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Even-Parity States of the Five-Nucleon System and Deuteron-Triton Photocapture*

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The non-normal-parity states for the five-nucleon system are obtained in the shell-model with particle-hole interactions. The residual force used is the separable smooth potential due to Tabakin. Coupling of the shell-model wave functions to deuteron and single-particle channels is calculated semiquantitatively, and the results are used in an R-matrix calculation of the cross section for photocapture of deuterons by tritons. A broad peak in the cross section at about 21-MeV excited energy due to capture through a $\frac{5}{2}^+$ state is predicted.

I. INTRODUCTION

At present, there are four experimentally known levels of the five-nucleon system.[1] The ground state of ^{5}He has been identified as a $\frac{3}{2}^-$ state and is unbound against the breakup into ^{4}He$+n$ by 0.96 MeV. The next state of this nucleus is a broad resonance which occurs 2.6 MeV higher and has spin $\frac{1}{2}^-$. The first even-parity state for this system lies 16.7 MeV above the ground state[2] and is known to have spin $\frac{3}{2}^+$. Further positive-parity structure has been observed around 20 MeV.[3] Since the state at 20 MeV is excited by ^{7}Li(p, ^{3}He)-^{5}He but not ^{7}Li(p, t)^{5}Li, Cerny, Detraz, and Pehl[4] argue the state is ^4D_J, $J^\pi = \frac{3}{2}^+$ or $\frac{5}{2}^+$. Several more levels have been tentatively detected in the region between 20–25 MeV.[5]

Simple shell-model considerations indicate there should be a sizable number of even-parity states beginning near the $t+d$ threshold which should dominate reactions between tritons and deuterons. In this paper we calculate the shell-model spectrum and $E1$ transition strength to the $\frac{3}{2}^-$ state for these even-parity states. We apply the results to a study of the reaction

$$d+t \rightarrow {}^5\text{He(g.s.)}+\gamma, \qquad (1)$$

where the final nuclear state is considered as the ground state of ^{5}He. The particle widths are estimated by calculating what are essentially the one- and two-body fractional-parentage coefficients and normalizing the widths with the known deuteron width of the $\frac{3}{2}^+$ state. The reaction theory we employ is the R-matrix theory of Lane and Thomas.[6] Crone and Werntz[7] have explicitly considered the case of many overlapping levels, such as we have here and given a formula for the angular correlation of the final γ ray with the incoming particle.

One might question the usefulness of a shell-model calculation with oscillator wave functions when applied to a system with few narrow resonances. From the experimental side, $d+t$ scattering might be viewed as a problem similar to pion-nucleon scattering – one would like to know the energy dependence of the real and imaginary parts of the phase shifts. In certain channels it might be expected that the energy dependence implies the existence of resonances, even though the internal symmetry of the state is such that no narrow resonances exist. The energy ordering of these resonances depends on the nuclear forces between pairs, and the coupling of the states to external channels depends on the internal symmetries, here $SU(4)$, of the states. The shell model with oscillator functions gives this kind of information just as the quark model does for baryon resonances.

It is true that the coupled-channel resonating-group method[8] (continuum cluster model) represents a unified approach to the problem of reactions between pairs of fragments. However, the numerical solution of the coupled equations is not trivial; while the general equations have been known for over nine years,[9] solutions for other than purely central forces have appeared only recently. Heiss and Hackenbroich[10] have solved the problem for the collision-matrix parameters of the $\frac{3}{2}^+$ partial wave taking into account the coupling between the $^2D_{3/2}$ ^{4}He$+n$ channel and the $^4S_{3/2}$ $t+d$ channel. Even though they obtained results in semiquantitive agreement with the known properties of the $\frac{3}{2}^+$ state, it is still true that the $^2D_{3/2}$ and $^4D_{3/2}$ $t+d$ channels were omitted. We will show that a shell-model calculation also yields information about the $\frac{3}{2}^+$ state that agrees with experiment, and we get estimates of the coupling to the $^4D_{3/2}$ and $^2D_{3/2}$ channels as well.

II. SHELL-MODEL STATES

We first solve for the five-nucleon spectrum by means of the shell model with particle-hole excitations. The model space for this calculation will include configurations which differ from the ^{5}He shell-model ground state $(s_{1/2})^4(p_{3/2})$ by the excitation of one particle through a single oscillator spacing of $1\hbar\omega$. Two types of such states exist: the states in which the valence particle is promoted through one oscillator shell, the spp (single-particle promoted) state, and states in which a particle from the ground-state core is promoted through an oscillator shell, the 2p−1h state.

We use the occupation-number (Fock space) representation to facilitate satisfying the requirements of the Pauli principle, and will adopt the j-j coupling scheme. In this representation our basis states become

$$|2\text{p-}1\text{h}\rangle_{\text{A.S.}} = \sum_{\substack{m_{\text{p}},m_{\text{v}}\\ m,m_{\text{h}}}} (j_{\text{p}}m_{\text{p}}j_{\text{v}}m_{\text{v}}|jm)(jmj_{\text{h}}-m_{\text{h}}|JM)\sum_{\substack{\tau_{\text{p}}\tau_{\text{v}}\\ \tau,\tau_{\text{h}}}}(\tfrac{1}{2}\tau_{\text{p}}\tfrac{1}{2}\tau_{\text{v}}|t\tau)(t\tau\tfrac{1}{2}-\tau_{\text{h}}|TM_\tau)$$
$$\times(-1)^{j_{\text{h}}-m_{\text{h}}+1/2-\tau_{\text{h}}}a^\dagger(nljm\tau)_{\text{p}}a^\dagger(nljm\tau)_{\text{v}}a(nljm\tau)_{\text{h}}|0\rangle\,,$$

$$N^2 = 1-(-1)^{j+t}\delta_{n_{\text{p}}n_{\text{v}}}\delta_{l_{\text{p}}l_{\text{v}}}\delta_{j_{\text{p}}j_{\text{v}}}\,,$$

and

$$|\text{spp}\rangle_{\text{A.S.}} = a^\dagger(nljm\tau)|0\rangle\,. \tag{2}$$

The core is the filled $1s_{1/2}$ shell. Here, and in the following, p refers to particle, v refers to valence particle, and h refers to hole. For a particular J, T, the shell-model space is spanned by all configurations in which p and v can be coupled to an intermediate j, t, and the result of this coupled to final J, T, with nonzero norm. For states with $T=\frac{1}{2}$ there is the additional spp state (we will solve for the spectrum for both $T=\frac{1}{2}$ and $T=\frac{3}{2}$).

We make the usual Tamm-Dancoff approximation and solve the secular equation

$$\sum_k(\langle\phi_{k'}|H|\phi_k\rangle - E_\lambda\delta_{k'k})A_{\lambda k}=0\,, \tag{3}$$

for the eigenvalues E_λ and for the states $\sum_k A_{\lambda k}|\phi_k\rangle$. The matrix elements of the Hamiltonian are taken to be

$$\langle\phi_{k'}|H|\phi_k\rangle = \Delta\epsilon_k\delta_{k'k}+\langle\phi_{k'}|V|\phi_k\rangle\,, \tag{4}$$

where the $\Delta\epsilon_k$ are the differences in single-particle energies, and the Tabakin potential[11] is used for $\langle\phi_{k'}|V|\phi_k\rangle$. The single-particle energies[1,8] are given below:

$1d_{3/2}$, 26.25 MeV;

$2s_{1/2}$, 23.25 MeV;

$1d_{5/2}$, 21.25 MeV;

$1p_{1/2}$, 2.60 MeV;

$1p_{3/2}$, 0.0 MeV;

$1s_{1/2}$, −21.54 MeV.

We approximate the reaction matrix to second order in the manner in which Barrett[12] treated ^{4}He and ^{16}O; i.e., oscillator states two shells higher in the relative system are taken as the intermediate states. The angular momentum coupling and recoupling and the two-body matrix elements which enter the secular equation have recently appeared in the literature.[13]

A final complication is the removal of the spurious center-of-mass motion which has been introduced into our representation by the inclusion of $3A$ ($A=5$) spatial coordinates rather than $3A-3$, and our use of experimental particle-hole energies. We have generated these states by the prescription of Baranger and Lee[14] and have projected them out by the Schmidt technique. The result of this treatment and its relevance to the remainder of the present paper is that the eigenfunctions X_λ have the center-of-mass of the five-nucleon system in a relative 1s state with respect to the center of the oscillator well.

III. DIPOLE MATRIX ELEMENTS

In the region of excitation that we are primarily interested in, the long-wavelength limit[15] for the transverse $E1$ transition amplitude can be applied

$$\langle f|T_{1\lambda}^{\mathrm{el}}|i\rangle_{k\to 0} = \omega(1/6\pi)^{1/2} D_{1\lambda}^{\mathrm{el}}\,, \tag{5}$$

where we define

$$D_{1\lambda}^{\mathrm{el}} = i\langle f|\sum_{j=1}^{A} \hat{\epsilon}_\lambda\cdot\vec{\mathrm{r}}_j\,[\tfrac{1}{2}(1+\tau_0)]_j\,|i\rangle\,.$$

Since we are in an oscillator basis, it will be a simplification to write the operator $\vec{\mathrm{r}}_j$ in terms of the oscillator raising and lowering operators

$$\vec{\mathrm{c}}_j^{\,\dagger} = \frac{\vec{\mathrm{p}}_j + iM\omega_{\mathrm{osc}}\vec{\mathrm{r}}_j}{(2M\omega_{\mathrm{osc}})^{1/2}}\,,\qquad \vec{\mathrm{c}}_j = \frac{\vec{\mathrm{p}}_j - iM\omega_{\mathrm{osc}}\vec{\mathrm{r}}_j}{(2M\omega_{\mathrm{osc}})^{1/2}}\,;$$

$$\vec{\mathrm{r}}_j = \frac{\vec{\mathrm{c}}_j^{\,\dagger} - \vec{\mathrm{c}}_j}{i(2M\omega_{\mathrm{osc}})^{1/2}}\,. \tag{6}$$

This simplifies the calculation of the transition amplitude, since the reduced matrix elements of the oscillator raising and lowering operators have been tabulated by Gartenhaus and Schwartz.[16]

IV. PHOTOCAPTURE CROSS SECTION

For application to the photocapture of deuterons by tritons we require an expression for the cross section for the emission of a γ ray with wave vector $\vec{\mathrm{k}}_\gamma$ and energy ω_γ from an excited nuclear state of the compound system. The general expression, where $J(\vec{\mathrm{k}}_\gamma)$ is the Fourier transform of the nuclear current and dN_f is the density of final states, is

$$d\sigma = 2\pi\frac{4\pi\alpha}{2\omega_\gamma}\sum_{M_i}\sum_{M_f}[J_+(\vec{\mathrm{k}}_\gamma)J_+^*(\vec{\mathrm{k}}_\gamma) + J_-(\vec{\mathrm{k}}_\gamma)J_-^*(\vec{\mathrm{k}}_\gamma)]\,dN_f\,. \tag{7}$$

In the dipole approximation we may identify the current with the matrix element of Eq. (5)

$$J_\lambda(\vec{\mathrm{k}}_\gamma) = \omega D_{1\lambda}^{\mathrm{el}}\,. \tag{8}$$

An expression for $J_\lambda(\vec{\mathrm{k}}_\gamma)$ for the case of photon absorption has been obtained by Crone and Werntz.[6] The approach used is that of Wigner R-matrix theory[5] in which the scattering state in the region of nuclear interaction is expanded in terms of a complete set of eigenfunctions of the nuclear Hamiltonian with certain boundary conditions. We will make the ansatz that the complete set of eigenstates is approximated by the set of shell-model solutions of Sec. II. The eigenfunction for a particular J_i are denoted X_λ, with corresponding eigen energy E_λ. Then if we require that our initial state be a Coulomb wave in the $\vec{\mathrm{k}}_\alpha$ direction with unit incoming flux, we obtain, after taking the squared modulus and performing the spin sums, an expression for the cross section of Eq. (7):

$$\frac{d\sigma}{d\Omega_\gamma} = \frac{\pi\alpha\omega_\gamma\lambda\!\!\!^{-}{}_\alpha^{\,2}}{(2s_1+1)(2s_2+1)}\sum_{\substack{J_i J_i'\\ l,l'}}\sum_{\substack{JJ'\\ sj}}(-i)^{J-J'}(-1)^{3J_f-J_i-J_i'-s+j}\,\overline{Z}(lJ_il'J_i';sj)Z(JJ_iJ'J_i;J_fj)P_j(\cos\theta_\gamma)$$
$$\times 2[e^{i\Omega_{\alpha l}}(\Gamma^{J_i}_{\lambda',\alpha s l})^{1/2}A^{J_i}_{\lambda\lambda'}\langle J_f\|T_J\|J_i\lambda'\rangle][e^{i\Omega_{\alpha l'}}(\Gamma^{J_i'}_{\beta,\alpha s l'})^{1/2}A^{J_i'}_{\beta\beta'}\langle J_f\|T_{J'}\|J_i',\beta'\rangle]^*\,. \tag{9}$$

It is understood that $2\langle\|T_J\|\rangle\langle\|T_{J'}\|\rangle^*$ stands for

$$[1+(-1)^{J+J'-j}][\langle\|T_J^{\mathrm{el}}\|\rangle\langle\|T_{J'}^{\mathrm{el}}\|\rangle^* + \langle\|T_J^{\mathrm{mag}}\|\rangle\langle\|T_{J'}^{\mathrm{mag}}\|\rangle^*] - [1-(-1)^{J+J'-j}][\langle\|T_J^{\mathrm{el}}\|\rangle\langle\|T_{J'}^{\mathrm{mag}}\|\rangle^* + \langle\|T_J^{\mathrm{mag}}\|\rangle\langle\|T_{J'}^{\mathrm{el}}\|\rangle^*]\,. \tag{10}$$

The particle-γ correlation is, of course, the same for photocapture as for photoabsorption. We have written the formula explicitly for photocapture by making use of the relation[15]

$$i^J\langle J_f\|T_J\|J_i\rangle = i^{-J}(-1)^{J_i-J_f+1}\langle J_i\|T_J\|J_f\rangle\,.$$

The particle and photon angular correlation coefficients, $\overline{Z}$ and Z, are defined by Ferguson.[17] If we consider only transitions to the $\frac{3}{2}^-$ ground state of ^{5}He and restrict the formula to $E1$ photons, the possible initial states are $J^\pi{}_i = \frac{1}{2}^+, \frac{3}{2}^+, \frac{5}{2}^+$. For the $t+d$ channel the channel spin $s = \frac{1}{2}, \frac{3}{2}$, and for the type of states we study the orbital angular momentum in the $t+d$ channel is limited to $l = 0, 2$. Then parity and angular momentum rules limit j to $j = 0, 2$ giving

$$d\sigma/d\Omega_\gamma = A_0P_0 + A_2P_2$$

for the correlation between the γ ray and the incoming deuterons. The definition of the phase angles $\Omega_{\alpha l}$, the physical widths $\Gamma^{J}_{\lambda,\alpha s l}$, and the inverse level matrix $A^{J}_{\lambda\lambda'}$, follow all the conventions of Refs. 6 and 7.

V. COUPLING TO PHYSICAL CHANNELS

When we talked about internal symmetry earlier we had in mind the $SU(4)$ symmetry of the spin and isospin functions. In an earlier paper[18] on ^{4}He the importance of the $SU(4)$ multiplicities of the internal wave functions and the channel wave functions was stressed. If a product representation contained in the asymptotic wave is a major component of an excited-state wave function, then the width for decay into these two fragments should be large. One sees this directly in the expression from R-matrix theory for the reduced width.

$$\gamma_{\lambda c}=\left(\frac{1}{2M_cA_c}\right)^{1/2}\int dS_c\,\phi_c^* X_\lambda\,, \tag{11}$$

where X_λ is the internal resonance wave function and ϕ_c is the asymptotic channel wave function. The integral is over all coordinates except the relative separation of the two fragments and, more importantly, all spins and isospins are summed over. If ϕ_c does not contain any $SU(4)$ representation in X_λ, then $\gamma_{\lambda c}$ vanishes.

The shell-model states we have used contain representations of dimensionality $\{4\}$, $\{20\}$, and $\{36\}$. The spp states contain only $\{4\}$ but all three are contained in the 2p–1h states. Figure 1 illustrates the group properties of the states. The coupling of these states to the physical channels can be discussed from the view point of group theory because of the fact that one can also make up representations of $SU(4)$ from the asymptotic form of the wave functions for these channels. For the case of the deuteron or singlet deuteron-plus-triton channels, in a notation in which the parentheses contain the multiplicities $(2S+1,\,2T+1)$, we have for the spin, isospin part of the wave functions,

$$[(3,1)+(1,3)]\times[(2,2)]=(4,2)+(2,2)+(2,4)+(2,2)\,.$$

That is, this case yields the $\{20\}$ and $\{4\}$ representations. For the case of the physical channel consisting of ^{4}He$+n$ we have for the spin, isospin part of the wave function

$$[(1,1)\times(2,2)]=(2,2)\,.$$

This case yields only the $\{4\}$ representation. At this point it is explicit how a particular shell-model state can be coupled to both single-particle and deuteron channels. Furthermore, we can make some qualitative remarks about the coupling even at this point. For example, if the $\{4\}$ component in the shell function is small, then that state will be only weakly coupled to the ^{4}He$+n$ channel. Also, the $\{36\}$ component can couple only to a channel in which two nucleons are emitted in a relative p state. Since the p-state interaction is small, this channel is suppressed. Then if we have a low-lying shell-model state with a large $\{36\}$ component, it can have little coupling to the remaining physical channels, and so can have only a relatively narrow total width. It also follows that the small reduced width for $^2D_{3/2}$ ^{4}He$+n$ of the $\frac{3}{2}^+$ state implies a small $\{4\}$ component in the wave function. A recent calculation of the five-body spectrum using an $SU(4)$ basis has yielded just this result.[19]

It remains to write down the wave functions which occur in Eq. (11) so that the indicated contraction can be performed to obtain a quantitative measure of the coupling. Neglecting all but the $(1s)^2$, $(1s)^3$, and $(1s)^4$ configurations in ^{2}H, ^{3}He, and ^{4}He, the surface wave function is decomposed as follows:

$$\phi_c=(i^l/a_c)Y_{lm}(\Omega_c)\Psi_{\text{cluster}}\Phi\,, \tag{12}$$

where Ψ_{cluster} contains the internal spatial structure of the fragments, a_c is the channel radius, and Φ is the spin and isospin function formed from the fragments. The angular momentum couplings are in the order $(\vec{s}_1\times\vec{s}_2)\times\vec{l}$, where s_1 is the spin of the target particle. Explicitly, the following spin-isospin functions occur in our available channels, viz., five nucleons clustered into two nucleons plus three nucleons, or into four nucleons plus one nucleon:

$$\Phi(^3\mathrm{H}+d)=\sum_{m'}(\tfrac{1}{2}m'1m-m'|Sm)\frac{1}{\sqrt{2}}[\chi_{1m-m'}(45)_S\,\eta_{00}(45)_A][\chi_{\frac{1}{2}m'}((12)_A,3)\eta_{\frac{1}{2}-\frac{1}{2}}((12)_S,3)-\chi_{\frac{1}{2}m'}((12)_S,3)\eta_{\frac{1}{2}-\frac{1}{2}}((12)_A,3)],$$

$$\Phi(^4\mathrm{He}+n)=\frac{1}{\sqrt{2}}[\chi_0((12)_A,(34)_A)\eta_0((12)_S,(34)_S)-\chi_0((12)_S,(34)_S)\eta_0((12)_A,(34)_A)]\,\chi_{\frac{1}{2}\frac{1}{2}}(5)\eta_{\frac{1}{2}-\frac{1}{2}}(5)\,,$$

$$\Phi(t+{}^1(np))=\frac{1}{\sqrt{2}}[\chi_{\frac{1}{2}m}((12)_A,3)\eta_{\frac{1}{2}-\frac{1}{2}}((12)_S,3)-\chi_{\frac{1}{2}m}((12)_S,3)\eta_{\frac{1}{2}-\frac{1}{2}}((12)_A,3)][\chi_{00}(45)_A\eta_{10}(45)_S]\,,$$

$$\Phi(^3\mathrm{He}+{}^1(nn))=\frac{1}{\sqrt{2}}[\chi_{\frac{1}{2}m}((12)_A,3)\eta_{\frac{1}{2}\frac{1}{2}}((12)_S,3)-\chi_{\frac{1}{2}m}((12)_S,3)\eta_{\frac{1}{2}\frac{1}{2}}((12)_A,3)][\chi_{00}(45)_A\eta_{1-1}(45)_S]\,, \tag{13}$$

where $^1(np)_{01}$ and $^1(nn)_{01}$ represent the singlet deuteron (two nucleons coupled to spin zero, isospin one). In these expressions $\chi_{1/2}((12)_S, 3)$ and $\chi_{1/2}((12)_A, 3)$ are mixed symmetry three-nucleon spin functions with $S=\frac{1}{2}$ and are either symmetric or antisymmetric in nucleons 1 and 2. The two-nucleon spinors $\chi_1(45)_S$, $\chi_0(45)_A$ with $S=1$ or 0, respectively, are paired to form the four-nucleon spinors $\chi_0((12)_A, (34)_A)$, $\chi_0((12)_S, (34)_S)$ which have total spin zero. The isospin functions are similarly defined. The complete product expressions are antisymmetrized by writing down $\tilde{\Phi}=(N_P)^{-1/2}\times\sum(-)^P P\Phi$, where N_P is the number of independent permutations P not already contained implicitly in the expressions.

In order to evaluate Eq. (11) we first transform our jj basis states to an LS basis (see Appendix)

$$X_\lambda=\sum_k A_{\lambda k}\phi_k(jj)=\sum_{k,j} A_{\lambda k}\chi_j(LS)\langle\chi_j(LS)|\phi_k(jj)\rangle\,. \tag{14a}$$

Then,

$$\gamma_{\lambda c}=\sum_{k,j} A_{\lambda k}\gamma_{c,j}\langle\chi_j(LS)|\phi_k(jj)\rangle\,, \tag{14b}$$

$$\gamma_{c,j}=\left(\frac{1}{2M_cA_c}\right)^{1/2}\int dS_c\phi_c\chi_j(LS)\,.$$

The antisymmetrized channel wave function is obtained directly from the ϕ previously defined. In evaluating Eq. (14b) care must be taken with respect to the ordering of the angular momentum coupling. From Eq. (2) we see that in the jj coupling scheme our angular momenta in the two-particle–one-hole states are coupled in the order $(l_p\times s_p)\times(l_v\times s_v)\times s_h$. The LS basis is defined by $[(s_p\times s_v)\times s_h]\times[(l_p\times l_v)]$ and the transformation coefficient $\langle\chi_j(LS)|\phi_k(jj)\rangle$ is derived in the Appendix. On the other hand, the angular momenta in the external channels occur in the order $[S_h\times(s_p\times s_v)]\times(l_h\times l_v)$. By the same token, sign problems can occur even in the spp states because the usual shell-model coupling order, $l\times s$, is opposite to the R-matrix convention, $s\times l$.

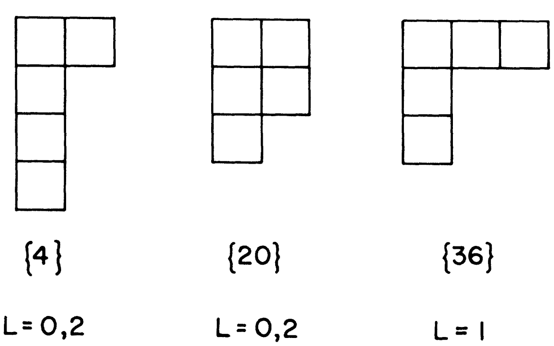

FIG. 1. Young tableaus for spin, isospin parts of shell-model wave functions for the five-nucleon problem.

For each LS basis wave function the reduced widths were evaluated. The antisymmetrized five-nucleon wave functions were explicitly written down and the space variables for each individual nucleon were replaced by relative coordinates appropriate to the external channel clusters and the center-of-mass coordinate. The spin and isospin sums were easily expressed in terms of Wigner $6j$ and $9j$ coefficients. Then the widths are all expressible in the form

$$\gamma_{\alpha s l,k}=\eta_{\alpha s l,k}N\frac{1}{a^{1/2}}\frac{a_c^3}{a^3}\frac{1}{(ma_c)^{1/2}}e^{-\mu_\alpha(a_c^2/2a^2)}\times\int d\vec{\rho}_1 d\vec{\rho}_2 d\vec{\rho}_3\Psi^*_{\text{cluster}}\Psi_{\text{Gaussian}}\,. \tag{15}$$

Here $a=1/(m\omega_{\text{osc}})^{1/2}$, m is the nucleon mass, $\mu_\alpha=M_c/m$. $N=\pi^{-1/4}(\frac{4}{5})^{5/4}$, and $\eta_{\alpha s l,k}$ is a positive or negative coefficient. The values of these coefficients are listed in the Appendix. It is noteworthy that two-particle–one-hole states can have significant single-nucleon width and that spp states have significant two-nucleon cluster widths. The function Ψ_{Gaussian} in Eq. (15) is a normalized function of the three relative coordinates that arise from the exponential part of the five-body oscillator functions.

Because of the use of the oscillator wave function we have an unrealistic Gaussian function of a_c appearing in Eq. (15). This term was ignored and the over-all normalization of all widths was fixed by the S-wave deuteron reduced width of the $\frac{3}{2}^+$ state which was taken to be $\gamma^2=2.0$ MeV, with $a_c=5.0$ fm. The integral in Eq. (15) was evaluated with Gaussian wave functions which yielded the proper rms radii for d, t, and ^{3}He and we found a ratio of 0.84 for the $t+d$ clusters with respect to the ^{4}He$+n$ in evaluating the integral. This meant the single-nucleon widths were effectively multiplied by $(0.84)^{-1}$. An oscillator energy of 22.4 MeV was used for the five-body wave function. In this

TABLE I. The 10 lowest even-parity levels of ^{5}He as calculated and used in this work.

Energy (MeV)	Spin	Isospin	Energy (MeV)	Spin	Isospin
17.7	$\frac{3}{2}$	$\frac{1}{2}$	26.0	$\frac{5}{2}$	$\frac{1}{2}$
19.6	$\frac{1}{2}$	$\frac{1}{2}$	26.2	$\frac{3}{2}$	$\frac{1}{2}$
20.8	$\frac{5}{2}$	$\frac{1}{2}$	27.1	$\frac{1}{2}$	$\frac{1}{2}$
21.9	$\frac{3}{2}$	$\frac{1}{2}$	28.6	$\frac{1}{2}$	$\frac{3}{2}$
22.8	$\frac{7}{2}$	$\frac{1}{2}$	28.8	$\frac{5}{2}$	$\frac{3}{2}$

way a complete table of values for $\gamma_{\alpha s l,k}$ was specified with only one experimental quantity being introduced.

The threshold or Q values for break-up into the various allowed channels are required for the calculation of the shift functions and penetration factors and were taken from experiment.[1] We followed the rather arbitrary scheme of Werntz and Meyerhof[18] for choosing the boundary conditions in each channel. The energy shift $\Delta(E)$ was made to vanish for the lowest level

$$\Delta_{\lambda\lambda'}(E_1) = \sum_c \gamma_{\lambda c}\gamma_{\lambda' c}[S_c(E_1) - B_c] = 0\,. \tag{16}$$

For all higher resonances we set the boundary condition B_c for each channel equal to the shift function $S_c(E_1)$ evaluated at the first resonance and used the shell-model energies as the E_λ.

VI. CALCULATION

The shell-model part of this problem was solved on the NASA IBM 360/91 at Greenbelt, Maryland with programs written for the general 2p-1h problem. The programs were checked against the work of Easlea[20] on ^{13}C for the eigenvalue problem and dipole-strength calculation, and against the work of Barrett[12] and Clement and Baranger[21] for the two-body matrix elements with the Tabakin potential. The agreement was excellent.

The R-matrix part of this work was carried out using a package of programs which constructs the complex inverse level matrix A from the levels, channel thresholds, and reduced-width amplitudes.

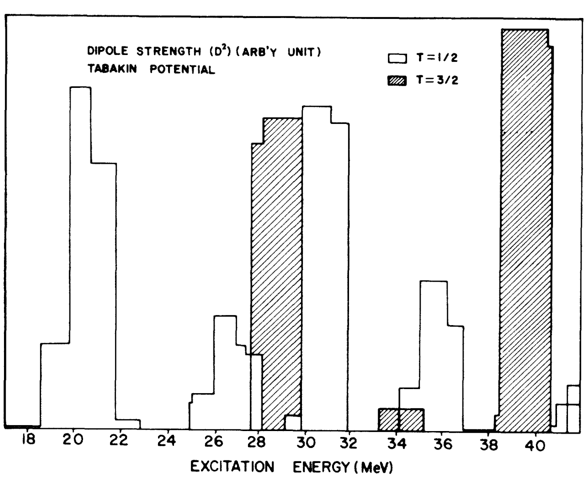

FIG. 2. Histogram of dipole strengths (D^2) carried by shell-model states for ^{5}He. States were given an arbitrary width of 2 MeV.

VII. RESULTS

Our results for the energy spectrum of the five-nucleon system are given in Table I. The dipole strengths associated with these states are given by the histogram of Fig. 2 in which each state was arbitrarily assigned a width of 2 MeV. States with $T=\frac{3}{2}$ are shown shaded and were not made to accumulate with the $T=\frac{1}{2}$ states since there is no way to excite such states.

Our lowest eigenvalue comes at 17.7 MeV and has $J^\pi,\ T=\frac{3}{2}^+,\ \frac{1}{2}$ which is in good agreement with the experimental energy of 16.7 for the first resonance of ^{5}He. This level has been identified as a $\frac{3}{2}^+$ state. This level could have been brought lower by increasing the well size; we used parameters appropriate to ^{4}He for this calculation. Figure 3 shows how the lowest levels are affected by the choice of the oscillator parameter. Recently a particle-hole calculation on the five-nucleon system has been used to investigate the dependence of the spectrum on this parameter more fully.[13]

This result may be compared with the work of Spicer and Fraser[22] on the five-nucleon system. Their calculation was performed with the Soper mixture for the δ-function potential. However, instead of placing the s-d shell in the region of 25-MeV excitation where it should occur naturally, they raised it to 45 MeV. This was done because the work of Hoop and Barschall[23] indicated that there is no significant single-particle s-wave phase shift below 24 MeV in $n+{}^4$He scattering.

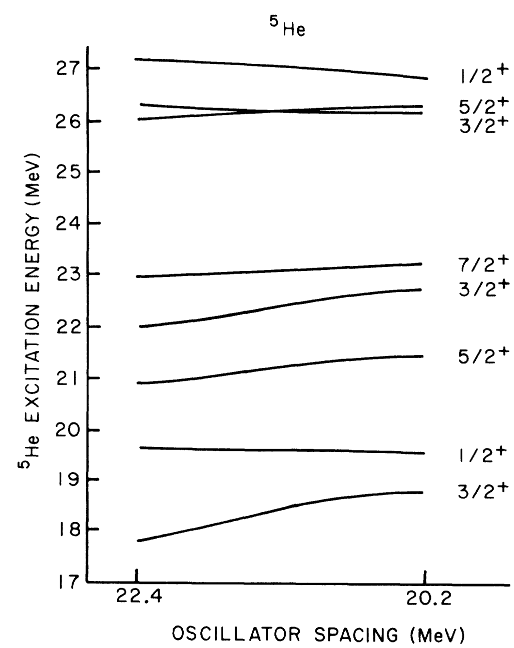

FIG. 3. Dependence of even-parity states of ^{5}He on oscillator spacing $\hbar\omega$.

Raising the s-d shell to 45 MeV was necessary to keep the single-particle shell-model component weak in their eigenfunctions below 24 MeV. However, as our calculation of the reduced widths indicates, the single-particle shell-model state is not to be identified with the physical single-particle channel. We repeated the δ-function calculation with the s-d shell in a more natural position and found that this brings a level with $J^\pi = \frac{1}{2}^+$ an MeV lower than the lowest $J^\pi = \frac{3}{2}^+$ level found with the elevated s-d shell. The tensor force seems to be necessary for the correct ordering of levels. We note that the first $T=\frac{3}{2}$ state occurs at ~28 MeV indicating ^{5}H is unbound by about 8 MeV against ^{3}H + (nn) decay.

In Table II we have listed the calculated reduced widths for the lowest four states. Since all the states are of the order of at least 17 MeV above the ^{4}He + n threshold, for a state to be narrow it must have essentially zero reduced width for this channel. It is seen that only the lowest $\frac{3}{2}^+$ state satisfies this requirement. However, the $\frac{7}{2}^+$ state (23 MeV) can only have $t+d$ cluster structure and consists entirely of $^4D_{7/2}$, so it is tempting to identify this state with the anomaly seen by Cerny, Detraz, and Pehl.[4] The second $\frac{3}{2}^+$ state (21.8 MeV) is, of course, another candidate for this structure.

Finally, we have calculated the excitation function and angular correlation for $t(d,\gamma)^5$He g.s. using Eq. (9). The results for the 90° cross section are shown in Fig. 4 and compared with a curve supplied to us by Meyerhof[24] of Stanford which represents preliminary results of an ^{3}He-$(d,\gamma)^5$Li(g.s.) experiment. The low-energy portion represents a composite of earlier work[25–28] with an attempt at removing γ rays going to the $\frac{1}{2}^-$ first excited state. When the γ_0 transitions are isolated a definite broad peak is seen which we associate with transitions occuring chiefly through the $\frac{5}{2}^+$ state at ~20.8 MeV. The lower curve gives our prediction for A_2/A_0 as a function of energy. Due to the interference between the $\frac{1}{2}^+$ and $\frac{5}{2}^+$ states the ratio changes sign at about 20-MeV excitation. This feature is apparently observed in the experimental angular distribution but at low energies we do not get good agreement, because our $\frac{3}{2}^+$ (16.7-MeV) state has essentially no dipole strength instead of ~1/300 of the dipole sum.

TABLE II. Calculated reduced widths for the lowest four levels of ^{5}He. Normalized such that $\gamma_d{}^2 = 2.0$ for the $^4S_{3/2}$, $t+d$ width.

Channel		Reduced widths (MeV) $\frac{3}{2}^+$	$\frac{1}{2}^+$	$\frac{5}{2}^+$	$\frac{3}{2}^+$
^{4}He + n	^2S	···	1.44	···	···
	^2D	0.067	···	1.33	0.64
$t+d$	^2S	···	0.24	···	···
	^4S	2.0	···	···	0.58
	^2D	0.48	···	0.52	0.46
	^4D	0.86	0.19	0.25	0.32
^{3}H + $(pp)_0$	^2S	···	0.21	···	···
	^2D	0.06	···	0.08	0.01
^{3}He + $(np)_0$	^2S	···	0.42	···	···
	^2D	0.12	···	0.15	0.02

VIII. CONCLUSION

There are several conclusions we can draw. (1) For very light nuclei center-of-mass effects are so large that estimates of channel widths can only be made after transforming to appropriate relative and center-of-mass coordinates. Upon doing this, the simple connection between the ssp and single-neutron channels etc. disappears. Internal symmetry becomes the dominant factor. (2) The largest dipole strength in the low-energy region is in the $\frac{5}{2}^+$ and $\frac{1}{2}^+$ states which also have large single-particle widths. The peak seen in ^{3}He$(d,\gamma)^5$Li(g.s.) is the result of the interference of at least two states, since the observed angular distribution, $A_2/A_0 = -0.1 \pm 0.03$, is not consistent with the limits obtained from Eq. (9) for a pure $\frac{5}{2}^+$ state. (These are $A_2/A_0 > -0.4$, lower limit entirely ^2D, and $A_2/A_0 < -0.143$, entirely ^4D.) (3) We cannot explain the additional relatively narrow peaks seen by Baker *et al.*[5,29] in the ^{7}Li$(d,{}^4$He$)^5$He and ^{6}Li$(^3$He$,{}^4$He$)^5$Li reactions. (4) More generally, it is not necessary to do continuum calculations to derive semiquantitative information about nu-

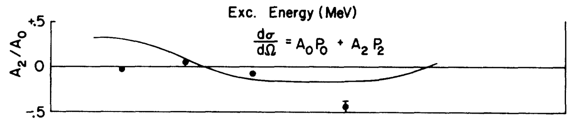

FIG. 4. Energy and angular dependence of deuteron-triton photocapture cross section. The upper curve compares the calculated 90° cross section to the observed $(d\sigma/d\Omega_\gamma)$ (90°) for the ^{3}He$(d,\gamma)^5$Li(g.s.). The open points are those of Meyerhof *et al*. (see Ref. 24), and represent the most systematic attempt to separate γ_0 transitions from the γ_1. The points around the $\frac{3}{2}^+$ resonance are selected from Refs. 25–28. The lower curve gives the computed ratio A_2/A_0 and compares it with the measured angular distribution (see Ref. 24).

clear reactions from nuclear forces in the resonance region of nuclei.

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APPENDIX

To calculate the jj-LS coupling transformation coefficients[30] for our basis states $|2\text{p-}1\text{h}\rangle_{\text{A.S.}}$ of Eq. (2), this equation is first rewritten with the isospin coupling suppressed for the moment, and the coupling of l_{p} with s_{p} to form j_{p} and similarly for j_{v} written explicitly. Since the core for He^5 has orbital angular momentum zero, we can write s_{h} for j_{h}. Then Eq. (2) becomes

$$|\phi\rangle = \sum_{m's} N_{jj}^{-1}(l_{\text{p}}m_{\text{p}}s_{\text{p}}m_s|j_{\text{p}}m_j)(l_{\text{v}}m_{\text{v}}s_{\text{v}}m_s'|j_{\text{v}}m_j')(j_{\text{p}}m_j j_{\text{v}}m_j'|jm)(jms_{\text{h}}-m_{\text{h}}|JM)(-1)^{s_{\text{h}}-m_{\text{h}}}a_{\text{p}}^{\dagger}a_{\text{v}}^{\dagger}a_{\text{h}}|0\rangle . \quad \text{(A1)}$$

A basis for the same space of antisymmetric functions with total angular momentum J and projection M can be written in an L-S representation in the form

TABLE III. Values of $\eta_{\alpha s l,k}$ for $J^{\pi}=\frac{1}{2}^{+}$. D-wave entry should be multiplied by $(\frac{2}{5})^{1/2}$.

Channel	LS basis states ($LstS$)			
	$001\frac{1}{2}$	$010\frac{1}{2}$	$210\frac{3}{2}$	$^2s_{1/2}$
$^4\text{He}+n$ ^2S	$-\frac{1}{2}$	$-\frac{1}{2}$	...	$-\frac{3}{4}$
$t+d$ ^2S	$+(\frac{3}{2})^{3/4}$	$-(\frac{2}{3})^{1/4}$	...	$+3^{1/4}/2^{7/4}$
$t+d$ ^4D	...	...	$+\frac{11}{5}(\frac{2}{3})^{1/4}$	...
$t+{}^1(np)$ ^2S	$-(\frac{1}{3})^{1/2}(\frac{2}{3})^{1/4}$	$(\frac{1}{3})^{1/2}(\frac{3}{2})^{3/4}$	...	$+(\frac{1}{3})^{1/2}3^{1/4}/2^{7/4}$
$^3\text{He}+{}^1(nn)$ ^2S	$+(\frac{2}{3})^{1/2}(\frac{2}{3})^{1/4}$	$-(\frac{2}{3})^{1/2}(\frac{3}{2})^{3/4}$	...	$-(\frac{2}{3})^{1/2}3^{1/4}/2^{7/4}$

TABLE IV. Values of $\eta_{\alpha s l,k}$ for $J^{\pi}=\frac{3}{2}^{+}$. D-wave entries should be multiplied by $(\frac{2}{5})^{1/2}$.

Channel	LS basis state ($LstS$)				
	$010\frac{3}{2}$	$201\frac{1}{2}$	$210\frac{1}{2}$	$210\frac{3}{2}$	$1d_{3/2}$
$^4\text{He}+n$ ^2D	...	$-\frac{1}{2}$	$-\frac{1}{2}$	...	$-\frac{3}{4}$
$t+d$ ^2D	...	$+(\frac{3}{2})^{3/4}$	$-(\frac{2}{3})^{1/4}$	...	$+3^{1/4}/2^{7/4}$
$t+d$ ^4S	$+\frac{11}{5}(\frac{2}{3})^{1/4}$	...	...	...	...
$t+d$ ^4D	...	...	...	$+\frac{11}{5}(\frac{2}{3})^{1/4}$	...
$t+{}^1(np)$ ^4D	...	$-(\frac{1}{3})^{1/2}(\frac{2}{3})^{1/4}$	$+(\frac{1}{3})^{1/2}(\frac{3}{2})^{3/4}$	...	$+(\frac{1}{3})^{1/2}3^{1/4}/2^{7/2}$
$^3\text{He}+{}^1(nn)$ ^4D	...	$+(\frac{2}{3})^{1/2}(\frac{2}{3})^{1/4}$	$-(\frac{2}{3})^{1/2}(\frac{3}{2})^{3/4}$	...	$-(\frac{2}{3})^{1/2}3^{1/4}/2^{7/2}$

TABLE V. Values of $\eta_{\alpha s l,k}$ for $J^{\pi}=\frac{5}{2}^{+}$. All entries should be multiplied by $(\frac{2}{5})^{1/2}$.

Channel	LS basis state ($LstS$)		
	$201\frac{1}{2}$	$210\frac{1}{2}$	$210\frac{3}{2}$	$1d_{5/2}$
$^4\text{He}+n$ ^2D	$-\frac{1}{2}$	$-\frac{1}{2}$	...	$+\frac{3}{4}$
$t+d$ ^2D	$+(\frac{3}{2})^{3/4}$	$-(\frac{2}{3})^{1/4}$	...	$-3^{1/4}/2^{7/4}$
$t+d$ ^4D	...	...	$+\frac{11}{5}(\frac{2}{3})^{1/4}$	...
$t+{}^1(np)$ ^2D	$-(\frac{1}{3})^{1/2}(\frac{2}{3})^{1/4}$	$+(\frac{1}{3})^{1/2}(\frac{3}{2})^{3/4}$	...	$-(\frac{1}{3})^{1/2}3^{1/4}/2^{7/4}$
$^3\text{He}+{}^1(nn)$ ^2D	$+(\frac{2}{3})^{1/2}(\frac{2}{3})^{1/4}$	$-(\frac{2}{3})^{1/2}(\frac{3}{2})^{1/4}$	...	$+(\frac{2}{3})^{1/2}3^{1/4}/2^{7/4}$

$$|\chi\rangle = \sum_{m's} N_{LS}^{-1}(l_{\rm p} m\, l_{\rm v} m_l' | L M_L)(s_{\rm p} m_s s_{\rm v} m_s' | sm)(sm s_{\rm h} - m_{\rm h} | S M_s)(S M_s L M_L | JM)(-1)^{s_{\rm h} - m_{\rm h}} a_{\rm p}^\dagger a_{\rm v}^\dagger a_{\rm h} |0\rangle \,. \tag{A2}$$

In taking the inner product $\langle \chi | \phi \rangle$ the six fermion operators contribute

$$\delta_{\rm pp'}\delta_{\rm vv'}\delta_{\rm hh'} - \delta_{{\rm p}v'}\delta_{v{\rm p}'}\delta_{\rm hh'} \,,$$

where p refers to all the quantum numbers of p, etc. The result of taking the m sums for the direct term is

$$\langle \chi | \phi \rangle = \hat{j}\hat{j}_v \hat{j}_{\rm h} \hat{S}\hat{s}\hat{L}(-1)^{1+s+s_{\rm h}+S} N_{LS}^{-1} N_{jj}^{-1} \begin{Bmatrix} l_{\rm p} & l_{\rm v} & L \\ s_{\rm p} & s_{\rm v} & s \\ j_{\rm p} & j_{\rm v} & j \end{Bmatrix} \begin{Bmatrix} s_{\rm h} & j & J \\ L & S & s \end{Bmatrix} . \tag{A3}$$

When the analogous factors for isospin space are included and the exchange term calculated the result is

$$\langle \chi | \phi \rangle = [\text{right-hand side of Eq. (A3)}] \times [\, 1 - (-1)^{L+s+t} \,] \,, \tag{A4}$$

where t is the intermediate isospin to which p and v are coupled. N_{LS} is always $1/\sqrt{2}$, since both p and v are in p orbits. N_{jj} was given earlier in Eq. (2).

We list below in Tables III–V the parameters $\eta_{\alpha s l, k}$ that describe the coupling between LS basis states and the physical channels. Factors of i^l for each single-particle state have been included as well as an i^L for external channels. The two-particle-one-hole states are characterized by L, s, t, and S, where s and t are the intermediate spin and isospin of the two particles. The factor $(2/5)^{1/2}$ that occurs for D-wave states is, in some sense, a quirk of the oscillator wave function. If one takes the overlap of a cluster wave function with the shell-model states, the clusters also being in an oscillator state relative to one another, the numerical values are identical for S and D waves. For this reason the $(2/5)^{1/2}$ factor was dropped. The effect of this is to make the sum of the reduced widths for the $\frac{1}{2}^+$ and $\frac{5}{2}^+$ states about equal.

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