Structure of the first excited state of ^{4}He

Dean Halderson, Ming Yu, and Jiang Yu
Physics Department, Western Michigan University, Kalamazoo, Michigan 49008
(Received 11 October 1988)

The inelastic electron scattering form factor for the 0^+ first excited state of ^{4}He has been calculated with wave functions from the recoil corrected continuum shell model. The use of a realistic interaction and proper boundary conditions lead to excellent agreement with experiment. These results demonstrate that this state can be understood as a superposition of simple 1p-1h excitations in the internal coordinates.

I. INTRODUCTION

The first excited state of ^{4}He is a 0^+ state and is, therefore, a candidate for the breathing mode of the nucleus. It has been of interest for some time because inelastic electron scattering experiments[1–3] have shown that it accounts for a very small percentage of the energy weighted sum rule. Indeed, it has been demonstrated that shell-model calculations, which assume a $1s0s^{-1}$ $(J=0,T=0)$ configuration, overpredict the strength by a factor of 20.[4] The inclusion of higher order shell-model configurations reduces this factor to between 5 and 10.

The exact nature of the 0^+ state has therefore been somewhat puzzling and has led to speculations that the shell model is inappropriate for describing this light system. Two other types of calculations have been performed for this state. A resonating group calculation with a central interaction and bound-state approximation was reported in Ref. 5. The resulting form factor was approximately three times larger than that observed. A calculation employing hyperspherical harmonics was reported in Ref. 6. Here good agreement with the experimental form factor was obtained. Since the calculated state turned out to be pure hyper-radial excitation, the authors concluded that it was a collective excitation of the ground state.

In this paper it is shown that it is possible to describe this state in a 1p-1h shell-model context, provided one employs a realistic interaction, translationally invariant wave functions, and proper boundary conditions. The recoil corrected continuum shell model[7] (RCCSM) incorporates these three important conditions.

II. THEORY

The RCCSM in the 1p-1h approximations has been very successful in describing low energy nucleon scattering phenomena for the four-nucleon systems. The formalism for this model is thoroughly described in Ref. 8. Briefly, the model employs the translationally invariant Hamiltonian

$$T+V=(2m)^{-1}\sum_i p_i^2-T_{\mathrm{c.m.}}+\sum_{i<j} v_{ij} \ , \tag{1}$$

where the two-body interaction is the Coulomb potential plus the g-matrix interaction $M3Y$,[9] which includes noncentral forces. The basis consists of one-particle excitations in the harmonic oscillator wave functions for the internal coordinate ϵ_3 in Fig. 1. Proper boundary conditions are imposed by R-matrix techniques at a matching radius of $a_c=7.2$ fm. A smooth joining to Coulomb functions is accomplished by allowing particle excitations up to $2n+l=14$, where n begins at zero. The core states of ^{3}H and ^{3}He are taken as pure $0s^3$. The oscillator constant, $\nu=m\omega/\hbar$, is chosen as 0.36 fm^{-2} to reproduce the mean-squared radius of ^{3}H.

Previous attempts to describe inelastic scattering of pions and electrons in the context of the recoil corrected continuum shell model have treated the ^{4}He ground state as a pure $0s^4$ and considered only the coordinate $\mathbf{r}_4$ in Fig. 1.[10,11] This is because the wave function of the coordinate ϵ_3 is readily available. However, one sees that if the coordinate $\mathbf{r}_4$ is excited, the coordinates $\mathbf{r}_1$, $\mathbf{r}_2$, and $\mathbf{r}_3$ will also move slightly with respect to the center of mass. This constitutes a target recoil or center of mass correction which was omitted from previous work. In the present work all coordinates are considered as well as $ns0s^{-1}$ correlations in the ground-state wave function.

Therefore, one is looking at a transition between an initial state and a final state of the same form:

$$\phi_{i,f}=a_0|0s^4\rangle|S=0\rangle + \sum_{n,t} a_{nt}(1-P_{34})|0s^3(\epsilon_1,\epsilon_2) \times ns(\epsilon_3)_t 0s(\mathbf{R})\rangle|S=0\rangle \ , \tag{2}$$

with components of $0s^4$ plus $ns0s^{-1}$ correlations. But because they are solutions to the same Hamiltonian at different energies, the states are orthogonal. This orthogonality is crucial in describing the shape of the form factor.

The great advantage of the RCCSM was its ability to provide matrix elements of translationally invariant operators in the internal coordinates by calculating matrix elements in normal shell-model coordinates, $\mathbf{x}_i$, with a fixed origin. This was very convenient for operators such as the two-body interaction, the kinetic energy, and transition operators for which a long wavelength approximation could be made. However, at high momentum

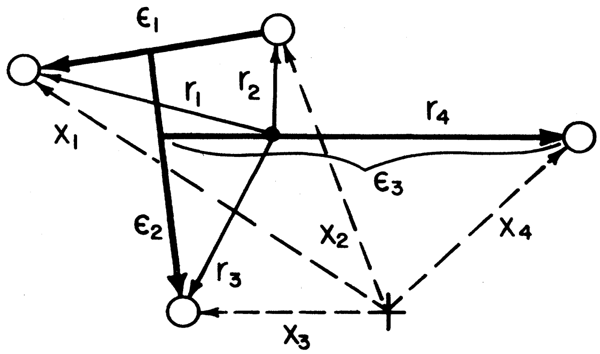

FIG. 1. The RCCSM coordinate system.

transfer q, operators such as $j_l(qr_i)=j_l(q|x_i-R|)$ do not lend themselves to a simple decomposition in terms of the shell-model coordinates, $\mathbf{x}_i$. Therefore, the matrix element of interest for the present problem,

$$M_0=\left\langle \phi_f \left|\left| \sum_i Y_0(\hat{r}_i) j_0(qr_i) \right|\right| \phi_i \right\rangle , \quad (3)$$

must be done explicitly in the relative coordinates. This involves a number of terms, most of which require three-dimensional integrals.

Cross sections for excitation of states above particle emission threshold are given by the expression[12]

$$d^2\sigma/d\Omega\, dE=(1/2\pi\hbar^2) \sum_{c,J_B} (\mu_c/k_c)(d\sigma_{c,J_B}/d\Omega) , \quad (4)$$

where μ is the nucleon reduced mass, k_c is the nucleon asymptotic relative momentum in the channel c, and $d\sigma_{c,J_B}/d\Omega$ is a fictitious Born cross section, calculated for nucleon wave functions with flux v_c in channel c. The index c stands for $\alpha J_c jl$ with J_c and j coupled to J_B, where J_c is the angular momentum of ^{3}H or ^{3}He, l and j are the nucleon orbital and total angular momentum, and α distinguishes between ^{3}H and ^{3}He.

The form factor is defined as

$$F=(d\sigma/d\Omega)/4\pi\sigma_M , \quad (5)$$

where σ_M is given by

$$\sigma_M=\left[\frac{Ze^2}{2E_0}\right]^2 \frac{\cos^2\frac{1}{2}\theta}{\sin^4\frac{1}{2}\theta} \frac{1}{1+2E_0\sin^2\frac{1}{2}\theta/Mc^2} . \quad (6)$$

III. RESULTS

The first comparison shown in Fig. 2 is the calculated ground-state form factor with the measured form factor.[13] The theoretical curves are multiplied by the finite proton size correction factor, $1/(1+0.0533q^2)^4$. Two theoretical curves are plotted. The dashed line results from assuming a pure $0s^4$ configuration with $\nu=0.36$ fm^{-2}. Because of the expansion

$$4\pi F(q)=(1 \cdots \langle r^2\rangle q^2+ \cdots \langle r^4\rangle q^4+ \cdots)^2 , \quad (7)$$

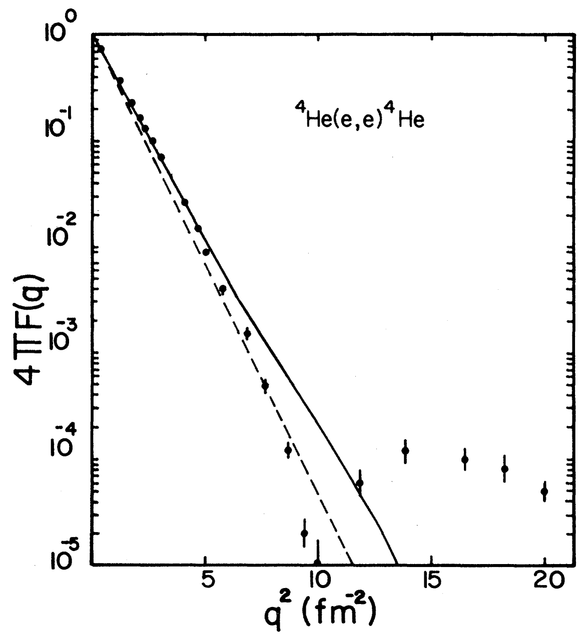

FIG. 2. The elastic form factor for ^{4}He. The dashed line is the result of a calculation with the $0s^4$ configuration. The solid curve is the result of a calculation that includes ground-state correlations. The data are from Ref. 13.

a comparison of data with calculated results at the first shoulder of the elastic form factor will test agreement between calculated and measured rms radii. The dashed cure clearly shows too large a radius because the oscillator constant was chosen to fit ^{3}H. The solid line, which is the result of including the ground-state correlations, does quite well in the first shoulder region, demonstrating that the $M3Y$ interaction produces an rms radius in agreement with experiment.

Comparison with three measured sets of form factors for the first excited state is made by the following procedure. This procedure does not eliminate the confusion among data sets. The reduced matrix elements $B(C0,q)$, given in Ref. 2, are converted to the above definition of form factors via $F(q)=B(C0,q)/4$. The form factors in Ref. 1 are converted to the definition in Eq. (5) by squaring them, dividing by $4\pi Z^2$, and dividing by a correction factor[2] of 3.2. The cross sections in Ref. 3 were converted to form factors according to Eqs. (5). Cross sections were used in this case because the reported form factors did not appear to be quite consistent with the definitions in Eqs. (5) and (6).

The calculated form factors are shown in Fig. 3 as compared to the data. The short-dash line is the calculated result, assuming a pure $0s^4$ ground state and discarding $0s^4$ components in the excited state. The dashed line is the calculated form factor with the complete wave functions included. One sees a 30% reduction in strength due to ground-state correlations. The solid line results from finite proton size correction of the dashed line and is the final result. Both the size and shape of the form factor is reasonable.

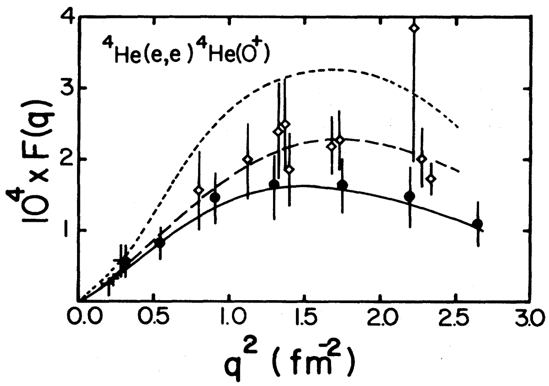

FIG. 3. The inelastic form factor for ^{4}He (0^+). The short-dash line is the result of a calculation with the $0s^4$ configuration for the ground state. The dashed line is the result of a calculation with ground-state correlations. The solid line is the same calculation as the dashed line, but corrected for proton finite size. The data are represented by diamonds, circles, and crosses and are from Ref. 3, 1, and 2, respectively.

TABLE I. Percentage of particle configuration in the wave function at $E_p = 6.0$ MeV.

Particle orbit	Percentage proton excitation	Percentage neutron excitation
$0s^4$	2.4	
$1s$	19.2	14.4
$2s$	12.1	7.6
$3s$	15.0	9.8
$4s$	9.8	6.2
$5s$	5.7	3.8
$6s$	2.0	1.4
$7s$	0.2	0.1

A three-dimensional graph of the calculation corresponding to the dashed line in Fig. 3 is shown in Fig. 4. Here it is pointed out that the theoretical points in Fig. 3 were obtained by integrating over the energy region, $E_p = 0$–1.2 MeV. This may not have been the correct procedure, since some of the 0^+ strength might have been assumed to be background in the experimental papers. This effect would be difficult to estimate since the different experimental papers showed different shapes and ranges for the background.

Another sources of uncertainty that could increase the calculated form factor can be seen in Fig. 4. Here one sees that part of the resonance appears to be cut away. The beginning of this cut coincides with the opening of the neutron threshold. With no Coulomb barrier the s-wave neutron escapes easily and produces a very broad resonance. An asymmetric shape was predicted earlier from the work of Crone and Werntz.[14] The reduction in strength due to this effect is probably greater in the calculation than reflected in the data for two reasons. First, the $M3Y$ interaction places the 0^+ resonance slightly higher than its observed location, and the neutron threshold is calculated to be 0.69 MeV instead of 0.76 MeV as observed experimentally. This difficulty with the Coulomb energy difference between ^{3}H and ^{3}He occurs in most binding energy calculations that use only the Coulomb potential to break charge symmetry. The two effects combine to produce a cut that begins approxi-

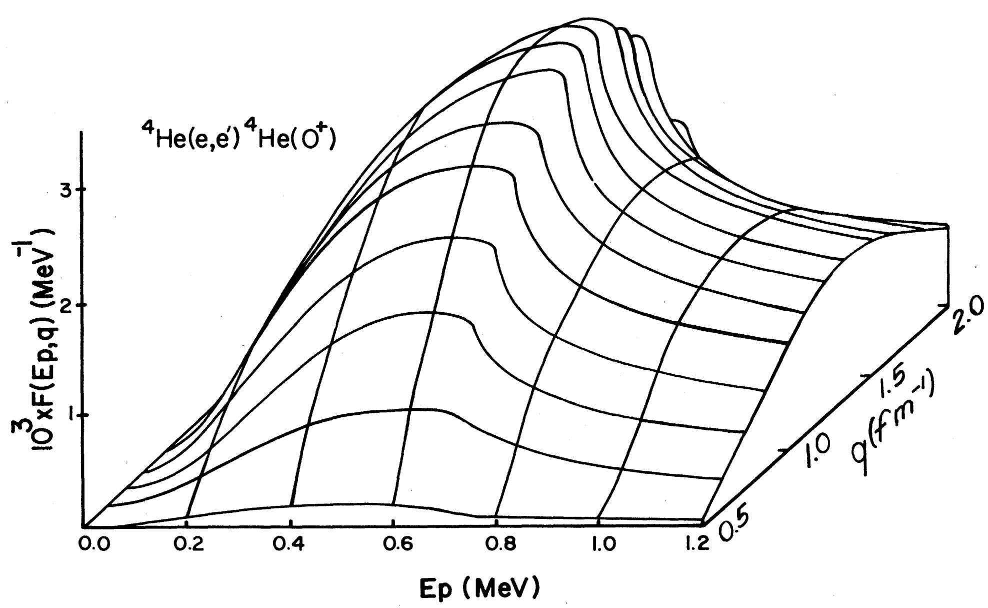

FIG. 4. Form factor per MeV as a function of q and proton energy. Calculation includes ground-state correlations but not proton finite size corrections.

mately 0.2 MeV early in the calculation and therefore steals some strength. Indeed, the later experiments did not report an asymmetric shape to their peaks.[2,3]

Finally, shown in Table I are the components of the wave function for $E_p = 0.60$ MeV. Here one can see that many configurations contribute to the wave function. The mixing of these configurations was not due primarily to the interaction, but to satisfying the continuum boundary conditions.

IV. CONCLUSION

In conclusions, the inelastic electron scattering form factor for the first excited state of ^{4}He appears to be well-described 1p-1h shell model terms, provided one employs a realistic interaction, translationally invariant wave functions, and proper continuum boundary conditions. The wave function at resonance is not a simple $2\hbar\omega$ excitation, but requires a linear combination of s-state particle excitations of high oscillator principle quantum number in order to satisfy the boundary conditions. However, because this linear combination is necessary only to produce the correct shape of the wave function for the one-particle excitation, this calculation suggests that the 0^+ can best be described as a 1p-1h excitation and not a collective state.

This work was supported in part by the National Science Foundation under Grant PHY-8604375.

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