J dependence in the reaction $^{16}O(\pi^+,2p)^{14}N$ at 116 MeV

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A distorted wave impulse approximation formalism is described for $(\pi^+,2p)$ reactions. The reaction is assumed to proceed via absorption on a deuteron-like p-n pair projected from shell-model wave functions. The spin dependence of this primary interaction is assumed to be dominated by s-wave Δ-nucleon terms. Spin-orbit distortions of the emitted protons, and the coherent addition of different orbital angular momentum transfers, L, are properly taken into account. For $L=2$, the calculations show a significant dependence on the total angular momentum transfer, $J=1^+$, 2^+, or 3^+. These new calculations are compared with experimental data for the $^{16}O(\pi^+,2p)^{14}N$ reaction.

I. INTRODUCTION

Recently we reported measurements of the reaction $^{16}O(\pi^+,2p)^{14}N$ at $T_\pi=116$ MeV with sufficiently good energy resolution that several discrete states in the residual nucleus could be resolved.[1] These data were compared with distorted wave impulse approximation (DWIA) calculations in which the reaction is assumed to take place on a deuteron-like cluster in the target nucleus. In these calculations, "form factors" representing the motion of the deuteron c.m. were obtained by projecting the target wave function onto the product of the residual nuclear state and deuteron ground-state wave functions. Aside from this refinement, the procedures outlined by Roos, Rees, and Chant[2] were followed.

As pointed out by Gouweloos and Thies,[3] a number of questionable approximations are used in Ref. 2 for calculational simplicity. In particular, we assumed that the spin dependence of the $\pi^+ + d \rightarrow 2p$ amplitudes is such that the corresponding cross section enters as a multiplicative factor. In the absence of spin-orbit terms in the emitted proton optical potentials, this assumption leads to a final expression which is correct for orbital angular momentum transfer $L=0$. However, in general, there is a more complicated coherent summation of amplitudes which leads to significant effects,[3] arising from an effective "tensor polarization" of the deuteron cluster participating in the reaction.

In the present paper we present an improved DWIA treatment in which the above tensor polarization effects, spin-orbit forces for the emitted nucleons and the coherent addition of different L transfers for a single transition are all taken into account. The resulting calculations are compared with the 116 MeV $^{16}O(\pi^+,2p)^{14}N$ data. We shall see that, in some cases, there results a significant dependence of the predicted cross sections on the value of J, the total angular momentum transfer.

II. DWIA FORMALISM

If we assume that the reaction $A(\pi^+,cd)B$ proceeds via a single-step direct process, the transition amplitude from an initial nuclear state with total angular momentum and isospin J_AM_A,T_AN_A to a final nuclear state of B with quantum numbers J_BM_B,T_BN_B is

$$T_{BA}=\begin{bmatrix} A \\ 2 \end{bmatrix}^{1/2} \sum_{MN}^{[n_1l_1j_1][n_2l_2j_2]LSJT} \mathcal{J}_{AB}([n_1l_1j_1][n_2l_2j_2];JT)(J_BM_BJM|J_AM_A)(T_BN_BTN|T_AN_A) \times \begin{bmatrix} l_1 & l_2 & L \\ \frac{1}{2} & \frac{1}{2} & S \\ j_1 & j_2 & J \end{bmatrix} \langle [\phi_c\phi_d]_{AS}|t|\phi_\pi\psi_{MN}^{l_1l_2LSJT}\rangle \ , \quad (1)$$

where $[n_1l_1j_1][n_2l_2j_2]$ are the quantum numbers of the two participating nucleons, $\mathcal{J}_{AB}$ is a two-nucleon coefficient of fractional parentage,[4] the quantity [ ] is a normalized 9-J symbol and the $(j_1m_1j_2m_2|j_3m_3)$ are vector coupling coefficients. The antisymmetrized two-nucleon wave function, resulting from projecting the target onto the residual state, is $\psi_{MN}^{l_1l_2LSJT}$, which is expressed in an LS basis with orbital angular momentum L, spin S, total angular momentum J (projection M) and isospin T (projection N). The incident pion wave function is ϕ_π and $[\phi_c\phi_d]_{AS}$ is the (antisymmetrized) wave function for the emitted nucleons. In terms of $X_{\Sigma N}^{ST}$, a two-nucleon spin/isospin wave function, we can write

$$\psi_{MN}^{l_1l_2LSJT}=\sum_{\Lambda\Sigma}(L\Lambda S\Sigma|JM)\tilde{\phi}_{\Lambda}^{l_1l_2L}X_{\Sigma N}^{ST} , \tag{2}$$

where

$$\tilde{\phi}_{\Lambda}^{l_1l_2L}=N\{\phi_{\Lambda}^{l_1l_2L}(1,2)+(-)^{1+S+T}\phi_{\Lambda}^{l_1l_2L}(2,1)\} \tag{3}$$

is an appropriately symmetric/antisymmetric spatial wave function. The quantity N is a normalization constant and

$$\phi_{\Lambda}^{l_1l_2L}(p,q)=[\phi^{[n_1l_1j_1]}(\mathbf{r}_p)\times\phi^{[n_2l_2j_2]}(\mathbf{r}_q)]_{\Lambda}^{L} .$$

Consider now an expansion of the two-nucleon wave function in terms of relative orbital angular momentum l and any other necessary quantum numbers α.

$$\tilde{\phi}_{\Lambda}^{l_1l_2L}=\sum_{\alpha lL'}[H^{\alpha lL'}(\mathbf{R})\times\phi^{\alpha l}(\mathbf{r})]_{\Lambda}^{L} , \tag{4}$$

where $\mathbf{R}=\frac{1}{2}(\mathbf{r}_1+\mathbf{r}_2)$ and $\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$ and the origin of coordinates is the c.m. of B. It is clear from Eq. (3) that $\phi^{\alpha l}$ must be space symmetric, if $S+T=1$ and $\phi^{\alpha l}$ must be space antisymmetric, if $S+T\neq 1$. Thus, we define $G^{\alpha lL'}(\mathbf{R})$ through

$$\phi_{\Lambda}^{l_1l_2L}(\mathbf{r}_1,\mathbf{r}_2)=\sum_{\alpha lL'}[G^{\alpha lL'}(\mathbf{R})\times\phi^{\alpha l}(\mathbf{r})]_{\Lambda}^{L} \tag{5}$$

and introduce the quantity

$$\begin{aligned} g &=2N=1 \ \text{ if } [n_1l_1j_1]=[n_2l_2j_2] \\ &=\sqrt{2} \ \text{ if } [n_1l_1j_1]\neq[n_2l_2j_2] . \end{aligned} \tag{6}$$

Inserting Eqs. (2)–(6) into Eq. (1) we obtain

$$\begin{aligned} T_{BA}=\begin{bmatrix} A \\ 2 \end{bmatrix}^{1/2} \sum_{\alpha\lambda\Lambda'\Lambda\Sigma MN}^{[n_1l_1j_1][n_2l_2j_2]lL'LSJT} & g\mathcal{J}_{AB}([n_1l_1j_1][n_2l_2j_2];JT)(J_BM_BJM|J_AM_A)(T_BN_BTN|T_AN_A) \\ & \times\begin{bmatrix} l_1 & l_2 & L \\ \frac{1}{2} & \frac{1}{2} & S \\ j_1 & j_2 & J \end{bmatrix}(L\Lambda S\Sigma|JM)(l\lambda L'\Lambda'|L\Lambda) \\ & \times\langle[\phi_c\phi_d]_{\mathrm{AS}}|t|\phi_\pi G_{\Lambda'}^{\alpha lL'}(\mathbf{R})\phi_{\lambda}^{\alpha l}(\mathbf{r})X_{\Sigma N}^{ST}\rangle . \end{aligned} \tag{7}$$

We next introduce distorted waves for the incident and emitted particles and employ an impulse approximation so that the $\pi^{+}+(pn)\to c+d$ amplitude can be taken outside the distorted-wave integral. The resultant expression for the differential cross section σ_{BA} is

$$\begin{aligned} \sigma_{BA}=\begin{bmatrix} \frac{2\pi}{\hbar v} \end{bmatrix}\omega_B\frac{1}{2J_A+1}\begin{bmatrix} A \\ 2 \end{bmatrix}\sum_{\substack{M_AM_B \\ \sigma_c\sigma_d}}\Bigg| \sum_{\alpha\lambda\Lambda'\Lambda\Sigma MN\Sigma'}^{[n_1l_1j_1][n_2l_2j_2]lL'LSJTS'} & (J_BM_BJM|J_AM_A)(\tfrac{1}{2}\sigma_c\tfrac{1}{2}\sigma_d|S'\Sigma')(L\Lambda S\Sigma|JM)(l\lambda L'\Lambda'|L\Lambda) \\ & \times g\mathcal{J}_{AB}([n_1l_1j_1][n_2l_2j_2];JT)(T_BN_BTN|T_AN_A)\begin{bmatrix} l_1 & l_2 & L \\ \frac{1}{2} & \frac{1}{2} & S \\ j_1 & j_2 & J \end{bmatrix} \\ & \times\langle\mathcal{A}\chi_c^{(-)}(\mathbf{r}_1')\chi_d^{(-)}(\mathbf{r}_2')X_{\Sigma'N'}^{S'T'}|t|\chi^{(+)} \\ & \times(\mathbf{r}_{\pi A})X_{\nu_\pi}^{1}\phi_{\lambda}^{\alpha l}(\mathbf{r}')G_{\Lambda}^{\alpha lL'}X_{\Sigma N}^{ST}\rangle\Bigg|^2 , \end{aligned} \tag{8}$$

where the χ are distorted waves, X_{ν}^{1} is the pion isospin wave function, $\mathcal{A}$ is an antisymmetrizer, v is the incident pion velocity and ω_B is the density of final states. Expressions for v and ω_B are given in Ref. 5. Making the impulse approximation leads to the result

$$\begin{aligned} \sigma_{BA}=\frac{2\pi}{\hbar v}\omega_b c^2\sum_{\substack{JM \\ \rho_c'\rho_d''}}\frac{1}{2J+1}\Bigg| \sum_{\alpha\Lambda'm}^{[n_1l_1j_1][n_2l_2j_2]lL'jLS} & (-)^{L'+l+J+S}gS_{AB}^{1/2}([n_1l_1j_1][n_2l_2j_2];JT)\hat{L}'\hat{L}\hat{j} \\ & \times\begin{bmatrix} l_1 & l_2 & L \\ \frac{1}{2} & \frac{1}{2} & S \\ j_1 & j_2 & J \end{bmatrix}\begin{Bmatrix} L' & l & L \\ S & J & j \end{Bmatrix}(L'\Lambda'jm|JM) \\ & \times T^{\alpha lL'\Lambda'}\langle\mathbf{k}';\rho_c'\rho_d'';\tau_c\tau_d|t^{\alpha}|\mathbf{k};lSjm;TN\rangle\Bigg|^2 , \end{aligned} \tag{9}$$

where $\hat{x}=\sqrt{2x+1}$ and { } is a 6-J symbol. The quantity c is the isospin Clebsch-Gordan coefficient, $S_{AB}^{1/2}$ is a two-nucleon spectroscopic amplitude,[4] the distorted-wave amplitude is

$$T^{\alpha lL'\Lambda'}=\frac{1}{(2L'+1)^{1/2}}\int \chi_c^{(-)*}\chi_d^{(-)*}\chi^{(+)}G_{\Lambda'}^{\alpha lL'}d^3R \tag{10}$$

and $\langle \mathbf{k}';\rho_c'\rho_d'';\tau_c\tau_d|t^\alpha|\mathbf{k};lSjm;TN\rangle$ is the amplitude for pion absorption on a p-n pair in the state $|lSjm\rangle$ leading to nucleons with spin projections ρ_c',ρ_d'' and with isospin projections τ_c and τ_d. Equation (9) is essentially identical to the expression given in Ref. 8. If spin-orbit distortions are included, the distorted waves become matrices in spin space with the result

$$\sigma_{BA}=\frac{2\pi}{\hbar v}\omega_B c^2\sum_{\substack{JM\\ \rho_c'\rho_d''}}\frac{1}{2J+1}\left|\sum_{\alpha\Lambda' m\sigma_c'\sigma_d''}^{[n_1l_1j_1][n_2l_2j_2]lL'jLS}(-)^{L'+l+J+S}gS_{AB}^{1/2}([n_1l_1j_1][n_2l_2j_2];JT)\hat{l}\hat{L}'\hat{\mathcal{L}}\hat{j}\right.$$
$$\times\begin{bmatrix} l_1 & l_2 & L\\ \frac{1}{2} & \frac{1}{2} & S\\ j_1 & j_2 & J\end{bmatrix}\begin{Bmatrix} L' & l & L\\ S & J & j\end{Bmatrix}(L'\Lambda'jm|JM)$$
$$\left.\times T_{\substack{\sigma_c'\sigma_d''\\ \rho_c'\rho_d''}}^{\alpha lL'\Lambda'}\langle \mathbf{k}';\sigma_c'\sigma_d'';\tau_c\tau_d|t^\alpha|\mathbf{k};lSjm;TN\rangle\right|^2 \tag{11}$$

and

$$T_{\substack{\sigma_c'\sigma_d''\\ \rho_c'\rho_d''}}^{\alpha lL'\Lambda'}=\frac{1}{(2L'+1)^{1/2}}\int \chi_{\sigma_c'\rho_c'}^{(-)*}\chi_{\sigma_d''\rho_d''}^{(-)*}\chi^{(+)}G_{\Lambda'}^{\alpha lL'}d^3R\ , \tag{12}$$

where, as noted above, the result is in terms of an amplitude for absorption on a nucleon pair with quantum numbers $\alpha lSjm$. The quantum numbers j and m are the total pair angular momentum and its projection, respectively.

In order to proceed we must calculate the quantity $G_{\Lambda'}^{\alpha lL'}(\mathbf{R})$. Making a multipole expansion of the two-nucleon wave function we may write

$$\phi_\Lambda^{l_1l_2L}(\mathbf{r}_1,\mathbf{r}_2)=\sum_{lL'}f_{lL'}^L(r,R)[Y_l(\hat{r})\times Y_{L'}(\hat{R})]_\Lambda^L\ . \tag{13}$$

In order to determine the expansion coefficients, we define radial functions through

$$G_{\Lambda'}^{\alpha lL'}(\mathbf{R})=g^{\alpha lL'}(R)Y_{L'\Lambda'}(\hat{R}) \tag{14}$$

and

$$\phi_\lambda^{\alpha l}(\mathbf{r})=p^{\alpha l}(r)Y_{l\lambda}(\hat{r})\ . \tag{15}$$

Next we rewrite Eq. (5) in the form

$$\phi_\Lambda^{l_1l_2L}(\mathbf{r}_1,\mathbf{r}_2)=\sum_{\alpha lL'}g^{\alpha lL'}(R)p^{\alpha l}(r)\times[Y_l(\hat{r})\times Y_{L'}(\hat{R})]_\Lambda^L \tag{16}$$

and thus identify the function

$$g^{\alpha lL'}(R)=\int f_{lL'}^L(r,R)p^{\alpha l*}(r)r^2dr \tag{17}$$

from which $G_{\Lambda'}^{\alpha lL'}(R)$ can be constructed using Eq. (14).

In order to complete the calculation two technical matters must be addressed. Firstly, as discussed for (p,2p) reactions,[6] it is convenient to introduce different quantization axes for particles a, c, and d. As a result, we write

$$\langle \mathbf{k}';\sigma_c'\sigma_d'';\tau_c\tau_d|t^\alpha|\mathbf{k};lSjm;TN\rangle=\sum_{\sigma_c\sigma_d}D_{\sigma_c\sigma_c'}^{1/2*}(R_{ac})D_{\sigma_d\sigma_d''}^{1/2*}(R_{ad})\langle \mathbf{k}';\sigma_c\sigma_d;\tau_c\tau_d|t^\alpha|\mathbf{k};lSjm;TN\rangle\ , \tag{18}$$

where the matrix element is now expressed in a coordinate system with the z axis parallel to the incident beam, and the rotations R_{ac} and R_{ad} defined in Ref. 6 relate this system with coordinates having z axes parallel to the emitted directions of c and d, respectively. Since, the incident beam is not necessarily parallel to the pion direction for the $\pi^++(pn)\rightarrow 2p$ process, additional rotations must also be introduced in order to evaluate the expression for the differential cross section given above.

Secondly, it is necessary to invert the multipole expansion defined in Eq. (13). This is carried out using techniques familiar from earlier studies of two-nucleon transfer reactions.[7] Specifically,

$$f_{lL'}^L(r,R)=\frac{\hat{l}_1\hat{l}_2\hat{l}}{2\hat{\mathcal{L}}}\sum_{\substack{m\\ \lambda_1\lambda_2}}(-)^{l-\lambda_1}(Lml-m|L'0)(l_1\lambda_1l_2\lambda_2|Lm)$$
$$\times\int_{-1}^{1}u^{[n_1l_1j_1]}(r_1)u^{[n_2l_2j_2]}(r_2)d_{\lambda_10}^{l_1}(\theta_1)d_{\lambda_20}^{l_2}(\theta_2)d_{m0}^{l}(\cos^{-1}x)dx\ , \tag{19}$$

where the $u^{[n_i l_i j_i]}(r_i)$ are the radial parts of the nucleon single-particle wave functions, and the $d^l_{mn}(\theta)$ are reduced rotation matrices. The arguments of the functions are related by

$$r_1=\left[R^2+\frac{r^2}{4}+Rrx\right]^{1/2},\quad r_2=\left[R^2+\frac{r^2}{4}-Rrx\right]^{1/2},$$
$$\cos\theta_1=\left[R+\frac{rx}{2}\right]\Big/r_1,\quad \cos\theta_2=\left[R-\frac{rx}{2}\right]\Big/r_2\ . \tag{20}$$

In the calculations which follow we consider only a single term $\phi^{\alpha l}$ which is approximated by the deuteron ground-state wave function. Furthermore, we ignore the D-state component so that $l=0$ and $j=S$. In our calculations the corresponding wave function is taken as a simple Hülthen function. We hope to explore other approximations in a future publication. For the present we simply note that, in the calculations of Ohta, Thies, and Lee,[8] the cross section is dominated by $^3S_1$ contributions and the inclusion of other configurations for the p-n relative motion lead only to a 20% increase in cross section. Thus Eq. (19) can be simplified and the required $f^L_{0L}(r,R)$ can be easily evaluated.[9] The use of the general expression[10] for $l>0$ detailed in Eq. (19) is also straightforward. With the restriction to $l=0$ we can simplify Eqs. (11) and (12) to the expressions

$$\sigma_{BA}=\frac{2\pi}{\hbar v}\omega_B c^2\sum_{\substack{JM\\ \rho'_c\rho''_d}}\frac{1}{2J+1}\left|\sum_{\Lambda\Sigma\sigma'_c\sigma''_d}^{[n_1l_1j_1][n_2l_2j_2]LS} gS^{1/2}_{AB}([n_1l_1j_1][n_2l_2j_2];JT)\begin{bmatrix} l_1 & l_2 & L\\ \frac12 & \frac12 & S\\ j_1 & j_2 & J\end{bmatrix}\right.$$
$$\left.\times\hat{L}(L\Lambda S\Sigma|JM)T^{\alpha L\Lambda}_{\substack{\sigma'_c\sigma''_d\\ \rho'_c\rho''_d}}\langle \mathbf{k}';\sigma'_c\sigma''_d;\tau_c\tau_d|t^\alpha|\mathbf{k};S\Sigma;TN\rangle\right|^2 \tag{21}$$

and

$$T^{\alpha L\Lambda}_{\substack{\sigma'_c\sigma''_d\\ \rho'_c\rho''_d}}=\frac{1}{(2L+1)^{1/2}}\int \chi^{(-)*}_{\sigma'_c\rho'_c}\chi^{(-)*}_{\sigma''_d\rho''_d}\chi^{(+)}G^{\alpha 0L}_{\Lambda}d^3R\ , \tag{22}$$

where the two-nucleon quantum numbers $lSjm$ are simply replaced by the spin angular momentum $S\Sigma$.

III. DWIA CALCULATIONS

In the calculations which follow the two-nucleon "form-factor" $G^{\alpha lL'}_{\Lambda'}(\mathbf{R})$ defined through Eqs. (14), (17), and (19) is computed in a code MICRO, for $l=0$ only, using a simple Hülthen form for $p^{\alpha l}(r)$. The form factors are obtained using Woods-Saxon single-nucleon wave functions with parameters obtained from an analysis of electron elastic scattering[11] in combination with the spectroscopic amplitudes of Cohen and Kurath.[12] It is precisely these results which are used in our earlier publication[1] concerning the reaction $^{16}O(\pi^+,2p)^{14}N$ at 116 MeV.

If spin-orbit terms in the emitted nucleon optical potentials can be neglected, we have already noted that the expression for the differential cross section (11) reduces to the simpler form (9). Furthermore, with the restriction to $l=0$ and a single term α, for $L=0$ or for certain (nonphysical) assumptions concerning the spin dependence of t^α, we can further reduce the expression to the form

$$\sigma_{BA}=\frac{2\pi}{\hbar v}\omega_b c^2\sum_{\substack{\Lambda\\ \rho'_c\rho''_d}}\left|\sum^{[n_1l_1j_1][n_2l_2j_2]} gS^{1/2}_{AB}([n_1l_1j_1][n_2l_2j_2];JT)\begin{bmatrix} l_1 & l_2 & L\\ \frac12 & \frac12 & S\\ j_1 & j_2 & J\end{bmatrix}T^{\alpha L\Lambda}\right|^2$$
$$\times|\langle \mathbf{k}';\rho'_c\rho''_d;\tau_c\tau_d|t^\alpha|\mathbf{k};S\Sigma;TN\rangle|^2\ , \tag{23}$$

where the result can clearly be written in terms of the $\pi^+ + (pn)\rightarrow c+d$ differential cross section. Despite its limitations, this result is suggested as an ansatz for $L>0$ in Ref. 2 and is the expression employed by Schumacher *et al.*[1] by inserting interpolated values of the empirical $\pi^+ + d\rightarrow 2p$ cross section. We refer to this approach as the "simple product approximation" (SPA).

We shall first compare the simple product approximation with improved DWIA calculations for $^{16}O(\pi^+,2p)^{14}N$ at 116 MeV using a new computer code, PIPP, in which the expressions given in Eqs. (21) and (22) are evaluated with the spin dependence of the $\pi^+ + (pn)\rightarrow 2p$ amplitudes calculated by assuming only s-wave Δ-nucleon terms are important. In order to facilitate the comparison, the $\pi^+ + (pn)\rightarrow 2p$ amplitudes in PIPP are renormalized to reproduce the magnitude of the

$\pi^+ + d \rightarrow 2p$ cross sections at each angle and energy used in the calculations. As a result, if spin-orbit terms are set to zero, the improved DWIA results will be identical to the SPA predictions for $L=0$, and may or may not show changes for $L>0$.

A. Spin-orbit effects and J dependence

In the calculations which follow, Kisslinger-type optical potentials for the incident pions are taken from the work of Amman *et al.*,[13] and for the emitted protons from the global analysis of medium-energy proton scattering by Nadasen *et al.*[14] Results are shown for energy sharing distributions for the reaction $^{16}O(\pi^+,2p)^{14}N$ at an incident energy of 116 MeV. In Fig. 1 calculations for the $L=0$ transition to the 1^+ level at 3.95 MeV in ^{14}N are shown for a quasifree angle pair, an angle pair for which zero recoil of the residual nucleus is kinematically allowed. Results are presented both using the correct values for the emitted proton spin-orbit potentials as well as results in which these terms are set to zero. As noted above this latter calculation is identical to the SPA prediction. We see that the spin-orbit terms have relatively little effect and that our earlier calculations for this level should be satisfactory. This behavior is probably a consequence of the fact that the emitted proton spin projections are summed over, leading to averaging of the spin-orbit effects.

In Fig. 2 predictions are shown for the $L=2$ transition to the 3^+ level at ~ 11 MeV in ^{14}N. Here we illustrate the difference between the SPA prediction and the DWIA result with and without spin-orbit terms. We see that the spin-orbit effects are again modest. However, there are

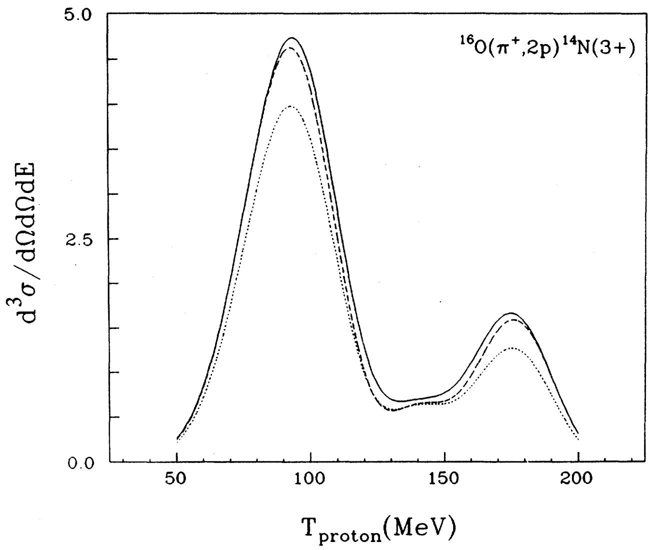

FIG. 2. Energy sharing cross sections ($\mu b/sr^2$ MeV) for the $^{16}O(\pi^+,2p)^{14}N$ (3^+, 11 MeV) reaction at $T_\pi = 116$ MeV and proton angles of $\theta_1 = 50°/\theta_2 = -107.5°$. The solid (dashed) curve is the DWIA $L=2$ calculation with (without) spin-orbit terms in the proton optical-model potentials. The dotted curve is the SPA cross section.

larger differences between DWIA and our SPA, so that the former calculations are certainly preferable.

In Fig. 3 DWIA calculations at four angle pairs are shown for the $L=2$ transitions to the 1^+, 2^+, and 3^+ levels of ^{14}N. For illustrative purposes, the spectroscopic amplitude and nucleon separation energies for all three

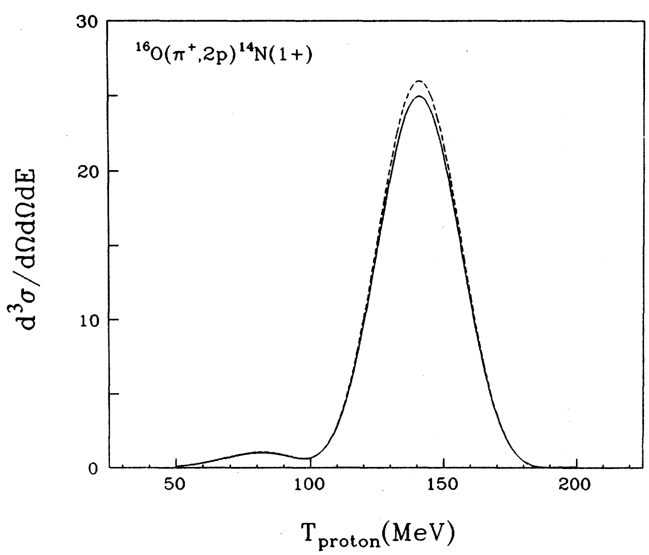

FIG. 1. Energy sharing cross sections ($\mu b/sr^2$ MeV) for the $^{16}O(\pi^+,2p)^{14}N$ (1^+, 3.95 MeV) reaction at $T_\pi = 116$ MeV and proton angles of $\theta_1 = 50°/\theta_2 = -107.5°$. The curves are DWIA calculations for a pure $L=0$ transition with (solid curve) and without (dashed curve) spin-orbit terms in the proton optical-model potentials.

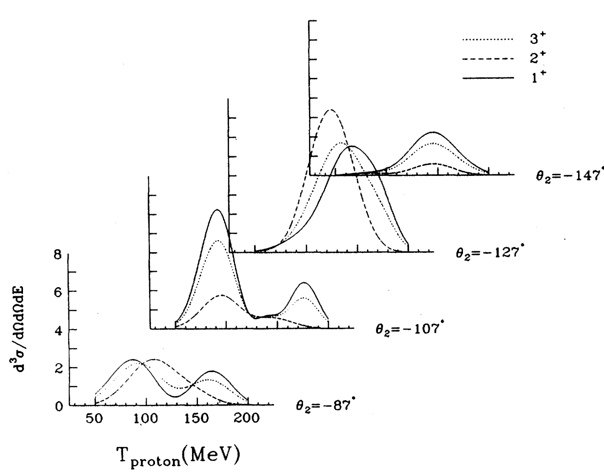

FIG. 3. Energy sharing cross sections ($\mu b/sr^2$ MeV) for the $^{16}O(\pi^+,2p)^{14}N$ reaction at $T_\pi = 116$ MeV. The outgoing proton angles are $\theta_1 = 50°$ and θ_2 as specified on the individual graphs. The curves are DWIA calculations using identical form factors and orbital angular momentum transfer $L=2$, but with total angular momentum transfer $J=1$ (solid), $J=2$ (dashed), or $J=3$ (dotted).

states are identical and taken to have the values used for the 3^+ calculations of Fig 2. As a result the three levels are degenerate and the SPA prediction for $-107°$, the quasifree angle, is identical to the curve shown in Fig. 2 for all three transitions. In DWIA, we see that there is pronounced J dependence. In particular, the 1^+ and 2^+ transitions are significantly different from the 3^+ case, which is more similar to the SPA result.

To examine the J dependence further, in Fig. 4 we present DWIA energy sharing distributions for three quasifree angle pairs ranging from $\theta_1=30°$ to 77°. Again the form factor is identical for each state. For comparison purposes we have included the SPA calculation in Fig. 4. As in Fig. 3 the J-dependent effects are quite pronounced for the 1^+ and 2^+ levels, whereas the 3^+ level is quite similar to the SPA calculation. Note that the effect of the J dependence changes sign from 30° to 50° and then back again at 77°.

This strong j dependence is a consequence of the "tensor polarization" effects reported in earlier calculations.[3] In essence, the largest angular momentum, 3^+, permits a more uniform sampling of the possible projections of the orbital angular momentum transfer, L and hence spin $S=1$. Thus, this calculation more nearly resembles the SPA result. In contrast, the more limited couplings possible in the $J=1^+$ and 2^+ cases lead to significant differences.

As discussed in Ref. 3 the tensor polarization effects persist even in the plane-wave limit. Thus, if we evaluate $T^{\alpha L\Lambda}$ in the plane-wave limit, we may rewrite Eq. (21), for a single L value, in the form

$$\sigma_{BA}\sim|R_L(P)|^2\sum_{\substack{M\\ \rho_c'\rho_d''}}\left|\sum_{\Lambda\Sigma}(L\Lambda S\Sigma|JM)Y_{L\Lambda}(\hat{P})\langle\rho_c'\rho_d'';\tau_c\tau_d|t^{\alpha}|\mathbf{k};S\Sigma;TN\rangle\right|^2 , \qquad (24)$$

where $\mathbf{P}$ is the c.m. momentum of the pn pair prior to absorption and $R_L(P)Y_{L\Lambda}(\hat{P})$ is the corresponding momentum wave function. In the approximation that the pion absorption is dominated by the s-wave Δ-nucleon term, Gouweloos and Thies have shown that the above expression can be reduced to the form

$$\sigma_{BA}\sim|R_L(P)|^2\{[1+\xi(L,J)][1+3(\hat{\mathbf{q}}\cdot\hat{\boldsymbol{\kappa}})^2] -3\xi(L,J)(3\hat{\mathbf{q}}\cdot\hat{\mathbf{P}}\,\hat{\mathbf{q}}\cdot\hat{\boldsymbol{\kappa}}-\hat{\mathbf{P}}\cdot\hat{\boldsymbol{\kappa}})^2\} \qquad (25)$$

[see Eq. (2.23) of Ref. 3], where $\mathbf{q}$ is the relative momentum of the emitted nucleons and $\boldsymbol{\kappa}$ the pion-pn pair relative momentum. The J-dependent effects seen in Figs. 3 and 4 are generally qualitatively reproduced by this plane-wave expression. The primary effect is due to a change in sign of $\xi(L,J)$ between the 1^+ and 2^+ transitions.

For the pion absorption on ^{16}O, possible values of ξ are tabulated in Table I. Thus, as observed, we expect roughly equal and opposite J-dependent effects for the $J=1^+$ and $J=2^+$ transitions whereas the $J=3^+$ transition is expected to show less modification and be closer to the SPA prediction.

It is interesting to note that for each L value in Table I the quantity $\sum_J(2J+1)\xi(L,J)=0$. Thus, given an appropriate average over different transitions, the tensor polarization effects cancel and the SPA expression is recovered. In terms of the expressions given in Sec. II, we may write schematically,

$$\sum_J(2J+1)\sigma_{LJ}^{\mathrm{DWIA}}=\sigma_L^{\mathrm{SPA}}\sum_J(2J+1) , \qquad (26)$$

where $\sigma_{LJ}^{\mathrm{DWIA}}$ and σ_L^{SPA} are the results of evaluating Eqs. (21) and (23), respectively, with the structure factor $X^{LJ}([n_1l_1j_1][n_2l_2j_2])=1$, where

$$X^{LJ}([n_1l_1j_1][n_2l_2j_2])=gS_{AB}^{1/2}([n_1l_1j_1][n_2l_2j_2];JT)\times\begin{bmatrix} l_1 & l_2 & L\\ \frac{1}{2} & \frac{1}{2} & S\\ j_1 & j_2 & J\end{bmatrix} . \qquad (27)$$

The result (26) is exact provided Q-value differences and the effects of spin-orbit terms in both the bound and emitted nucleon potentials can be ignored. In addition, the possibility of interference between different L values for a given J has been ignored.

For pion absorption on the closed-shell nucleus ^{16}O it is straightforward to show that, for $L=2$,

$$[X^{2J}(p_{1/2}^2)]^2+[X^{2J}(p_{1/2}p_{3/2})]^2+[X^{2J}(p_{3/2}^2)]^2=(2J+1) . \qquad (28)$$

Similarly, for $L=1$ transitions in which a pion is absorbed on an sp pair

$$[X^{1J}(s_{1/2}p_{1/2})]^2+[X^{1J}(s_{1/2}p_{3/2})]^2=2(2J+1) , \qquad (29)$$

where the additional factor of 2 is a consequence of our choice for the normalization of $G_{\Lambda}^{\alpha 0L}(\mathbf{R})$ [Eq. (5)]. Thus, calculations for experimental data which do not resolve individual levels in ^{14}N, may be carried out using the SPA approximation with the hope that, at least partially, tensor polarization effects will cancel. If configuration mixing is taken into account the same "sum rules" are obtained[15] in Eqs. (28) and (29) provided the summation is over the $(X^{LJ})^2$ for all states of a given J. Thus, to this same level of approximation, all shell-model calculations for ^{16}O/^{14}N restricted to an $(s^4)(p^{12})$ basis will yield identical results.

B. Reanalysis of ^{16}O$(\pi^+,2p)^{14}$N angular correlations

In view of the J dependence found for the $L=2$ transitions, it is of interest to repeat our analysis[1] of the

$^{16}O(\pi^+,2p)^{14}N$ reaction at 116 MeV. Angular correlations, $d^2\sigma/d\Omega_1 d\Omega_2$, for specific final states in ^{14}N were obtained by integrating over the energy range 60–175 MeV for one proton (detected in the SUSI spectrometer), the other proton (detected in a NaI telescope) having an energy between 30 and 200 MeV. Results were compared with SPA calculations using Cohen-Kurath wave functions, for the pure $L=2$ transitions to the 2^+, 7 MeV level and 3^+, 11 MeV level in ^{14}N, for the predominantly $L=0$ transition to the 1^+ level at 3.9 MeV in ^{14}N, and for the predominantly $L=2$ transition to the 1^+ ground state. For the transitions to excited levels of ^{14}N, fairly good agreement was obtained between theory and experiment. However, for the ground-state 1^+ transition, agreement was less satisfactory, suggesting that the Cohen-Kurath wave functions underestimate the $L=0$ contribution to the transition.

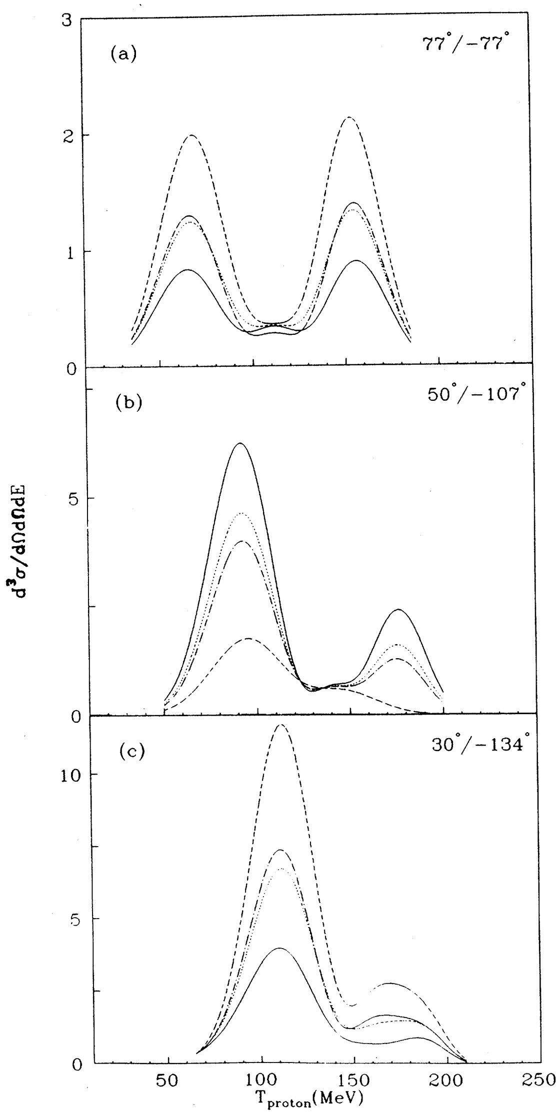

FIG. 4. Energy sharing cross sections (μb/sr^2 MeV) for the $^{16}O(\pi^+,2p)^{14}N$ reaction at $T_\pi=116$ MeV. The outgoing proton angle pairs are as indicated. All curves are DWIA calculations using identical form factors and orbital angular momentum transfer $L=2$, but with total angular momentum transfer $J=1$ (solid), $J=2$ (dashed), or $J=3$ (dotted). The dashed-dotted curve is the SPA result and is identical for each J.

TABLE I. Values of ξ in Eq. (25) for the allowed (L,J) transfer in $^{16}O \rightarrow ^{14}N$.

L	J	ξ
0	1	0
	0	-1
1	1	$\frac{1}{2}$
	2	$-\frac{1}{10}$
	1	$-\frac{1}{2}$
2	2	$\frac{1}{2}$
	3	$-\frac{1}{7}$

In Fig. 5 we show the results of a new DWIA analysis. In addition to the inclusion of spin-orbit distortions and a proper treatment of the tensor polarization effects as outlined above, we also treat correctly the coherent addition of the $L=0$ and $L=2$ contributions to the two 1^+ transitions. On the whole the results are encouraging. As was the case in the SPA analysis[1] the calculations must be renormalized and these normalizations are comparable. Presumably much of this renormalization results from an inadequate basis for the shell-model calculations and the restriction of the two-body t matrix to a Hülthen deuteron wave function and s-wave Δ-N interactions. However, agreement with the ground-state transition [Fig. 5(a)] is much improved, seemingly confirming the J dependence. Note that the need for additional $L=0$ strength in this transition is reduced. Although agreement with the 2^+ transition could be considered slightly worse, it must be noted that the region of the minimum ($-107°$) is exactly where the data are most unreliable. In this angular region the adjacent 1^+ level (3.95 MeV) is at a maximum, and separation of the 2^+ strength from the tail is extremely difficult. For the other two levels (1^+, 3.95 MeV and 3^+, 11.0 MeV) the differences between DWIA and SPA calculations are relatively small, so that the agreement is as before.

IV. CONCLUSIONS

We have described a formalism for DWIA calculations of $(\pi^+,2p)$ reactions in terms of absorption on a (pn) pair in a state $|lSjm\rangle$ projected from shell-model wave functions. Sample calculations, for the $^{16}O(\pi^+,2p)^{14}N$ reaction at 116 MeV, were carried out for the deuteron-like quantum numbers $^3S_1$, $l=0$. It was found, for both energy sharing distributions and for angular correlations, that there were significant J-dependent effects arising from the anticipated tensor polarization of the (pn) pair. The effects of including spin-orbit terms in the emitted nucleon optical potentials were examined and found to be

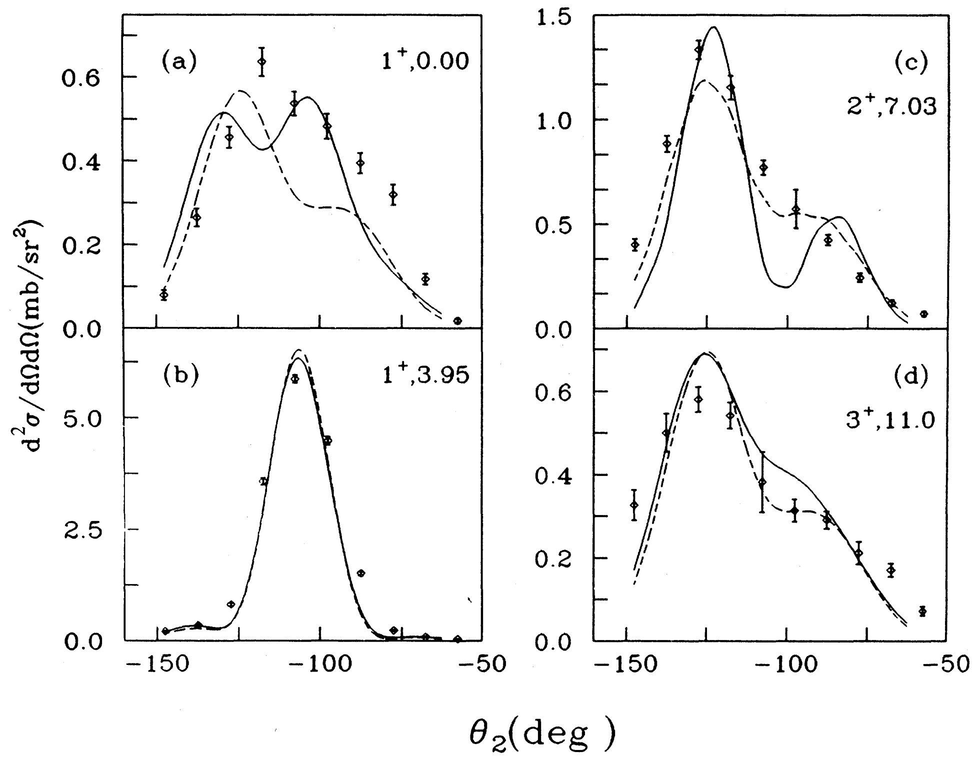

FIG. 5. Angular distributions for the $^{16}O(\pi^+,2p)^{14}N$ (J^π) reaction at $T_\pi=116$ MeV with one proton detected at 50°. The data are from Schumacher *et al.* (Ref. 1): (a) 1^+, ground state, (b) 1^+, 3.95 MeV, (c) 2^+, 7.0 MeV, and (d) 3^+, 11.0 MeV. The solid curves are DWIA calculations normalized to the experimental data. The DWIA calculations have been multiplied by (a) 3.0, (b) 6.7, (c) 4.2, and (d) 1.9. The dashed curves are the SPA calculations with the same normalization factors.

small. Comparisons with earlier (SPA) calculations which ignored both these effects showed that there were some significant changes for $L=2$, $J=1,2$ transitions, modest changes for $L=2$, $J=3$, and negligible differences for $L=0, J=1$.

For the closed-shell target considered, it was shown that sum rules could be applied. As a result, in the approximation that both spin-orbit effects and binding-energy differences could be neglected, the earlier SPA should not lead to serious error in describing experimental data which summed over all possible final states.

Using the improved DWIA calculations, our 116 MeV $^{16}O(\pi^+,2p)^{14}N$ experimental data to specific final states was reanalyzed. In general there was somewhat improved agreement between theory and experiment, particularly for the ground-state transition. However, while it is clear that it is important to use the more correct DWIA expressions, the new analysis does not change our earlier conclusions based upon an SPA analysis.

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