Peripheral reactions induced by a 2.1-GeV/nucleon ^{12}C beam on a C target using a simple combinatorial model

A. J. Cole,* P. J. Lindstrom, H. J. Crawford,† and B. G. Harvey
Nuclear Science Division, Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720
(Received 28 July 1988)

Recently acquired data on peripheral reactions induced by a ^{12}C beam at 2.1 GeV/nucleon on a C target have been used to create a distribution of exclusive yields ("mass patterns") in which the charge of detected fragments is ignored and the data supplemented by (presumed) missing nucleons. A simple multinomial expression using probabilities obtained from inclusive mass yields gives a satisfactory account of the revised data.

I. INTRODUCTION

The study of the decay of highly excited nuclei has in recent years received much attention due mainly to the availability of intermediate energy (10–100 MeV/nucleon) heavy-ion beams which have been employed to provoke complete or incomplete fusion reactions.[1–3] Decay of heavy target residues has also been studied[4] using relativistic heavy-ion beams. (References to the main theoretical models can be found in Ref. 3.) In such reactions the decay of the excited parent(s) is best studied with detection systems adapted to the large multiplicities involved. Such multidetectors are somewhat difficult to operate due to the large dynamic range and angular coverage necessary to detect complete events. In this respect measurement of projectile fragments produced in peripheral reactions induced by high-energy beams presents some important advantages, not the least of which is the strong forward focusing provided by the high beam momentum.

The HISS spectrometer project takes advantage of this and, in a recent experiment, succeeded in measuring yields for 404 coincidence channels produced by bombarding targets of C and CH_2 with a 2.1 GeV/nucleon ^{12}C beam at the LBL Bevalac. While providing multidetector capability (at least for near beam rapidity reaction products), the detection system is insensitive to neutrons and is somewhat inefficient for protons. For all heavier isotopes the system is essentially 100% efficient. The experiment is described in detail in Ref. 5 and the "raw" data are currently available in an internal report.[6]

The range of excitation energy for projectile-like fragments extends from a few MeV up to a few hundred MeV.[6,7] Thus we may have to deal with several simultaneously contributing reaction mechanisms ranging from evaporation of cold light fragments from a statistically equilibrated parent though the emission of excited fragments (which in turn decay) from a parent in, or out of, thermal equilibrium up to total explosion or shattering of the parent complex. The parent itself may be produced by inelastic scattering, charge exchange[8] (via delta production), nucleon knockout, or indeed a combination of these three mechanisms. The excitation energy is understood to be produced by nucleon-nucleon collisions, which in the projectile rest frame cause nucleons to drive into the projectile (perpendicular to the beam axis) with average energies of several tens of MeV,[9] or by the delta mediated charge-exchange reaction.

The experimental measurements[10] show that the inclusive observation of beam rapidity fragments from $A=2$ to 12 corresponds to a cross section of 1174 mb in the case of the C target and, by subtraction, to a cross section of 530 mb for a hydrogen (CH_2) target. The relative inclusive isotopic yields estimated from the coincidence data are rather similar for the C and CH_2 targets and agree quite well with the inclusive yields measured independently and reported by Olson *et al.*[10]

The limitation in detection efficiency for protons is partially due to a limitation in momentum acceptance in the Y (vertical) direction, which is produced by the dimensions of the drift chamber outside the HISS magnet. As already mentioned, this limitation was not observed for deuterons and heavier isotopes. Calculations[6] suggest that undetected protons originate mainly from the initial (cascade) phase of the reaction.

We attempt in the present work to gain some understanding of the coincidence event patterns using an extremely simple probabilistic formalism (Sec. III). Before such an analysis may be attempted, however, we "supplement" the data to correct for missing nucleons (neutrons and protons). This procedure is crucial to our understanding of the experiment and is discussed in Sec. II. Despite the success obtained in predicting yields of mass patterns (Sec. IV) we feel that the present analysis provides only a starting point for a more detailed investigation of the reaction mechanism(s).

Accordingly, in Sec. V, besides discussing possible reasons for the success of our simple formalism, we discuss its limitations and investigate its relevance to the sequential binary breakup mechanism. Finally in Sec. VI we present a brief summary of our results, together with suggestions for further study.

II. CORRECTING THE DATA FOR UNDETECTED NUCLEONS

As mentioned in the Introduction, the experimental setup detects some fraction of emitted protons and no emitted neutrons. In Fig. 1 this effect is illustrated for missing protons by plotting series of observed cross sections for X, $X+p$, $X+2p$, etc., where X represents one or more isotopes with mass ≥ 1. As can be seen in the figure, the cross sections are well approximated by allowing nucleons chosen at random from a Gaussian momentum distribution to fall on a rectangular momentum aperture whose width is some fraction f of the Gaussian width.

The similarity of the observed and calculated curves in Fig. 1 leads us to conjecture that all events of the form X, $X+p$, $X+2p$, etc., are, in fact, manifestations of the same event in which missing protons fell outside the momentum detection aperture. Extending this conjecture to include undetected neutrons, we have supposed that all detected events consisted, in fact, of a total mass of 12 units corresponding to the incident projectile. A modified data set has thus been produced by ignoring charge and constructing exclusive events (mass patterns) in which the unobserved mass (12-observed mass) is supposed to consist of free nucleons.

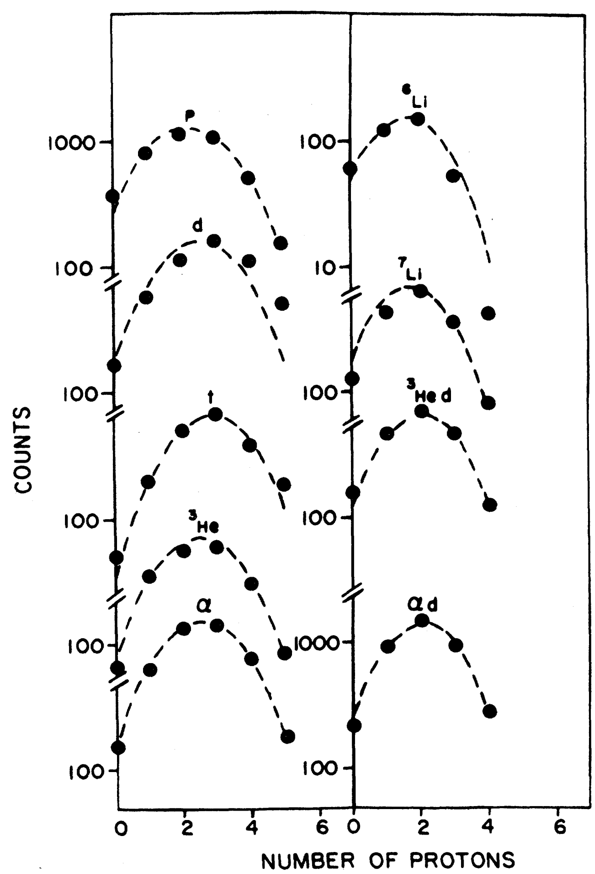

FIG. 1. Cross sections for X, $+k$ protons from the $^{12}C+^{12}C$ reaction at 2.1 GeV/nucleon (Ref. 6). The solid lines are binomial distributions representing the probability that out of K protons chosen at random from a Gaussian momentum distribution (of width σ), k protons fall inside a centered rectangular aperture of width $f\sigma$. In all cases f lies between 0.5 and 0.8.

Because of the presence of the charge-exchange reaction, it is difficult to extend this method to determine the proportion of protons in the missing mass.

The procedure clearly involves only minimal assumptions as to the nature of the reaction mechanism. Projectile nucleons, or heavier constituents, initially moving with beam velocity, may be knocked out of the projectile in the primary reaction phase or may appear as secondary products from excited precursors. It does, however, suppose (in agreement with observation) that no precursor of mass >12 is produced. It also supposes (again in agreement with observation) that all fragments of mass >1 are detected with 100% efficiency.

The modified data set, which thus consists of a set of experimental counts corresponding to each distinct mass pattern, is presented in Table I for all exclusive events with >0 counts.

III. COMBINATORIAL MODEL

Consider a nucleus of mass M which breaks into fragments such that the probability p_m for observing a fragment of mass m is given by the inclusive mass yield σ_m,

$$p_m = \sigma_m / \sum \sigma_m \ . \tag{1}$$

If the probability of emission of a fragment of given mass is unaffected by what happens to the rest of the nucleus (whether or not this emission is sequential) the probability of observing a given mass pattern for an event with multiplicity N is given by the multinomial distribution.

$$P(i_1, i_2, \ldots, i_M) = N! \frac{p_1^{i_1}}{i_1!} \frac{p_2^{i_2}}{i_2!} \frac{p_3^{i_3}}{i_3!} \cdots \frac{p_M^{i_M}}{i_M!} \ , \tag{2}$$

with $\sum m i_m = M$, $\sum i_m = N$.

Before comparing the predictions of Eqs. (2) with the experimental data we should, perhaps, emphasize its limitations:

(a) Dynamical effects are ignored.

(b) No distinction is made between primary and secondary phases of production (as would be true, for example, of a prompt fragmentation and secondary decay mechanism).

(c) The probabilities p_m for producing cold fragments are supposed to be constant or at least in some sense well-defined average values, the average being taken over the ensemble of initial systems.

None of these conditions would be expected to be well satisfied in a practical case. At low excitation energies separation energies for the emission of light particles should induce considerable variation in the emission probabilities along the decay chain. At higher excitation energies, even if we adopt a prompt shattering mechanism,[11] the decay probabilities would be expected to depend on excitation energy, a quantity which in the situation under study is thought to vary over more than an order of magnitude (see Ref. 5). Nonetheless, we expect that Eq. (2) may provide us with a useful baseline estimate so that effects cited above can be observed as the origin of discrepancies between the predictions of Eq. (2) and the experimental observations.

TABLE I. Data for exclusive events (supplemented by missing nucleons) in the reaction $^{12}C+C$ at 2.1 GeV/nucleon arranged in order of decreasing counts.

Channel	Counts	Mass pattern	Channel	Counts	Mass pattern
1	12466	1 11	29	640	1 1 3 7
2	6388	1 1 1 1 4 4	30	574	1 1 1 1 1 1 2 2 2
3	5352	1 1 1 1 1 1 1 1 1 1 2	31	492	1 1 1 1 8
4	5063	1 1 10	32	453	1 1 3 3 4
5	4609	1 1 1 1 1 1 1 1 4	33	266	2 10
6	4210	1 1 1 1 1 1 1 1 1 1 1 1	34	251	1 2 9
7	4195	1 1 1 1 1 1 6	35	141	1 1 1 3 3 3
8	4052	1 1 1 1 1 1 1 2 3	36	135	1 1 1 2 2 2 3
9	4038	1 1 1 1 1 1 1 1 1 3	37	100	1 1 2 2 3 3
10	3950	12	38	84	1 1 2 8
11	3948	1 1 1 1 1 1 2 4	39	73	1 2 2 3 4
12	3910	1 1 1 1 1 3 4	40	73	1 2 2 7
13	3022	1 1 1 1 1 7	41	72	1 1 2 2 2 4
14	2689	1 1 1 1 1 1 1 1 2 2	42	61	1 1 2 2 6
15	1703	1 1 1 9	43	57	1 2 3 6
16	1643	1 1 4 6	44	57	2 4 6
17	1466	1 1 1 1 1 1 3 3	45	51	3 9
18	1456	1 1 2 4 4	46	47	1 1 1 1 2 2 2 2
19	1394	1 4 7	47	31	1 3 8
20	1281	1 1 1 2 3 4	48	22	6 6
21	1278	1 1 1 1 2 3 3	49	22	2 2 4 4
22	1136	1 1 1 1 2 6	50	21	2 3 3 4
23	1109	1 1 1 1 1 2 2 3	51	20	1 2 3 3 3
24	1046	1 1 1 2 7	52	14	2 3 7
25	899	1 1 1 3 6	53	14	4 8
26	852	1 1 1 1 2 2 4	54	5	2 2 2 2 4
27	791	1 3 4 4	55	4	3 3 6
28	678	4 4 4	56	3	2 2 2 6
			57	2	2 2 8

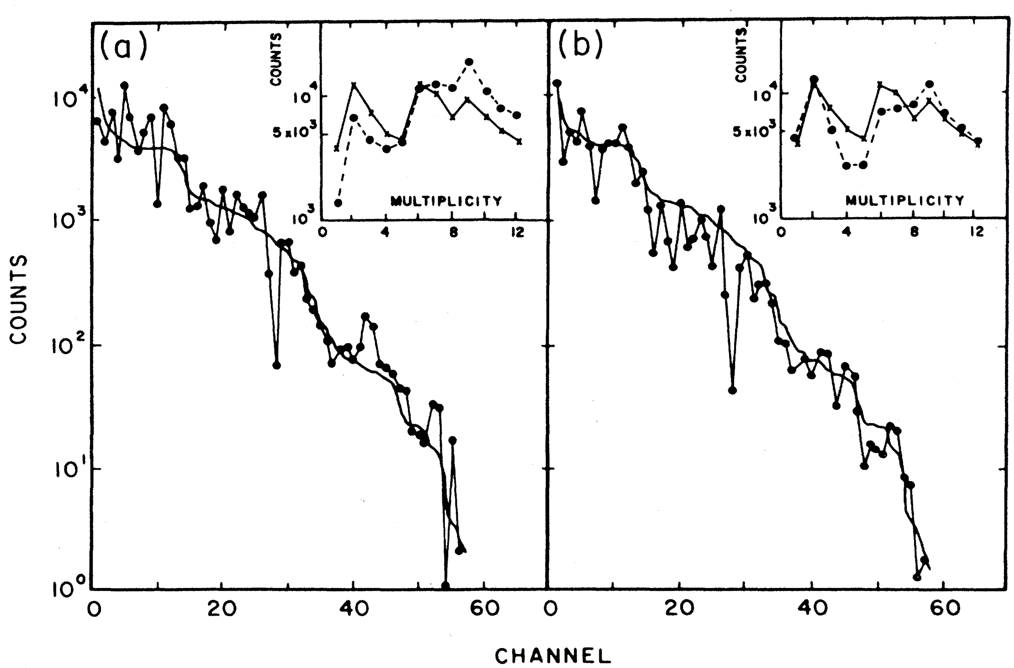

FIG. 2. Cross sections for distinct mass patterns obtained as described in the text. The sold line represents the data in order of decreasing counts as given in Table I. The dotted lines were calculated using Eq. (2). Part (a) depicts the calculation using probabilities obtained from the inclusive mass yields. Part (b) shows the result of searching the probabilities. In each case the calculated multiplicity distribution is shown with the observed distribution (solid line) as an inset.

IV. COMPARISON WITH EXPERIMENT

Predictions were carried out for multiplicities between 1 and 12 using, as input, the inclusive mass distribution observed in the experiment. The result is shown in Fig. 2(a) for the ^{12}C target. In making the predictions the overall normalization (counts/probability) was left as a parameter. It will be clear that the overall trends of the data are well reproduced by the calculation. Indeed, despite the occurrence of outlying data (notably the three-alpha channel) 90% of the predictions agree to within better than a factor of 2 with the data over a range of cross section of 4 orders of magnitude. A similar result for a hydrogen (CH_2) target has already been reported.[12]

Given the uncertainties inherent in our approach we have also tried to fit the data by varying the input probabilities. Figure 2(b) shows that a slight improvement in the description of the data can be obtained mainly due to an increase in the probabilities for high masses. Naturally we have also calculated the multiplicity distribution by direct summation of the subset of probabilities for each multiplicity. The results are shown as insets in Fig. 2. The nucleon probability seems to control the slope of the multiplicity distribution although the reason for this effect is not immediately obvious. The probability distributions themselves are shown in Fig. 3. The searched probabilities are quite close to those obtained using the inclusive data from Eq. (1).

V. DISCUSSION AND COMPARISON WITH A SEQUENTIAL BINARY DECAY CALCULATION

The analysis of the experimental measurements has revealed the surprising result that a first-order treatment, in which all dynamical correlations and ensemble averaging are ignored, reproduces the data on average to within better than a factor of 2 over four orders of magnitude.

It seems certain that the cold fragment production probabilities taken from the observed inclusive mass yields represent in some sense average values over an ensemble of systems in which the most important parameter is probably the excitation energy. Thus it is not surprising that variation of the inclusive probabilities somewhat improves the description of the data. It is indeed remarkable that the fitted probabilities are quite close to the observed relative inclusive yields.

Of course, the method employed here can, in principle, be applied in a straightforward way to data including charge in the event description. In the present case however, a serious obstacle arises from consideration of the charge-exchange contributions which render uncertain the data correction procedure (the proportion of the missing mass that is added as protons or neutrons).

It is, of course, interesting to try to categorize various

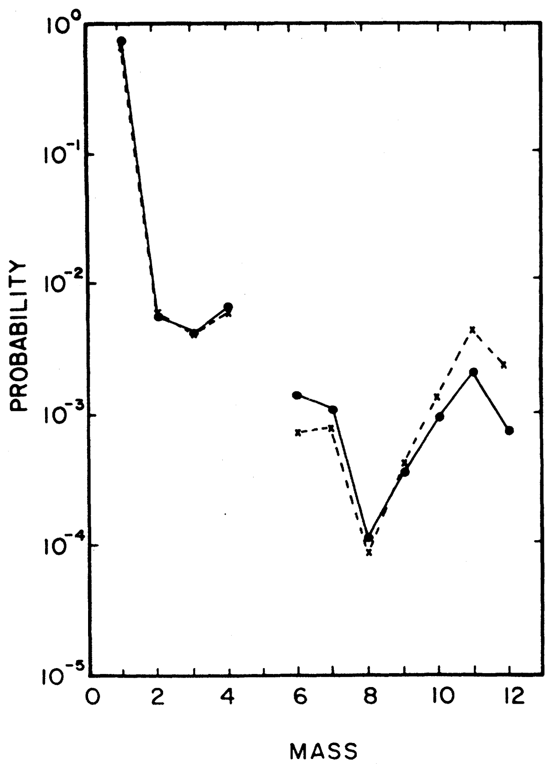

FIG. 3. Probabilities for observing a given mass as a function of mass number (used in the calculations of Fig. 2). The solid line was obtained from the inclusive mass yields using Eq. (1). The dotted line shows the result of searching the probability distribution.

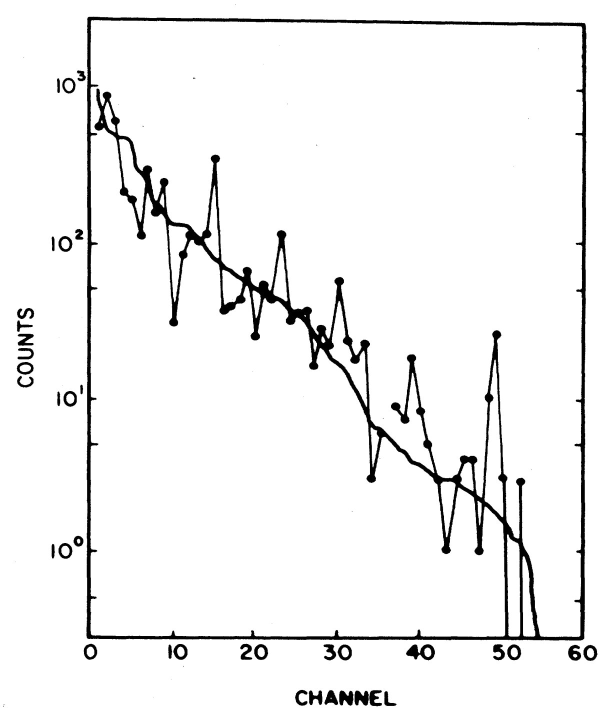

FIG. 4. Same as Fig. 2 except that the solid line represents the calculation [Eq. (2)] generated using inclusive mass yields from the sequential binary decay code (Table II). The dotted line is the binary breakup prediction of exclusive yields (mass patterns). For channel 36, the mass 8–mass 4 decay channel is absent in the decay calculation due to the instability of ^8Be.

TABLE II. Identification of channels (in order to decreasing counts) predicted using Eq. (2) using inclusive probabilities obtained from a binary sequential decay calculation as explained in the text.

Channel	Identification	Channel	Identification
1	1 1 1 1 4	30	2 10
2	12	31	1 1 1 1 1 2 2 3
3	1 1 1	32	1 1 1 1 8
4	1 1 1 1 1 1 1 1 4	33	1 1 1 3 3 3
5	1 1 1 1 1 3 4	34	1 2 2 3 4
6	1 1 1 1 1 1 2 4	35	2 4 6
7	1 1 10	36	4 8
8	1 1 2 4 4	37	1 2 9
9	1 3 4 4	38	1 2 3 6
10	1 1 1 2 3 4	39	2 2 4 4
11	1 1 4 6	40	3 9
12	1 4 7	41	1 1 1 1 1 1 2 2 2
13	1 1 1 1 1 7	42	1 3 8
14	1 1 1 1 1 1 1 1 1 3	43	1 1 2 2 2 4
15	4 4 4	44	2 3 3 4
16	1 1 3 3 4	45	1 1 2 8
17	1 1 1 1 1 1 1 2 3	46	1 1 2 2 6
18	1 1 1 1 1 1 6	47	2 3 7
19	1 1 1 1 1 1 3 3	48	1 2 2 7
20	1 1 1 1 2 2 4	49	6 6
21	1 1 1 1 1 1 1 1 1 1 2	50	1 1 2 2 3 3
22	1 1 3 7	51	3 3 6
23	1 1 1 1 1 1 1 1 1 1 1 1	52	1 1 1 2 2 2 3
24	1 1 1 3 6	53	1 2 3 3 3
25	1 1 1 2 7	54	1 1 1 1 2 2 2 2
26	1 1 1 9	55	3 3 3 3
27	1 1 1 1 2 6	56	2 2 8
28	1 1 1 1 2 3 3	57	2 2 2 6
29	1 1 1 1 1 1 1 1 2 2		

theoretical models of nuclear disintegration in terms of statistical independence for the emission of a given fragment. The so-called quantum statistical models[13] and their classical equivalents[14,15] would be good candidates to conform to Eq. (2). In particular, the work of Fai and Randrup[15,16] is interesting because exclusive data are constructed from inclusive yields. However, the exact connection is difficult to discern because in Ref. 16 the factorization of the exclusive probability leads to successive terms which are, in principle, conditioned by preceding factors [i.e., not independent as in Eq. (2)].

Another problem arises from consideration of the fact that statistical disassembly may produce fragments with excitation energies above particle emission threshold and, therefore, must be supplemented by evaporation cascades in order to produce the final cold products. As explained in Sec. III, sequential evaporation is not, *a priori*, a good candidate for statistical independence of decay probabilities. In order to investigate this point we have performed calculations of the "sequential binary" type[17-19] for the decay of ^{12}C excited with an exponential spectrum $P(E_x)\sim e^{-E_x/75}$ (in accord with that observed in the experiment).[6-19] The inclusive mass yield from the calculation was used to create exclusive yields [using Eq. (2)], and the results compared with exclusive yields obtained directly from the (Monte Carlo) binary decay code. Figure 4 shows the rather surprising result that to within an average factor of <3 the multinomial formula again reproduces the decay code results. (The channel definitions for this figure are given in Table II.) The agreement is somewhat less good than that obtained using the HISS data [Fig. 2(a)], but the two figures are certainly comparable. One exclusive channel (mass 4–mass 8) is absent from the decay calculation due to the instability of ^{8}Be. All other channels predicted by Eq. (2) are present (to within the statistical accuracy) in the binary decay calculation. (It should perhaps be emphasized that the decay calculation was used here simply to investigate a model situation. An attempt to reproduce the data directly from the decay code would involve including parent nuclei produced by charge exchange as well as the ^{12}C itself. Preliminary results of a detailed investigation along these lines can be found in Ref. 19.)

We are thus led to the conclusion that the presence of sequential decay does not, necessarily, perturb the simple picture implied by Eq. (2). This is, of course, something of a disadvantage since it implies that it will be necessary to look beyond the static aspects of the experimental observations (inclusive and exclusive yields) in order to gain a detailed understanding of the reaction mechanism.

Other models for the disassembly of hot nuclei[20-23] have, of course, been proposed. However, none of them would be expected, *a priori*, to yield exclusive cross sections based only on inclusive yields.

VI. SUMMARY AND CONCLUSIONS

In this work we have investigated mass pattern yields for near beam rapidity fragments produced by a 2.1 GeV/nucleon ^{12}C beam impinging on a C target. By completing the data with (presumed) undetected nucleons the multiplicity distribution as well as the yields of exclusive events were obtained.

The analysis of the data has revealed the unexpected fact that a first-order treatment, which assumes the independence of decay probabilities, and which thus provides a multinomial probability distribution for exclusive yields, is quite successful in reproducing the exclusive measurements.

Since the relation between input (inclusive) and calculated (exclusive) probabilities is, in general, nonlinear, it is quite gratifying that searching on the inclusive probabilities produces results which are quite close to the observed inclusive yields.

An extension of the present work is planned in order to include charge as well as mass in the event descriptor. A detailed survey of the various theoretical models currently available would clearly be useful and might lead us to understand the (dynamical) circumstances under which the multinomial prediction might be expected to be relevant.

Finally, of course, we need to employ a well-defined model of the disintegration process (or a set of models) which allows us to predict dynamical properties of the final cold products for a wide range of initial excitation energy. Work along these lines is in progress.[19] The findings of the present investigation strongly suggest that the decay mechanism will be difficult to elucidate unless dynamical (energy, momentum) correlations are studied in detail.

ACKNOWLEDGMENTS

One of the authors (A.J.C.) thanks Lawrence Berkeley Laboratory and the IN2P3 institute of the Centre National de la Recherche Scientifique for support of this project. This work was supported by the U.S. Department of Energy under Contract DE-AC03-76SF00098, and by NASA under Grant NGR-05-003-513.

*Permanent address: Institut des Sciences Nucléaires (IN2P3), 53 Ave. des Martyrs 38026, Grenoble CEDEX, France.

†Permanent address: Space Science Laboratory, University of California, Berkeley, CA 94720.

[1]C. B. Chitwood, D. J. Fields, C. K. Gelbke, D. R. Klesch, W. G. Lynch, M. B. Tsang, R. L. Ferguson, F. E. Obenshain, F. Plasil, R. L. Robinson, and G. R. Young, Phys. Rev. C **34**, 858 (1986); F. Rami, J. P. Coffin, G. Guillaume, B. Heusch, P. Wagner, A. Fahli, and P. Fintz, Nucl. Phys. **A444**, 325 (1985).

[2]U. Lynen, M. Ho, W. Kohn, D. Pelte, U. Winkler, W. F. J. Muller, Y. T. Chu, P. Doll, A. Gobbi, K. Hildenbrand, A. Olmi, H. Sann, H. Stelzer, R. Bock, H. Lohner, R. Glasgow, and R. Santo, Nucl. Phys. **A387**, 129C (1982).

[3]B. Borderie, J. Phys. (Paris) **47**, C4-251 (1986).

[4]A. I. Warwick, H. H. Wieman, H. H. Gutbrod, M. R. Maier, J. Peter, H. G. Ritter, H. Stelzer, and F. Weik, Phys. Rev. C **27**, 1083 (1983).

[5]J. Engelage, H. J. Crawford, I. Flores, O. Hashimoto, S. Nagamiya, I. Tanihata, M. E. Baumgartner, E. Beleal, F. Bieser, M. Bronson, D. E. Greiner, L. Greiner, P. J. Lindstrom, C. McParland, T. J. M. Symons, R. Wada, B. L. Berman, D. L. Olson, P. M. Kirk, C. L. Ruiz, F. P. Brady, J. L. Romero, M. L. Webb, J. B. Carroll, and G. Igo, Lawrence Berkeley Laboratory Internal Report LBL-23867, 1988 (unpublished).

[6]H. J. Crawford, M. E. Baumgartner, J. Engelage, D. E. Greiner, P. J. Lindstrom, D. L. Olson, R. Wada, and M. L. Webb, Lawrence Berkeley Laboratory Internal Report LBL-24380, 1988 (unpublished).

[7]M. L. Webb, H. J. Crawford, J. Engelage, M. E. Baumgartner, D. E. Greiner, P. J. Lindstrom, D. L. Olson, and R. Wada, Phys. Rev. C **36**, 193 (1987).

[8]P. J. Lindstrom, M. E. Baumgartner, H. J. Crawford, J. Engelage, D. E. Greiner, D. L. Olson, R. Wada, and M. L. Webb, Proceedings of the 8th High Energy Heavy-Ion Study, Lawrence Berkeley Laboratory, 1987.

[9]J. Hufner, K. Schafer and B. Schurman, Phys. Rev. C **12**, 1888 (1975).

[10]D. L. Olson, B. L. Berman, D. E. Greiner, H. H. Heckman, P. J. Lindstrom, and H. J. Crawford, Phys. Rev. C **28**, 1602 (1983).

[11]G. Fai and A. Z. Mekjian, Phys. Lett. B **196**, 281 (1987).

[12]A. J. Cole, H. J. Crawford, P. J. Lindstrom, and B. G. Harvey, contributed paper to Third International Conference on Nucleus-Nucleus Collisions, Saint Malo (1988).

[13]H. Stocker, G. Buchwald, G. Graebner, P. Subramanian, J. A. Maruhn, W. Greiner, B. V. Jacak, and G. D. Westfall, Nucl. Phys. **A400**, 63C (1983).

[14]J. Randrup and S. E. Koonin, Nucl. Phys. **A356**, 223 (1981).

[15]G. Fai and J. Randrup, Nucl. Phys. **A381**, 557 (1982).

[16]J. Randrup and G. Fai, Phys. Lett. **115B**, 281 (1982).

[17]W. J. Swiatecki, Aust. J. Phys. **36**, 641 (1983).

[18]F. Auger, B. Berthier, A. Cunsolo, A. Foli, W. Miltig, I. M. Pascaud, E. Plagnol, J. Queber, and J. P. Wieleczko, Phys. Rev. C **35**, 190 (1987).

[19]B. G. Harvey, A. J. Cole, H. J. Crawford, and P. J. Lindstrom, contibuted paper to Third International Conference on Nucleus-Nucleus Collisions, Saint Malo (1988).

[20]W. A. Friedman and W. G. Lynch, Phys. Rev. C **28**, 16 (1983).

[21]D. H. E. Gross and H. Massman, Nucl. Phys. **A471**, 339C (1987).

[22]J. P. Bondorf, R. Donangelo, I. N. Mishustin, C. J. Pethick, H. Schulz, and K. Sneppen, Nucl. Phys. **A443**, 321 (1985).

[23]C. Gregoire, B. Remaud, F. Sebille, L. Vinet, and Y. Raffray, Nucl. Phys. **A465**, 317 (1987).