Detailed-balance test of time-reversal symmetry for a pair of close-lying resonances

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We present a thorough analysis of tests of detailed balance involving a pair of interfering resonances. For two close-lying (overlapping) compound-nucleus resonances in a regime where the *average* resonance spacing is much larger than the *average* resonance width, we find enhancement factors as large as 10^3-10^4.

I. INTRODUCTION

Nuclear reactions have long been used to test detailed balance and, thus, time-reversal symmetry (see, e.g., Ref. 1 and other references therein). Several experiments[2,3] were carried out in the domain of isolated resonances for which a theoretical analysis[4] is available. After the observation was made[5] that, in the two-channel case, detailed balance follows from unitarity alone, theoretical[6–10] and experimental[1,11] interest shifted to the domain of many open channels and strongly overlapping compound nuclear resonances.

It is the purpose of this paper to focus attention once again onto the region of weakly overlapping resonances, because this domain does not seem to have been fully explored as yet, either theoretically or experimentally. In the theoretical analysis, two attitudes can be taken. One may study violation of detailed balance in the *energy-averaged* quantities, or one may investigate explicitly a case involving a few (in our case, a pair of) compound nucleus resonances, and ask for the optimal conditions for time-reversal breaking to be observed. The first approach will yield expressions for observables measuring violation of detailed balance that depend only on simple, well-understood average nuclear properties, and on the strength of the time-reversal-symmetry breaking part H' of the Hamiltonian. Such expressions can usefully be employed in the analysis of experimental data to establish unambiguous upper bounds on H'. This approach was taken recently.[12] The authors identified an observable which shows a strong enhancement of symmetry violation in the domain of weakly overlapping resonances. Experiments in this domain may lower existing upper bounds on H' by at least an order of magnitude. The second approach identifies the special situation most sensitive to a possible detection of time-reversal-symmetry violation. In this approach, a negative result is somewhat more difficult to interpret than in the first one: The analysis of the data involves a number of parameters (resonance energies, partial widths, scattering phase shifts, etc.) which are poorly known in part, and which are difficult to determine precisely. Moreover, the analysis—even if successful—will only yield a bound on a single matrix element of H'. Because of the expected stochastic behavior of H' in the compound-nucleus regime, such a bound carries limited information. (We quantify this statement at the end of the paper.) On the other hand, the sensitivity of the analysis is, by the very nature of the method, bigger in the second than in the first approach. Therefore the two approaches complement each other, and the present paper can be viewed as a complement to Ref. 12.

In this paper, we consider the case of a pair of resonances and take account of interference terms not considered in Ref. 4. It has been shown in Ref. 13 that such terms might cause a significant "dynamical" enhancement of order $N^{1/2}\cong(D_0/D)^{1/2}$. Here, N is the number of simple-structure components typically building up the compound-nucleus wave function, and D_0 and D are the average spacings of the single-particle and compound states, respectively. We show that under suitable conditions the enhancement might even be bigger. We also investigate the optimal choice of theoretical measures of symmetry violation, and the statistical significance of upper bounds deduced from experiments of this type.

The simplest quantity to use in tests of detailed balance is

$$\Delta(E)=2\frac{\sigma_{ab}(E)-\sigma_{ba}(E)}{\sigma_{ab}(E)+\sigma_{ba}(E)} . \tag{1}$$

Here, a and b denote channels, and $\sigma_{ab}(E)$ is the cross section for the reaction $a\rightarrow b$ at energy E. Because of the difficulty to measure with sufficient accuracy absolute cross sections, it is advisable to consider instead of $\Delta(E)$ the observable

$$\tilde{\Delta}(E_{\rm I},E_{\rm II})=\frac{\sigma_{ab}(E_{\rm I})\sigma_{ba}(E_{\rm II})}{\sigma_{ab}(E_{\rm II})\sigma_{ba}(E_{\rm I})}-1 , \tag{2}$$

which involves only relative cross sections and can therefore be determined much more precisely. To first order in $\Delta(E)$, we have

$$\tilde{\Delta}(E_{\rm I},E_{\rm II})\cong\Delta(E_{\rm I})-\Delta(E_{\rm II}) . \tag{3}$$

Therefore we proceed to work with the simpler form (1).

In Sec. II, we consider an idealized case—two well-separated resonances without background, and investigate when $\tilde{\Delta}$ becomes maximal. In Sec. III, we include in the analysis both an incoherent and a coherent background. We are led to the conclusion that for well-separated resonances a sizeable resonance enhancement does not exist, due to background problems. This leads us to consider in Sec. IV the case of two close-lying (overlapping) resonances and the identification of a resonance enhancement. Our result is analyzed in Sec. V in terms of an enhancement factor and a statistical significance investigation.

II. TWO WELL-SEPARATED RESONANCES WITHOUT BACKGROUND

To work out $\Delta(E)$, we use the general expression obtained in Ref. 14 for the difference $\delta S_{ab}=S_{ab}-S_{ba}$ between the S-matrix elements connecting channels a and b caused by the presence of a time-reversal-symmetry breaking (T-breaking) part H' in the Hamiltonian. To first order in H', and in the case of two resonances with complex resonance energies ξ_1 and ξ_2, partial widths Γ_{1a} and Γ_{2a}, and resonance wave functions Ψ_1 and Ψ_2, we have from Eq. (14) of Ref. 14

$$\delta S_{ab}=-2(2\pi)^{1/2}\left[\sum_{j=1}^{2}(E-\xi_j)^{-1}(\Gamma_{jb}^{1/2}\operatorname{Im}\langle\Psi_E^{(a)}|H'|\Psi_j\rangle-\Gamma_{ja}^{1/2}\operatorname{Im}\langle\Psi_E^{(b)}|H'|\Psi_j\rangle)\right.$$
$$\left.+2i(E-\xi_1)^{-1}(E-\xi_2)^{-1}(\Gamma_{2a}^{1/2}\mathcal{H}'_{21}\Gamma_{1b}^{1/2}-\Gamma_{2b}^{1/2}\mathcal{H}'_{21}\Gamma_{1a}^{1/2})\right]. \tag{4}$$

The quantities $\Psi_E^{(a)}$ are the channel wave functions, and $\mathcal{H}'_{21}=\langle\Psi_2|H'|\Psi_1\rangle$ defines the mixing of compound resonances 1 and 2 caused by the perturbation H'. We have[14] $\mathcal{H}'_{21}=-\mathcal{H}'_{12}$. The matrix elements $\langle\Psi_E^{(a)}|H'|\Psi_j\rangle$ describe compound-nucleus decay via the T-violating interaction. In Eq. (4) we have considered two resonances in the absence of any background originating from direct reactions, or from far-lying resonances. This restriction is lifted in the next section.

It is our aim to maximize $\Delta(E)$ as expressed in terms of δS_{ab} and $S_{ab}^{(0)}=\frac{1}{2}(S_{ab}+S_{ba})$. To this end, we introduce some simplifications. Since only the last term on the right-hand side of Eq. (4) contains the resonance enhancement,[6,13,14] we focus attention on this term and omit the terms proportional to the matrix elements $\langle\Psi_E^{(a)}|H'|\Psi_j\rangle$. Moreover, we write $\sigma_{ab}=|S_{ab}|^2$. The omission of kinematical factors in this relation is justified because we evaluate the normalization-independent quantity $\tilde{\Delta}$ in Eq. (3). The restriction to a single entrance and a single exit channel is lifted in Sec. III.

We have, to first order in H',

$$\sigma_{ab}-\sigma_{ba}=+2[\operatorname{Re}(\delta S_{ab})\operatorname{Re}(S_{ab}^{(0)})+\operatorname{Im}(\delta S_{ab})\operatorname{Im}(S_{ab}^{(0)})]$$
$$=2\prod_{j=1}^{2}|E-\xi_j|^{-2}\operatorname{Re}\{\mathcal{H}'_{21}(E-\xi_2^*)[|\Gamma_{1a}|(\Gamma_{1b}^*\Gamma_{2b})^{1/2}-|\Gamma_{1b}|(\Gamma_{1a}^*\Gamma_{2a})^{1/2}]$$
$$+\mathcal{H}'_{21}(E-\xi_1^*)[|\Gamma_{2b}|(\Gamma_{1a}\Gamma_{2a}^*)^{1/2}-|\Gamma_{2a}|(\Gamma_{1b}\Gamma_{2b}^*)^{1/2}]\} \tag{5}$$

and, to zeroth order in H',

$$\tfrac{1}{2}(\sigma_{ab}+\sigma_{ba})=|S_{ab}^{(0)}|^2$$
$$=\prod_{j=1}^{2}|E-\xi_j|^{-2}|(\Gamma_{1a}\Gamma_{1b})^{1/2}(E-\xi_2)+(\Gamma_{2a}\Gamma_{2b})^{1/2}(E-\xi_1)|^2\,. \tag{6}$$

We consider the case of very weakly overlapping resonances. It is characterized by the assumption that the "external" coupling between compound resonances via the open channels can be neglected in comparison with the "internal" coupling via the matrix elements of the Hamiltonian taken with respect to the resonance wave functions. Formally, this means that we omit the last term on the right-hand side of Eq. (3) of Ref. 14 in the determination of the orthogonal matrix introduced in Eq. (9) of the same reference. Then this orthogonal matrix is real and, as a consequence, all $\Gamma_{ma}^{1/2}$ carry, for all m but fixed a, the *same* (potential scattering) phase shift. Moreover, the matrix $\mathcal{H}'_{mn}$ is purely imaginary, and can be written, for $m<n$, as $\mathcal{H}'_{mn}=iv_{mn}$. Then v_{mn} is real and symmetric. Expressions (5) and (6) simplify to

$$\Delta(E)=v_{12}(|\Gamma_{1b}\Gamma_{2a}|^{1/2}-|\Gamma_{2b}\Gamma_{1a}|^{1/2})(\Gamma_1|\Gamma_{2a}\Gamma_{2b}|^{1/2}+\Gamma_2|\Gamma_{1a}\Gamma_{1b}|^{1/2})$$
$$\times\{[(E-E_2)|\Gamma_{1a}\Gamma_{1b}|^{1/2}+(E-E_1)|\Gamma_{2a}\Gamma_{2b}|^{1/2}]^2+\tfrac{1}{4}(\Gamma_2|\Gamma_{1a}\Gamma_{1b}|^{1/2}+\Gamma_1|\Gamma_{2a}\Gamma_{2b}|^{1/2})^2\}^{-1}\,. \tag{7}$$

We have used $\xi_j=E_j-\frac{1}{2}i\Gamma_j$, $j=1,2$. Equation (7) shows that $\Delta(E)$ is maximal when E is at the interference minimum of the cross section $|S_{ab}^{(0)}|^2$, i.e., when

$$E=E_0=\frac{|\Gamma_{1a}\Gamma_{1b}|^{1/2}E_2+|\Gamma_{2a}\Gamma_{2b}|^{1/2}E_1}{|\Gamma_{1a}\Gamma_{1b}|^{1/2}+|\Gamma_{2a}\Gamma_{2b}|^{1/2}}\,. \tag{8}$$

With $\Gamma_1 \simeq \Gamma_2 \cong \Gamma$ where Γ is the average resonance width (a condition often met for isolated resonances where the many open gamma channels contribute more strongly to the total width than the particle channels) this yields for $\Delta(E_0)$

$$\Delta(E_0) \cong 4\frac{v_{12}}{\Gamma} \frac{|\Gamma_{2b}\Gamma_{1a}|^{1/2} - |\Gamma_{1b}\Gamma_{2a}|^{1/2}}{|\Gamma_{1a}\Gamma_{1b}|^{1/2} + |\Gamma_{2a}\Gamma_{2b}|^{1/2}} . \tag{9}$$

For simplicity, we sometimes consider the special choice $|\Gamma_{1a}\Gamma_{1b}| \cong |\Gamma_{2a}\Gamma_{2b}|$. Then,

$$\Delta(E_0) = 2\frac{v_{12}}{\Gamma} \left[\left| \frac{\Gamma_{2b}}{\Gamma_{1b}} \right|^{1/2} - \left| \frac{\Gamma_{2a}}{\Gamma_{1a}} \right|^{1/2} \right] \tag{10}$$

and $E_0 \cong \frac{1}{2}(E_1 + E_2)$. This essentially is the result of Ref. 14 where the same choice was made. We observe that the maximum value (9) of $\Delta(E_0)$ is attained only in a narrow neighborhood $|E - E_0| \le \Gamma$ of E_0. Outside of this interval, $\Delta(E)$ decreases rapidly, and at most other energies in $|E - \frac{1}{2}(E_1 + E_2)| \le D$ is reduced by a factor of order $(\Gamma/D)^2 \ll 1$ (we assume here that $|E_1 - E_2| \approx D$). The function $\Delta(E)$ thus shows a "resonance enhancement" near $E = E_0$. This enhancement is correlated with a sharp minimum of $\sigma_{ab}(E)$ at $E = E_0$, where, with $\gamma_{ab} = |\Gamma_{1a}\Gamma_{1b}|^{1/2} = |\Gamma_{2a}\Gamma_{2b}|^{1/2}$, we have

$$\sigma_{ab}(E_0) = 2^4 \left[\frac{\Gamma}{D} \right]^2 \left[\frac{\gamma_{ab}}{D} \right]^2 . \tag{11}$$

This value is extremely small in comparison with the value $2^4(\gamma_{ab}/D)^2$ typically attained outside the minimum. We note that the parameter D is conspicuously absent in Eqs. (9) and (10). This is counterintuitive inasmuch as for isolated resonances the energy scale is defined by D. We now show that this unphysical feature results from the neglect of the background contribution.

III. TWO WELL-SEPARATED RESONANCES WITH BACKGROUND

We investigate the condition under which $\Delta(E)$ attains a maximum in the more realistic situation where a background is included. We take this background amplitude T_{ab} to be independent of energy, and to be T conserving, $T_{ab} = T_{ba}$, since T_{ab} cannot show any resonance enhancement. With γ_{ab} the mean value of $|\Gamma_{ja}\Gamma_{jb}|^{1/2}$ over many resonances j, we estimate the magnitude $|T_{ab}|$ of T_{ab} as γ_{ab}/D. Note that we consider cases where, for the two resonances under consideration, $|\Gamma_{1a}\Gamma_{2b}|^{1/2}$ and/or $|\Gamma_{2a}\Gamma_{2b}|^{1/2}$ may differ from γ_{ab}.

In preparation of a more realistic treatment, we first consider the case of an incoherent background. It might be due, for example, to systematic noise in the detectors, or to incoherent channels free of resonance contributions. We see from Eq. (11) that because of the very small size of $\sigma_{ab}(E_0)$, the inclusion of an incoherent background $|T_{ab}|^2$ reduces the estimate (10) by a factor $\sigma_{ab}(E_0)/|T_{ab}|^2$ which even for $|T_{ab}|^2 \simeq \gamma_{ab}^2/D^2$ (a condition which ensures that the resonance curves can be observed over an interval of length $\simeq D$) destroys the resonance enhancement factor $(D/\Gamma)^2$ in this estimate.

However, inspection of expression (7) modified by the inclusion of an incoherent background shows that a resonance enhancement still persists, provided that we consider a more asymmetric situation for the partial widths. If $\Gamma_{2a}\Gamma_{2b} = (\Gamma/2D)^2\Gamma_{1a}\Gamma_{1b}$ then the maximum in $\Delta(E)$ is shifted to

$$E_{\max} = (2DE_2 + \Gamma E_1)/(2D + \Gamma) \cong E_2 ,$$

the minimum value of σ_{ab} at $E = E_{\max}$ is given by $\Gamma_{1a}\Gamma_{1b}/D^2$ and thus is much larger than in the previous case (we put again $|T_{ab}|^2 \simeq |\Gamma_{1a}\Gamma_{1b}|/D^2$), but the estimate (10) (now at $E = E_{\max}$) remains the same and so does the resonance enhancement factor $(D/\Gamma)^2$.

We now consider the case of a *coherent* background, originating from the tails of far-lying resonances. The amplitude $S_{ab}^{(0)}$ contains the additional term $T_{ab}\exp(i\delta_a + i\delta_b)$ with T_{ab} positive and $T_{ab} \simeq \gamma_{ab}/D$, while δ_a, δ_b are the potential phase shifts. The expression for $\Delta(E)$ is now

$$\Delta(E) = \mathrm{Re} \left[\frac{\delta S_{ab}}{S_{ab}^{(0)}} \right] = \mathrm{Re} \left[\frac{2v_{12}(|\Gamma_{1b}\Gamma_{2a}|^{1/2} - |\Gamma_{2b}\Gamma_{1a}|^{1/2})}{|\Gamma_{1a}\Gamma_{1b}|^{1/2}(E - \xi_2) + |\Gamma_{2a}\Gamma_{2b}|^{1/2}(E - \xi_1) + T_{ab}(E - \xi_1)(E - \xi_2)} \right] . \tag{12}$$

We see that $\Delta(E)$ will attain its maximum value when the absolute value of the denominator on the right-hand side of Eq. (12) has its minimum, i.e, in the minimum of the cross section. A more detailed analysis of Eq. (12) shows that for

$$|\Gamma_{1a}\Gamma_{1b}| \cong |\Gamma_{2a}\Gamma_{2b}| \cong \gamma_{ab}^2 \gtrsim T_{ab}^2 D^2$$

the minimum is shifted by $\cong T_{ab}D/(4\gamma_{ab})$ from its original position E_0. For $T_{ab} \lesssim \gamma_{ab}/D$ the maximum value of $\Delta(E)$ is still given by Eq. (10). More importantly, the minimum of the cross section has the small value (11). Hence, the addition of any sizable *incoherent* background (which is always present) to the coherent background just considered will wipe out the enhancement in the same manner as discussed above for the case of a purely incoherent background.

This situation can once again be changed by considering the asymmetric case

$$|\Gamma_{2a}\Gamma_{2b}| \cong \tfrac{1}{4}(\Gamma/D)^2 |\Gamma_{1a}\Gamma_{1b}| .$$

Then the interference minimum shifts to $E_{\max} \cong E_2$, and for $|T_{ab}| \lesssim \gamma_{ab}/D$ the cross section in this minimum increases to a value γ_{ab}^2/D^2, while $\Delta(E)$ at $E=E_{\max}$ retains its value [(9) and (10)] and, hence, the resonance enhancement factor $(D/\Gamma)^2$. For $T_{ab} \gg \gamma_{ab}/D$ there appear two minima at $E \cong E_1$ and $E \cong E_2$ where the cross sections are $\sigma_{ab} \cong |T_{ab}|^2$. Therefore, the maximum of $\Delta(E)$ at these points is reduced by a factor $\gamma_{ab}/TD \ll 1$.

In the presence of direct reactions, the matrix element T_{ab} acquires an additional phase. The situation remains qualitatively the same as for T_{ab} real, provided that $\mathrm{Im}T_{ab} \lesssim \mathrm{Re}T_{ab} \lesssim \gamma_{ab}/D$: In the symmetric case $|\Gamma_{1a}\Gamma_{1b}| \cong |\Gamma_{2a}\Gamma_{2b}|$ the cross section minimum is smaller by a factor $(\Gamma/D)^2$ than the value elsewhere, and the incoherent background therefore destroys the enhancement of $\Delta(E)$. In the strongly asymmetric case

$$|\Gamma_{1a}\Gamma_{1b}| \cong (D/\Gamma)^2 |\Gamma_{2a}\Gamma_{2b}|$$

the enhancement of $\Delta(E)$ persists.

In summary, we see that the introduction of a coherent background with $T_{ab} \lesssim \gamma_{ab}/D$ does not qualitatively change the picture obtained by studying the incoherent background alone: For the symmetrical case, the minimum value of the cross section is so small that the enhancement of $\Delta(E)$ is destroyed upon the inclusion of an incoherent background. One needs to go to the exotic asymmetrical case in order to improve the situation. In other words, the case of two well-separated resonances does not offer much hope for meaningful experimental tests of detailed balance.

IV. TWO CLOSE-LYING RESONANCES

The physical reason for the negative result found in Sec. III is the very smallness of σ_{ab} in the minimum, suppressed in comparison to its value elsewhere by the factor $(\Gamma/D)^2 \ll 1$. A remedy may be found by consideration of a pair of resonances for which this factor accidentally is larger than expected on average. Since the case of two resonances with unusually large widths appears fortuitous, we here focus attention on the case of two close-lying resonances for which the spacing $|E_1 - E_2| = d$ is significantly smaller than the average spacing D. (In this situation one may obviously also allow for a larger background than given by $|T_{ab}| = \gamma_{ab}/D$.) However, this leads us into the situation of two overlapping resonances ($\Gamma \lesssim d$) for which the number of parameters in S_{ab} increases drastically since each of the $\Gamma_{ja}^{1/2} = |\Gamma_{ja}^{1/2}| \exp(i\hat{\delta}_{ja})$ in $S_{ab}(E)$ becomes independently complex. (We do not consider the case $\Gamma > d$ since the repulsion of the two resonances makes this situation rather unlikely.) However, the analysis of this case with $\Gamma \sim d$ and $\sin\hat{\delta}_{ja} \cong \cos\hat{\delta}_{ja}$ shows that for *any* choice of $\Gamma_{1a}\Gamma_{1b}/(\Gamma_{2a}\Gamma_{2b})$ one still gets the maximal effect [(9) and (10)] for $\Delta(E)$ at the minimum $E = E_{\min}$ of the cross section, defined by

$$E_{\min} = \frac{|\Gamma_{1a}\Gamma_{1b}|E_2 + |\Gamma_{2a}\Gamma_{2b}|E_1 + |\Gamma_{1a}\Gamma_{1b}\Gamma_{2a}\Gamma_{2b}|^{1/2}[(E_1+E_2)\cos\phi + \frac{1}{2}(\Gamma_2 - \Gamma_1)\sin\phi]}{|\Gamma_{1a}\Gamma_{1b}| + |\Gamma_{2a}\Gamma_{2b}| + 2|\Gamma_{1a}\Gamma_{1b}\Gamma_{2a}\Gamma_{2b}|^{1/2}\cos\phi} \tag{13}$$

where $\phi = \phi_1 - \phi_2$ and $\phi_j = \hat{\delta}_{ja} + \hat{\delta}_{jb}$. The value of σ_{ab} at the minimum is typically γ^2/Γ^2 allowing for a much larger background. We note that the minimum is not very pronounced for $\Gamma \sim d$ and that $\Delta(E)$ takes a value close to (9) and (10) in the whole range $|E - \frac{1}{2}(E_1+E_2)| \lesssim \Gamma$ of the two overlapping resonances. Hence, the point E_{I} (E_{II}) in Eqs. (2) and (3) should be chosen inside (outside) this interval, respectively, with $|E_{\mathrm{II}} - \frac{1}{2}(E_1+E_2)| \gtrsim D$. Then, $\Delta(E_{\mathrm{II}}) \approx (\Gamma/D)\Delta(E_{\mathrm{I}})$, and $\tilde{\Delta}(E_{\mathrm{I}}, E_{\mathrm{II}}) \cong \Delta(E_{\mathrm{I}})$ shows an enhancement by the factor $D/\Gamma \gg 1$. This factor is not as large as the factor $(D/\Gamma)^2$ found in Secs. II and III, but its usefulness is not put into jeopardy by the unavoidable incoherent background.

It is interesting to compare our resonance enhancement factor D/Γ for $\tilde{\Delta}(E_{\mathrm{I}}, E_{\mathrm{II}})$ and for $\Delta(E_{\mathrm{I}})$ with the result of Ref. 12. There, detailed balance violation is calculated for several observables, one of which can be rewritten as

$$F = \frac{\langle (\delta\sigma_{ab})^2 \rangle}{\langle [\frac{1}{2}(\sigma_{ab} + \sigma_{ba})]^2 \rangle}$$

(the brackets denote energy averages) and thus bears some formal analogy to Δ^2 [cf. Eq. (1)]. The authors of Ref. 12 show that for $(\Gamma/D) \to 0$, the quantity $F^{1/2}$ carries a resonance enhancement factor $(D/\Gamma)^{1/2}$. (This factor appears to be little affected by incoherent background contributions.) Our resonance enhancement factor D/Γ is, for $D \gg \Gamma$, substantially bigger than $(D/\Gamma)^{1/2}$ but it comes into existence only in the rather special situation considered in the present section. This is in keeping with the remarks in the Introduction.

V. ENHANCEMENT FACTOR AND SIGNIFICANCE OF EXPERIMENTAL RESULTS

We use the estimates (9) and (10) for the maximum value of $\tilde{\Delta}(E_{\mathrm{I}}, E_{\mathrm{II}})$ to introduce and evaluate the enhancement factor for the observation of T violation in detailed balance. Following Ref. 13, we introduce the root-mean-square matrix element v_T for T violation (aspects of this step are analyzed below), and write Eq. (10) (on which we focus attention) as

$$\tilde{\Delta}_{\max} \cong \Delta_{\max} \cong \frac{v_T}{\Gamma} f \frac{v_{12}}{v_T} , \tag{14}$$

where

$$f=2\left[\left|\frac{\Gamma_{2b}}{\Gamma_{1b}}\right|^{1/2}-\left|\frac{\Gamma_{2a}}{\Gamma_{1a}}\right|^{1/2}\right].$$

We relate v_T to the root-mean-square matrix element V of the strong interaction by

$$v_T=\phi V . \tag{15}$$

The quantity ϕ (or, more modestly, an upper bound on ϕ) is the object of experiments on detailed balance. It is largely independent of excitation energy E and mass number A (while v_T and V are not). This can be seen by writing $\phi=(\Gamma^{(u)}/\Gamma^{\downarrow})^{1/2}$ where $\Gamma^{(u)}=2\pi v_T^2/D$ and $\Gamma^{\downarrow}=2\pi V^2/D$ are the spreading widths for T violation and for the strong interaction, respectively—quantities which are known[9,10] to depend little on E and A. This yields

$$\tilde{\Delta}_{\max}\cong\phi f\frac{V}{\Gamma}\frac{v_{12}}{v_T} . \tag{16}$$

We identify $V/\Gamma=(2\pi D\Gamma^{\downarrow})^{1/2}/\Gamma$ as the enhancement factor. (This quantity, of central interest for the theoretical analysis, should not be confused with the "resonance enhancement factor" introduced above to quantify the amplification of $\tilde{\Delta}_{\max}$ due to resonance effects.) In the ideal case of infinite energy resolution and arbitrarily small Γ this factor might be very large but in practice we have to account for the finite energy resolution ΔE which might be bigger than Γ, the average width of isolated resonances. It is easy to see that in this case the estimate for $\tilde{\Delta}_{\max}$ reads

$$\tilde{\Delta}_{\max}=\phi f\frac{V}{\Delta E}\frac{v_{12}}{v_T} . \tag{17}$$

The best energy resolution known to us for protons is[15] $\Delta E=300$ eV. The Triangle Universities Nuclear Laboratory (TUNL) experiments were carried out on nuclei with $A=50-60$ and proton energies near the Coulomb barrier. Because of the negative Q values, the α channel (the best candidate for the entrance channel of the inverse reaction) was closed or at least unobserved in these nuclei. The situation is more favorable near $A=40$. [References 16–18 report resonance studies with the (p,α) reaction on ^{39}K, ^{41}K, and ^{37}Cl.] Assuming that a value $\Delta E=1$ keV can be attained for both proton and alpha channels, using a typical resonance spacing[16–18] of $D=50$ keV, and a spreading width $\Gamma^{\downarrow}\cong 1$ MeV, we find for the enhancement factor

$$\frac{V}{\Delta E}=\frac{(2\pi\Gamma^{\downarrow}D)^{1/2}}{\Delta E}\cong\tfrac{1}{2}\times 10^3 . \tag{18}$$

The range of variation of the observed[16–18] values of Γ_p and Γ_α allows for a value of $f\cong 10$. We therefore estimate

$$\tilde{\Delta}_{\max}\cong(10^3/10^4)\phi\frac{v_{12}}{v_T} . \tag{19}$$

The energy resolution of 1 keV is obviously not easily attainable for α beams, but the alternative kind of T-noninvariance test with the same amount of enhancement[13] involves rather complicated experiments with polarized neutrons and oriented targets.

To determine v_T, many experiments yielding $\tilde{\Delta}$ and the calculation of the variance of v_{12} would be necessary. In practice, only one (or a few) upper bounds on $\tilde{\Delta}$ are expected to become available. To derive an upper bound on v_T (or better, on ϕ) from this kind of data, we return to Eq. (17), and following Ref. 10 in a modified way, consider

$$\beta=\frac{v_{12}}{v_T}=\frac{\tilde{\Delta}_{\max}\Delta E}{\phi f V}$$

as a Gaussian-distributed random variable with zero mean value. Then a single upper bound, $|\tilde{\Delta}_{\max}|\le\Delta_0$, implies[10] a statistical upper bound ϕ_0 on ϕ with confidence

$$P(\phi\le\phi_0)=1-\operatorname{erf}\left[\frac{\Delta_0\Delta E}{\phi_0 V f}\right] . \tag{20}$$

Taking[2] $\Delta_0\cong 5\times 10^{-3}$ we obtain from Eqs. (20) and (19) the bound $\phi\le 5.5\times 10^{-6}$ at a 90% confidence level. Another independent measurement with the same bound increases the confidence level to 99%. A more conservative estimate of $f=1$ or $\Delta E=10$ keV increases the bound by an order of magnitude, which is still a factor 20 better than the best obtained so far with the same confidence level.[11,10] The experiment proposed in the present paper thus offers a viable alternative to the test advocated in Ref. 12. We hasten to add, however, that only a confidence level well above 99% would yield a physically relevant statement, and that in the actual experiment values of $|f|<1$ must be excluded to avoid a decrease in the enhancement factor.

In summary, we have shown that existing bounds on the T-violating interaction can be much improved for the realistic case of two close-lying resonances. The improvement is due to specific resonance-enhancement factors typical for detailed balance experiments on compound-nucleus resonances.

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