Nilsson parameter set in the $A \approx 120-140$ region

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New Nilsson (κ,μ) parameter sets for the proton $N=4$, 5, and 6 harmonic oscillator shells are deduced for the $A \approx 120-140$ mass region by fitting a potential-energy-surface calculation, using the Nilsson-Strutinsky method, to 28 experimental bandhead energies. The results from both the standard and the fitted Nilsson (κ,μ) parameter sets are compared with experimental bandhead energies. The fitted (κ,μ) parameter sets achieve a significant improvement in the bandhead energies over this region. In addition, calculated single-particle levels using both the Woods-Saxon and the Nilsson potentials are compared.

Considerable experimental and theoretical interest has developed over a period of time in regard to the influence of rotation towards nuclear structure, pairing correlations and shapes of nuclei including the interplay among them. In this regard, the modified harmonic-oscillator potential introduced by Nilsson *et al.*[1] has been extensively employed in the interpolation of deformed nuclei.[2] There are two shell model parameters, κ and μ, which appear in the potential

$$V=-\kappa\hbar\omega_0[2\mathbf{l}_t\cdot\mathbf{s}+\mu(\mathbf{l}_t^2-\langle\mathbf{l}_t^2\rangle_N)]\ ,$$

where ω_0 is the harmonic-oscillator parameter which incorporates the principle of volume conservation for nuclei deformed from spherical shapes.[1] The intrinsic nucleon spin is represented by $\mathbf{s}$, while $\mathbf{l}_t$ represents the orbital angular momentum in the stretched coordinate basis.

The approach of Nilsson *et al.*[1] was to find a given (κ,μ) parameter pair for the deformed potential that reproduced the experimental level scheme for the extensively studied nuclei of the deformed rare-earth region ($A \approx 165$) and another (κ,μ) pair for those nuclei in the strongly deformed actinide region ($A \approx 240$). For other regions where only limited experimental data were available at the time, the appropriate (κ,μ) pair was formed by extrapolations from the above sets of parameters assuming linear dependences on the mass number A. Better fits to the experimental results for such extrapolated regions were obtained by Bengtsson and Ragnarsson[3,4] using a (κ,μ) dependence related to the main oscillator quantum number N, rather than A. Although the resulting so-called "standard set" of (κ,μ) values for each oscillator shell are better in the extrapolated mass regions, improvements could still be made in the (κ,μ) parameter set by searching for the best theoretical fit to experimental bandhead energies, as was done[5] in the Au-Pt region.

Recent interest in the deformed $A \approx 130$ mass region has developed in regard to the collective and quasiparticle-structure properties as a function of rotational frequency. An extensive set of spectroscopic information[6] has resulted in the availability of systematic bandhead energies over the region $53 \le Z \le 63$ and $64 \le N \le 80$. These data on the relative positions of bands built on the $g_{7/2}$, $g_{9/2}$, and $h_{11/2}$ proton Nilsson orbitals allow the extraction of an improved (κ,μ) set for protons in the $N=4$, 5, and 6 oscillator shells from theoretical fits in the $A \approx 130$ region. Since the $N=5$ and 6 neutron oscillator shells, and to some extent the $N=4$ shell, have already been fitted for appropriate (κ,μ) parameters with the extensive data available in the heavier rare-earth region, no significant improvements could be made to the (κ,μ) parameters for the neutron orbitals from the existing data in the $A \approx 130$ region.

Our approach to achieving an improved (κ,μ) parameter set for the $A \approx 130$ region was to search for the best theoretical fit to the available experimental bandhead energies in odd-proton nuclei over the whole $A \approx 120-140$ region in order to provide a uniform emphasis. High-spin orbitals that can be observed above the Fermi surface are very sensitive to the (κ,μ) parameters. Thus we used the bandhead energies for the $g_{9/2}$ and $h_{11/2}$ proton orbitals relative to the ground states, which in most cases involved $g_{7/2}$ proton orbitals. These fits, which were made over the region defined by $53 \le Z \le 63$ and $64 \le N \le 80$, included 28 experimental bandhead energies.[6] The resulting (κ,μ) parameter set for the best fit is given in Table I. The new (κ,μ) parameter set involves changes in the $N=4$, 5, and 6 proton shells. The mean-square-root deviation between the calculated and experimental values for these 28 cases, which is 389 keV for the standard (κ,μ) set, was significantly reduced to 155 keV for the fitted (κ,μ) parameter set.

The theoretical fitting of this (κ,μ) parameter set involved a minimization calculation of the potential-energy surface using the Nilsson-Strutinsky method[1] for the different proton configurations in this mass region. The mesh points for the calculation covered the deformation parameters ϵ_2 from -0.28 to 0.32 and ϵ_4 from -0.04 to 0.08 (both with step size of 0.04) under the assumption that the triaxial asymmetry parameter $\gamma=0°$. More than 30 sets of proton (κ,μ) parameters for the $N=4$, 5, and 6 oscillator shells were tried in order to achieve the best fit to the experimental bandhead energies. The set of (κ,μ) parameters which gave the best fit is shown in Table I.

TABLE I. (κ,μ) values as a function of the oscillator shell N for protons and neutrons. The values of Bengtsson and Ragnarsson (Ref. 4) are given, along with the values obtained from the current analysis, which are italicized in the table.

	Protons				Neutrons	
	Standard		Fitted		Standard	
N	κ	μ	κ	μ	κ	μ
0	0.120	0.00	0.120	0.00	0.120	0.00
1	0.120	0.00	0.120	0.00	0.120	0.00
2	0.105	0.00	0.105	0.00	0.105	0.00
3	0.090	0.30	0.090	0.30	0.090	0.25
4	0.065	0.57	*0.070*	*0.48*	0.070	0.39
5	0.060	0.65	*0.056*	*0.54*	0.062	0.43
6	0.054	0.69	*0.054*	*0.52*	0.062	0.34
7	0.054	0.69	0.054	0.69	0.062	0.26
8	0.054	0.69	0.054	0.69	0.062	0.26

To demonstrate the improvement made with the fitted (κ,μ) parameter set for this $A \approx 120-140$ region, the calculated bandhead energies for both the $h_{11/2}$ and $g_{9/2}$ proton orbitals using the two sets of parameters are compared with the experimental values as a function of neutron number for the I ($Z=53$) isotopes in Fig. 1. This isotope chain represents one of the poorer fitted se-

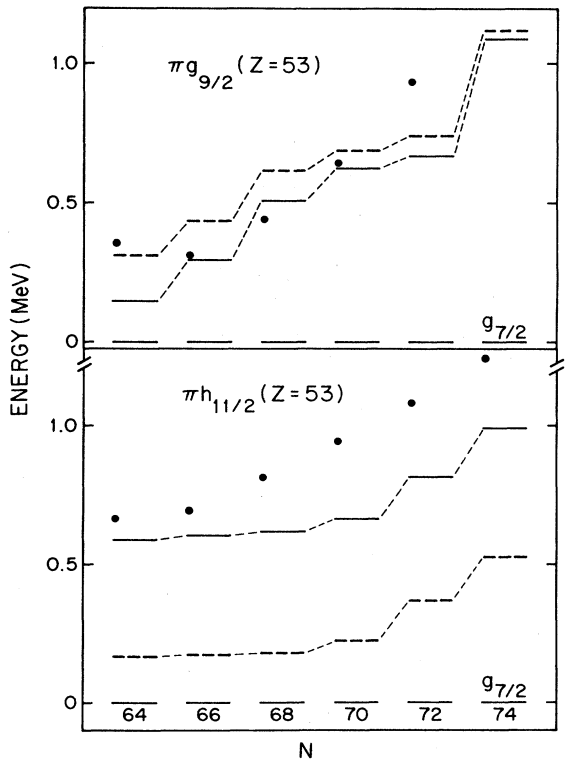

FIG. 1. Calculated bandhead energies of both $h_{11/2}$ and $g_{9/2}$ orbitals for I ($Z=53$) isotopes with two different Nilsson potential parameters, the new fitted κ,μ (solid line) and the standard κ,μ (dashed line). Experimental values (filled circle) are also presented in the figure.

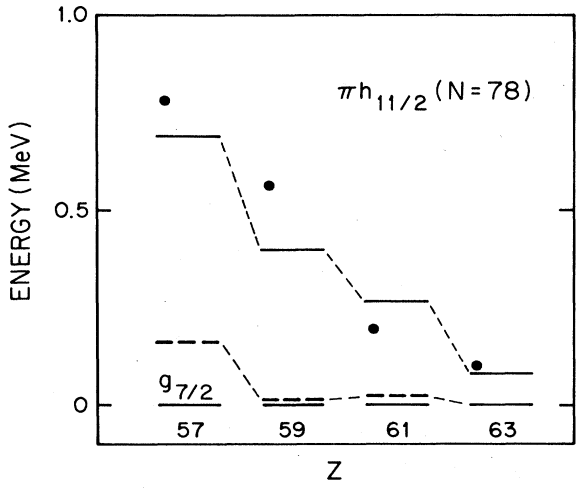

FIG. 2. Calculated bandhead energies of the $h_{11/2}$ orbital for the $N=78$ isotones with two different Nilsson potential parameters, the new fitted κ,μ (solid line) and the standard κ,μ (dashed line). Experimental values (filled circle) are also presented in the figure.

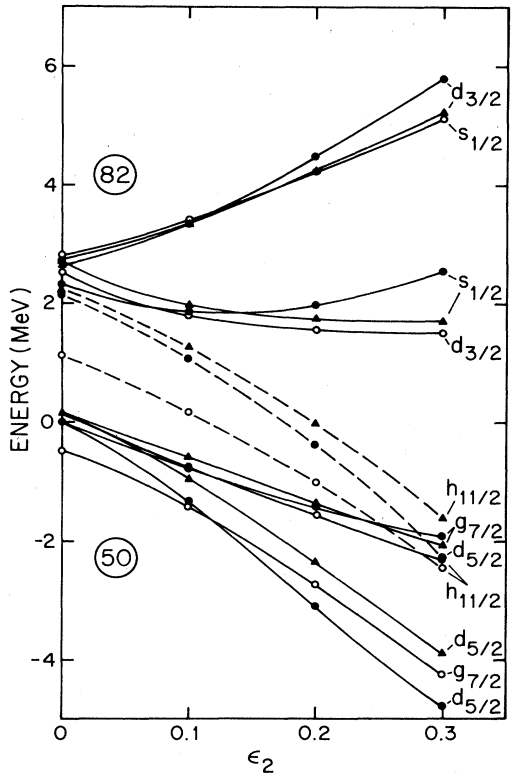

FIG. 3. Comparison of the energies of proton single-particle levels as a function of quadrupole deformation calculated for Nilsson and Woods-Saxon potentials. The filled circles are the results from the Woods-Saxon potential, while the open circles and the filled triangles are the results from the Nilsson potential with standard κ,μ and fitted κ,μ, respectively. For simplicity, only the $K=\frac{1}{2}$ levels are shown in the comparison. The energies from the Nilsson potential are normalized to the energy of the $d_{5/2}$ state at $\beta_2=\beta_4=0$ from the Woods-Saxon potential.

TABLE II. Conversion (Ref. 8) of the (ϵ_2,ϵ_4) parameter set used for the Nilsson potential to the (β_2,β_4) parameter set used in the Woods-Saxon potential calculation.

ϵ_2	ϵ_4	β_2	β_4
0	0	0	0
0.1	0	0.106	0.004
0.2	0	0.215	0.016
0.3	0	0.329	0.040

quences of this region. To document the Z dependence of the fitting, the theoretical and experimental comparison for the $h_{11/2}$ proton orbital in the $N=78$ isotones is shown in Fig. 2. As can be seen from these two figures, the fitted (κ,μ) parameter set produces a systematic improvement over the standard set.

Because there are some uncertainties about the theoretical treatment such as with pairing correlations and because the experimental single-particle levels cannot be directly extracted for a theoretical comparison, it is appropriate to compare the calculated single-particle structure using different theoretical potentials. Both the Woods-Saxon and Nilsson potentials have been successfully used in the well deformed rare-earth and actinide regions. In addition, the universal parameters for the Woods-Saxon potential are known to reproduce the nuclear behavior and single-particle structure in many regions. Thus a comparison of the Woods-Saxon single-particle level scheme with the Nilsson scheme in the $A\approx 120-140$ region is useful as a further check on the current fitted (κ,μ) parameter set. Figure 3 shows proton single-particle energies as a function of quadrupole deformation calculated from the Woods-Saxon potential. These values are compared in the same figure with those calculated from the Nilsson potential using both the standard set and the fitted (κ,μ) parameters. The conversion[8] from the (ϵ_2,ϵ_4) parameters used in the Nilsson potential to the (β_2,β_4) parameters used in the Woods-Saxon potential is listed in Table II. Only $K=\frac{1}{2}$ levels are shown in the comparison for simplicity. It can be seen that the fitted (κ,μ) parameter set agrees considerably better with the Woods-Saxon result as the deformation increases than the standard set. The standard set seems to reverse the role between the $g_{7/2}$ and $d_{5/2}$ proton orbitals as well as the $d_{3/2}$ and $s_{1/2}$ orbitals. This theoretical comparison corroborates the systematic improvement in the experimental comparison for the fitted (κ,μ) parameter set of the Nilsson potential.

As a further evaluation of the fitted Nilsson (κ,μ) parameter sets, a systematic study[7] of rotational crossing frequencies, alignment processes, and interactions between the ground-state bands and the multiquasiparticle bands, is currently being undertaken. Theoretical calculations for this study are being made with the cranked shell model using these (κ,μ) parameters. These results will be compared to the extensive experimental information available in this mass region.

In summary, new Nilsson (κ,μ) parameter sets for the $N=4$, 5, and 6 proton harmonic oscillator shells are deduced for the $A\approx 120-140$ mass region. These Nilsson (κ,μ) parameter sets reproduce the experimental bandhead energies with the mean-square-root deviation better by more than a factor of 2 compared to that from the standard parameter set. In addition, the Nilsson potential with the new parameter set compared well with the Woods-Saxon potential in regard to calculated single-particle energy levels in this mass region.

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[1]S. G. Nilsson, C. F. Tsang, A. Sobiczewski, Z. Szymanski, S. Wycech, C. Gustafson, I. Lamm, P. Möller, and B. Nilsson, Nucl. Phys. **A131**, 1 (1969).

[2]A. Bohr and B. R. Mottelson, *Nuclear Structure* (Benjamin, New York, 1975), Vol. II.

[3]I. Ragnarsson, in *Future Directions in Studies of Nuclei far from Stability,* edited by J. Hamilton *et al.* (North-Holland, Amsterdam, 1980), p. 367.

[4]T. Bengtsson and I. Ragnarsson, Nucl. Phys. **A436**, 14 (1985).

[5]Jing-ye Zhang, A. J. Larabee, and L. L. Riedinger, J. Phys. G **13**, L75 (1987).

[6]R. E. Shroy, D. M. Gordon, M. Gai, D. B. Fossan, and A. K. Gaigalas, Phys. Rev. C **26**, 1089 (1982); M. Gai, D. M. Gordon, R. E. Shroy, D. B. Fossan, and A. K. Gaigalas, *ibid.* **26**, 1101 (1982); U. Garg, T. P. Sjoreen, and D. B. Fossan, *ibid.* **19**, 217 (1979); E. S. Paul, C. W. Beausang, D. B. Fossan, R. Ma, W. F. Piel, Jr., N. Xu, L. Hildingsson, and G. A. Leander, Phys. Rev. Lett. **58**, 984 (1987); J. Chiba, R. S. Hayano, M. Sekimoto, H. Nakayama, and K. Nakai, J. Phys. Soc. Jpn. **43**, 1109 (1977); L. Hildingsson, C. W. Beausang, D. B. Fossan, and W. F. Peil, Jr., Phys. Rev. C **37**, 985 (1988); T. M. Semkow, D. G. Sarantites, K. Honkanen, V. Abenante, L. A. Adler, C. Baktash, N. R. Johnson, I. Y. Lee, M. Oshima, Y. Schutz, Y. S. Chen, J. X. Saladin, C. Y. Chen, O. Dietzsch, A. J. Larabee, L. L. Riedinger, and H. C. Griffin, *ibid.* **34**, 523 (1986); N. Xu, C. W. Beausang, R. Ma, E. S. Paul, W. F. Piel, Jr., P. K. Weng, D. B. Fossan, and L. Hildingsson, Bull. Am. Phys. Soc. **32**, 1563 (1987); E. Drägulescu, M. Ivascu, M. Ionescu-bujoy, A. Iordaghescu, C. Petrache, D. Popsecu, G. Pascovici, G. Semenescu, M. Duma, I. Gurgu, F. Baciu, R. Meyer, V. Paar, S. Brant, D. Vorkapic, D. Vretenar, and H. Ajazaj, Roum. Phys. **32**, 485 (1987); R. Aryaeinejad and Wm. C. McHarris, Michigan State University Cyclotron Laboratory Report No. MSUCL-619, 1987; M. Piiparien, M. Kortelahti, T. Komppa, and R. Komu, Nucl. Phys. **A342**, 53 (1980); N. Xu, C. W. Beausang, E. S. Paul, W. F. Piel, Jr., P. K. Weng, and D. B. Fossan, Phys. Rev. C **36**, 1649 (1987); L. K. Peker, Nucl. Data Sheets **45**, 47 (1985); N. Xu, K. Ahn, Y. Liang, R. Ma, E. S. Paul, W. F. Piel, Jr., and D. B. Fossan, Bull. Am. Phys. Soc. **33**, 981 (1988); S. Lunardi, F. Sormel, W. Starzecki, W. Meczynski, R. Julin, M. Lach, A. Ercan, and P. Kleinheinz, Z. Phys. A **324**, 433 (1986); L. K. Peker, Nucl. Data Sheets **48**, 769 (1986).

[7]N. Xu *et al.* (unpublished).

[8]I. Ragnarsson, private communication (1985).