described in more detail in Ref. 11. Four different sets of effective-interaction parameters between the  $1f_{7/2}$  particle and the  $1d_{3/2}$  hole have been used in the calculations and are shown in Table IV. These values have been derived from the existing data on the  $d_{3/2}f_{7/2}$  interaction<sup>11, 14, 15</sup> through the use of the Pandya relation<sup>16</sup> modified by isospin as follows:

$$
E(f_{7/2}d_{3/2}^{-1})_{JT} = \frac{1}{16} \sum (2J' + 1)(2T' + 1)E(f_{7/2}d_{3/2})_{J'T'} - \sum (2J' + 1)(2T' + 1)W(\frac{7}{2}\frac{3}{2}\frac{3}{2}\frac{7}{4}|JJ')
$$
  
× $W(\frac{11}{2}\frac{11}{2}\frac{1}{2}|TT')E(f_{7/2}d_{3/2})_{J'T'}$ .

Channel	V (MeV)	W (MeV)	$W_{\text{D}}$ (MeV)	$V_{\rm so}$ (MeV)	$W_{\rm so}$ (MeV)	$r_{\rm n}$ (fm)	a (fm)	$r'_0$ (fm)	$\mathfrak{a}'$ (fm)	$r_{C}$ (fm)	β
$^{41}K + ^3He$ a	162.0	13.0				1,0	0.8	1.62	0.8	1.4	
$42$ Ca + d <sup>a</sup>	112.0		18.0			1.0	0.9	1.55	0.47	1.3	
$^{41}K + p^{a}$				$\lambda = 25$		1.2	0.65			1.25	
$^{41}K + ^3He$ b	142.6		24.3			1.07	0.882	1.382	0.809	1.4	0.2
${}^{42}Ca + d{}^{b}$	111.8		10.2	8.7	0.6	$1.0\,$	0.835	1.498	0.672	1.3	0.54
$^{41}K + p^{b}$				$\lambda = 25$		1.25	0.65			1.25	0.85
$^{41}K + ^3He$ b	142.6		24.3			1.07	0.882	1,382	0.809	1.4	0.2
${}^{42}Ca + d{}^{b}$	113.6		16.3			1.0	0.831	1.416	0.552	1.3	0.54
$^{41}K + p^{b}$				$\lambda = 25$		1.25	0.65			1.25	0.85

TABLE III. Optical-model parameters reported on literature for  $\binom{3}{1}$ e, d) reaction on potassium region.

 $P^a$ From Ref. 7. bFrom Ref. 4.

The interaction of the  $1f_{7/2}$  particles shown in Table V was inferred from the  $^{42}Ca-^{42}Sc$  spectra<sup>17</sup> and has been kept constant throughout the calculation. The numerical evaluation of the matrix elements and the subsequent diagonalization of the matrices were performed at the CDC 6600 computer of the Centro Calcolo Interuniversitario of Bologna. The results are displayed in Fig. <sup>5</sup> together with experimental results. The experimental spectrum includes the six 3<sup>-</sup> states below 7-MeV excitation known from inelastic  $\alpha$  scattering<sup>18</sup> and the T = 2 states from the <sup>41</sup>K(<sup>3</sup>He,  $dy$ )<sup>42</sup>Ca reaction of Tellez et al.<sup>19</sup> In the calculated spectra we have reported the five lowest eigenvalues of the  $3$ ,  $T = 1$  states, while for the others only the lowest eigenvalue. From Fig. 5 we see that there are slight differences among the calculated spectra, and the  $T = 2$  states are practically insensitive on the choice of the  $(1f_{7/2}1d_{3/2}^{-1})$  interaction. Generally we have found that for the lowest eigenvalu $\epsilon$ the  $1f_{7/2}$  nucleons coupled to  $J = \frac{7}{2}$ ,  $T = \frac{3}{2}$  have amplitudes larger than those coupled to  $J = \frac{7}{2}$ ,  $T = \frac{1}{2}$ . It has been remarked by  $Zamik^{20}$  that this is a consequence of the fact that the interaction of a hole and several particles is strongest when the particles are coupled to the largest possible isospin. However it must be stressed that, although the

calculated spectra are not very sensitive on the  $(1f_{7/2}1d_{3/2}$ <sup>-1</sup>) interaction, there might be a greater sensitivity on the choice of the  $(1f_{7/2})^2$  interaction, especially on the amplitudes of the wave functions. For instance Dieperink and Brussaard<sup>11</sup> using a different  $(1f_{7/2})^2$  interaction found that the  $1f_{7/2}$  nucleons in the lowest  $3<sup>-</sup>$  state are predominantly (60%) coupled to  $J = \frac{7}{2}$ ,  $T = \frac{1}{2}$ .

# 8. Spectroscopic Factors

A single-proton-transfer reaction on a target nucleus of spin  $J_i$  and isospin  $T_i = \frac{1}{2}(N-Z)$  excites states of the residual nucleus with isospin  $T_f$  $=T_i \pm \frac{1}{2}$ . Summation over final states  $J_f, T_f$  excited by transfer of a proton in the  $j$  orbit yields the simple sum rules $^{21}$ 

$$
\sum \frac{2J_f+1}{2J_i+1} C^2(T_f) S_{1f}(J_f T_f) = \frac{\langle \text{neutron holes} \rangle_f}{2T_i+1}
$$
  
for  $T_f = T_i + \frac{1}{2}$ ,  

$$
\sum \frac{2J_f+1}{2J_i+1} C^2(T_f) S_{1f}(J_f T_f) = \langle \text{proton holes} \rangle_f
$$

$$
- \frac{\langle \text{neutron holes} \rangle_f}{2T_i+1}
$$
  
for  $T_f = T_i - \frac{1}{2}$ .

TABLE IV. Matrix elements in MeV of the  $(f_{7/2} d_{3/2}^{-1})_{\text{JT}}$  interaction expressed by



 $E(f_{7/2} d_{3/2}^{-1})_{JT} - \frac{1}{16} \sum (2J' + 1)(2T' + 1)E(f_{7/2} d_{3/2})_{J'T'}$ .

The  $(f_{7/2} d_{3/2}^{-1})_{JT}$  interaction has been taken from Ref. 14.

The  $(f_{7/2} d_{3/2}^{-1})_{JT}$  interaction has been taken from Ref. 11.

The  $(f_{7/2}d_{3/2} ^{-1})_{JT}$  interaction has been derived from the <sup>40</sup>Ca spectrum of Ref. 17.

The  $(f_{7/2} d_{3/2}^{-1})_{\text{JT}}$  interaction has been taken from Ref. 15.

$\epsilon$ $\bm{\tau}$	01		41	61	10	30	50	70
	$-3.09$	$-1.58$	$-0.33$	0.10	$-2.47$	$-1.58$	$-1.65$	$-2.49$

TABLE V.  $(f_{1/2}^2)_{JT}$  Matrix elements (in MeV).

Application of these sum rules in the  $^{41}K(^{3}He, d)$ -<sup>42</sup>Ca reaction for the transfer of a  $d_{3/2}$  proton give 1. For the transfers of a  $1f_{7/2}$  and  $2p_{3/2}$  proton the sum rules give 6.5 and 3 for  $T_f = T_i - \frac{1}{2}$  states and 1.5 and 1 for the  $T_f = T_i + \frac{1}{2}$  states

# 1.  $1d_{3/2}$  Proton Transfer

If we assume pure configuration for <sup>41</sup>K, the  $d_{3/2}$ transition may be represented as

$$
\begin{aligned} \left[1f_{7/2}^{\,2}(01)1d_{3/2}^{\,7}(\frac{3}{2\,2})\right]_{3/2\,3/2} + 1d_{3/2} \\ &\quad \ \left. \qquad \qquad + \left[1f_{7/2}^{\,2}(01)1d_{3/2}^{\,8}(00)\right]_{01}. \end{aligned}
$$

The corresponding spectroscopic factor  $S(01)$  is the following:

$$
S(01)=8\langle d^800 \left| \, d^7\, \tfrac{3}{2}\tfrac{1}{2}\rangle^2 U^2 \big(\tfrac{3}{2}\tfrac{3}{2}00 \left| \, 0\tfrac{3}{2}\right) U^2 \big(\tfrac{1}{2}\tfrac{1}{2}11 \left| \, 0\tfrac{3}{2}\right.\big)=\tfrac{16}{3}\,,
$$

where  $\langle \cdot | \cdot \rangle$  is a coefficient of fractional parentage and  $U($  ) are the Racah coefficients which take into account the different coupling of spins and isospin in target and residual nuclei. Now, the  $4K$  nucleus also in the pure configuration contains in its ground state and admixture of  $[1f_{7/2}^2(21)1d_{3/2}^{\phantom{3}}]$ 

configuration. Thus the resulting wave function is  $\alpha(01)[1_{7/2}^{2}(01)1_{3/2}^{-1}]_{3/2\,3/2}$ 

# $+\alpha(21)\left[1_{7/2}^{2}(21)1d_{3/2}^{-1}\right]_{3/2\,3/2}$ .

The coefficients  $\alpha(01)$  and  $\alpha(21)$  have the value +(0.84)<sup>1/2</sup> and -(0.16)<sup>1/2</sup>.<sup>22</sup> With this wave functi  $+(0.84)^{1/2}$  and  $-(0.16)^{1/2}$ .<sup>22</sup> With this wave function the corresponding spectroscopic factor  $S(0^+, 1)$  is 4.48, which is much larger than the experimental value (see Table II). However, the <sup>42</sup>Ca ground state is not a pure 2p-Oh configuration. The recent work of Federman and Pittel' showed that the ground state of  $^{42}$ Ca contains only 58% of 2p-0h and approximately 32% 4p-2h configurations, with holes and particles restricted to the  $1d_{3/2}$  and  $1f_{7/2}$ orbits. With this admixture the spectroscopic factor for the ground-state transition is reduced from 4.48 to 2.60, which is in a closer agreement with the experimental value. For the excited 0' state at 1.84 MeV we can get an higher limit of 8% for the 2p-Oh configuration. In the work of Ref. <sup>2</sup> the authors calculated that the 1.84-MeV 0' state contains less than  $1\%$  of 2p-0h configuration. If the  $2^*$  states of  $^{42}$ Ca at 1.52 and 2.42 MeV are

TABLE VI. Calculated and observed spectroscopic factors for the  $T = 1$  states of <sup>42</sup>Ca coming from the  $(1f_{7/2}^{3} 1d_{3/2}^{-1})_{JT}$  configuration.

	Ref. a		Ref. b		Ref. c		Ref. d		Experimental	
JT	$E_{\rm r}$ (MeV)	S	$E_{\rm r}$ (MeV)	S	$E_{\rm r}$ (Mev)	S	$E_x$ (MeV)	S	$E_{x}$ (MeV)	S
$2^-1$	4.23	0.10	4.69	0.01	4.51	0.07	3.82	0.02		
$2^-1$	4.52	0.04	5.00	0.10	5.25	0.19	4.33	0.08	4.39	0.28
$2^-1$	5.44	0.72	5.58	0.76	6.36	0.56	5.46	0.76		
$3^-1$	3.47	0.18	3.63	0.20	3.10	0.20	2.68	0.11		
$3^-1$	4.04	0.40	4.04	0.41	3.88	0.04	3.34	0.05	3.44	0.33
$3 - 1$	4.41	0.26	4.41	0.23	4.06	0.57	3.82	0.42		
$4 - 1$	4.10	0.44	4.19	0.08	3.73	0.003	3.33	0.002		
$4^-1$	4.28	0.03	4.29	0.43	4.36	0.52	3.77	0.33	3.95	0.51
$4^-1$	5.04	0.35	5.27	0.28	5.29	0.15	4.79	0.44		
$5^-$ 1	4.19	0.16	3.93	0.20	3.79	0.16	3.25	0.12		
$5^-$ 1	4.95	0.58	5.06	0.59	4.90	0.10	4.41	0.01	4.10	0.55
$5 - 1$	5.25	0.04	5.29	0.02	5.07	0.53	4.78	0.66		

<sup>a</sup> The spectroscopic factors have been calculated with eigenfunctions generated by the  $(1f_{7/2} 1d_{3/2}$ <sup>-1</sup>) interaction derived from Ref. 14.

<sup>b</sup> The spectroscopic factors have been calculated with eigenfunctions generated by the  $(1f_{7/2} 1d_{3/2} ^{-1})$  interaction derived from Ref. 11.

<sup>c</sup>The spectroscopic factors have been calculated with eigenfunctions generated by the  $(1f_{7/2} 1d_{3/2}^{-1})$  interaction derived from Ref. 15.

<sup>d</sup>The spectroscopic factors have been calculated with eigenfunctions generated by the  $(1f_{7/2}1d_{3/2}^{-1})$  interaction derived from Ref. 17.



FIG. 5. Experimental and calculated level schemes for  ${}^{42}$ Ca. The spectra labeled (a), (b), and (c) employ the  $(1f_{7/2}1d_{3/2}^{-1})$  matrix elements of Erne (Ref. 14), of Dieperink and Brussaard (Ref. 11), and of Moinester and Alford (Ref. 15). The spectrum labeled (d) employs the  $(1f_{7/2}1d_{3/2}^{-1})$  interaction derived from the experimental <sup>40</sup>Ca spectrum of Ref. 17.



<sup>a</sup>The total strengths in brackets are those obtained by Forster  $et al.$  (Ref. 4).

described by a fraction  $\beta_i$  of pure  $1f_{7/2}^2(21)$  configuration, the calculated spectroscopic factor is given by  $S(2^+1) = 0.16\beta_i^2$ . Comparing this value with the experimental one we find that each 2' state contains approximately less than  $25-30\%$  of the  $1f_{7/2}^2(21)$  configuration.

# 2.  $1f_{7/2}$  Proton Transfer

The transition involved in the  $1f_{7/2}$  proton transfer is

 $(0.84)^{1/2} [1f_{7/2}^{\ 2}(01)1d_{3/2}^{\ \, -1}]_{3/2\;3/2}$  +  $1f_{7/2}$  $-\alpha(\frac{7}{2}T')[1f_{7/2}^3(\frac{7}{2}T')]d_{3/2}^{-1}]_{IT}$ ,

where we have neglected the small component (16%) of  $1f_{7/2}^2$ (21) in the <sup>41</sup>K ground-state configuration. For the  $(J, T=1)$  states of <sup>42</sup>Ca coming mainly from the  $1f^3(\frac{7}{2}\frac{1}{2})$ , the spectroscopic factor is

$$
S(J1) = 0.84 \alpha^2 (\frac{7}{2} \frac{1}{2}) (f^3 \frac{7}{2} \frac{1}{2}) (f^2 01)^2 U^2 (\frac{7}{2} 0 J \frac{3}{2}) (\frac{7}{2} \frac{3}{2})
$$
  
 
$$
\times U^2 (\frac{1}{2} 11 \frac{1}{2}) (\frac{13}{2} \frac{3}{2})
$$
  
= 0.84  $\alpha^2 (\frac{7}{2} \frac{1}{2})$ .

For the  $(J, T = 1)$  states coming mainly from the  $1f^3(\frac{7}{2}, \frac{3}{2})$  the spectroscopic factor is

$$
\begin{array}{cccc}\n\frac{1}{3}\frac{d_{3/2}}{2} & \frac{1f_{7/2}}{2} & \frac{2p_{3/2}}{2} & S(J1) = 0.84\alpha^2 \left(\frac{7}{2}\frac{3}{2}\right) \left(f^3 \frac{7}{2}\frac{3}{2}\right) \left(f^2 01\right)^2 U^2 \left(\frac{7}{2}0 \cdot J\frac{3}{2}\right) \left(\frac{7}{2}\frac{3}{2}\right) \\
& \frac{52}{10} & 0.54\right)^{\frac{3}{4}} & 2.69 & (2.62) & 0.62 & (0.76) \\
& 0.55 & 3 & 0.07\alpha^2 \left(\frac{7}{2}\frac{3}{2}\right).\n\end{array}
$$

From the above computation we observe that the spectroscopic factors for the  $1f_{7/2}$  proton transfer are pronounced when the  $1f_{7/2}$  nucleons are couare pronounced when the  $1/7/2$  nucleons are coupled to the lowest isospin  $T' = \frac{1}{2}$ . It is clear from this observation that the lowest eigenvalues will have small spectroscopic factors. This is so because in the calculated spectra we have found that the lowest eigenvalues contain generally small amplitudes when the  $1f_{7/2}$  nucleons are coupled to the lowest isospin. In Table VI are shown the eigenvalues and the calculated spectroscopic factors of the three lowest states for  $2^-, 3^-, 4^-, 5^-,$  $T = 1$  states and the experimental ones. From analysis of the tables we see that, although the experimental spectroscopic factors have large uncertainties, there is clear evidence that the lowest eigenvalue has spectroscopic factors much smaller than the observed ones. As previously noted, the amplitudes of the wave functions may be more sensitive to the  $(1f_{7/2})^2$  interaction which is not well known. However, if the interaction of a hole and several particles is generally strongest when the particles are coupled to the largest possible isospin, these shell-model calculations do not reproduce the experimental strength found in the  $^{41}$ K(<sup>3</sup>He, d)<sup>42</sup>Ca reaction.



EXCITATION ENERGY (MeV)

FIG. 6. Proton spectrum for the  ${}^{39}K(\alpha, p)^{42}$ Ca reaction at  $\theta_{1ab} = 40^\circ$ .



FIG. 7. Optical-model fits to the elastic scattering data with potential described in Table VIII. The continuous and dashed lines correspond to diffuseness parameters  $a = 0.500$  and  $a = 0.588$ . The compound elastic scattering estimated by Hauser-Feshbach calculations (H-F) is also shown.

# 3.  $2p_{3/2}$  Proton Transfer

The transition involved in the  $2p_{3/2}$  proton transfer gives origin to states with  $J^{\pi} = 0^{-}$ , 1<sup>-</sup>, 2<sup>-</sup>, 3<sup>-</sup> and  $T = 1, 2$ . The corresponding spectroscopic factor, assuming pure configuration, is equal to 1 for all states involved. Experimentally we observe that the  $(2p_{3/2}1d_{3/2}^{-1})$  quartet of  $T=1$  states is highly fragmented. Below 6-MeV excitation energy we found seven  $l_{\rm o}$  = 1 transitions, and in the experiment of Ref. 4 ten  $l_{\rho} = 1$  transitions have been observed. However, since the spins of only a few of the levels are known it is difficult to say much about the way the members of the quartet are split. Finally, in Table VII the total observed

strength relative to the excitation of the  $T = 1$ states of <sup>42</sup>Ca for the  $1d_{3/2}$ ,  $1f_{7/2}$ , and  $2p_{3/2}$  proton transfers are reported.

# V.  $^{39}K(\alpha,p)^{42}Ca$ . EXPERIMENTAL RESULTS

As mentioned in the introduction, the  $^{39}\text{K}(\alpha,p)$ -<sup>42</sup>Ca reaction has been investigated in order to have a comparison between ( ${}^{3}$ He, d) and  $(\alpha, p)$  reactions. For the  $(\alpha, p)$  reaction mechanism there are few available experimental data and they are mostly for light-medium nuclei. They indicate<sup>23</sup> angular distributions typical of direct interaction for  $\alpha$  energies above 20 MeV and target nuclei with  $A > 19$ . In our case the angular distributions observed are not forward peaked and indicate that the  $(\alpha, \beta)$  reaction goes predominantly through a compound-nucleus reaction mechanism. A spectrum of the  $^{39}K(\alpha,p)^{42}$ Ca reaction is shown in Fig. 6. The peaks are labeled by their excitation energies and, if known, by their spins and parities. The error associated with the  $42$ Ca excitation energy is  $\pm 40$  keV. The angular distributions of elastically scattered particles are shown in Fig. 7, together with optical-model fits and with the compound elastic scattering estimated by Hauser-Feshbach (H-F) calculation. Finally, in Fig. 8 are shown the proton angular distributions and the curves reported are those obtained by H-F calculations. The code used for H-F calculations permitted the evaluation of compound-nucleus contributions to the differential cross sections through the use of level densities<sup>24</sup> and of an appropriate optical potential for incoming and outgoing particles. At this bombarding energy only the  $(\alpha, \beta)$ ,  $(\alpha, n)$ , and  $(\alpha, \alpha')$  channels are open. The  $(\alpha, d)$  channel has the large negative Q value of -9.<sup>37</sup> MeV and it has been neglected. In Table VIII the optical potentials used for the three channels considered in the calculation are shown. The  $\alpha$  parameters were those obtained by John *et al.*<sup>25</sup>  $\alpha$  parameters were those obtained by John et al.<sup>25</sup> in the analysis of  $\alpha$  elastic scattering from  $^{40}Ca$ in the energy range 5 to 12.5 MeV. The proton parameters were taken by Marinov, Lee, and meters were taken by Marinov, Lee, and<br>Schiffer,<sup>26</sup> who analyzed proton elastic scattering at 9 and 12 MeV from calcium isotopes. Finally, the optical-model parameters of Perey and Buck as compiled by  $Rosen^{27}$  were used for the neutrons. Table IX lists the excitation energies,  $J^{\pi}$  values,

TABLE VIII. Optical-model parameters used for the Hauser-Feshbach calculations.

Channel	(MeV)	W (MeV)	$W_{\rm D}$ (MeV)	$\boldsymbol{V}_{\boldsymbol{\mathrm{so}}}$ (MeV)	(f <sub>m</sub> )	$a_{0}$ (f <sub>m</sub> )	$r_{0}$ (fm)	a' (fm)	$r_{\rm C}$ (fm)
$39K + \alpha$ $42$ Ca + p $42$ Sc + n	142 52.4 $49.3 - 0.33E$	5.75	30 6.9	3.75 5.5	1,533 1.209 1.25	0.588 0.678 0.65	1.474 1.363 1.25	0.3 0.305 0.70	1.533 1.25

and integrated experimental and calculated cross sections. The agreement between experimental and estimated (H-F) cross sections is very good. This is surprising because the H-F expression for the cross section of a statistical reaction is valid when the compound nucleus is excited into

the region of strongly overlapping levels  $(\Gamma/D \gg 1)$ . In our case the total width  $\Gamma = \Gamma_{\alpha} + \Gamma_{p} + \Gamma_{n}$  divided by the compound-nucleus spacing  $D$  was equal to 4.17, which is not large enough to fulfill the condition of strongly overlapping levels. In addition, if we compare the target thickness  $(\sim 40 \text{ keV})$  to



FIG. 8. Proton angular distributions for the  $^{39}K(\alpha, p)^{42}$ Ca reaction. The continuous lines are the theoretical curves predicted by Hauser-Feshbach calculations.

Excitation		Integrated cross section					
energy (MeV)		Expt	Calc.				
$\pm 40~\mathrm{keV}$	$J^{\pi}$	(m <sub>b</sub> )	(m <sub>b</sub> )				
0	$^{0+}$	$4.49 \pm 0.7$	3.98				
1.52	$2^+$	$14.78 \pm 2.2$	14.17				
1.84	$0^+$	$2.86 \pm 0.4$	2.84				
2.43	$2^+$	$11.03 \pm 1.6$	11.68				
2.76	$4^+$	$19.38 \pm 2.9$	17.71				
3.20	$6+$	$18.46 \pm 3.7$	19,47				
3.26	$4^+$	$18.01 \pm 3.6$	15.99				
3.39 3,44	$2^+$ $3^{-}$	$14.80 \pm 2.2$	20.31				
3.68	$(2^{+})$	$9.55 \pm 1.4$	8.68				
3.91	$1^-$	$3.82 \pm 0.8$	4.49				
4.26	17	$4.68 \pm 0.7$	4.49				
4.40	$(2^-)$	$9.97 \pm 2.0$	6.52				

TABLE IX. Summary of experimental results from  $^{39}K(\alpha, p)^{42}$ Ca reaction.

the compound-nucleus spacing  $(-4.5 \text{ keV})$  we see that the level density is not sufficiently large to justify completely the statistical assumption. stify completely the statistical assumption.<br>In the recent work of Lawley *et al.*<sup>28</sup> who mea-

sured angular correlation on the excited states of <sup>42</sup>Ca by the <sup>42</sup>Ca( $p, p'\gamma$ )<sup>42</sup>Ca reaction, unique spin assignments were obtained for states at 3.26 MeV  $(J^{\pi} = 4^+)$ , 3.91 MeV  $(J^{\pi} = 1^-)$ , and 4.26 MeV  $(J^{\pi} = 1^-)$ . The natural parity for these states is in agreement with the fact that the  $(t, p)$  angular distributions of Ref. 1 show a forward-peaked angular distribution. From Table IX we observe for these states a good agreement between the integrated experimental and calculated cross section, thus confirming these spin assignments. For the negative-parity states with  $J^{\pi} = (4^{-})$  at 3.99 MeV and  $J^{\pi} = 5^{-}$  at 4.13 MeV, the corresponding peaks could not be accurately evaluated. In fact, between the 3.91- and 4.26-MeV states shown in Fig. 6 are five known levels<sup>28</sup> which in the present experiment could not be resolved.

### VI. CONCLUSIONS

The negative-parity states of  $^{42}$ Ca observed in the  ${}^{41}K({}^{3}\text{He}, d)^{42}\text{Ca}$  reaction can be described mainly by configurations in which a  $1d_{3/2}$  hole is coupled to particles in the  $fp$  shell. Within the experimental uncertainty and differences in the distorted-wave analysis the results obtained in the present  $(^{3}$ He, d) experiment agree with those previousem ( ne, *a*) experiment agree with those previour but the value of  $at.4$  at 11 MeV. Standard shell-model calculations for the negative-parity states of <sup>42</sup>Ca arising from the  $(1f_{7/2}^{3}1d_{3/2}^{2-1})_{JT}$ configuration reproduce the high density of negative-parity states but fail to account for the experimental strength of the  $T = 1$  states in the <sup>41</sup>K - $(^{3}He, d)^{42}$ Ca reaction. The main reason for this failure is that in the lowest eigenvalues the  $1f_{7/2}$ nucleons coupled to the highest isospin  $T$  =  $\frac{3}{2}$  have much larger amplitudes than those coupled to  $T = \frac{1}{2}$ . Since the corresponding spectroscopic factors are hindered by a factor of 12 when the  $1f_{7/2}$  nucleons are coupled to  $T = \frac{3}{2}$  with respect to those with T  $=\frac{1}{2}$ , the calculated strength fails to reproduce the observed one. Finally the  $^{39}{\rm K}(\alpha,\rho)^{42}{\rm Ca}$  reaction has been found to proceed dominantly through a compound-nucleus mechanism, and good agreement has been obtained between the integrated experimental and theoretical cross section, based on the Hauser- Feshbach calculations. The agreement achieved between experiment and theory is surprisingly good, although the experimental conditions for the  $(\alpha, p)$  experiment were not such to justify completely the statistical assumption. Due to this agreement, the present investigation supports the recent spin assignments of Ref. 28 for the <sup>42</sup>Ca states at 3.26 MeV ( $J^{\pi} = 4^{+}$ ), 3.91 MeV  $(J^{\pi} = 1^{-})$ , and 4.26 MeV  $(J^{\pi} = 1^{-})$ .

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# PHYSICAL REVIEW C VOLUME 4, NUMBER 1 VILY 1971

# Stopping of 1- to 1.8-keV  $^{73}$ Ge Atoms in Germanium\*

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Measurements of the amount of energy lost to ionization in the stopping of low-energy  $^{73}$ Ge atoms, produced by inelastic neutron scattering in a germanium  $\gamma$ -ray detector, have been extended down to energies of about 1 keV. The results are about 35% higher than theoretical values of the Lindhard theory found from an extrapolation to this energy region. The energy of the 73Ge third excited state was measured for this experiment and found to be 68.752  $± 0.007 keV.$ 

### I. INTRODUCTION

The amount of ionization produced in the stopping of a heavy atom has been studied theoretically by Lindhard et  $al.$ <sup>1</sup> A number of experimental  $\mu$ ,  $\mu$ , lar case of Ge atoms stopping in a germanium crystal with the energy of the Ge atoms ranging from 10 keV to 1 MeV. The agreement of these measurements with theory is reasonable over the entire range of energies.

In order to search further for deviations from the theory it seems reasonable to try to carry out measurements at very low energies. This should be a good test of the validity of the theoretical calculations at low energies, and it should be a sensitive indicator of any threshold of channeling effects in the ionization process. We have previously shown<sup>5</sup> that if threshold effects occur, they may be important for recoil energies of 2 to 5 keV or less but are not important at higher energies.

In the present paper, we present the results of measurements at about 1-keV germanium ion energy and compare the results with the theoretical predictions of Lindhard et  $al.$ <sup>1</sup> Some new information on the level schemes of  $73$ Ge and  $73$ As is also presented as a by-product of the primary experiment.

# II. EXPERIMENTAL TECHNIQUE

Much of the experimental technique used in the present experiment has been described in our previous papers. $3-5$  This experiment is mainly made possible by the great improvement in the resolution of lithium-drifted germanium detectors at low energies resulting from recent improvements in fabrication and in electronics. Since, in stopping, a 1-keV atom should lose about 200 eV of its energy to ionization and the balance to atomic scattering, excellent resolution is a sine qua non for a successful experiment.

The 1-keV Ge atoms were produced by the inelastic scattering of neutrons from the third excited state of  $73$ Ge. Figure 1 shows the variation of recoil energy with incident neutron energy. If the experiment is carried out just over the inelastic scattering threshold, then a beam of 0.9-keV <sup>73</sup>Ge atoms, with an energy spread of  $\pm 250$  eV, is obtained. At such a low neutron energy the scat-