Inverted signature dependence of $B(M1)$ in natural-parity rotational bands

Masayuki Matsuzaki
Department of Radioisotopes, Japan Atomic Energy Research Institute, Tokai, Naka, Ibaraki 319-11, Japan

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The conditions for occurrence of the "inverted" signature dependence of $B(M1)$, that is, the enhancement of transitions from the unfavored states, in natural-parity rotational bands are discussed based on the rotating shell model. An example of coexistence of the inverted and the normal signature dependence in an axially symmetric nucleus due to the change in $g_{\rm RPA}$ associated with the band crossing is presented.

Magnetic-dipole transition rates between signature-partner states have been studied as a sensitive probe for wave functions of rotating nuclei.[1] Properties of the transitions in unique-parity bands, where the single-j approximation holds well, have been investigated extensively: the phase rule of signature dependence in connection with the signature splitting of quasiparticle energy,[2] effects of the static and the dynamic triaxialities,[3-9] and the change in absolute values caused by the band crossings.[10,5-7] On the other hand, the signature dependence of $B(M1)$ in natural-parity rotational bands, which are classified by the asymptotic quantum numbers rather than j, has not been fully studied yet. This is partly because of lack of experimental data that show the staggering in $B(M1)$ except for the $\Omega=\frac{1}{2}$ case.[11] In another paper[12] we reported the experimental data for the ground-state rotational band of ^{163}Dy which show apparent signature dependence in $B(M1)$ and analyzed its microscopic origin. In the present paper, we discuss the signature dependence of $B(M1)$ in natural-parity bands based on the picture of a single quasiparticle moving in a rotating, deformed mean field[6,7] and show the conditions under which the transitions from the unfavored states become stronger than those form the favored states.

The magnetic-dipole operator acting on the single-quasiparticle wave functions is given by

$$\hat{\mu}_{-1}=(g_l-g_{\rm RPA})\hat{l}_{-1}+(g_s^{(\rm eff)}-g_{\rm RPA})\hat{s}_{-1}\ , \quad (1)$$

with

$$g_{\rm RPA}=\frac{\langle\hat{\mu}_x\rangle_{\rm RPA}}{\langle\hat{J}_x\rangle_{\rm RPA}}\ , \quad (2)$$

where expectation values are evaluated with respect to the RPA vacuum, i.e., the even-even reference. Expression (1) is reduced, in the single-j approximation, to

$$\hat{\mu}_{-1}=(g_j-g_{\rm RPA})\hat{J}_{-1}\ , \quad (3)$$

where g_j is the Schmidt value. The matrix elements of **J** have a property

$$\Delta E\langle\chi_{I-1}|\hat{J}_z|\chi_I\rangle=(-1)^{I-1/2}\hbar\omega_{\rm rot}\langle\chi_{I-1}|i\hat{J}_y|\chi_I\rangle\ , \quad (4)$$

where $|\chi_I\rangle$ denotes the intrinsic wave function which corresponds to the state with spin I. Furthermore the signature splittings of quasiparticle energy have a phase rule

$$\Delta E=E_{+i}-E_{-i}=(-1)^{j-1/2}|\Delta E|\ , \quad (5)$$

when the intrinsic system is axially symmetric. Combining these relations we obtain the usual signature dependence of $B(M1)$; the transitions from the favored states, where $I-j=$even, are enhanced in the high-j, unique-parity bands.

The correlation between ΔE and matrix elements of $i\hat{J}_y$ and $\hat{J}_z$, are given by Eqs. (4) and (5), persists also when the odd quasiparticle occupies a natural-parity orbital if one of the spherical shell-model states j is dominant in the deformed wave function. The matrix elements of $i\hat{\mu}_y$ and $\hat{\mu}_z$, however, may have the opposite relative phase, which leads to the different staggering from the one in the high-j bands. We know two examples of such "inverted" signature dependence of $B(M1)$. One is the $\nu[523\frac{5}{2}]$ band of ^{163}Dy (Figs. 5 and 8 of Ref. 12) and the other is the $\pi[411\frac{1}{2}]$ band of ^{169}Tm (Ref. 11) (Fig. 1). In these examples $\nu h_{9/2}$ and $\pi d_{3/2}$ are dominant respectively. The signs of ΔE [Eq. (5)] and absolute magnitudes of $B(M1)$, determined by g_j in Eq. (3), support the above assignments.

As was pointed out in Ref. 12, the "inverted" signature dependence is originated from the breaking of the proportionality between $\hat{\mu}_{-1}$ and $\hat{J}_{-1}$:

$$\left[\frac{i\mu_y(\bar{1}1)}{\mu_z(\bar{1}1)}\right]\Bigg/\left[\frac{iJ_y(\bar{1}1)}{J_z(\bar{1}1)}\right]<0\ , \quad (6)$$

where indices $\bar{1}$ and 1 denote the lowest-energy natural-parity states with $r=+i$ and $r=-i$, respectively, and the quasiparticle matrix elements between them are given by $\mathcal{O}(\bar{1}1)$. This breaking is brought about by the j mixing in deformed wave functions. If we look at these matrix elements more closely, we can find the fact that relative magnitudes of the orbital and the spin contributions to $\mu_{-1}(\bar{1}1)=(-i/\sqrt{2})[i\mu_y(\bar{1}1)+\mu_z(\bar{1}1)]$ are inverted between the y and z components (Table II of Ref. 12 for ^{163}Dy and Table I for ^{169}Tm). It should be noted that we can also treat the $\Omega=\frac{1}{2}$ bands on the same footing with

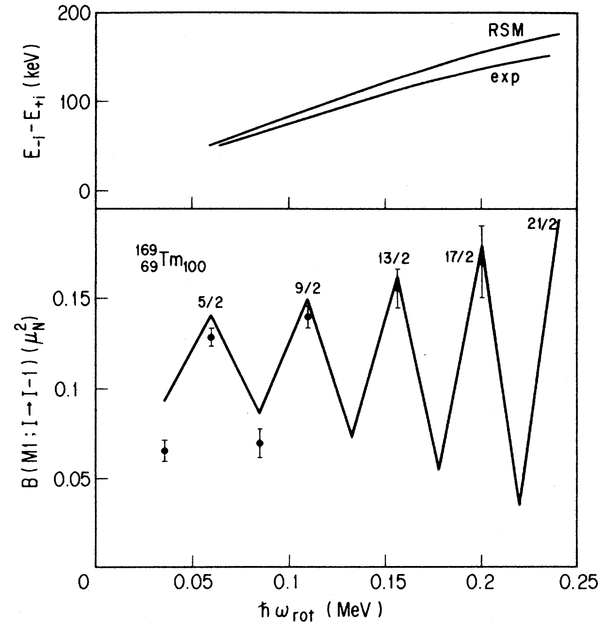

FIG. 1. Signature splitting of quasiparticle energy (upper portion) and $B(M1)$ (lower portion) in the ground-state band of ^{169}Tm as functions of $\hbar\omega_{\rm rot}$. Parameters used are $\delta=0.28$ ($\beta^{(\rm pot)}=0.287$), $\gamma^{(\rm pot)}=0$, $\Delta_n=0.803$ MeV, $\Delta_p=0.901$ MeV, $\lambda_n=6.645\hbar\omega_0$, $\lambda_p=5.902\hbar\omega_0$ and $g_s^{(\rm eff)}/g_s^{(\rm free)}=0.79$ (Ref. 13).

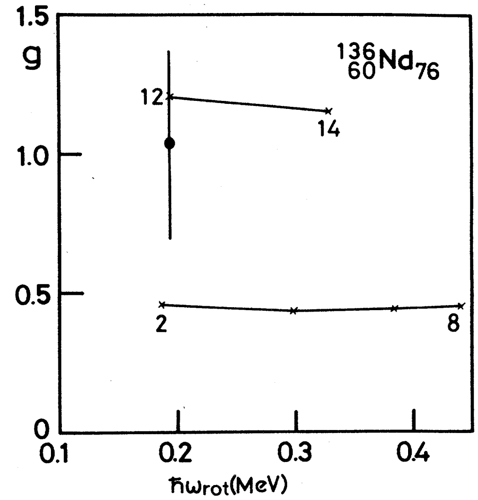

FIG. 2. Calculated and experimental g factors of ^{136}Nd. The experimental value is an average of g factors of 10_s^+ and 12_s^+ states (Ref. 14). The first band crossing in this nucleus is caused by $(\pi h_{11/2})^2$. The calculated deformations are $\beta^{(\rm pot)}=0.15\sim0.20$ and $\gamma^{(\rm pot)}=-5°\sim+40°$ for the g band, and $\beta^{(\rm pot)}\simeq0.18$ and $\gamma^{(\rm pot)}=-24°\sim-16°$ for the s band in calculated frequency range using the isotropic velocity distribution condition (Ref. 15) where the definition of the sign of γ conforms to that of Ref. 13. Effective spin g factor $g_s^{(\rm eff)}=0.7g_s^{(\rm free)}$ is used.

other bands without introducing phenomenological decoupling parameters.

The signature dependence of $M1$ transitions in odd-A nuclei is determined by a sort of interplay between the property of the odd quasiparticle, the matrix elements of l and s, and that of the RPA vacuum, $g_{\rm RPA}$. The necessary conditions for occurrence of the enhanced $B(M1;u\rightarrow f)$ can be summarized as follows: (i) difference in degree of alignment between the orbital and the spin angular momenta:

$$\frac{il_y(\bar{1}1)}{l_z(\bar{1}1)}\neq\frac{is_y(\bar{1}1)}{s_z(\bar{1}1)}\ ,$$

TABLE I. Orbital and spin contributions to the matrix elements of angular momentum and magnetic-dipole operators between the $\pi[411\frac{1}{2}]$ signature-partner states of ^{169}Tm. These values were calculated at $\hbar\omega_{\rm rot}=0.157$ MeV. Parameters used are the same as in Fig. 1.

	y component				z component		
il_y	−0.421	$(g_l-g_{\rm RPA})il_y$	−0.285	l_z	0.887	$(g_l-g_{\rm RPA})l_z$	0.600
iS_y	0.016	$(g_s^{(\rm eff)}-g_{\rm RPA})iS_y$	0.066	s_z	−0.378	$(g_s^{(\rm eff)}-g_{\rm RPA})s_z$	−1.544
iJ_y	−	$i\mu_y$	−	J_z	+	μ_z	−

TABLE II. Same as Table I but for both before and after the $(\nu i_{13/2})^2$ crossing in the $\nu[521\frac{3}{2}]$ band of ^{155}Gd. When two values are given, the upper denotes the one before crossing, the lower denotes the one after crossing. These values were calculated at $\hbar\omega_{\rm rot}=0.2$ MeV. Parameters used are the same as in Fig. 3.

	y component				z component		
il_y	0.381	$(g_l-g_{\rm RPA})il_y$	−0.146 0.018	l_z	1.519	$(g_l-g_{\rm RPA})l_z$	−0.584 0.071
is_y	−0.060	$(g_s^{(\rm eff)}-g_{\rm RPA})is_y$	0.204 0.178	s_z	−0.029	$(g_s^{(\rm eff)}-g_{\rm RPA})s_z$	0.100 0.087
iJ_y	+	$i\mu_y$	+ +	J_z	+	μ_z	− +

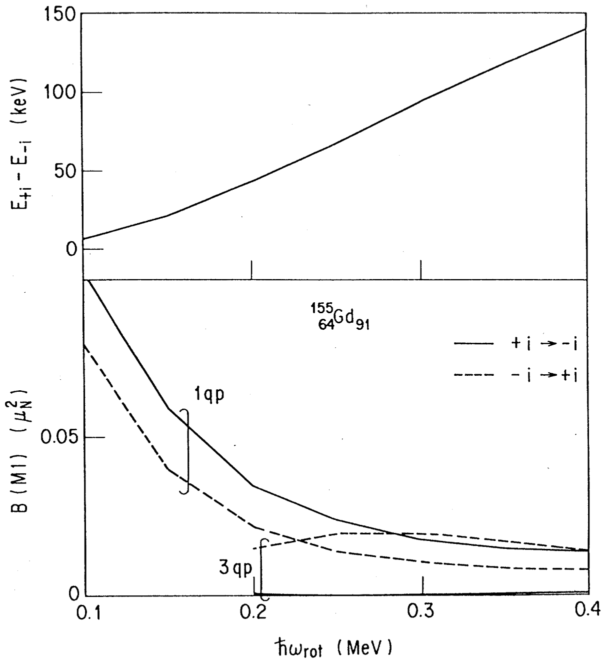

FIG. 3. Calculated $B(M1)$ for the one-quasiparticle and the three-quasiparticle bands of ^{155}Gd (lower portion) and signature splitting of quasiparticle energy which is common to both bands (upper portion). Solid and broken lines denote the transitions from $r=+i$ and $r=-i$, respectively. Parameters used are $\delta=0.25$ ($\beta^{(pot)}=0.257$), $\gamma^{(pot)}=0$, $\Delta_n=1.05$ MeV, $\Delta_p=1.05$ MeV, $\lambda_n=6.410\hbar\omega_0$, $\lambda_p=5.723\hbar\omega_0$ and $g_s^{(eff)}/g_s^{(free)}=0.79$ (Ref. 13).

and (ii) positive sign of the g-factor ratio $(g_l-g_{RPA}/(g_s^{(eff)}-g_{RPA})$ for the spin-down ($\Omega=\Lambda-\frac{1}{2}$) orbitals or negative one for the spin-up ($\Omega=\Lambda+\frac{1}{2}$) orbitals. This ratio is always positive for one-quasiparticle bands where the RPA vacua coincide with the "rotor;" $g_{RPA}\simeq 0.4$. On the other hand, this ratio may become negative for three-quasiproton or three-quasineutron bands where the RPA vacua are the rotor plus aligned two quasiparticles; g_{RPA} may become larger than unity after the $(\pi h_{11/2})^2$ crossing (Fig. 2) or negative after the $(\nu i_{13/2})^2$ crossing (Fig. 2 of Ref. 7). The "inverted" staggering of $B(M1)$, therefore, may take place in the $(\Omega=\Lambda-\frac{1}{2})^1$ bands or $(\Omega=\Lambda+\frac{1}{2})^1(\text{high-}j)^2$ bands where three quasiparticles have the same isospin.

From the above conditions we can expect that some nuclei show the different signature dependence in $B(M1)$ between before and after the band crossing even if the nuclei keep the same shape. We present a numerical example for the $[521\frac{3}{2}]$ band of ^{155}Gd. This orbital contains both the $\Omega=\Lambda-\frac{1}{2}$ and the $\Omega=\Lambda+\frac{1}{2}$ components in comparable magnitudes.[13] Although the sign of $s_z(\bar{1}1)/l_z(\bar{1}1)$ is positive and the absolute magnitude of $B(M1)$ is larger at $\hbar\omega_{rot}\lesssim 0.15$ MeV, the properties of the higher spin states, the signature splitting of quasiparticle energy and the absolute magnitude of $B(M1)$, can be understood as a spin-down orbital at the used deformation $\delta=0.25$ which approximately reproduces the observed quadrupole moment.[16] Calculated $M1$-transition rates are shown in the lower part of Fig. 3 both for the one- and the three-quasiparticle bands as functions of the rotational frequency.

Since we used the same mean-field parameters for both bands, the inversion of the signature dependence is brought about by the change in g_{RPA} (Table II). Here we note that the interband interactions were removed by making use of the diabatic basis[7] in these calculations.

The signature splitting of quasiparticle energy is the same in both bands; $r=-i$ is favored (the upper part of Fig. 3). The pattern in the one-quasiparticle band can be regarded as the inverted one in this respect. The same pattern is obtained in the one-quasiparticle band of ^{157}Gd.[17]

In summary, we have clarified the necessary conditions for occurrence of the inverted signature dependence of $B(M1)$ in natural-parity rotational bands based on the rotating shell model and presented an example of coexistence of two kinds of signature dependence. The experimental information for the natural-parity bands after the band crossing is desired to test our prediction.

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