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D-state effects in the radiative capture reaction $d+\alpha \rightarrow {}^6\mathrm{Li}+\gamma$

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The ${}^4\mathrm{He}(\vec{d},\gamma){}^6\mathrm{Li}$ reaction at low energies is discussed using a direct capture model and assuming that the process is predominantly $E2$. A good description of the total cross section for 1 MeV $< E_{\mathrm{c.m.}} <$ 9 MeV is obtained. The inclusion of a D-state component into the $\alpha+d$ cluster model wave function of ${}^6\mathrm{Li}$ originates nonvanishing tensor analyzing powers. T_{20} depends linearly on the D/S asymptotic ratio ρ of ${}^6\mathrm{Li}$ and increases in magnitude with energy. It is suggested that the measurement of T_{20} or A_{yy} can determine the sign and magnitude of ρ.

In the last few years there has been an increasing interest in the experimental and theoretical study of radiative capture reactions induced by polarized deuterons in the three and four nucleon bound systems. Measurements of the tensor analyzing powers (TAP) in the ${}^1\mathrm{H}(\vec{d},\gamma){}^3\mathrm{He}$ and ${}^2\mathrm{H}(\vec{d},\gamma){}^4\mathrm{He}$ reactions obtained by several groups at different energies, indicate that the D-state component in the ^{3}He and ^{4}He ground states plays a significant role in the explanation of the data.[1–4] With such experiments it is possible to understand quantitatively the nature of the D-state effects in the reaction and also to extract information on the magnitude of the D-state component of the few nucleon bound-state wave functions. Asymptotic D/S-state parameters in the three and four nucleon systems are also currently obtained from transfer reactions induced by polarized deuterons. Performing a distorted-wave Born approximation (DWBA) analysis of the TAP of $(\vec{d},t)$, $(\vec{d},{}^3\mathrm{He})$, and $(\vec{d},{}^4\mathrm{He})$ reactions[5,6] we can determine the D_2 parameter[7] which can be compared with detailed calculations of the bound-state wave functions using realistic N-N interactions.

It is well known that ^{6}Li is a deformed nuclei like the triton and helium. Although the $\alpha+d$ cluster component of the ^{6}Li wave function is predominantly an S state, a small percentage of D state is introduced by the tensor part of the triplet n-p interaction between clusters. In a cluster model of ^{6}Li the asymptotic D/S-state ratio ρ is predicted[8] to be negative in order to account for the negative quadrupole moment of ^{6}Li. However, a detailed study of ^{6}Li by Lehman *et al.*[9,10] using a three-body (αNN) model, where the contribution of the correlated (np) pair in the continuum is neglected, leads to a positive value for the ^{6}Li qudarupole moment and also to a positive value for ρ in contradiction with the cluster model result.

In this paper we show that the ${}^4\mathrm{He}(\vec{d},\gamma){}^6\mathrm{Li}$ reaction can be used to study the structure of ^{6}Li, particularly its D-state component. Because of its simple spin structure, the reaction is quite interesting from the theoretical point of view and reveals some similarities with the ${}^2\mathrm{H}(\vec{d},\gamma){}^4\mathrm{He}$ reaction. Assuming a direct capture model, we show that the TAP are very sensitive to the presence of a D-state component in ^{6}Li, and are directly related to the value of ρ.

The probability amplitude for a transition from a continuum α-d initial state $|\phi^{\sigma_d}\phi_\alpha;\mathbf{k}\rangle$ to the ground state of ^{6}Li $|{}^6\mathrm{Li};1M'\rangle$ with the emission of a photon with momentum $\mathbf{k}_\gamma$ and polarization $\boldsymbol{\varepsilon}_n$ is

$$T(\alpha\sigma_d,\mathbf{k}\rightarrow {}^6\mathrm{Li}\mathbf{k}_\gamma\boldsymbol{\varepsilon}_n) = \langle {}^6\mathrm{Li};1M'|H_e(\mathbf{k}_\gamma,\boldsymbol{\varepsilon}_n)|\phi^{\sigma_d}\phi_\alpha;\mathbf{k}\rangle , \quad (1)$$

where σ_d is the spin projection of the incident deuteron, $\mathbf{k}$ its asymptotic momentum, and M' the spin projection of ^{6}Li. Using a spherical basis in which $n=\pm 1$, the interaction Hamiltonian H_e is given in first-order perturbation theory by[11]

$$H_e(\mathbf{k}_\gamma,\boldsymbol{\varepsilon}_n) = -\sum_{LM\pi} n^\pi T_{LM}(\pi)^+ \mathcal{D}^{L*}_{Mn}(R) , \quad (2)$$

where $T_{LM}(\pi)$ are the multipole operators of rank L for the electric $(\pi=0)$ and magnetic $(\pi=1)$ transitions, and R is a rotation taking the z axis into the direction of $\mathbf{k}_\gamma$. We use the Madison Convention coordinate system where the z axis is along $\mathbf{k}$ and the y axis along $\mathbf{k}\times\mathbf{k}_\gamma$. Under the assumption that the nucleon-nucleon interaction is charge independent, and hence isospin is a good quantum number, both initial and final states have total isospin $T=0$. Thus the capture proceeds only through the isoscalar components of the operators T_{LM}. This implies that in the long-wavelength approximation (LWA), the spin-independent part of the $E1$ transition is forbidden, and the $M1$ transition is strongly suppressed.[12] The reaction is therefore expected to proceed predominantly through the $E2$ transition. Experimental results obtained for low-energy deuterons,[13] indeed show that the differential cross section exhibits a distinct $\sin^2 2\theta$ shaped angular distribution, characteristic of the $E2$ transition, with a small asymmetry. This asymmetry is interpreted as arising from the interference between the $E2$ multipole and the spin-dependent part of the $E1$ multipole. Here we shall consider only the $E2$ multipole. In the LWA the $E2$ operator is given by

$$E_{2M} = -c_\gamma \sum_i \frac{e}{2}[1+\tau_z(i)]r_i^2 Y_2^{M*}(\hat{\mathbf{r}}_i) , \quad (3)$$

where the sum is over all nucleons, τ_z is the z component of the isospin operator for the i particle, $c_\gamma = k_\gamma^2(\pi/15)^{1/2}$, and e is the elementary charge. Meson exchange currents are taken into account within the context of the LWA

(Siegert's theorem). In the calculation of the $E2$ matrix elements, we assumed[4] that the nucleon coordinates $\mathbf{r}_i$ in the E_{2M} operator are proportional to the vector $\mathbf{r}=\mathbf{r}_d-\mathbf{r}_\alpha$ joining the center of mass of the deuteron and the α particle. With this approximation

$$E_{2M}=-c_\gamma \tfrac{2}{3} r^2 Y_2^{M*}(\hat{\mathbf{r}}) , \tag{4}$$

and the resulting $E2$ amplitudes depend on the internal structure of ^{6}Li through the overlap

$$\langle \phi^{\sigma_d}\phi_\alpha | {}^6\mathrm{Li};1M'\rangle = \sum_{\substack{\mathcal{L}=02\\ \lambda}} (\mathcal{L}\lambda 1\sigma_d | 1M')u_{\mathcal{L}}(r)Y_{\mathcal{L}}^{\lambda}(\hat{\mathbf{r}}) , \tag{5}$$

where $u_{\mathcal{L}}(r)$ are the radial wave functions of the $\alpha+d$ cluster component of the ^{6}Li ground-state wave function. Neglecting the coupling between channels with different L we can write for the initial scattering state, which asymptotically is characterized by the spin projection σ_d,

$$\langle \xi_\alpha \xi_d \mathbf{r} | \Psi^{\sigma_d};{}^{2S+1}L_J\rangle = \sum_{\substack{m\sigma_d'\\ m'M''}} (Lm1\sigma_d|JM'')(Lm'1\sigma_d'|JM'')\phi^{\sigma_d'}(\xi_d)\phi_\alpha(\xi_\alpha)Y_L^{m*}(\hat{\mathbf{k}})Y_L^{m'}(\hat{\mathbf{r}})\Psi_{JL}(r) . \tag{6}$$

Because of parity and angular momentum selection rules, transitions will occur from the $|{}^3S_1\rangle$, $|{}^3D_J\rangle$, and $|{}^3G_3\rangle$ scattering states to the ${}^3S_1+{}^3D_1$ bound state of ^{6}Li, and can be expressed as

$$\langle {}^3\mathcal{L}_1;1M'|E2|{}^3L_J;J\sigma_d\rangle = \sum_{Mn}(-)^M(L01\sigma_d|J\sigma_d)(J\sigma_d 2-M|1M')\mathcal{C}_{\mathcal{L}LJ}\mathcal{J}_{\mathcal{L}L}\mathcal{D}_{Mn}^{2*}(0,\theta,0) , \tag{7}$$

where the coefficients

$$\mathcal{C}_{\mathcal{L}LJ}=\hat{J}\hat{L}^2\frac{\sqrt{5}}{6\pi}W(2\mathcal{L}J1;L1)(L020|\mathcal{L}0) \tag{8}$$

describe the angular momentum coupling. The dynamics of the reaction is present in the integrals $\mathcal{J}_{\mathcal{L}L}$ which, in the present model, are given by

$$\mathcal{J}_{\mathcal{L}L}=\int u_{\mathcal{L}}(r)\Psi_L(r)r^4dr , \tag{9}$$

where the J dependence of the scattering functions was neglected. Due to the presence of the r^4 factor in the integrand, the reaction probes the asymptotic region of $u_{\mathcal{L}}$. For large values of r

$$u_{\mathcal{L}}(r)\xrightarrow[r\to\infty]{} N_{\mathcal{L}} i^{\mathcal{L}} h_{\mathcal{L}}(i\beta r) , \tag{10}$$

where $h_{\mathcal{L}}$ are Hankel functions and $\beta=0.3066\ \mathrm{fm}^{-1}$ is the wave number corresponding to the $\alpha d-{}^6$Li separation energy $B=1.473$ MeV. The asymptotic D/S-state ratio is $\rho=N_2/N_0$. We generate the u_0 and u_2 radial functions as the $2S$ and $1D$ normalized wave functions obtained in a Wood-Saxon potential well with a fixed geometry ($r_0=1.90$ fm, $a=0.65$ fm) and with the depth adjusted to the experimental binding energy. The S-state radial wave function has a node close to 2 fm and both functions are chosen to have the same phase in the asymptotic region. This model has been successfully used in the analysis of transfer reactions involving ^{6}Li (Ref. 14) and also to study the deformation effects in the aligned ^{6}Li scattering.[8] Furthermore, it provides a good representation of the $\alpha+d$ cluster component of three body ^{6}Li wave functions.[9] For the S scattering state, the radial wave function $\Psi_0(r)$ was generated by solving a two-body equation with separable potentials constrained to give the energy dependence of the phase shifts of α-d scattering obtained with a one-channel RGM calculation.[15] For the D and G waves, in view of small distortion at very low energies, we used plane waves.

The calculated differential cross section shown in Fig. 1 is in good agreement with data[13] for the ^{2}H$(\alpha,\gamma)^6$Li reaction at a c.m. energy of $E_{\mathrm{c.m.}}=1.63$ MeV. The small

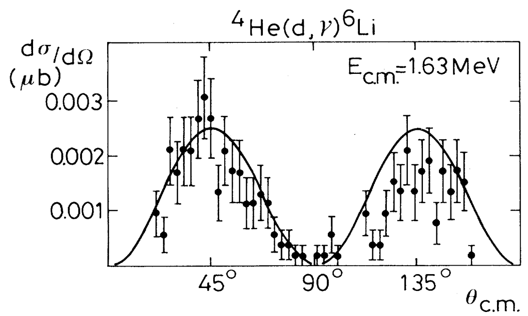

FIG. 1. Angular distribution of the differential cross section of ^{4}He$(\vec{d},\gamma)^6$Li reaction at $E_{\mathrm{c.m.}}=1.63$ MeV. The curve is a result of the calculation described in the text for $P_{\alpha d}=1$ and the data are from Ref. 13.

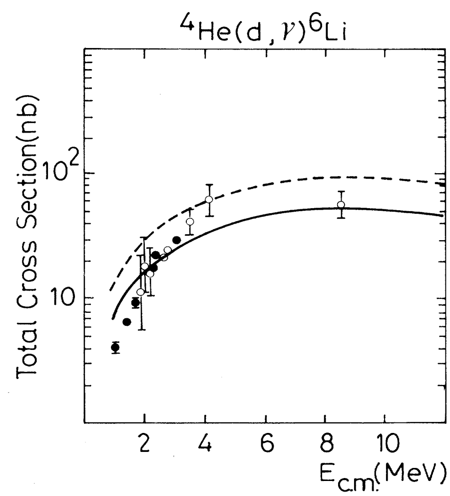

FIG. 2. Calculated total cross section σ_t for the ^{4}He$(\vec{d},\gamma)^6$Li reaction as a function of center-of-mass energy. The curves correspond to a $P_{\alpha d}=0.40$ (full curve) and $P_{\alpha d}=0.70$ (dash curve). The data are from Ref. 13.

asymmetry in the data cannot be reproduced by the $E2$ amplitudes. In Fig. 2 we show the results of the calculation of the total cross section as a function of energy for 1 MeV $< E_{c.m.} <$ 12 MeV. The two curves correspond to different values of the probability P_{ad} of the α-d cluster component in ^{6}Li, which is the only free parameter in the calculation. Theoretical estimates of that probability are $P_{ad}=0.42$ in Plattner,[16] $P_{ad}=0.54$ in Noble,[17] $P_{ad}=0.65$ in Lehman,[9] and $P_{ad}=0.85$ in Robertson.[13] In the present calculations better agreement with the data[13] is obtained for a lower probability of $P_{ad}=0.40$, although for low energies the calculated cross section overestimates the data. Using Eq. (7) we find that the tensor analyzing powers can be expressed as $T_{kq}=\mathcal{T}_{kq}/\mathcal{T}_{00}$, where

$$\mathcal{T}_{kq}=\sum_{LL'ff'n}(-)^{k-n-f'}\frac{20}{\sqrt{3}}\frac{\hat{\mathcal{L}}^2\hat{\mathcal{L}}'^2}{(4\pi)^2}\hat{f}'^2\frac{\sqrt{4\pi}}{\hat{f}}Y_f^q(\theta)(2-n2n|f0)(L0f'q|fq)(L'0f'q|kq)(L020|\mathcal{L}0)(L'020|\mathcal{L}'0) \times\begin{Bmatrix} L & \mathcal{L} & 2\\ 2 & f & f'\end{Bmatrix}\begin{Bmatrix} 1 & k & 1\\ \mathcal{L} & 1 & \mathcal{L}'\end{Bmatrix}\begin{Bmatrix} L' & k & f'\\ \mathcal{L} & 2 & \mathcal{L}'\end{Bmatrix}\mathcal{J}_{\mathcal{L}L}\mathcal{J}^*_{\mathcal{L}'L'} . \tag{11}$$

If in the above expression we consider only the S state in ^{6}Li ($\mathcal{L}=\mathcal{L}'=0$), we obtain $\mathcal{T}_{kq}=\delta_{k0}$. Under this assumption the only nonvanishing observable is the cross section which is proportional to $\mathcal{T}_{00}$. Thus the tensor analyzing powers arise exclusively from the D-state component while $T_{11}=0$ in the present model. Considering now S and D states, and neglecting the very small terms corresponding to the interference between D states ($\mathcal{L}=\mathcal{L}'=2$), we obtain from Eq. (11)

$$\mathcal{T}_{00}=\frac{20}{(4\pi)^2}|\mathcal{J}_{02}|^2\sin^2 2\theta , \tag{12}$$

$$\mathcal{T}_{20}=\frac{20}{(4\pi)^2}[\tfrac{8}{7}\,\mathrm{Re}(\mathcal{J}_{02}\mathcal{J}^*_{22})+\tfrac{2}{5}\,\mathrm{Re}(\mathcal{J}_{02}\mathcal{J}^*_{20})]\sin^2 2\theta , \tag{13}$$

and therefore

$$T_{20}=\frac{8}{7}\frac{\mathcal{J}_{22}}{\mathcal{J}_{02}}+\frac{2}{5}\frac{\mathcal{J}_{20}}{\mathcal{J}_{02}} . \tag{14}$$

This shows that T_{20} is expected to be approximately isotropic. Calculations of the T_{20} angular distribution including all terms in Eq. (11) are shown in Fig. 3(a) for two values of ρ. The value of $\rho=-0.014$ corresponds to the negative mixing parameter $b_d=-0.08$ predicted by Nishioka, Tostevin, and Johnson.[8] We find that, to a very good approximation, T_{20} is proportional to ρ and has the sign of ρ, except in the angular region close to $\theta=\pi/2$. The peak at $\theta=\pi/2$ is due to the small terms in the cross section which do not have a $\sin^2 2\theta$ angular distribution. Figure 3(b) shows that T_{21} is much smaller than T_{20}, and is sensitive to ρ only in the angular region of $\theta=\pi/2$ where the cross section is very small. T_{22} is almost isotropic as T_{20} but of much smaller magnitude. Consequently $A_{yy}=-(T_{20}/\sqrt{2}-\sqrt{3}T_{22})$ has a magnitude comparable to T_{20} and the same kind of sensitivity to ρ. Figure 4 shows that in the energy range 1 MeV $< E_{c.m.} <$ 12 MeV, A_{yy} increases with energy for negative ρ. Notice that since T_{22} is very small, A_{yy} and T_{20} have opposite signs. The observables reverse sign for a symmetric value of ρ.

The experimental determination of ρ is currently of considerable interest particularly because of contradictory predictions based on different models for the ^{6}Li internal structure. The present calculations for the $^4\mathrm{He}(\vec{d},\gamma)^6\mathrm{Li}$

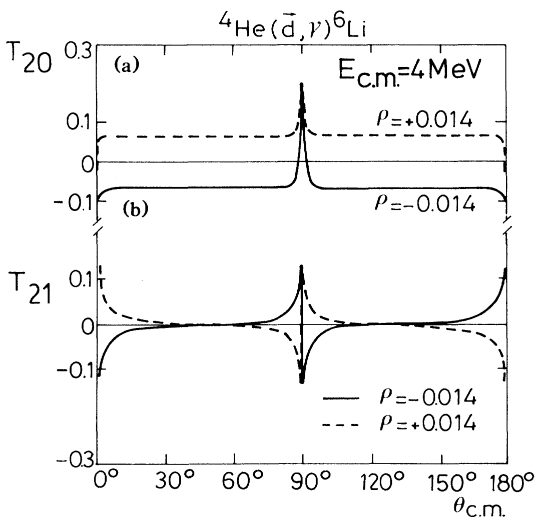

FIG. 3. (a) Calculated tensor analyzing powers $T_{20}(\theta)$ for $^4\mathrm{He}(\vec{d},\gamma)^6\mathrm{Li}$ reaction at $E_{c.m.}=4$ MeV, for the indicated values of ρ. (b) As for (a) for the tensor analyzing power $T_{21}(\theta)$.

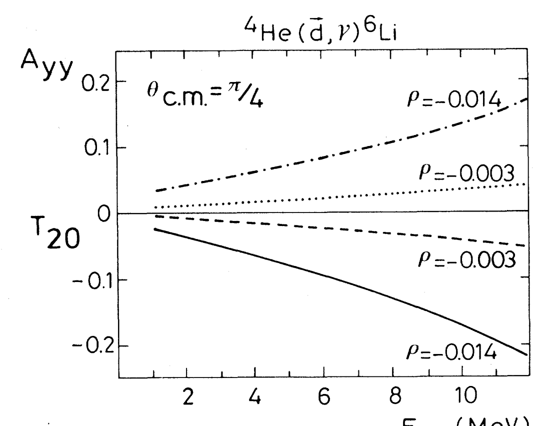

FIG. 4. Calculated tensor analyzing powers T_{20} and A_{yy} for the $^4\mathrm{He}(\vec{d},\gamma)^6\mathrm{Li}$ reaction as a function of center-of-mass energy. For negative values of ρ, A_{yy} is positive and T_{20} negative. The upper (lower) curves correspond to A_{yy} (T_{20}) for the indicated values ρ.

reaction provide a satisfactory description of the available cross section data and show that the measurement of A_{yy} or of T_{20} can determine the magnitude and sign of ρ, and becomes more favorable as the energy increases. Calculations for higher energies including the distortion in the scattering D waves and the effect of other multipoles are in progress.

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