Calculation of the first-order s-wave optical potential in pionic atoms

C. García-Recio* and L. L. Salcedo†
Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

E. Oset
Departamento de Física Teórica, Facultad de Física, Universidad de Valencia, Burjassot (Valencia), Spain
(Received 9 March 1988)

A microscopic calculation of the first-order s-wave optical potential in pionic atoms which allows the inclusion of the off-shell dependence in momentum of the s-wave πN amplitude is carried out. Binding effects are also properly taken into account. The results obtained, together with those of the second-order optical potential, are in disagreement with phenomenological optical potentials. The results of the paper suggest the need of new and precise experiments on πN scattering at low energies and other experiments which can provide precise values of the πN scattering lengths.

I. INTRODUCTION

The local part (s-wave) of the optical potential of pionic atoms has been studied repeatedly,[1-4] trying to obtain models in agreement with the semiempirical potential which fits the pionic atoms data. But these theoretical potentials, including the most complete of Ref. 4 are not repulsive enough to explain the empirical results.

The s-wave part of the optical potential has been traditionally parametrized[1] on theoretical grounds as

$$-\frac{2\omega V_{\rm opt}^{(s)}(\mathbf{r})}{4\pi}=(1+\epsilon)[\bar{b}_0\rho(\mathbf{r})+\bar{b}_1\delta\rho(\mathbf{r})]+\left[1+\frac{\epsilon}{2}\right]B_0\rho^2(\mathbf{r}) , \tag{1.1}$$

where ω is the pion energy, $\epsilon=\omega/M_N$, ρ is the whole nuclear density and $\delta\rho=\rho_n-\rho_p$ is the difference between neutron and proton densities. Such a potential can be obtained within the scope of the crudest local density approach, i.e., taking for $V_{\rm opt}^{(s)}(\mathbf{r})$ the value of the optical potential computed in infinite nuclear matter at constant density $\rho(\mathbf{r})$. In this approximation, $\bar{b}_0$ equals $b_0+\delta b_0^P$ and $\bar{b}_1\simeq b_1$ where the parameters b_0 and b_1 are the isoscalar and isovector πN scattering lengths, the δb_0^P term takes into account the Pauli correction to the rescattering term[1,4] and the $B_0\rho^2$ term accounts for π nucleus processes which involve two nucleons of the nucleus.

The potential in Eq. (1.1) has been fitted[5,6] to numerous experimental energy shifts and widths of pionic atoms, providing empirical sets of parameters $\bar{b}_0$, $\bar{b}_1$, and B_0 to test the theoretical predictions. Despite the theoretical efforts to calculate these parameters, mainly B_0, they do not match with the empirical ones.

The current situation of the problem is the following: If we take for b_0 the experimental isoscalar scattering length ($b_0=-0.013m_\pi^{-1}$)[8] and for δb_0^P the value $-0.014m_\pi^{-1}$ from Ref. 4, then $\bar{b}_0=-0.027m_\pi^{-1}$. The value of B_0 which fits the pionic atom data is

$$B_0^{\rm fit}\simeq(-0.010-i\,0.042)m_\pi^{-4} , \tag{1.2}$$

while the $\mathrm{Re}B_0$ found theoretically has opposite sign and is about[4]

$$B_0^{\rm th}=(0.030-i\,0.039)m_\pi^{-4} . \tag{1.3}$$

Thus while $\mathrm{Im}V_{\rm opt}^{(s)}$ is reasonably well understood microscopically, the same does not hold for the real part of the s-wave optical potential. It should be pointed out that $\bar{b}_0$ and $\mathrm{Re}B_0$ are not independently given by the fits to the empirical data [Ref. 5 and Eq. (3.8)]. In a previous work[4] we did a thorough computation of the second-order parameter B_0. In this paper we aim at studying the first-order parameter $\bar{b}_0$ beyond the simplest scattering lengths plus Pauli blocking terms.

At least two other physical mechanisms will contribute to $\bar{b}_0$. First, the nucleons in the nucleus are bound, implying the πN reaction is below the *free* threshold. One should then expect that the effective value of b_0 needed in this situation will differ from that of the scattering length. Second, the momentum is not a good quantum number in the nucleus, so the pion in its propagation in the nucleus can be off shell in intermediate situations due to the momentum transfer. The isoscalar πN s-wave amplitude has large cancellations at threshold and increases quite fast going to off-shell situations. So the off-shell effects need not be negligible. As we shall see, each one of both mechanisms gives a contribution to the real part of the optical potential of the same order as other pieces which involve two nucleons excitation and which are customarily included in $\mathrm{Re}B_0$.

We must first clarify what is the precise meaning that we give to the first-order optical potential. For us this is the contribution to the optical potential from the interaction of the pion with only one nucleon of the nucleus. In this way this first order optical potential, together with the mechanisms of Ref. 4 which involve two nucleons,

would provide the full optical potential. (Three-body mechanisms seem to be already negligible for pionic atom energies in base of the calculations of Ref. 19). The first order then contains the usual impulse approximation, with inclusion of binding energy and off-shell effects, and the Pauli correction to the rescattering term δb_0^P. Note, however, and it will become more clear later on, that this definition does not imply that the first order only contains terms linear in $\rho(r)$ and the second-order terms quadratic in $\rho(r)$.

The π-nucleus optical potential, in momentum space, $V_{\rm opt}(\mathbf{p}',\mathbf{p};\omega)$ is related to the pion self-energy $\Pi(\mathbf{p}',\mathbf{p};\omega)$ by

$$2\omega V_{\rm opt}(\mathbf{p}',\mathbf{p};\omega)=\Pi(\mathbf{p}',\mathbf{p};\omega)\ . \tag{1.4}$$

The self-energy of a pion interacting with a nucleus with A nucleons in states $n=1,2,\ldots A$, is given in the impulse approximation by

$$\Pi(\mathbf{p}',\mathbf{p};\omega)\equiv\langle \mathbf{p}'|\hat{\Pi}(\omega)|\mathbf{p}\rangle = \sum_{n=1}^{A}\langle \pi(\mathbf{p}'),N(n)|\hat{T}'_{\pi N}(\omega+E_n)|\pi(\mathbf{p}),N(n)\rangle\ , \tag{1.5}$$

where $\mathbf{p},\mathbf{p}',\omega$ are respectively the incoming and outgoing momenta and energy of the pion in the pion-nucleus frame, and $\hat{T}'_{\pi N}$ is the free πN amplitude. This amplitude is normalized in such a way that its on-shell value is related to the $\hat{S}$ matrix as:

$$\langle \mathbf{p}',\mathbf{q}'|(\hat{S}-I)|\mathbf{p},\mathbf{q}\rangle = -i(2\pi)\delta(p'^0+q'^0-p^0-q^0)\sqrt{M_N/E_N(\mathbf{q}')}\sqrt{M_N/E_N(\mathbf{q})}\frac{\langle \mathbf{p}',\mathbf{q}'|\hat{T}'_{\pi N}(E)|\mathbf{p},\mathbf{q}\rangle}{\sqrt{2\omega(\mathbf{p}')}\sqrt{2\omega(\mathbf{p})}}\ ,$$
$$= -i(2\pi)^4\delta^4(p'+q'-p-q)\frac{1}{(2\pi)^6}\sqrt{M_N/E_N(\mathbf{q}')}\sqrt{M_N/E_N(\mathbf{q})}\frac{T(\mathbf{p}',\mathbf{q}';\mathbf{p},\mathbf{q};E)}{\sqrt{2\omega(\mathbf{p}')}\sqrt{2\omega(\mathbf{p})}}\ , \tag{1.6}$$

where $T(\mathbf{p}',\mathbf{q}';\mathbf{p},\mathbf{q};E)$ is the πN invariant matrix element.[7] We are interested in the pion self-energy in coordinate space, so using the normalizations of Appendix A, for the position and momentum eigenstates we find from (1.5) and (1.6)

$$2\omega V_{\rm opt}(\mathbf{r}',\mathbf{r};\omega)\equiv\Pi(\mathbf{r}',\mathbf{r};\omega)\equiv\langle \mathbf{r}'|\hat{\Pi}(\omega)|\mathbf{r}\rangle$$
$$=\int\frac{d^3\mathbf{P}\,d^3\mathbf{k}\,d^3\mathbf{Q}}{(2\pi)^6}e^{i\mathbf{P}\cdot(\mathbf{r}'-\mathbf{r})+i\mathbf{k}\cdot(\mathbf{r}+\mathbf{r}')/2}$$
$$\times\sum_{n=1}^{A}\bar{\psi}_n^*\left[\mathbf{Q}-\frac{\mathbf{k}}{2}\right]\bar{\psi}_n\left[\mathbf{Q}+\frac{\mathbf{k}}{2}\right]T\left[\mathbf{P}+\frac{\mathbf{k}}{2},\mathbf{Q}-\frac{\mathbf{k}}{2};\mathbf{P}-\frac{\mathbf{k}}{2},\mathbf{Q}+\frac{\mathbf{k}}{2};\omega+E_n+M_N\right]\ . \tag{1.7}$$

Here E_n is the energy of the nucleon in state $|n\rangle$, its mass excluded. We can see here that the optical potential is local if and only if the πN amplitude

$$T\left[\mathbf{P}+\frac{\mathbf{k}}{2},\mathbf{Q}-\frac{\mathbf{k}}{2};\mathbf{P}-\frac{\mathbf{k}}{2},\mathbf{Q}+\frac{\mathbf{k}}{2};E\right]$$

does not depend on $\mathbf{P}$, i.e., T only depends on incoming and outgoing pion momenta through its difference. Indeed, if T does not depend on $\mathbf{P}$, the integral of $\mathbf{P}$ in (1.7) is $\delta(\mathbf{r}'-\mathbf{r})$ and the remainder is a function of $\mathbf{r}$, so the potential is local.

The paper is organized as follows. Section II summarizes the model of Londergan, McVoy, and Moniz (LMM) for the πN amplitude comparing it with other models in the literature. The calculation is done in three steps. Section III analyzes the momentum transfer effect on $\bar{b}_0$ by assuming a local πN amplitude. Section IV develops the standard impulse approximation including both momentum transfer and binding effects. Section V shows that the use of the free πN amplitude plus binding effects in the standard impulse approximations runs into inconsistency and a modified impulse approximation without this drawback is proposed. The definitive results are obtained in this section, but use is made of the results obtained in the former sections. Finally, Sec. VI summarized our conclusions.

II. s-WAVE πN OFF-SHELL EXTRAPOLATION

The s-wave isoscalar and isovector scattering lengths b_0 and b_1, respectively, have experimentally[8] the values

$$b_0=-0.013m_\pi^{-1}\ ,$$
$$b_1=-0.092m_\pi^{-1}\ . \tag{2.1}$$

Note that b_0 is about one order of magnitude lower than b_1. This fact is not accidental, indeed the soft pion model provides a b_0 exactly null. Due to this fact, the first order in ρ of the optical potential, the term $b_0\rho(\mathbf{r})$ in (1.1), is quite small and other corrections, such as the Pauli corrections, and higher-order terms, such as the $B_0\rho^2$ term of the potential, are important.

Let us look at the off-shell behavior of the s-wave isoscalar T matrix for three different models in the literature.

In the first model, due to Hamilton,[9] the πN s-wave interaction has an inner structure based on particle exchange. It is assumed that the isoscalar πN coupling comes from the $\pi\pi$ coupling to a "σ-meson" (0^+0) and from a $\pi N\bar{N}$ coupling.

The explicit expression for the isoscalar invariant amplitude, in this model, is

$$T^+(t)=-4\pi(1+\epsilon)\left[a_{sr}+a_\sigma\frac{m_\sigma^2}{m_\sigma^2-t}\right] \tag{2.2}$$

with $a_\sigma=0.220m_\pi^{-1}$,[10] $m_\sigma\simeq 550$ MeV, and $a_{sr}=-0.233m_\pi^{-1}$ in order to reproduce the on-shell value at threshold, and t is the squared four-momentum transfer. In this model $T^+(t)$ shows a partial cancellation at $t=0$, but it increases very fast for $t<0$.

Now we summarize the hereafter called LMM model.[11] The LMM model assumes a separable T-matrix for each partial wave, which reproduces the on-shell values and provides a well-defined off-shell extrapolation. For the πN elastic channel in a partial wave $\alpha(\alpha=L_{2J,2I})$ the T matrix is given by

$$T_\alpha(p',p;E)=\lambda_\alpha v_\alpha(p')v_\alpha(p)/\mathcal{D}_\alpha^+(E) \tag{2.3}$$

here $E=\sqrt{s}$ is the total c.m. energy and p',p are the moduli of the incoming and outgoing c.m. momenta. These functions v_α and $\mathcal{D}_\alpha^+$ can be obtained from the experimental phase-shifts δ_α and the inelasticity parameters η_α by means of

$$\mathcal{D}_\alpha^+(E)=\exp\left[\frac{1}{\pi}\int_{M_N+m_\pi}^{\infty}dx\frac{\delta'_\alpha(x)}{E-x+i\epsilon}\right], \tag{2.4a}$$

$$\tan\delta'_\alpha=\tan\delta_\alpha\left[1+\frac{1-\eta_\alpha}{2\eta_\alpha\sin^2\delta_\alpha}\right], \tag{2.4b}$$

$$\lambda_\alpha v_\alpha^2(k)=\mathcal{D}_\alpha^+(E(k))T_\alpha(k,k;E(k)), \tag{2.4c}$$

$$\lambda_\alpha=\pm 1, \tag{2.4d}$$

$$E(k)=(M_N^2+k^2)^{1/2}+(m_\pi^2+k^2)^{1/2}. \tag{2.4e}$$

Note that $\mathcal{D}_\alpha^+$ is defined such that the right-hand side of (2.4c) is real, because the phases of $\mathcal{D}_\alpha^+$ and T_α are equal but of opposite sign. So v_α is always real choosing for λ_α the appropriate sign.

The isoscalar (isovector) s-wave matrix, $T^+(T^-)$, are related to the partial waves $\alpha=S_{11}$ and $\alpha=S_{13}$ by

$$\begin{aligned}T^+&=\tfrac{1}{3}(T_{S_{11}}+2T_{S_{13}}),\\ T^-&=-\tfrac{1}{3}(T_{S_{11}}-T_{S_{13}}).\end{aligned} \tag{2.5}$$

Note that the LMM model provides a nonlocal T-matrix.

We compare the LMM and Hamilton models in the case at threshold $E=M_N+m_\pi$, where all the particles are on shell with the exception of outgoing pion, which has momentum $\mathbf{q}$. This kinematic situation is depicted in Fig. 1.

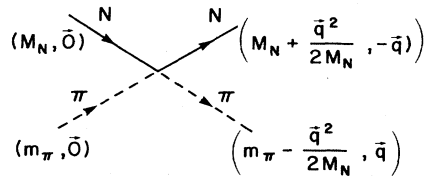

FIG. 1. Half-off-shell kinematic variables for πN scattering at threshold; only the outgoing pion is allowed to be off-shell.

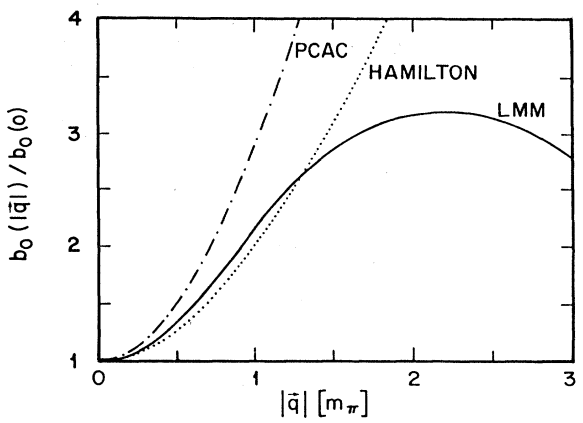

FIG. 2. Comparison of isoscalar-half-off-shell extrapolations for different models.

The T^+ values as a function of $|\mathbf{q}|$ are depicted in Fig. 2 for the Hamilton, LMM, and PCAC (partial conservation of the axial current) models. This last model is constructed through an interpolation between known points from current algebra and has been taken from Ref. 12, choosing the parameters such that they reproduce the empirical on-shell T matrix at threshold (see Ref. 13 for details).

For off-shell values of q near $q=0$ all the models show an important rise of T^+. Note that the $q\to 0$ behavior of T^+ agrees quantitatively for all the models. On the other hand, for higher momenta the discrepancy between different models is expected not to be very relevant due to the finite momentum transfer allowed in finite nuclei.

III. AMPLITUDE DEPENDING ON THE MOMENTUM TRANSFER

Let us suppose that the s-wave isoscalar πN interaction amplitude depends only on the momentum transfer, as in the Hamilton model. We can include the t dependence in a function $b_0(t)$, such that $b_0(0)$ equals b_0.

$$\begin{aligned}&T^+(\mathbf{p}',\mathbf{q}';\mathbf{p},\mathbf{q};E)=-4\pi(1+\epsilon)b_0(t),\\ &t=(\omega-\omega)^2-(\mathbf{p}'-\mathbf{p})^2\equiv-\mathbf{k}^2.\end{aligned} \tag{3.1}$$

Then Eq. (1.7) provides the following local optical potential

$$2\omega V_{\rm opt}^{(s)}(\mathbf{r}',\mathbf{r};\omega)=\delta^3(\mathbf{r}'-\mathbf{r})F(\mathbf{r}), \tag{3.2a}$$

$$F(\mathbf{r})=-4\pi(1+\epsilon)\int\frac{d^3\mathbf{k}}{(2\pi)^3}e^{i\mathbf{k}\cdot\mathbf{r}}b_0(-\mathbf{k}^2)\tilde{\rho}(\mathbf{k}), \tag{3.2b}$$

where $\tilde{\rho}(\mathbf{k})$ is the Fourier transform of the nuclear density, and Eqs. (A10) and (A11) have been used. Expression (3.2) shows that in the impulse approximation one must replace $b_0\rho(r)$ by $F(\mathbf{r})$ in order to account for the off-shell effects in the πN amplitude.

A different way of writing the potential $F(\mathbf{r})$ is

$$F(\mathbf{r})=-4\pi(1+\epsilon)\int d^3\mathbf{r}'\tilde{b}_0(\mathbf{r}-\mathbf{r}')\rho(\mathbf{r}'), \tag{3.3}$$

where $\tilde{b}_0(\mathbf{r})$ is the Fourier transform of $b_0(-\mathbf{k}^2)$:

$$\tilde{b}_0(\mathbf{r})=\int \frac{d^3\mathbf{k}}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{r}} b_0(-\mathbf{k}^2) \ . \tag{3.4}$$

We can interpret Eq. (3.3) as a way of taking into account the finite range of the interaction.

We can easily see that for a constant density ρ, $F(r)$ coincides with $b_0\rho$; however, for a finite nucleus, $\rho(r)$ is not constant and $F(\mathbf{r})$ can differ appreciably from $b_0\rho(\mathbf{r})$ especially at the nuclear surface.

The effect of the off-shell extrapolation in the optical potential for an amplitude depending only on the momentum transfer was studied in detail Ref. 13. We summarize the results here since they will be used in the next sections.

In this reference $b_0(-\mathbf{k}^2)$ was parametrized in the following way:

$$b_0(-\mathbf{k}^2)=b_0\,(1+\beta_1\mathbf{k}^2+\beta_2\mathbf{k}^4+\beta_3\mathbf{k}^6)\,e^{-\beta\mathbf{k}^2} \tag{3.5}$$

with the parameter values $\beta_1=4.98$ fm^2, $\beta_2=-0.726$ fm^4, $\beta_3=0.203$ fm^6, and $\beta=0.462$ fm^2. The function (3.5) allows one to get an analytic expression for the optical potential $F(\mathbf{r})$ in (3.2) using harmonic oscillator densities for the nuclei.

The function $F(\mathbf{r})$ is a complicated function of r. However, it would be very useful to find an equivalent optical potential written in the standard form (1.1) which would give rise to the same results as using the function $F(\mathbf{r})$. To that purpose the following was done: the s-wave optical potential was taken to be

$$2\omega V^{(s)}_{\rm opt}(\mathbf{r})=F(\mathbf{r})-4\pi(1+\epsilon)\delta b_0^P\rho(\mathbf{r}) -4\pi\left[1+\frac{\epsilon}{2}\right]B_0\rho^2(\mathbf{r}) \tag{3.6}$$

with $\delta b_0^P=-0.14m_\pi^{-1}$ (see Ref. 4), and a best fit to the pionic atom data of ^{12}C and ^{16}O with B_0 as a free complex parameter was carried out. The B_0 value obtained by using (3.6) is then[13]

$$\begin{aligned}&{\rm Re}B_0=-0.030m_\pi^{-4} \ ,\\ &{\rm Im}B_0=-0.042m_\pi^{-4} \ .\end{aligned} \tag{3.7}$$

If there is not extrapolation included, the parameter $\bar{b}_0$ is given by $b_0+\delta b_0^P=-0.027m_\pi^{-1}$ and the best fit to the same data gives ${\rm Re}B_0=-0.010m_\pi^{-4}$.

The parameter ${\rm Re}B_0$ obtained by using the potential (3.6) to fit the data is more negative than that obtained without off-shell extrapolation. This shows that the off-shell effect is attractive, being necessary more repulsion in the ρ^2 term in order to balance this new attractive effect.

The former discussion shows that the effect of the off-shell extrapolation can be simulated by an increase in the ${\rm Re}B_0$ parameter of $\delta\,{\rm Re}B_0=0.020m_\pi^{-4}$. However, since we include these off-shell effects in the first-order optical potential it is useful to express that in terms of an equivalent change in the parameter $\bar{b}_0$. This is easily accomplished by means of the linear correlation between ${\rm Re}B_0$ and ${\rm Re}\bar{b}_0$ established in Ref. 5,

$$\begin{aligned}&\delta\bar{b}_0=\frac{1+\epsilon/2}{1+\epsilon}\rho_{\rm eff}\delta\,{\rm Re}B_0 \ ,\\ &\rho_{\rm eff}=\rho_0/2 \ .\end{aligned} \tag{3.8}$$

Hence the effect of the off-shell extrapolation would be simulated with an increase of the ${\rm Re}\bar{b}_0$ parameter

$$\delta\bar{b}_0^{\rm extr.}=0.005m_\pi^{-1} \ . \tag{3.9}$$

Since $|b_0(-\mathbf{k}^2)|>|b_0|$ for $\mathbf{k}\neq 0$, one would naively expect that $F(\mathbf{r})$ would yield an effective $|b_0|$ bigger than before, hence the extrapolation effect would be repulsive in contrast to the numerical result in Eq. (3.9).

In order to understand better the attractive effect of the off-shell extrapolation, let us remark that the spacial average of the difference between the potentials with and without extrapolation is zero, i.e.,

$$\begin{aligned}&2m_\pi\delta V_{\rm opt}(\mathbf{r})=F(\mathbf{r})-4\pi(1+\epsilon)b_0\rho(\mathbf{r}) \ ,\\ &\int 2m_\pi\delta V_{\rm opt}(\mathbf{r})d^3\mathbf{r}=0 \ .\end{aligned} \tag{3.10}$$

We plot $2m_\pi\delta V_{\rm opt}(\mathbf{r})\cdot\mathbf{r}^2$ vs r in Fig. 3. $\delta V_{\rm opt}(\mathbf{r})$ is positive for $r<3.4$ fm and negative for bigger values of r. The pion density in a pure Coulomb potential for a $1s$ level would be monotonously decreasing and it would weigh more than the repulsive part of $\delta V_{\rm opt}$ and the net effect of $\delta V_{\rm opt}$ on the pion would be repulsive. However, the s-wave π nucleus strong interaction is repulsive and it pushes the pion outside the nucleus. The pionic density in the level $1s$ of an ^{16}O pionic atom, $|\phi^{1s}(r)|^2$, is depicted in Fig. 3. We see that, due to the strong repulsion, the pion feels more the attractive part of $\delta V_{\rm opt}$ and the net effect of the off-shell extrapolation is to produce attraction in the potential. These conclusions are in agreement with the findings of Refs. 5 and 15.

It is worthwhile to note that the consideration of momentum transfer in intermediate steps in the pion-nucleus interaction, not only does not solve the problem of the missing repulsion but makes it worse because it introduces an extra attraction.

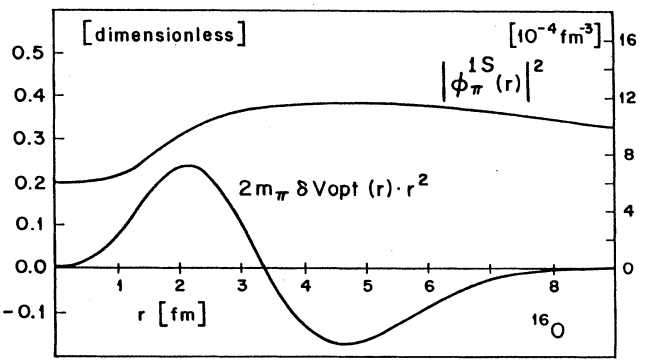

FIG. 3. Off-shell extrapolation correction to the optical potential, $2m_\pi\delta V_{\rm opt}(\mathbf{r})\cdot\mathbf{r}^2$, compared to the pionic density for the level $1s$ in ^{16}O $|\phi^{1s}(r)|^2$. Due to the s-wave repulsion the pion sees a net attractive effect.

IV. AMPLITUDE DEPENDENCE ON ENERGY AND MOMENTA

We have considered in Sec. III that the πN interaction in a pionic atom is a process at threshold. This is, evidently, an approximation. The nucleons of the nucleus are bound and, therefore, the πN interaction takes place below the free πN threshold.

In order to take into account the binding energy of the nucleons, we need a model which provides us with the interacting πN T matrix below threshold.

The Hamilton model is constructed to reproduce only the scattering lengths and does not give the energy dependence. The models based on PCAC give the T matrix around $\omega_\pi=0$ and we want an expansion around $\omega_\pi=m_\pi$. We choose the LMM model which provides the energy dependence of the T-matrix and reproduces the right on-shell amplitude for any energy. As we have seen, the invariant amplitude T of (2.3) depends, in this model, on the incoming and outgoing momenta and the total energy of the system πN in the c.m. frame.

Here we are going to consider the off-shell and off-threshold effects by expanding the amplitude around threshold in first order of the energy variable and the momentum squared. Hence we make an expansion of $T(\mathbf{p}'_{\text{c.m.}},\mathbf{p}_{\text{c.m.}};E_{\text{c.m.}})$ around $\mathbf{p}'^2_{\text{c.m.}}=\mathbf{p}^2_{\text{c.m.}}=0$, $E_{\text{c.m.}}=M_N+m_\pi$.

There are some reasons to stop at this order in the expansion: (1) The different off-shell models agree quite well in the momentum transfer dependence until second order but not further. (2) Higher orders in momentum are expected not to be very relevant because large momentum transfer are severely suppressed in not too light nuclei. (3) The variation of the energy is of the order of the kinetic energy of the nucleons and keeping first order in this is consistent with the first order in squared momenta.

Thus we shall fit the LMM T-matrix below threshold with the following form

$$T_s^+(\mathbf{p}',\mathbf{p};E)=-4\pi\frac{E}{M_N}[b_0+b_2(\mathbf{p}^2+\mathbf{p}'^2)+b_3(E-M_N-m_\pi)]\ , \tag{4.1}$$

where $\mathbf{p}',\mathbf{p},E$ refer to the πN c.m. frame and the parameters are

$$\begin{aligned} b_0&=-0.013m_\pi^{-1}\ ,\\ b_2&=-0.017m_\pi^{-3}\ ,\\ b_3&=+0.055m_\pi^{-2}\ . \end{aligned} \tag{4.2}$$

Note that the value quoted here for b_2 differs from that of Ref. 13 due to the use of different values for the empirical phase shifts and inelasticities. We are now using Arndt's phase shifts.[16] However, at small energies these phase shifts do not match the measured scattering length of Ref. 8. Hence, in order to be consistent with the value of b_0 that we are using[8] we have done a smooth extrapolation from $T_\pi\approx 50$ MeV to threshold of those phase shifts in order to match the scattering lengths. The effect of the energy dependence of the πN amplitude is determined by the parameter b_3, which gives a term of the same sign as b_0 to the amplitude T_s^+. Strictly speaking, an expansion of T_s^+ around threshold, done in Appendix C, yields not only a term going as $E-M_N-m_\pi$ but also another one as $\sqrt{M_N+m_\pi-E}$, which above threshold gives the imaginary part of the T matrix required by unitarity. We shall use here only a linear term in E which should account effectively for the two terms at an average nucleon binding energy. In the next chapter, however, we shall make modifications to account for medium corrections to the T matrix arising from the binding energies of the nucleons and we shall see that the term with the square root will not provide any modification to the optical potential. We use the amplitude in (4.1) to get the s-wave isoscalar part of the pion-nucleus optical potential for pionic atoms. The explicit calculation has been done in Appendix B. The result found for the potential is:

$$-\frac{2\omega V_{\text{opt}}^{(s)}(\mathbf{r},\omega)}{4\pi(1+\epsilon)}=[b_0+b_3(\omega-m_\pi)]\rho(\mathbf{r}) \tag{4.3a}$$

$$+b_3V(\mathbf{r})\rho(\mathbf{r}) \tag{4.3b}$$

$$+b_3\epsilon\frac{3}{5}\frac{k_F^2(\mathbf{r})}{2M_N}\rho(\mathbf{r}) \tag{4.3c}$$

$$-\left[b_2(1-\epsilon)+\frac{b_3}{8(M_N+\omega)}\epsilon\right]\nabla^2\rho(\mathbf{r}) \tag{4.3d}$$

$$-\left[2b_2(1-2\epsilon)-\frac{b_3}{2(M_N+\omega)}\right]\times\nabla\cdot\rho(\mathbf{r})\nabla\ , \tag{4.3e}$$

where ω is the pion energy and $V(\mathbf{r})$ is the one-body average potential which one nucleon feels in the point $\mathbf{r}$ due to the other nucleons in the nucleus.

We see that this optical potential, associated to the πN interaction in the s-wave, is not local but it has a term of the type $\nabla\rho(\mathbf{r})\nabla$. This part comes from the nonlocality of the separable LMM model used for the amplitude. The $b_0\rho(\mathbf{r})$ term corresponds to the case of interaction at threshold.

The terms with the parameter b_3 are related with the fact that the πN c.m. energy differs from that at threshold, so there is a term with $V(\mathbf{r})$ which takes into account the binding energy of the nucleon and the other terms with b_3 account for the kinetic energies of the pion and nucleon. The terms with b_2 come from the dependence in momentum of the amplitude and take care of the pion and nucleon momenta.

Let us apply our potential (4.3) to the pionic atom case, $\omega=m_\pi$, neglecting the binding energy of the pion which is less than 1 MeV in most cases and is quite smaller than the nucleon binding energy. We take the nucleon potential $V(\mathbf{r})$ to be proportional to the nuclear density at each point,

$$V(\mathbf{r})\approx -54\ \text{MeV}\frac{\rho(\mathbf{r})}{\rho_0} \tag{4.4}$$

with ρ_0 being the normal nuclear matter density, $\rho_0=0.483m_\pi^3$. This potential is consistent with infinite nuclear matter results.[14] Using it, the term (4.3b) becomes

$$b_3V(\mathbf{r})\rho(\mathbf{r})=\delta B_0^{(V_N)}\frac{(1+\epsilon/2)}{(1+\epsilon)}\rho^2(\mathbf{r}) , \tag{4.5}$$

where

$$\delta B_0^{(V_N)}=-0.048m_\pi^{-4} . \tag{4.6}$$

This term turns out to be quadratic in density although it was generated in a process involving only one nucleon. This is because the potential energy $V(\mathbf{r})$ is due to the existence of other nucleons. As a consequence, we interpret this term as a correction to the second-order parameter B_0 [see Eq. (1.1)] in (4.6). Note that $\delta B_0^{(V_N)}$ is of the same order and opposite sign as $\mathrm{Re}B_0^{\mathrm{th}}$ coming from processes involving two nucleons of the medium, Eq. (1.3).

The local term of the potential (4.3c), which goes as $k_F^2(\mathbf{r})\rho(\mathbf{r})$, is very small due to the factor ϵ and we shall neglect it. As far as the term (4.3d) is concerned we will postpone its discussion until the introduction of the ATT (angular transformation term) correction, which has the same $\nabla^2\rho(\mathbf{r})$ functional form.

The remaining term is the nonlocal one of (4.3e). This can be interpreted as a change in the linear density parameter $\bar{c}_0$ of the nonlocal part of the optical potential[5] (the p-wave part). The result is:

$$\delta\bar{c}_0^{\mathrm{extr.}}=-0.04m_\pi^{-3} , \tag{4.7}$$

with the parametrization of the p-wave optical potential of Ref 6.

Now we calculate the optical potential in the impulse approximation associated to the p-wave πN interaction. Because of the transformation of the reference frame it will give a contribution of order ϵ to the local part of the optical potential. Remaining in second order in the c.m. momenta, in order to be consistent with the previous calculation, the πN isoscalar amplitude in p wave around threshold is

$$T_p^+(\mathbf{p}'_{\mathrm{c.m.}},\mathbf{p}_{\mathrm{c.m.}};E_{\mathrm{c.m.}})=-4\pi(1+\epsilon)c_0\mathbf{p}'_{\mathrm{c.m.}}\cdot\mathbf{p}_{\mathrm{c.m.}} , \tag{4.8}$$

where c_0 is the scattering volume in this channel,

$$c_0=0.20m_\pi^{-3} . \tag{4.9}$$

The impulse optical potential associated to this amplitude [see Appendix B] is:

$$-\frac{(1+\epsilon)2\omega V_{\mathrm{opt}}^{(p)}(\mathbf{r};\omega)}{4\pi}=-c_0\nabla\cdot\rho(\mathbf{r})\nabla \tag{4.10a}$$

$$+\frac{\epsilon}{2}c_0\nabla^2\rho(\mathbf{r}) \tag{4.10b}$$

The term (4.10a) is usual in the p-wave optical potential. The local term (4.10b) is known as ATT in the literature[5,15] and has an attractive effect for pionic atoms. It has the same form as (4.3d), and its inclusion, together with the term (4.3d) gives

$$-\left[b_2(1-\epsilon)+\frac{b_3}{8(M_N+m_\pi)}\epsilon-\frac{c_0}{(1+\epsilon)^2}\frac{\epsilon}{2}\right]\nabla^2\rho(\mathbf{r})$$

$$=+0.026m_\pi^{-3}\nabla^2\rho(\mathbf{r}) . \tag{4.11}$$

This term is local, but it is not easily interpreted in terms of the usual parameters $\bar{b}_0$ and B_0 since $\nabla^2\rho(\mathbf{r})$ is a function without defined sign. The method to interpret such a term as a variation of an effective $\bar{b}_0$ was described in Sec. III. Actually, a Laplacian approximation (as done throughout all this section) in Eq. (3.10) yields

$$-\frac{2\omega\delta V_{\mathrm{opt}}(\mathbf{r})}{4\pi(1+\epsilon)}=0.029m_\pi^{-3}\nabla^2\rho(\mathbf{r}) . \tag{4.12}$$

The coefficients in Eqs. (4.11) and (4.12) are close enough to use the result in Eq. (3.9) as an estimation of the effect of the Laplacian term of (4.11). Hence we can take into account this Laplacian term, in an approximate way, by means of a correction to the $\bar{b}_0$ parameter given by

$$\delta\bar{b}_0^{\mathrm{Lap}}\approx 0.005m_\pi^{-1} . \tag{4.13}$$

We have shown the effects of the allowed pion momentum transfer due to the nucleus finite size and of the binding energy of the nucleon which places the πN processes under threshold for the s-wave optical potential associated to the pion interaction with only one nucleon of the nucleus. One can ask how these corrections will affect the processes which involve two nucleons of the nucleus, i.e., how the theoretical results for the parameter B_0, Eq. (1.3), coming from two-nucleons processes would be affected by the inclusion of these facts. The answer is simple: in the calculation[4] of the B_0^{th}, off-shell effects for the intermediate pions are included. However, inclusion of the off-shell effects in the external pions, as we have done here for the lowest-order terms, would not change the results appreciably because, as shown in Ref. 4, B_0^{th} goes approximately as $\{b_0^2+b_0b_1+\frac{1}{2}b_1^2\}$ and the largest contribution comes from the b_1^2 term, which is not much affected by the off-shell extrapolation.

Summarizing the results of this section we have obtained, for pionic atoms, from the impulse approximation

$$\frac{-2\omega V_{\mathrm{opt}}^{(s)}(\mathbf{r},\omega)}{4\pi(1+\epsilon)}=b_0\rho(\mathbf{r})+\delta\bar{b}_0^{\mathrm{Lap}}\rho(\mathbf{r})$$

$$+\delta B_0^{(V_N)}\frac{(1+\epsilon/2)}{(1+\epsilon)}\rho^2(\mathbf{r}) \tag{4.14}$$

with $\delta\bar{b}_0^{\mathrm{Lap}}$ and $\delta B_0^{(V_N)}$ given by (4.13) and (4.6), respectively. In addition, we have obtained a correction to the p-wave optical potential coming from the off-shell extrapolation of the s-wave πN scattering amplitude while at the same time we have included in the term $\delta\bar{b}_0^{\mathrm{Lap}}$ a correction coming from the c.m. to laboratory transformation of the p-wave πN amplitude. However, we should not take these results as definitive yet, since as we shall see in the next section, the medium corrections to the T amplitude will modify the numerical results of (4.14) although not its structure.

V. MODIFIED IMPULSE APPROXIMATION

In the previous section we have developed a strict impulse approximation as stated in Eq. (1.7), where T stands for the free πN T matrix. Among the different pieces obtained, the one due to the nuclear potential energy in Eq. (4.3b), plays a special role for it goes as ρ^2 rather than as ρ. Actually it turns out to be of the same order of magnitude as the second-order pieces of the s-wave optical potential. One can ask whether such a big correction, unaccounted for in previous calculations, is a genuine one or rather an artifact due to an over-simplistic application of the impulse approximation.

A clue is given by the Pauli blocking piece of Fig. 4(b), which is computed in nuclear matter (Appendix A of Ref. 4). This Feynman diagram produces an imaginary part corresponding to the quasielastic process $p+q\to p''+q'$, where $\mathbf{q}$ and $\mathbf{q}'$ are below the Fermi surface and $p=(m_\pi,\mathbf{0})$ for pionic atoms. In the absence of any nuclear binding energy, this imaginary part exactly cancels the one coming from ImT in Fig. 4(a). This is as it should be because the quasielastic process is forbidden, due to Pauli's principle, for pions at threshold. Above threshold the exact cancellation would not hold.

Now, if we switch on the nuclear potential in the hole line in Fig. 4(a), as was done in Sec. III, we should do the same in the hole line q in Fig. 4(b). Furthermore, in nuclear matter the one particle Hamiltonian can depend at most on the momentum, and the line q' runs over the same set of momenta as q, then we must include the nuclear potential also in the "particle" line q'. Both shifts in their energies partially cancel leaving the Pauli blocking piece essentially unchanged. This is not so for the impulse approximation graph of Fig. 4(a): once the binding energy is included, the πN collision is below threshold and any imaginary part disappears. The previous cancellation between graphs no longer exists and the combined imaginary part turns out to be positive.

The way out of this puzzle is to take into account that the T matrix is not an elementary vertex and is renormalized in the nuclear medium because of the binding of the nucleons. Indeed the πN T matrix will contain, among other pieces, the iterated rescattering of the πN pair, as depicted in Fig. 5. For consistency we should put in the intermediate nucleon lines the same binding as we put for the external lines.

To this end we need a definite model for the πN T matrix and we shall take the LMM model.[11] The basic assumptions of this model are that the nonelastic channels mixing with πN in s-wave (depicted as boxes in Fig. 5) correspond to two particle states, and that the elementary interaction vertices are separable

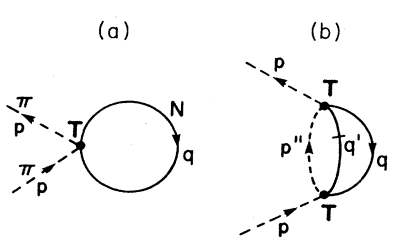

FIG. 4. (a) Lowest-order graph of the optical potential in nuclear matter. (b) Pauli's correction to graph (a).

FIG. 5. A typical term in the expansion of the free πN T matrix in elementary processes is shown. The boxes stand for intermediate states other than πN.

$$\mathcal{V}_{i,j;\alpha}(p',p)=\lambda_{i,j;\alpha}v_{i;\alpha}(p')v_{j;\alpha}(p)\ , \tag{5.1}$$

where i,j denote both pairs of reacting particles, $\alpha=L_{2J,2I}$ and p,p' the moduli of the incoming and outgoing momenta in the c.m. system.

The explicit T matrix as predicted by this model was already given in Eqs. (2.3) and (2.4) where $v_\alpha(p),\lambda_\alpha$ refer to that of the elastic channel. On the other hand, by performing the implicit summation of graphs in Fig. 5, one easily establishes the relationship[11]

$$\mathcal{D}_\alpha^+(E)=1-\sum_i\int\frac{d\mathbf{s}}{(2\pi)^3}\frac{\lambda_{i,i;\alpha}v_{i;\alpha}^2(s)}{E-E_i(s)+i\eta}\ , \tag{5.2}$$

where $E_i(s)$ is a generalization of Eq. (2.4e) for other channels, namely, the relativistic energy of the two particles of channel i with momentum $\mathbf{s}$. It was already pointed out in Sec. II that $v_\alpha(s)$ are given by the model once $\delta_\alpha(E)$ and $\eta_\alpha(E)$ are provided. The other $v_{i;\alpha}(s)$ vertices for nonelastic channels are not determined in this way, but they are not separately needed.

In order to get the medium modified T matrix due to nucleon binding we shall assume, rather naturally, that the elementary vertices $v_{i;\alpha}(p)$ are not affected by the nuclear medium. Also, we shall suppose that the only channel feeling a nuclear potential is the elastic one, mainly due to our ignorance about how the nuclear medium can alter other, far off shell, particles. Under these assumptions the binding energy effects are easily accounted for by modifying the nucleon dispersion relation. For the sake of simplicity we just shift the nucleon mass by its (local) nuclear potential, $M_N\to M_N+V$, thus keeping the formal relativistic invariance in the formulae. Equation (2.4e) changes to

$$E^{(V)}(s)=(m_\pi^2+s^2)^{1/2}+[(M_N+V)^2+s^2]^{1/2}\ . \tag{5.3}$$

Similarly, the medium corrected T matrix will be

$$T_\alpha^{(V)}(p',p;E)=\frac{\lambda_\alpha v_\alpha(p')v_\alpha(p)}{\mathcal{D}_\alpha^{+(V)}(E)}\ , \tag{5.4}$$

$$\mathcal{D}_\alpha^{+(V)}(E)=\mathcal{D}_\alpha^+(E)-\mathcal{H}_\alpha^{(V)}(E)|_{V=0}+\mathcal{H}_\alpha^{(V)}(E) \tag{5.5}$$

with $\mathcal{H}_\alpha^{(V)}(E)$ the V-corrected elastic part of $\mathcal{D}_\alpha^+$

$$\mathcal{H}_\alpha^{(V)}(E)=-\int\frac{d\mathbf{s}}{(2\pi)^3}\frac{\lambda_\alpha v_\alpha^2(s)}{E-E^{(V)}(s)+i\eta}\ . \tag{5.6}$$

In principle as $v_\alpha(s)$ is known, so are $\mathcal{H}_\alpha^{(V)}$, $\mathcal{D}_\alpha^{+(V)}$, and

$T_\alpha^{(V)}$. However some technical comments are in order here. We have use the empirical phase shifts and inelasticities of Arndt[16] below 750 MeV of momentum in c.m. Above this energy, Regge's model[17] is needed, obtaining $\delta_{S_{11}}(\infty)=\pi$, $\delta_{S_{13}}(\infty)=0$, $\eta_{S_{11}}(\infty)=\eta_{S_{13}}(\infty)=1$. Then the integral in Eq. (2.4a) has a logarithmic divergence for the S_{11} channel. Usually,[11] form factors are included in $\delta_\alpha(E)$ and $\eta_\alpha(E)$ in both channels thus removing high-energy parts so as to improve the convergence. The function $\mathcal{D}_\alpha^+(E)$ is made convergent by means of the cutoff. So is the function $v_\alpha(p)$. However, for energies well below the cutoff energy, the ratio $v_\alpha(p)v_\alpha(p')$ divided by $\mathcal{D}_\alpha^+(E)$, which provides the T_α matrix, is rather independent of that cutoff. On the other hand $\mathcal{H}_\alpha^{(V)}(E)$, given by (5.6) requires the knowledge of $v_\alpha(s)$ at large momenta, where $v_\alpha(s)$ is cutoff dependent. Fortunately the high-momentum part of the integral in Eq. (5.6) yields a contribution weakly dependent on V (far below the cutoff) and cancels out in Eq. (5.5).

Besides introducing a cut off, we have also regularized Eq. (2.4a) without cutoff by taking

$$\mathcal{D}_\alpha^+(E)=\exp\int_{M_N+m_\pi}^{\infty}\frac{dx}{\pi}\frac{\delta'_\alpha(x)}{E-x+i\eta}\frac{E-M_N-m_\pi}{x-M_N-m_\pi} \tag{5.7}$$

which is always finite and only differs from Eq. (2.4a) by a constant factor. In conclusion, we have used both regularizations and different values for the cutoff, and observed that the results are essentially unique near threshold.

By recalling Eqs. (5.4) and (2.3) and the fact that $v_\alpha(p)$ does depend on the energy we can include the effects due to the binding at threshold by multiplying the T matrix of Eq. (2.3) by the factor

$$\bar{F}_\alpha(V)=\frac{\mathcal{D}_\alpha^+(M_N+m_\pi)}{\mathcal{D}_\alpha^{+(V)}(M_N+m_\pi+V)},\quad \alpha=1,3\ . \tag{5.8}$$

Note that we are including the binding effects both, in the external nucleon energy, through the argument in the denominator, $M_N+m_\pi+V$, and in the medium modified T matrix through $\mathcal{D}_\alpha^{+(V)}$. The results of Sec. IV would be obtained by multiplying the T matrix of Eq. (2.3) by the factor

$$F_\alpha(V)=\frac{\mathcal{D}_\alpha^+(M_N+m_\pi)}{\mathcal{D}_\alpha^+(M_N+m_\pi+V)},\quad \alpha=1,3\ . \tag{5.9}$$

The functions $F_\alpha(V),\bar{F}_\alpha(V)$ are plotted in Fig. 6. Not surprisingly the new results show a far more moderate dependence on V than before, due to the partial cancellation between the binding energies in external and internal nucleons lines in Fig. 5. Actually, if we had taken the same binding as for nucleons in the fermions of the inelastic channels, we would have obtained

$$T_\alpha^{(V)}(p',p;E)=T_\alpha(p',p;E-V)\ ,$$

which holds when V is just additive in the c.m. and only πN intermediate states are allowed in the T matrix. Then the nucleon binding energy would have had no effect whatsoever on the optical potential, because we would have to take $T_\alpha^{(V)}(p',p;E+V)$ which equals $T_\alpha(p',p;E)$.

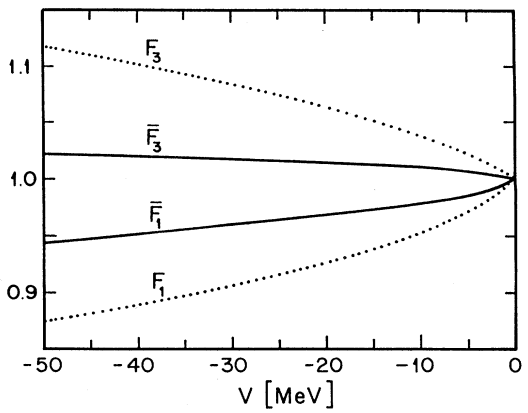

FIG. 6. Multiplicative binding corrections to the S_{11} and S_{13} channels as a function of the nuclear potential. Dotted line: free T-matrix [Eq. (5.9)], solid line: binding-corrected T-matrix, [Eq. (5.8)].

By construction, the imaginary part from the modified impulse approximation graph exactly cancels that coming from the Pauli blocking piece, because the binding has now been put both in the external and internal nucleon lines in Fig. 5, the problem of the imaginary part which arose in Sec. IV and which was mentioned at the beginning of this section does not appear. Furthermore, the Pauli correction can be easily implemented in the nuclear medium corrected T-matrix scheme by just introducing the appropriate step function in Eq. (5.6) in order to remove nucleons produced below the Fermi surface. We have checked that this method yields the same results as the two graphs in Fig. 4. We have also checked that the Pauli correction is not appreciably modified by the binding correction of the modified impulse approximation.

The new result amounts to making the following substitution in Eq. (4.3b):

$$b_3V(\mathbf{r})\to\bar{b}_3V(\mathbf{r})=-\tfrac{1}{3}[a_1(\bar{F}_1(V)-1)+2a_3(\bar{F}_3(V)-1)] \tag{5.10}$$

[the same $b_3\to\bar{b}_3$ substitution should be done in all other terms in (4.3) but the other terms proportional to b_3 are actually very small].

The functions $\bar{F}_\alpha(V)-1$ are approximately linear in V (see Fig. 6). According to Ref. 5, we will obtain an equivalent optical potential by taking $\bar{b}_3$ frozen to its value at the s-wave effective density $\rho_{\text{eff}}\simeq 0.5\rho_0$. This gives

$$\bar{b}_3=0.022m_\pi^{-2} \tag{5.11}$$

which is about half the value quoted in Eq. (4.2).

One more word should be said about the term with the square root of the energy mentioned in the previous section. As can be seen in Appendix C, the energy dependence of the free T matrix near threshold goes as

$$b_0 + A\sqrt{-\mathcal{E}} + \bar{b}_3'\mathcal{E} + O(\mathcal{E}^{3/2}) ,$$
$$\mathcal{E} = E - M_N - m_\pi . \tag{5.12}$$

Above threshold the optical theorem directly relates the (imaginary) second term to the scattering lengths, implying that such a term has a purely elastic origin. As a consequence, with the prescription of the modified impulse approximation of including the binding in external and internal nucleons in the T matrix, the potential energy should be included both in E and M_N in (5.12) and hence there are no changes from this square root term. The linear term in $\mathcal{E}$, however, would contain contributions from intermediate inelastic channels and hence would lead to some correction since no binding has been associated to these inelastic channels.

With the new value of $\bar{b}_3$ of (5.11), the new optical potential would be given again by (4.14) but $\delta B_0^{(V_N)}$ is now

$$\delta B_0^{(V_N)} = -0.019 m_\pi^{-4} \tag{5.13}$$

instead of $-0.048 m_\pi^{-4}$ which we get in the previous section.

If we had taken also the same binding in the inelastic channels as in the elastic one we would have obtained once more Eq. (4.14) but with $\delta B_0^{(V_N)} = 0$. We would like to see these two results as indicative of the intrinsic theoretical uncertainties of our approach.

VI. SUMMARY AND CONCLUSIONS

We carried out a thorough study of the lowest order s-wave optical potential for pionic atoms in order to include the effect of the off-shell extrapolation and binding effects. All the efforts have been concentrated in the isoscalar part of the optical potential since the isoscalar πN amplitude is very much affected by the off-shell extrapolation, which is not the case for the isovector πN amplitude.

We have proceeded in three steps. In the first one we have worried only about the off-shell extrapolation in momentum. In the second step we have also taken into account the nuclear binding. However, a closer look at the T matrix and its internal structure shows that a straightforward inclusion of binding effects in the variables of the T matrix is inconsistent and such binding effects have to be incorporated simultaneously in the variables of any theoretical model for the T matrix. We have done so in the context of the LMM model and have found that the results depend somewhat on the nuclear binding of the baryonic states of the inelastic intermediate states in the T matrix expansion. While this binding is obviously difficult to assess for all the possible baryonic components, two limiting assumptions have been done: First, no binding for these baryonic components, other than the nucleon; second, the same binding for these baryonic components as for the nucleon. We take these two results as an indication of the intrinsic theoretical uncertainties of our study.

The results that we obtain for the first-order s-wave optical potential in pionic atoms are given by

$$-\frac{2\omega V_{\rm opt}^{(s)}(\mathbf{r},\omega)}{4\pi(1+\epsilon)} = b_0\rho(\mathbf{r}) + \delta b_0^P \rho(\mathbf{r}) + \delta\bar{b}_0^{\rm Lap}\rho(r) + \delta B_0^{(V_N)}\frac{1+\epsilon/2}{1+\epsilon}\rho^2(\mathbf{r}) , \tag{6.1}$$

where the last two terms are coming from the off-shell extrapolation and binding effects (part of $\delta\bar{b}_0^{\rm Lap}$ also comes from the angular transformation term of the πN p-wave amplitude).

Our numerical results are the following:

$$b_0 = -0.013 m_\pi^{-1} \quad \text{(experiment, Ref. 8)} ,$$
$$\delta b_0^P = -0.014 m_\pi^{-1} ,$$
$$\delta\bar{b}_0^{\rm Lap} = 0.005 m_\pi^{-1} , \tag{6.2}$$
$$\text{case (a):} \quad \delta B_0^{(V_N)} = -0.020 m_\pi^{-4}$$
$$\text{(with no binding energy for inelastic states)} ,$$
$$\text{case (b):} \quad \delta B_0^{(V_N)} = 0$$
$$\text{(with same binding for inelastic states and nucleons)} .$$

We take these last two values of $\delta B_0^{(V_N)}$ as indicative of the intrinsic theoretical uncertainties of our approach. We can bring the results of Eq. (6.2) in a more handy form by means of the correlation between the $\bar{b}_0$ and B_0 parameters of Eq. (3.8). We obtain

$$\delta\bar{b}_0^{\rm Lap} = 0.005 m_\pi^{-1} ,$$
$$\text{case (a):} \quad \delta\bar{b}_0^{(V_N)} = -0.0045 m_\pi^{-1} , \tag{6.3}$$
$$\text{case (b):} \quad \delta\bar{b}_0^{(V_N)} = 0 ,$$

where $\delta\bar{b}_0^{(V_N)}$ is the corresponding term to the $\delta B_0^{(V_N)}$. In addition, we can also write in terms of $\delta\bar{b}_0$ the theoretical result from the second order calculation of Eq. (1.3), which corresponds to

$$\delta\bar{b}_0^{(2)} = 0.0067 m_\pi^{-1} . \tag{6.4}$$

Note that in case (a) the two effects from the off-shell extrapolation and nuclear binding roughly cancel each other. In case (b) we get a net attraction from the off-shell extrapolation, making the problem of the missing repulsion mentioned at the beginning even worse.

For the sake of comparison and assuming that the problem lies in the experimental value of b_0, if we take the theoretical values that we have obtained from our calculations and the one of δb_0^P, we would need a value of the parameter $b_0 = (-0.022 \sim -0.027) m_\pi^{-1}$ in order to fit the pionic atom data [the present value, used here is $-0.013 m_\pi^{-1}$, given in (6.2)].

We carried the analysis in the LMM model, which provides the coefficients b_2 and $\bar{b}_3$ of the off-shell extrapolation and energy dependence, respectively, but we should mention that the results are quite model independent. Indeed, the parameters b_2 and $\bar{b}_3$ are not independent since a linear combination of them [see Eq. (C6)] is relat-

ed to the behavior of the πN phase shifts around threshold. On the other hand we saw in Sec. II that different models gave similar off-shell extrapolations.

We have seen that we have some theoretical uncertainties, but the discrepancies with the present experimental values of b_0 remain in any case. While one cannot disregard the presence of some other nuclear corrections not taken into account in the present work, together with the work in Ref. 4, we should call the attention to the fact that the quantity b_0 is very small, coming from the cancellation between two scattering lengths one order of magnitude bigger, i.e., a 15% error in these would result in a 100% change in b_0.

It is clear to us that a more precise determination of the πN phase shifts at low energy and of the scattering lengths is needed. Some recent results from the analysis of $\pi^- p$ atoms give[20] a $\pi^- p$ scattering length in disagreement with the results of Ref. 8. The present values of the scattering lengths might also be at the heart of the problems with the σ-term, as noted in Ref. 21. In summary we believe that the time has come for a serious reanalysis of the low energy πN data before more efforts are devoted to the nuclear problem.

ACKNOWLEDGMENTS

We are very grateful to R. Seki and K. Masutani for clarifying discussions related to Sec. III. This work was supported in part by funds provided by the U.S. Department of Energy (DOE) under Contract No. DE-AC02-76ER03069, and by Comisión Asesora de Investigación Científica y Técnica and Consejo Superior de Investigaciones Científicas.

APPENDIX A: WIGNER TRANSFORMATION

Let us suppose a nucleus with A nucleons. Its state is described in the independent particle model by A single-particle eigenstates $|n\rangle$ with $n=1,2,\ldots,A$. For such systems the one-body density operator is:

$$\hat{\rho}=\sum_{n=1}^{A}|n\rangle\langle n|\ ,\qquad \langle n|n'\rangle=\delta_{nn'}\ . \tag{A1}$$

This operator $\hat{\rho}$ allows one to calculate the expectation value of any one-body operator $\hat{O}$ as $\langle\hat{O}\rangle=\mathrm{Tr}(\hat{O}\hat{\rho})$. Let $|\mathbf{r}\rangle$ be an eigenstate in coordinate representation, with the normalization

$$\langle\mathbf{r}|\mathbf{r}'\rangle=\delta^3(\mathbf{r}-\mathbf{r}')\ , \tag{A2}$$

the density matrix is

$$\rho_R(\mathbf{r}',\mathbf{r})\equiv\langle\mathbf{r}'|\hat{\rho}|\mathbf{r}\rangle=\sum_{n=1}^{A}\psi_n(\mathbf{r}')\psi_n^*(\mathbf{r})\ ,\qquad \psi_n(\mathbf{r})\equiv\langle\mathbf{r}|n\rangle\ . \tag{A3}$$

We will also use the momentum representation. The eigenstate $|\mathbf{q}\rangle$ with eigenvalue of momentum $\mathbf{q}$ is defined by

$$\langle\mathbf{r}|\mathbf{q}\rangle=\frac{e^{i\mathbf{q}\cdot\mathbf{r}}}{(2\pi)^{3/2}}\ . \tag{A4}$$

In this representation the states $|n\rangle$ are given by the wave function $\bar{\psi}_n(\mathbf{q})$ and the $\hat{\rho}$ matrix is

$$\rho_Q(\mathbf{q}',\mathbf{q})\equiv\langle\mathbf{q}'|\hat{\rho}|\mathbf{q}\rangle=\sum_{n=1}^{A}\bar{\psi}_n(\mathbf{q}')\bar{\psi}_n^*(\mathbf{q})\ ,\qquad \bar{\psi}_n(\mathbf{q})\equiv\langle\mathbf{q}|n\rangle\ . \tag{A5}$$

With the normalization (A4), the wave functions in coordinate and momentum representation are related by

$$\psi_n(\mathbf{r})=\int d^3\mathbf{q}\frac{e^{i\mathbf{q}\cdot\mathbf{r}}}{(2\pi)^{3/2}}\bar{\psi}_n(\mathbf{q})\ . \tag{A6}$$

There is a hybrid representation in coordinate and momentum space for the $\hat{\rho}$ matrix, it is given by the Wigner transform $\rho_W(\mathbf{R},\mathbf{Q})$ defined as follows:

$$\rho_W(\mathbf{R},\mathbf{Q})\equiv\int\frac{d^3\mathbf{s}}{(2\pi)^3}e^{-i\mathbf{s}\cdot\mathbf{Q}}\left\langle\mathbf{R}+\frac{\mathbf{s}}{2}\left|\hat{\rho}\right|\mathbf{R}-\frac{\mathbf{s}}{2}\right\rangle\ . \tag{A7}$$

This Wigner transform $\rho_W(\mathbf{R},\mathbf{Q})$ contains the whole information of the $\hat{\rho}$ operator.

This representation has some advantages. For any one-body operator $\hat{O}$ we define its Wigner representation

$$O(\mathbf{R},\mathbf{Q})=\int d^3\mathbf{s}\,e^{-i\mathbf{s}\cdot\mathbf{Q}}\left\langle\mathbf{R}+\frac{\mathbf{s}}{2}\left|\hat{O}\right|\mathbf{R}-\frac{\mathbf{s}}{2}\right\rangle\ , \tag{A7'}$$

which is a real function if $\hat{O}$ is Hermitian.

So the expectation value of this operator $\hat{O}$ in this system is

$$\langle\hat{O}\rangle=\int d^3\mathbf{R}\,d^3\mathbf{Q}\,O(\mathbf{R},\mathbf{Q})\rho_W(\mathbf{R},\mathbf{Q})\ . \tag{A7''}$$

This expression is totally similar to the expectation value of a classical observable $O(\mathbf{R},\mathbf{Q})$ for a classical system with a probability $\rho_W(\mathbf{R},\mathbf{Q})$ in the phase space of the system. A difference between the classical case and the quantum case is that $\rho_W(\mathbf{R},\mathbf{Q})$ is not necessarily positive.

Some other interesting properties of the Wigner transform are

$$\rho_W(\mathbf{R},\mathbf{Q})=\int\frac{d^3\mathbf{k}}{(2\pi)^3}e^{i\mathbf{k}\cdot\mathbf{R}}\left\langle\mathbf{Q}+\frac{\mathbf{k}}{2}\left|\hat{\rho}\right|\mathbf{Q}-\frac{\mathbf{k}}{2}\right\rangle\ , \tag{A8}$$

which is equivalent to the definition given in (A7). It also verifies that

$$\int\rho_W(\mathbf{R},\mathbf{Q})d^3\mathbf{Q}=\sum_n\psi_n(\mathbf{R})\psi_n^*(\mathbf{R})\equiv\rho(\mathbf{R})\ , \tag{A9}$$

where $\rho(\mathbf{R})$ is the density of particles in coordinate space. And

$$\int\rho_W(\mathbf{R},\mathbf{Q})d^3\mathbf{R}=\sum_n\bar{\psi}_n(\mathbf{Q})\bar{\psi}_n^*(\mathbf{Q})\equiv\tau(\mathbf{Q})\ , \tag{A9'}$$

where $\tau(\mathbf{Q})$ is the probability density of particles in momentum space. The integration of $\rho_W(\mathbf{R},\mathbf{Q})$ over the whole phase space is the total number of particles in the system.

$$\int\rho_W(\mathbf{R},\mathbf{Q})d^3\mathbf{Q}d^3\mathbf{R}=A\ . \tag{A9''}$$

Another magnitude we are interested in is the Fourier transform of the density in coordinate space, $\tilde{\rho}(\mathbf{k})$

$$\tilde{\rho}(\mathbf{k}) \equiv \int d^3\mathbf{r}\, e^{-i\mathbf{k}\cdot\mathbf{r}}\rho(\mathbf{r}) \ . \tag{A10}$$

This quantity $\tilde{\rho}(\mathbf{k})$ is related to the probability of a momentum transfer $\mathbf{k}$ in the system and can be written as

$$\tilde{\rho}(\mathbf{k}) = \int d^3\mathbf{Q} \sum_{n=1}^{A} \bar{\psi}_n \left[\mathbf{Q}+\frac{\mathbf{k}}{2}\right] \bar{\psi}_n^* \left[\mathbf{Q}-\frac{\mathbf{k}}{2}\right] . \tag{A11}$$

One approximation to the Wigner transform which is often used is the Slater approximation given by

$$\rho_W(\mathbf{R},\mathbf{Q}) = \frac{4}{(2\pi)^3}\theta[k_F(\mathbf{R})-|\mathbf{Q}|] \ , \tag{A12}$$

being $k_F(\mathbf{R})$ defined by

$$\rho(\mathbf{R}) = \frac{2}{3\pi^2}k_F^3(\mathbf{R}) \ , \tag{A12'}$$

where a factor of 4 has been introduced to account for the spin and isospin, so we are already supposing that the fermions of the system are nucleons and the system is saturated in spin and isospin.

APPENDIX B: EXPLICIT CALCULATION OF THE OPTICAL POTENTIAL IN THE IMPULSE APPROXIMATION

We use the expression (1.7) which provides the optical potential in the impulse approximation from the invariant amplitude T expressed in the laboratory frame kinetic variables.

This invariant amplitude T is given in Eq. (1.6) as a function of the πN c.m. variables. Neglecting the binding energy of the nucleon against its mass and carrying out the change of π nucleus to πN frame in a non relativistic way (which is consistent with the amplitude expansion up to energy variation and squared momentum), we get for the amplitude,

$$T\left[\mathbf{P}+\frac{\mathbf{k}}{2},\mathbf{Q}-\frac{\mathbf{k}}{2};\mathbf{P}-\frac{\mathbf{k}}{2},\mathbf{Q}+\frac{\mathbf{k}}{2};M_N+E_n+\omega\right] = -4\pi(1+\epsilon)\left\{b_0+2b_2\left[(\alpha_1\mathbf{P}-\alpha_2\mathbf{Q})^2+\left[\frac{\mathbf{k}}{2}\right]^2\right]\right.$$
$$\left.+b_3\left[(\omega-m_\pi)+E_n-\frac{(\mathbf{P}+\mathbf{Q})^2}{2(M_N+\omega)}\right]\right\} \tag{B1}$$

with $\alpha_1 \equiv M_N/(M_N+\omega) \simeq (1-\epsilon)$; $\alpha_2 \equiv \omega/(M_N+\omega) \simeq \omega/M_N \equiv \epsilon$.

Substituting (B1) in (1.7) and using the Appendix A we can easily obtain

$$\frac{2\omega}{-4\pi(1+\epsilon)}V_{\text{opt}}^{(s)}(\mathbf{r}',\mathbf{r};\omega) = \{b_0+b_3(\omega-m_\pi)\}\rho(\mathbf{r})\delta(\mathbf{r}'-\mathbf{r}) \tag{B2a'}$$
$$+b_3\sum_n E_n\psi_n^*(\mathbf{r})\psi_n(\mathbf{r})\delta(\mathbf{r}'-\mathbf{r}) \tag{B2a''}$$
$$+\tfrac{1}{2}b_2\int\frac{d^3\mathbf{k}}{(2\pi)^3}\mathbf{k}^2 e^{i\mathbf{k}\cdot\mathbf{r}}\tilde{\rho}(\mathbf{k})\delta(\mathbf{r}'-\mathbf{r}) \tag{B2b}$$
$$+\left[2b_2\alpha_2^2-\frac{b_3}{2(M_N+\omega)}\right]\int d^3\mathbf{Q}\mathbf{Q}^2\rho_W(\mathbf{r},\mathbf{Q})\delta(\mathbf{r}'-\mathbf{r}) \tag{B2c}$$
$$+\left[2b_2\alpha_1^2-\frac{b_3}{2(M_N+\omega)}\right]\int\frac{d^3\mathbf{P}}{(2\pi)^3}e^{i\mathbf{P}\cdot(\mathbf{r}'-\mathbf{r})}\mathbf{P}^2\rho\left[\frac{\mathbf{r}+\mathbf{r}'}{2}\right] \tag{B2d}$$
$$+\left[-4b_2\alpha_1\alpha_2-\frac{b_3}{M_N+\omega}\right]\int\frac{d^3\mathbf{P}}{(2\pi)^3}e^{i\mathbf{P}\cdot(\mathbf{r}'-\mathbf{r})}$$
$$\times\int d^3\mathbf{Q}\,\mathbf{P}\cdot\mathbf{Q}\rho_W\left[\frac{\mathbf{r}'+\mathbf{r}}{2},\mathbf{Q}\right] . \tag{B2e}$$

The term (B2a') already has the phenomenological optical potential form. The integral in (B2b) is $-\nabla^2\rho(\mathbf{r})$. The (B2e) term is null. The terms (B2a'), (B2a''), (B2b), (B2c) are local. The term (B2d) is nonlocal because the separable T-matrix of the LMM model used is nonlocal. The integral of the (B2d) term can be formally written as $(-\nabla^2\delta)(\mathbf{r}'-\mathbf{r})$, and it is easy to show the following relationships:

$$\rho\left[\frac{\mathbf{r}'+\mathbf{r}}{2}\right]\int\frac{d^3\mathbf{P}}{(2\pi)^3}\mathbf{P}^2 e^{i\mathbf{P}\cdot(\mathbf{r}'-\mathbf{r})} = \rho\left[\frac{\mathbf{r}'+\mathbf{r}}{2}\right]\left[-\nabla^2\delta\right](\mathbf{r}'-\mathbf{r})$$
$$= \delta(\mathbf{r}'-\mathbf{r})[-\nabla\cdot\rho(\mathbf{r})\nabla-\tfrac{1}{4}(\nabla^2\rho(\mathbf{r}))] \ . \tag{B3}$$

The term (B2a'') needs a bit of work before showing a density dependence form. Let us do it:

$$\sum_{n=1}^{A} E_n \psi_n^*(\mathbf{r})\psi_n(\mathbf{r}) = \sum_{n=1}^{A} \psi_n^*(\mathbf{r}) \left[V(\mathbf{r}) - \frac{\nabla^2}{2M_N} \right] \psi_n(\mathbf{r})$$
$$= V(\mathbf{r})\rho(\mathbf{r}) - \frac{1}{2M_N} \lim_{\mathbf{r}' \to \mathbf{r}} \nabla_{\mathbf{r}}^2 \langle \mathbf{r}|\hat{\rho}|\mathbf{r}' \rangle \ . \tag{B4}$$

By using the Wigner representation introduced in Appendix A we get

$$\sum_{n=1}^{A} E_n \psi_n^*(\mathbf{r})\psi_n(\mathbf{r}) = V(\mathbf{r})\rho(\mathbf{r}) + \frac{1}{2M_N} \int d^3\mathbf{Q}\, \mathbf{Q}^2 \rho_W(\mathbf{r},\mathbf{Q}) - \frac{1}{8M_N} \nabla^2 \rho(\mathbf{r}) \ . \tag{B5}$$

So we can write the impulse approximation to the pion-nucleus s-wave optical potential, corresponding to processes where only one nucleon of the nucleus interacts with the pion, as follows:

$$V_{\text{opt}}(\mathbf{r}',\mathbf{r};\omega) = \delta(\mathbf{r}'-\mathbf{r}) V_{\text{opt}}(\mathbf{r};\omega) \ ; \tag{B6}$$

$$-\frac{2\omega}{4\pi(1+\epsilon)} V_{\text{opt}}^{(s)}(\mathbf{r};\omega) = [b_0 + b_3(\omega - m_\pi)]\rho(\mathbf{r}) + b_3 V(\mathbf{r})\rho(\mathbf{r}) + b_3 \epsilon \frac{1}{2M_N} \int d^3\mathbf{Q}\, \mathbf{Q}^2 \rho_W(\mathbf{r},\mathbf{Q})$$
$$- \left[b_2(1-\epsilon) + \frac{b_3}{8(M_N+\omega)} \epsilon \right] \nabla^2 \rho(\mathbf{r})$$
$$- \left[2b_2(1-2\epsilon) - \frac{b_3}{2(M_N+\omega)} \right] \nabla \cdot \rho(\mathbf{r}) \nabla \ , \tag{B7}$$

where we have neglected the terms in $\epsilon^2 \equiv (\omega/M_N)^2 \simeq \frac{1}{50}$.

The term with the integral in $\mathbf{Q}$ is multiplied by the factor ϵ which is small. We shall use the Slater approximation to evaluate it. By means of Eq. (A12), we obtain immediately

$$b_3 \epsilon \frac{1}{2M_N} \int d^3\mathbf{Q}\, \mathbf{Q}^2 \rho_W(\mathbf{r},\mathbf{Q}) = b_3 \epsilon \frac{3}{5} \frac{k_F^2(\mathbf{r})}{2M_N} \rho(\mathbf{r}) \ . \tag{B8}$$

The actual numerical calculation of this term gives a result of around one order of magnitude smaller than the other corrections and we shall neglect it.

Now, let us take as the isoscalar p-wave πN amplitude the one given in Eq. (4.8). By repeating the same steps done for the s-wave case,

$$-\frac{(1+\epsilon)2\omega}{4\pi} V_{\text{opt}}^{(p)}(\mathbf{r};\omega) = -c_0 \nabla\rho(\mathbf{r})\nabla + \frac{\epsilon}{2} c_0 \Delta\rho(\mathbf{r}) + \epsilon^2 c_0 \tfrac{3}{5} k_F^2(\mathbf{r})\rho(\mathbf{r}) \ , \tag{B9}$$

where we see that the change from πN c.m. frame to the laboratory frame gives a local contribution to the optical potential of the order of ϵ, associated to the πN interaction in p-wave. We will also neglect here ϵ^2 terms as usual[18] and take Eq. (4.10) as a result.

As the nucleus is very heavy in relation to the pion we will use the laboratory frame instead of the pion-nucleus c.m. frame, this is equivalent to neglecting ϵ/A terms, where A = mass number.

APPENDIX C: EXPANSION AROUND THRESHOLD

The phase shifts around the threshold, for any $L=0$ channel, go as

$$\delta(q) = aq[1 + \lambda q^2 + O(q^4)], \quad \eta(q) = 1 \ . \tag{C1}$$

These expressions together with the LMM model, allow one to analyze the behavior around threshold of all the functions $v_\alpha, \mathcal{D}_\alpha, \mathcal{H}_\alpha, \ldots$ involved in the off-shell extrapolation used in this paper.

Instead of the energy in c.m., E, we will use, for the sake of simplicity, the variable $\tilde{E}$ defined as

$$\tilde{E} = (E - M_N - m_\pi) 2m_\pi M_N / (M_N + m_\pi) \ . \tag{C2}$$

Above threshold but near it, $\tilde{E}$ is the squared on-shell momentum.

We obtain for the extrapolation in momentum:

$$\frac{v(q)}{v(0)} = 1 + B_2 q^2 + O(q^4) \ . \tag{C3}$$

For $\mathcal{D}^+(E)$ and $T(p',p;E)$ above threshold, the results obtained from (2.4a) and (C1) are

$$\frac{\mathcal{D}^+(0)}{\mathcal{D}^+(E)} = [1 + \bar{B}_3 \tilde{E} + O(\tilde{E}^2)] e^{i\delta(\tilde{E})}$$
$$= \left[1 + \left[\bar{B}_3 - \frac{a^2}{2} \right] \tilde{E} + O(\tilde{E}^2) \right]$$
$$+ i[a\tilde{E}^{1/2} + O(\tilde{E}^{3/2})] \ ;$$
$$T(p',p;E) = a \left[1 + B_2 \frac{(p'^2+p^2)}{2} + \bar{B}_3 \tilde{E} + O(\tilde{E}^2) \right] e^{i\delta(\tilde{E})}$$
$$= a \left\{ \left[1 + B_2 \frac{(p'^2+p^2)}{2} + \left[\bar{B}_3 - \frac{a^2}{2} \right] \tilde{E} \right.\right.$$
$$\left.\left. + O(\tilde{E}^2) \right] + i[a\tilde{E}^{1/2} + O(\tilde{E}^{3/2})] \right\} . \tag{C4}$$

Below threshold one gets:

$$\frac{\mathcal{D}^+(0)}{\mathcal{D}^+(E)}=\left[1-a(-\tilde{E})^{1/2}+\left[\bar{B}_3-\frac{a^2}{2}\right]\tilde{E}+O((-\tilde{E})^{3/2})\right];$$
$$T(p',p;E)=a\left\{\left[1+B_2\frac{(p'^2+p^2)}{2}-a(-\tilde{E})^{1/2}+\left[\bar{B}_3-\frac{a^2}{2}\right]\tilde{E}+O((-\tilde{E})^{3/2})\right]\right\}. \quad \text{(C5)}$$

Indeed Eq. (C5) is the analytic extension of (C4) by taking for $\sqrt{\tilde{E}}$ the value $i\sqrt{-\tilde{E}}$ when $\tilde{E}<0$.

The parameters B_2 and $\bar{B}_3$ are constrained by the behavior of the phase shifts around threshold by the following relationship:

$$2B_2+\bar{B}_3=\lambda-\frac{a^2}{6}, \quad \text{(C6)}$$

which is obtained by putting the T matrix on-shell.

---

*On leave from Departamento de Física Teórica, Facultad de Ciencias, Valladolid, Spain.
†On leave from Departamento de Física and Instituto de Física Corpuscular, Facultad de Física, Burjassot (Valencia), Spain.

[1]M. Ericson and T. E. O. Ericson, Ann. Phys. (N.Y.) **36**, 323 (1966).
[2]F. Hachenberg and H. J. Pirner, Ann. Phys. (N.Y.) **112**, 401 (1978).
[3]D. O. Riska, Nucl. Phys. **A377**, 319 (1982); D. O. Riska and H. Sarafian, Phys. Lett. **95B**, 185 (1980).
[4]C. García-Recio, E. Oset, and L. L. Salcedo, Phys. Rev. C **37**, 194 (1988).
[5]R. Seki and K. Masutani, Phys. Rev. C **27**, 2799 (1983).
[6]J. A. Carr, H. McManus, and K. Stricker, Phys. Rev. C **25**, 925 (1982).
[7]J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965).
[8]G. Höhler, F. Kaiser, R. Koch, and E. Pietarinen, *Handbook of Pion-Nucleon Scattering,* Vol. 12, No. 1 of *Physics Data Series* (Fachinformationszentrum Energie-Physik-Mathematik G.m.b.H., Karlsruhe, Germany, 1979).
[9]J. Hamilton, in *High Energy Physics,* edited by E. H. S. Burhop (Academic, New York, 1967), Vol. I.
[10]W. Weise, Nucl. Phys. **A278**, 402 (1977).
[11]G. T. Londergan, K. W. McVoy, and E. J. Moniz, Ann Phys. **86**, 147 (1974).
[12]S. A. Coon *et al.,* Nucl. Phys. **A317**, 242 (1979), and references therein.
[13]C. García-Recio, R Brockmann, E. Oset, and H. Toki, An. Fis. **83**, 127 (1987).
[14]J. P. Jeukenne, A. Lejeune, and C. Mahaux, Phys. Rep. **25**, 83 (1976).
[15]R. Seki, K. Masutani, and K. Yazaki, Phys. Rev. C **27**, 2817 (1983).
[16]R. A. Arndt, private communication.
[17]W. Rarita *et al.,* Phys. Rev. **165**, 1615 (1968).
[18]K. Striker, H. McManus, and J. A. Carr, Phys. Rev. C **19**, 929 (1979).
[19]E. Oset, Y. Futami, and H. Toki, Nucl. Phys. **A448**, 597 (1986).
[20]E. Bovet *et al.,* Phys. Lett. **135B**, 231 (1985).
[21]W. Kluge, in *Pion-Nucleus Physics: Future Directions and New Facilities at LAMPF,* Proceedings of the Los Alamos Conference on Pion-Nucleus Physics, AIP Conf. No. 163, edited by R. J. Peterson and D. D. Strottman (AIP, New York, 1987).