Energy of the First Two Excited States of $^{12}C^{\dagger}$

H. Stocker, A. A. Rollefson, and C. P. Browne
Department of Physics, University of Notre Dame, Notre Dame, Indiana 46556
(Received 14 June 1971)

Owing to its importance in stellar helium burning, we have carefully remeasured the excitation energy of the second excited state of ^{12}C with a tandem accelerator and a broad-range magnetic spectrograph. The excitation energy of this state is found to be 7655.9 ± 2.5 keV, in excellent agreement with two other recent measurements. This value lowers the calculated rate of the helium-burning process in stars by a factor of ≈ 3 from that previously assumed. A simultaneous measurement of the first excited state of ^{12}C gives 4442.2 ± 1.5 keV. A comparison is made with recent γ-ray measurements of this state.

INTRODUCTION

The second excited state in ^{12}C plays an important role in the helium-burning process in stars. In this process three α particles are fused together in two resonance stages to form a ^{12}C nucleus. In condensed notation these stages can be written: $^4He(\alpha)^8Be(\alpha)^{12}C^*(\gamma\gamma$ or $e^+e^-)^{12}C$, where the ground state of ^8Be and the second excited state of ^{12}C are the two resonances in the over-all process.[1] Burbidge *et al.*[2] have shown that the rate of the process (called the 3α process) depends exponentially on the energy difference between the second excited state of ^{12}C and three α particles; i.e.,

$$\text{Rate} \propto \exp\left[-\frac{(M_C - 3M_\alpha)c^2 + E_x(7.6)}{kT}\right],$$

where k is Boltzmann's constant, T is the temperature, and c is the velocity of light. M_C and M_α are the atomic masses of ^{12}C and ^4He, respectively, and $E_x(7.6)$ represents the excitation energy of the second excited state of ^{12}C. It is evident that the rate is a sensitive function of this energy difference because of the exponential dependence. Consequently, it is important to know $E_x(7.6)$ and the masses of ^{12}C and ^4He very accurately.

The *1964 Mass Table* of Mattauch, Thiele, and Wapstra,[3] based on a mass excess of $^{12}C \equiv 0$, gives ± 0.39 keV for the uncertainty in the mass excess of ^4He. Since the range of temperature T is[1] such that $kT \approx 10$ keV, the least well-known term in the expression for the reaction rate is the excitation energy of the second excited state of ^{12}C. The value of the energy difference

$$M_C c^2 + E_x(7.6) - 3M_\alpha c^2$$

used by Fowler, Caughlan, and Zimmerman[1] for their compilation of reaction rates was 370 ± 4 keV. This value, which corresponded to an excitation energy in ^{12}C of 7644 ± 4 keV, was based on the measurements of Cook *et al.*[4]

These authors measured the Q values of the decay $^{12}C^*(7.6) \rightarrow {}^8Be + \alpha$ following the β^- decay of ^{12}B, and of the $^8Be \rightarrow 2\alpha$ decay. Using the more precise value of $Q(^8Be \rightarrow 2\alpha)$ measured by Benn *et al.*[5] and the *1964 Mass Table*,[3] one deduces an excitation energy of 7644 keV, which is the value used by Fowler, Caughlan, and Zimmerman.

The same state was measured by Browne, Dorenbusch, and Erskine[6] using the reaction $^{10}B(^3He,p)^{12}C^*(7.6)$. When the result is corrected for the most precise ground-state Q value, an excitation energy of 7653.5 ± 6 keV is found.

Measurements of this level reported by several other investigators, though less precise than that of Browne, Dorenbusch, and Erskine, gave values higher than that of Cook *et al.* In fact, the weighted average of all measurements up to 1968 given by Ajzenberg-Selove and Lauritsen in their tabulation[7] is 7653 ± 3 keV.

In view of the importance of the $^{12}C(7.6$ MeV) state in stellar evolution, a precise remeasurement was in order. Several laboratories have recently undertaken careful measurements in order to resolve the question of the excitation energy. Austin, Trentelman, and Kashy[8] used the reaction $^{12}C(p,p')^{12}C^*$, momentum-analyzing the outgoing protons, whereas McCaslin, Mann, and Kavanagh[9] used the reaction $^{15}N(p, \alpha)^{12}C^*$, detecting the α_0^{++} and α_2^+ groups at the same magnetic rigidity.

The present work used inelastic scattering of proton and helium-3 beams with the outgoing particles being analyzed by a broad-range magnetic spectrograph. The technique is an extension of one used earlier by Stocker, Rollefson, Hrejsa, and Browne[10] in a comparison of the nuclear-reaction energy scale with the γ-ray scale. Incidental to the measurement of the energy of the second excited state, the energy of the first excited state was found with about the same over-all uncertainty, namely, ≈ 2 keV.

PROCEDURE

Protons of 14.5 and 15.0 MeV and $^3He^{++}$ of 19.0, 19.5, and 20.0 MeV were scattered from carbon foils and ^{197}Au evaporated on carbon foil. Scattering angles were 60 and 80° for the proton runs and 60, 60, and 65°, respectively, for the ^3He runs. Targets were mounted in the reflection position and the energies of the scattered particles were measured with the spectrograph. The spectrograph magnetic field was always taken through a standard cycle[11] before being set and was then allowed to stabilize for an hour or more before a run was started. Before data were taken the accelerated beam was tuned to balance simultaneously the current readings on sets of defining slits located in the target chamber and about 2 m ahead of the chamber. The latter point is ahead of the last quadrupole lens, and the beam here is about 1.5 cm wide. Because of the large width, the trace on a beam-profile monitor was centered, in addition to balancing the slit currents.

The half angle of the beam convergence at the target is about 0.5°. After tuning, no further adjustments in steering or focusing of the beam were made. While targets were inserted, the beam was interrupted by a shutter mounted just beyond the exit slits of the beam analyzer.

The particles elastically scattered from ^{197}Au were recorded, and then with the ^{12}C target in place and with the same spectrograph field, an exposure was made to record the scattered particles from the ground, first, and second excited states of ^{12}C. [Hereafter a run in which the elastically scattered group is present along with the excited state(s) is referred to as a *direct* measurement.]

A second run on ^{197}Au was then made to verify the constancy of the input energy. A new spectrograph field was then set to place only the particles scattered from the first and second excited states on the plate.

DATA REDUCTION

The particle groups were generally counted in $\frac{1}{4}$-mm intervals under a 20× objective. Data runs were usually long enough so that scanning of one or at most a few fields of view ($\frac{1}{2}$ mm × $\frac{1}{2}$ mm) at the geometrical center of the exposed zone gave sufficiently good statistical accuracy. When this was not possible, (as was usual in the case of the second excited state), a correction was made for the kinematic broadening of the group, because at tandem energies these kinematic effects are significant and must be corrected for in precise measurements.

To correct for the range in scattering angle when more than the center strip of the exposed zone was scanned, the scanning was done in 1.5-mm-wide strips at $\frac{1}{4}$-mm intervals. A shift in the distance scale was computed for each strip to compensate for the change in outgoing momentum with the change in angle. Counts were added from as many of these shifted strips as needed to give statistical accuracy.

Following standard procedure we converted the $\frac{1}{3}$-height point of the high-energy edge of the particle group to a trajectory radius, using the calibration obtained with ^{210}Po α particles. The magnetic rigidity, given by the product of this radius and the NMR-field-meter reading, was converted to particle energy using the fundamental constants of Cohen and DuMond.[12]

Because of the large convergence angle of the tandem beam at the target and the consequent possibility of variation in scattering angle, both the scattering angle and input energy were determined from the energy of the elastically scattered groups. The input energy and scattering angle required to give the observed output energies from the light target (^{12}C) and the heavy target (^{197}Au) were calculated for each run.

Using this measured energy and angle and the measured energies of the inelastic groups, we then calculated the excitation energies of the first and second excited states of ^{12}C. In these calculations the mass of the excited recoiling nucleus was used in the Q-value equation.

For runs in which the spectrograph field was changed so that only the first and second excited state groups were recorded (but no change was made in the beam energy), beam energy and scattering angle from the "direct-measurement" runs were used.

RESULTS

Eight measurements were made in which the ground state, first excited state (4.4 MeV), and second excited state (7.6 MeV) were simultaneously recorded. Four of these measurements were made with the reaction $^{12}C(p,p')^{12}C^*$ and four with the reaction $^{12}C(^3He, ^3He')^{12}C^*$. Average values of

TABLE I. Averages of excitation energy of the second excited state of ^{12}C determined from direct measurement of the energy difference from the ground state.

Number of runs	Beam particle	Average excitation energy (keV)	Standard deviation of mean (keV)
4	p	7655.9	0.6
4	^3He	7655.9	1.4
	Grand average	7655.9	0.7

TABLE II. Average excitation energy of the first excited state of ^{12}C.

Number of runs	Beam particle	Average excitation energy (keV)	Standard deviation of mean (keV)
6	p	4441.7	0.6
6	^3He	4442.7	0.3
	Grand average	4442.2	0.3

the results together with standard deviations of the mean are given in Table I. Six additional measurements giving only the separation energy of the two excited states were made. Finally, four measurements were made in which only the ground state and the 4.4-MeV state were observed.

The result from the 12 runs in which the g.s.-to-4.4-MeV separation was measured is 4442.2 keV with a standard deviation of the mean of 0.3 keV. Six of these runs were made with protons and six with $^3He^{++}$. These results are displayed in Table II.

An average for the separation of the 4.4- and 7.6-MeV states was calculated from all 14 runs in which these two groups were observed. The result is 3214.7 keV with a standard deviation of 0.7 keV. If one combines the value from the augmented set of 12 runs for the g.s.-4.4-MeV separation with that from the 14 runs for the 4.4-7.6-MeV separation, one has 7656.9 keV, which is in good agreement with the value of 7655.9 keV from the smaller set of direct runs in which all three states of interest were observed at the same time.

DISCUSSION

7.6-MeV State

The excitation energy found by direct observation of the separation of the 7.6-MeV state from the ground state is 1 keV lower than that found by adding the separation of the 7.6- and 4.4-MeV states to the measured excitation energy of the 4.4-MeV state. This difference is well within the uncertainties discussed below. One strongly favors the "direct" measurement as giving the best value because the result is unaffected by any small drifts in beam energy or magnetic field.

Thus the excitation energy given by this measurement is 7655.9 ± 2.5 keV where the uncertainty is the internal error given below. Table III shows earlier values, recent values, and the present results. Our value, that of Austin, Trentelman, and Kashy, and that of McCaslin, Mann, and Kavanagh agree within a fraction of a keV. These three recent measurements show that the excitation energy of the second excited state of ^{12}C is 7656 keV, within <1.5 keV.

Using the mass excesses from the *1964 Mass Table* and 7656 keV for the energy of the second excited state of ^{12}C, one finds the energy difference

$$M_C c^2 + E_x(7.6) - 3M_\alpha c^2$$

to be 382 keV rather than the 370 keV used by Fowler, Caughlan, and Zimmerman. The change of 12 keV from the earlier value reduces the rate of the 3α process by a factor of ≈ 3.3 at $kT \approx 10$ keV.

4.4-MeV State

The present result for the first excited state in ^{12}C given in Table II is 4442.2 keV, with a total uncertainty of ±1.5 keV. The most precise previous measurements of this state were made by Chasman *et al.*,[13] who reported 4439.79 ± 0.31 keV and Kolata, Auble, and Galonsky,[14] who gave 4440.0 ± 0.5 keV. These results are shown in Table IV. Both of the above γ-ray measurements were made with Ge(Li) detectors, and are seen to be in excellent agreement with one another.

The calibration for both measurements is based largely on the ^{56}Co γ rays of 3254 and 3452 keV. Chasman and Ristinen[15] made an independent measurement of these ^{56}Co γ rays, and their values are listed in the second column of Table IV. On the other hand, Kolata, Auble, and Galonsky used the values listed in the third column of Table IV for the measurement[14] of the 4.4-MeV state. Each calibration value differs by about 1 keV, yet the measurements of the 4.4-MeV state are reported to agree within 0.2 keV.

Since the double-escape peak for the 4440-keV

TABLE III. Summary of recent measurements of the excitation energy of the second state in ^{12}C. All values are keV.

Cook *et al.* (Ref. 4) (1957)	Browne, Dorenbusch, and Erskine (Ref. 6) (1962)	Energy levels of $A = 11-12$ nuclei (Ref. 7) (1968)	Austin, Trentelman, and Kashy (Ref. 8) (1970)	McCaslin, Mann, and Kavanagh (Ref. 9) (1970)	Present (1971)
7644 ± 4	7653.5 ± 6	7653 ± 3	7656.2 ± 2.1	7656 ± 3	7655.9 ± 2.5 (Direct)

TABLE IV. Energy values in previous γ-ray measurements for the 4.44-MeV state in ^{12}C. All values are keV.

Energy	Chasman, Jones, and Ristinen (Ref. 13)	Kolata, Auble, and Galonsky (Ref. 14)
^{56}Co calibration line	3253.7	3254.5
^{56}Co calibration line	3452.1	3451.1
$E_\gamma - 2m_0c^2$ for $^{12}C(4.44 \rightarrow 0)$	3416.9	3417.1
E_γ	4438.9 ± 0.3	4439.1 ± 0.5
Excitation energy in ^{12}C	4439.8 ± 0.3	4440.0 ± 0.5

γ ray lies close to the 3452-keV calibration line from ^{56}Co, the measured energy of the 4440 keV γ ray will be most sensitive to this particular line.

It would seem that a clarification of the γ-ray measurements is in order, including perhaps a remeasurement of the 4.4-MeV γ ray. Our own charged-particle measurements are higher than the most precise γ-ray measurements, with the difference being just outside the combined stated uncertainties.

Uncertainties

The estimates of uncertainties in the present work rely chiefly on the extensive investigations of O'Donnell,[11] O'Donnell and Browne,[16] and Stocker, Rollefson, Hrejsa, and Browne.[10] Even though the matching technique used in the beam-energy and scattering-angle determinations tends to reduce some of the usual uncertainties, we have been rather generous in their assignments.

The change in excitation energy, ΔE_x, associated with each uncertainty was calculated for each excitation energy, and the results are summarized in Table V. Two uncertainties were assigned to the trajectory radius of the scattered particles. The first corresponded to an over-all shift in the calibration curve of 0.1 mm, whereas the second was a possible random shift of ±0.04 mm for a particular trajectory radius. Careful comparisons of the magnetic field determined from a ^{210}Po α source and that given by the NMR fluxmeter indicate that the difference in field was generally of the order of 1.5 G, or less, and so we have assigned an uncertainty of 1.5 G to an individual run. Previous experience indicates an uncertainty in beam-spot position of 0.25 mm. A calibration uncertainty of 0.5 keV was assigned for the absolute energy uncertainty of the ^{210}Po α-particle standard. Despite the matching technique used to determine the reaction angle and input energy, an uncertainty of 5′ for the reaction angle and 0.1% for the input energy were assigned.

Combining the uncertainties listed in Table V in quadrature gives an uncertainty for each run of 5.1 and 7.2 keV, for the 4.4- and 7.6-MeV states, respectively. The internal error, defined as

$$e_{\text{int}} = \left[\sum_{i=1}^{N}\left(\frac{1}{\Delta E_i}\right)^2\right]^{-1/2}$$

TABLE V. Uncertainties.

Quantity	Uncertainty	4.4-MeV state ΔE_x (keV)	7.6-MeV state ΔE_x (keV)
Trajectory radius (random)	0.040 mm	3.39	3.12
Trajectory radius (whole-curve shift)	0.1 mm	1.24	2.39
Magnetic field	1.5 G	1.6	2.7
Object position	0.25 mm	2.38	4.41
Calibration	0.5 keV	0.12	0.23
Reaction angle	5′	2.2	3.0
Input energy	0.1%	0.02	0.15
Uncertainty in one run		5.11	7.16
Internal error		1.48	2.53

for the 12 runs on the g.s.-4.4-MeV separation is then 1.5 keV and for the 8 runs on the g.s.-7.6-MeV separation is 2.5 keV.

In conclusion, the result of the present measurement of the excitation energy of the second excited state of ^{12}C is in excellent agreement with two other recent measurements where other techniques have been employed. The excitation energy so determined reduces the theoretical reaction rate of stellar helium burning by a factor of ≈ 3. The energy of the first excited state of ^{12}C as determined by the present charged-particle technique differs from the best γ-ray measurements by slightly more than the combined uncertainties.

†Work supported by the National Science Foundation under Grant No. Gp-15560.

[1]W. A. Fowler, G. R. Caughlan, and B. A. Zimmerman, Ann. Rev. Astronomy Astrophys. 5, 525 (1967).
[2]E. M. Burbidge, G. R. Burbidge, W. A. Fowler, and F. Hoyle, Rev. Mod. Phys. 29, 547 (1957).
[3]J. H. E. Mattauch, W. Thiele, and A. H. Wapstra, Nucl. Phys. 67, 1 (1965).
[4]C. W. Cook, W. A. Fowler, C. C. Lauritsen, and T. Lauritsen, Phys. Rev. 107, 508 (1957).
[5]J. Benn, E. B. Dally, H. H. Muller, R. E. Pixley, H. H. Staub, and H. Winkler, Phys. Letters 20, 43 (1966).
[6]C. P. Browne, W. E. Dorenbusch, and J. R. Erskine, Phys. Rev. 125, 992 (1962).
[7]F. Ajzenberg-Selove and T. Lauritsen, Nucl. Phys. A114, 1 (1968).
[8]S. M. Austin, G. F. Trentelman, and E. Kashy, Bull. Am. Phys. Soc. 15, 1662 (1970).
[9]S. J. McCaslin, F. M. Mann, and R. W. Kavanagh, Bull. Am. Phys. Soc. 15, 1598 (1970).
[10]H. Stocker, A. A. Rollefson, A. F. Hrejsa, and C. P. Browne, Phys. Rev. C 4, 930 (1971).
[11]F. H. O'Donnell, Ph.D. dissertation, University of Notre Dame, 1966 (unpublished).
[12]E. R. Cohen and J. W. M. DuMond, Rev. Mod. Phys. 37, 537 (1965).
[13]C. Chasman, K. W. Jones, R. A. Ristinen, and D. E. Alburger, Phys. Rev. 159, 830 (1967).
[14]J. J. Kolata, R. Auble, and A. Galonsky, Phys. Rev. 162, 957 (1967).
[15]C. Chasman and R. A. Ristinen, Phys. Rev. 159, 915 (1967).
[16]F. H. O'Donnell and C. P. Browne, Phys. Rev. 158, 957 (1967).

PHYSICAL REVIEW C VOLUME 4, NUMBER 4 OCTOBER 1971

Real and Virtual Radiation in Electron-Nucleus Scattering*

W. W. Gargaro† and D. S. Onley
Ohio University, Athens, Ohio 45701
(Received 25 March 1971)

Expressions are developed for the virtual photon spectrum and for the bremsstrahlung cross section, using the distorted-wave method for relativistic electrons in a Coulomb field. Evaluation of the matrix elements is made in terms of generalized hypergeometric functions, and some numerical results are given for electrons of energy 10 and 100 MeV. Significant enhancement of the virtual radiation spectrum is found to result from the use of distorted waves as opposed to plane waves. Calculations of the contributions of $E1$, $M1$, $E2$, and $E3$ radiation to the bremsstrahlung cross section, $d^2\sigma/d\omega d\Omega$, are also made; enhancement, similar to that observed with the virtual spectrum, is found in this case also.

I. INTRODUCTION

Inelastic scattering of high-energy electrons has been the subject of extensive analysis in the plane-wave Born approximation (for a comprehensive review see Ref. 1). Although this treatment enjoys a most attractive feature, namely, the mathematical simplicity of its results, it is known that plane-wave analysis introduces considerable error when used for scattering from nuclei with large atomic numbers, at least in the calculation of angular distributions.[2] For such nuclei a distorted-wave treatment, using Dirac-Coulomb wave functions for the basis states of the electron, is more effective. Such a calculation can be a laborious task, the chief difficulty being the evaluation of the radial part of the electron matrix element, which requires extensive numerical integration. In some circumstances, particularly for high-angular-momentum components, the electron wave function has no appreciable amplitude in the region near the origin, which is occupied by the nucleus. It is then a reasonable approximation to suppose that the electron is moving in the field of a point charge, which opens up the possibility of expressing the matrix elements in some analytic