Calculation of the Separation-Energy Spectrum in the Reaction $^{12}C(p, 2p)$

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The cross section for the $^{12}C(p, 2p)$ reaction at an incident kinetic energy of 460 MeV is calculated in the framework of a model which takes account of nucleon-nucleon correlations and of the coupling to continuum channels of the residual nucleus.

Recently, a method for calculating the separation-energy spectrum in quasifree scattering and pick-up reactions has been proposed which takes account of nucleon-nucleon correlations and of the decay of the residual nucleus into continuum channels.[1] In the present note, this method is applied to the reaction $^{12}C(p, 2p)^{11}B$.

According to Ref. 1, the $(p, 2p)$ cross section for symmetric coplanar geometry is given, in the plane-wave impulse approximation and neglecting recoil energies, by (for notation cf. Ref. 1)

$$\frac{d^6\sigma}{dE_0' d\Omega_0' dE_1' d\Omega_1'} = \frac{4}{(\hbar c)^2}\frac{p'^2}{\hbar p}\frac{c^2p^2\sin^2\theta' + m^2c^4}{[c^2(2p'\cos\theta' - p_0)^2 + m^2c^4]^{1/2}} \times \frac{d\sigma^{fr}}{d\Omega}\frac{2}{(2\pi)^3}S(\vec{p}; W), \quad (1)$$

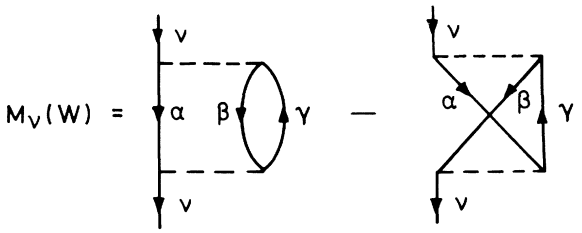
where $d\sigma^{fr}/d\Omega$ is the pp scattering cross section in the parametrized form used by Tyrén *et al.*,[2] and $cp_0 = (E_0^2 - m^2c^4)^{1/2}$, $cp' = cp_i' = (E_i'^2 - m^2c^4)^{1/2}$, $\theta' = \theta_i$, with $i = 0, 1$. The spectral function $S(\vec{p}; W)$ is the joint probability for finding a proton with momentum $\vec{p} = \vec{p}_0' + \vec{p}_i' - \vec{p}_0$ in the target nucleus and the separation energy $-W \equiv S = E_0 - E_0' - E_1' + mc^2$ in the residual nucleus after that proton has been removed.[1] It is computed from its shell-model representation $S_\nu(W)$, which is assumed to be diagonal:

$$S(\vec{p}; W) = \sum_\nu |\phi_\nu(\vec{p})|^2 S_\nu(W). \quad (2)$$

Here, the index ν refers to the two occupied shell-model states $1s_{1/2}$ and $1p_{3/2}$ (ground-state correlations in ^{12}C are neglected). The functions $\phi_\nu(\vec{p})$ are assumed to represent Hartree-Fock wave functions in momentum space, with the corresponding self-consistent energies denoted by ϵ_ν. The spectral function $S_\nu(W)$ is given by

$$S_\nu(W) = \frac{1}{\pi}\,\mathrm{Im}\frac{1}{W - i\eta - \epsilon_\nu - M_\nu(W - i\eta)} = \sum_i z_{\nu i}\delta(W - W_{\nu i}) + \frac{1}{\pi}\frac{\mathrm{Im}M_\nu(W)\theta(W_0 - W)}{[W - \epsilon_\nu - \mathrm{Re}M_\nu(W)]^2 + [\mathrm{Im}M_\nu(W)]^2}, \quad (3)$$

where $M_\nu(W)$ is the mass operator. The energies $W_{\nu i}$ are the positions of the real poles of the function $[W - \epsilon_\nu - M_\nu(W)]^{-1}$, and $z_{\nu i} = [1 - M_\nu'(W_{\nu i})]^{-1}$ are the corresponding residues (spectroscopic factors). In the variable $S \equiv -W$, these real poles lie to the left of the continuum part of the separation-energy spectrum which starts at $S_0 \equiv -W_0$ and runs to the right. The mass operator $M_\nu(W)$ is calculated in second order in the residual interaction (the first order vanishes owing to the self-consistency assumption):



(4)

The single-particle states used were computed in a Woods-Saxon well (plus Coulomb potential for protons); the energies for the discrete single-particle states (including a few single-particle resonances) are given in Table I. The parameters of the potential well were adjusted such as to yield single-particle energies near the Fermi level equal to the experimental separation energies for neighboring nuclei. For the deep-lying $1s_{1/2}$ level,

TABLE I. Shell-model states of ^{12}C. The superscript C refers to continuum states.

Protons		Neutrons	
nlj	ϵ_{nlj} (MeV)	nlj	ϵ_{nlj} (MeV)
$d^C_{3/2}$	6.28	$d^C_{5/2}$	3.39
$d^C_{5/2}$	1.62	$1d_{5/2}$	−1.10
$s^C_{1/2}$	0.42	$2s_{1/2}$	−1.86
$1p_{1/2}$	−1.94	$1p_{1/2}$	−4.95
$1p_{3/2}$	−15.96	$1p_{3/2}$	−18.72
$1s_{1/2}$	−30.86	$1s_{1/2}$	−35.10

the same well parameters as for the $2s_{1/2}$ level were chosen. The discrete 1p-2h states included in the calculation and their energies are listed in Table II. In addition, 1p-2h states with the particle in the $s_{1/2}$, $p_{1/2}$, $p_{3/2}$, $d_{3/2}$, $d_{5/2}$, $f_{5/2}$, and $f_{7/2}$ continua were taken into account. These give rise to the continuous part of the separation-energy spectrum. For the residual interaction we used the Lemmer-Shakin force[3]:

$$V_{12} = -V_0(\pi_t^\sigma + p\pi_s^\sigma)\delta(\vec{r}_1 - \vec{r}_2), \quad (5)$$

$$V_0 = 582 \text{ MeV fm}^3, \quad p = 0.46.$$

In the calculation of the mass operator the integration over the continuous energies of the intermediate particle states γ is avoided by using the continuum part of the radial single-particle Green function for particle γ.

Figure 1(a) shows the real part of the mass operator for the $1p_{3/2}$ state as a function of the separation energy $S = -W$ over the range $S = 29$ to 51 MeV ($S_0 = 31.92$ MeV). The points of intersection with the line $-(S - |\epsilon_\nu|)$ determine the real energies of the discrete spectrum (labeled 1 and 2) and the approximate positions of the resonances in the continuous spectrum (3 to 8), all corresponding to spin $J = \frac{3}{2}^-$ in the residual nucleus. The real part of the $1s_{1/2}$ mass operator is shown in Fig. 1(b). In the range of energies considered, only resonances occur (1 to 7), while the corresponding discrete states with $J = \frac{1}{2}^+$ lie farther to the left.

Figure 1(c) shows the calculated $^{12}C(p, 2p)$ cross

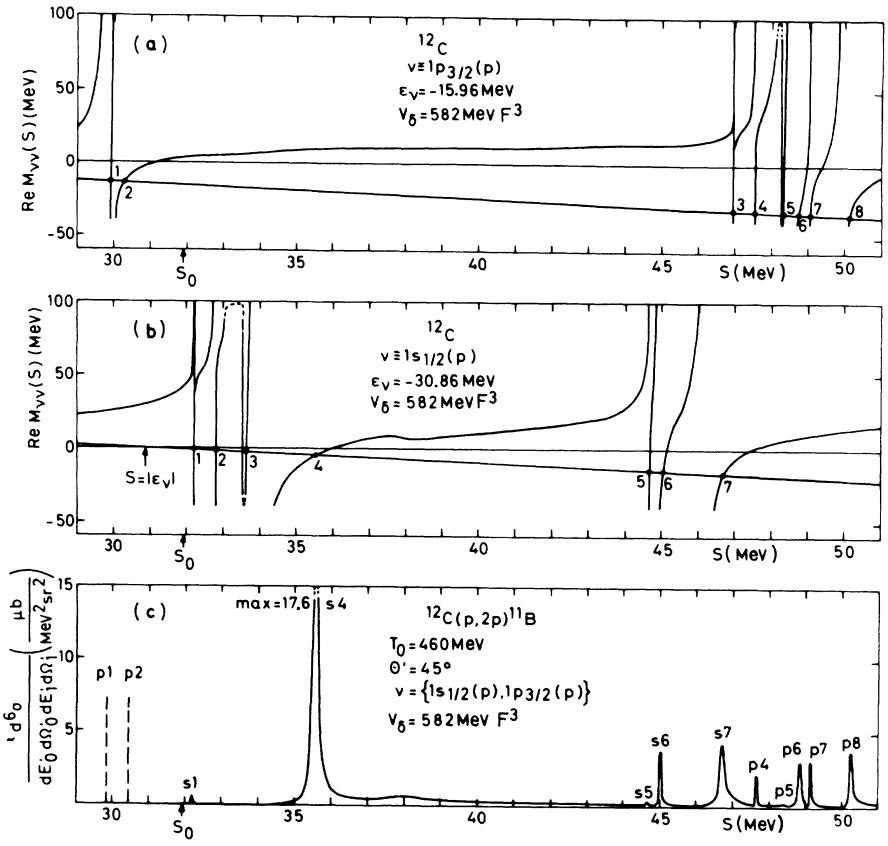


FIG. 1. (a) Real part of the $1p_{3/2}$ mass operator, (b) real part of the $1s_{1/2}$ mass operator, and (c) cross section for the $^{12}C(p, 2p)^{11}B$ reaction as a function of the separation energy $S = -W$. The numbers identify the points of intersection of Re$M_\nu(S)$ and $-(S - |\epsilon_\nu|)$.

TABLE II. One-particle–two-hole states included in the calculation of the mass operator. (a) Coupling to $1p_{3/2}$, (b) coupling to $1s_{1/2}$ hole. The superscript C refers to continuum states.

	(a)					(b)			
	α	β	γ	S (MeV)		α	β	γ	S (MeV)
1	$1p_{3/2}(p)$	$1p_{3/2}(n)$	$1p_{1/2}(n)$	29.73	1	$1p_{3/2}(p)$	$1p_{3/2}(p)$	$s^C_{1/2}(p)$	32.34
2	$1p_{3/2}(p)$	$1p_{3/2}(p)$	$1p_{1/2}(p)$	29.88	2	$1p_{3/2}(p)$	$1p_{3/2}(n)$	$2s_{1/2}(n)$	32.82
3	$1s_{1/2}(p)$	$1p_{3/2}(p)$	$s^C_{1/2}(p)$	47.24	3	$1p_{3/2}(p)$	$1p_{3/2}(p)$	$d^C_{5/2}(p)$	33.54
4	$1s_{1/2}(p)$	$1p_{3/2}(n)$	$2s_{1/2}(n)$	47.72	4	$1p_{3/2}(p)$	$1p_{3/2}(n)$	$1d_{5/2}(n)$	33.58
5	$1s_{1/2}(p)$	$1p_{3/2}(p)$	$d^C_{5/2}(p)$	48.44	5	$1p_{3/2}(p)$	$1p_{3/2}(n)$	$d^C_{3/2}(n)$	38.07
6	$1s_{1/2}(p)$	$1p_{3/2}(n)$	$1d_{5/2}(n)$	48.48	6	$1p_{3/2}(p)$	$1p_{3/2}(p)$	$d^C_{3/2}(p)$	38.20
7	$1p_{3/2}(p)$	$1s_{1/2}(n)$	$2s_{1/2}(n)$	49.20	7	$1s_{1/2}(p)$	$1p_{3/2}(n)$	$1p_{1/2}(n)$	44.63
8	$1p_{3/2}(p)$	$1s_{1/2}(n)$	$1d_{5/2}(n)$	49.96	8	$1s_{1/2}(p)$	$1p_{3/2}(p)$	$1p_{1/2}(p)$	44.88
9	$1s_{1/2}(p)$	$1p_{3/2}(n)$	$d^C_{3/2}(n)$	52.97	9	$1p_{3/2}(p)$	$1s_{1/2}(n)$	$1p_{1/2}(n)$	46.13
10	$1s_{1/2}(p)$	$1p_{3/2}(p)$	$d^C_{3/2}(p)$	53.10	10	$1s_{1/2}(p)$	$1s_{1/2}(p)$	$s^C_{1/2}(p)$	62.34
11	$1p_{3/2}(p)$	$1s_{1/2}(n)$	$d^C_{3/2}(n)$	54.45	11	$1s_{1/2}(p)$	$1s_{1/2}(n)$	$2s_{1/2}(n)$	64.10
12	$1s_{1/2}(p)$	$1s_{1/2}(n)$	$1p_{1/2}(n)$	61.01					

section for a fixed primary kinetic energy of 460 MeV and angle $\theta' = 45°$ of either proton relative to the incident beam (coplanar symmetric geometry). This case has been chosen in conformity with the data of Ref. 2. It is seen that the cross section (1), via the energy-momentum spectral function $S(\vec{p}; W)$, is composed of incoherent contributions from the energy and momentum distributions of the shell-model states ν [cf. formula (2)]. The calculated cross section, or separation-energy spectrum, shown in Fig. 1(c) exhibits a number of fairly isolated resonances. It appears that the strength of the pure $1p_{3/2}$ and $1s_{1/2}$ hole states is distributed over a wide range of 1p-2h excitations. It is interesting to note that the peak $s4$ lies at an energy ($S \approx 35$ MeV) where the maximum of the $s_{1/2}$ strength is found experimentally.[2,4] However, much of the strength of the $1p_{3/2}$ and $1s_{1/2}$ spectral functions lies in the discrete states at lower values of S (not shown in the figure).

This calculation of the $(p, 2p)$ cross section takes account of nucleon-nucleon correlations and continuum coupling in the simplest way: (i) Only single-nucleon continua have been included, thus omitting possibly important α channels, etc. (ii) Ground-state correlations in ^{12}C have been neglected; that is, in the mass operator (4) we have omitted graphs where the external lines γ correspond to states above the Fermi level ("particle states"), and where the intermediate states are 2p-1h states. The ground-state correlations affect most seriously the separation-energy spectrum near the Fermi level of ^{12}C ($S = 15.96$ MeV). Accordingly, we do not attach much significance to the positions (and spectroscopic factors) of the discrete states obtained in our calculation [e.g., the dashed lines $p1$, $p2$ in Fig. 1(c)]. The inclusion of the ground-state correlations in the mass operator is purely a numerical problem. (iii) The mass operator (4) has been computed only in second order in the residual interaction. Introducing interactions between the particle-hole states, i.e., coupling vibrations to the single-hole states, may have important effects. (iv) We have used a very simple energy-independent residual interaction. (v) We have identified the single-particle energies near the Fermi level with experimental separation energies. Actually, the former should be identified with the "centroid energies."[5] (vi) Finally, we have neglected initial- and final-state interactions.

For these reasons it would be premature to make a detailed comparison between our present results and the experimental data, the more so since the experimentally resolved widths seem to be much larger than the natural widths of the peaks in the continuum, so that an energy average of the calculated cross sections should be taken first. Such a comparison will become more meaningful as more and better experimental data become available and as the above-mentioned defects of the calculation are removed. It should be interesting to see whether the large splitting of the single-hole states and the relatively small natural widths ($\lesssim 200$ keV) obtained by us are confirmed by improved calculations or more accurate experimental data.

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Nuclear-Structure Calculations in ^{18}F

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The two intrinsic Hartree-Fock (HF) states, one characterized by isospin $T = T_z = \frac{1}{2}(N-Z)$ and the other containing an admixture of $T = T_z$ and $T = T_z + 1$, lie energetically very close in the case of light odd-Z–odd-N nuclei. As a result, the low-lying $T = T_z$ states of these nuclei have substantial "band mixing." The HF projection formalism is employed to investigate the effect of band mixing on the properties of nuclear states in ^{18}F by explicitly treating all the nucleons. The results of our calculations are in fair agreement with the experimental data.

1. INTRODUCTION

There are many shell-model calculations available in the literature[1] for simple odd-Z–odd-N nuclei such as ^{18}F and ^{14}N. With ^{16}O as a core, these nuclei provide the simplest system of a neutron-proton pair of particles or holes. The recent trend has been to employ, in these shell-model calculations, effective model interactions derived from the realistic nucleon-nucleon (NN) interactions.[2] The simplicity of these calculations is more than compensated for by difficulties and uncertainties involved in the derivation[3] of the model interaction. The first successful attempt to explain the two-nucleon (outside the ^{16}O core) energy spectra of ^{18}O and ^{18}F by employing the model interaction derived from the free NN interaction was by Kuo and Brown.[3] However, there were some errors[4] in calculating the contributions from the second-order tensor interaction and the core-polarization effects. When these are properly rectified in rather involved calculations, the computed energy spectrum of ^{18}F does not agree well with the experimental spectrum: The lowest state is predicted to have $J=3$, $T=0$ in contrast to the actual $J=1$, $T=0$ for the ground state. It is quite obvious[4] that the calculation of core-polarization effects is very laborious and it involves some approximations to take the core excitations into account. In view of this state of affairs in computing the renormalized effective NN interaction to incorporate the effect of core polarization, one cannot use the electromagnetic properties of the nuclear states to test the shell-model wave functions; rather one has to introduce an arbitrary "effective charge" for the "valence" nucleons. These undesirable features can be removed in the framework of Hartree-Fock (HF) projection formalism by explicitly treating all the nucleons in the system in a sufficiently large configuration space. The two intrinsic HF states, one characterized by $T = T_z$ and the other containing an admixture of $T = T_z$ and $T = T_z + 1$, lie very close in energy in the case of light odd-Z–odd-N nuclei. This necessitates a band-mixing calculation[5] which is rather involved. From the calculations reported in this paper, we find that the properties of the low-lying positive-parity levels of ^{18}F are quite sensitively dependent on the band-mixing matrix element between the good J and T states projected from the two intrinsic HF states.

2. METHOD OF CALCULATION

The HF method of solving for self-consistent single-particle wave functions and energies is well known and can be found, for example, in Ref. 6. The projection method for obtaining states