Electron scattering and ^{14}N wave functions

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Wave functions of ^{14}N deduced from electron scattering are shown to be in contradiction with the spin-orbit interaction on which the shell model is based.

In a recent paper[1] results of electron scattering were considered in detail. The authors, who used harmonic oscillator wave functions, could not fit their data with conventional wave functions. Instead, they looked for shell-model wave functions in the p^{-2} configuration which gave the best fit to their data. The wave functions obtained in Ref. 1 do not agree with results obtained during the last 40 years. In particular, some of their wave functions do not yield the very slow decay rate of ^{14}C.

The surprisingly long lifetime of ^{14}C posed a challenge to nuclear theory. According to Wigner's supermultiplet (SU_4) scheme it should have been a superallowed or favored transition with $\log ft \sim 3$ (such as the ^{6}He decay). Also in the *jj*-coupling shell model it should have been a fast transition. In a 1953 review article[2] Inglis discussed this difficulty. He concluded that no "accidental cancellation" of the Gamow-Teller matrix element could take place if central and spin-orbit interactions are used within the p^{-2} configuration. He suggested that configuration mixing is necessary to obtain such cancellation. He also stated that adding tensor forces will not lead to cancellation but there was a flaw in that argument.

In a 1954 paper it was demonstrated that if tensor forces are included it is possible to obtain cancellation of the Gamow-Teller matrix element within the p^{-2} configuration.[3] It was also explained that the similar attenuation, though to a smaller extent, of the mirror ^{14}O decay supports the idea of accidental cancellation. Precise measurements of that decay by Sherr *et al.*[4] were used to clinch this point and to determine phenomenological wave functions for the $T=1,J=0$ and $T=0,J=1$ states of ^{14}N. Tensor forces which gave better agreement with the data were then introduced.[5,6] Much later, a method was introduced for calculating nuclear energies using effective interactions determined consistently from experimental data.[7] This method was successfully applied to energies of nuclei in the $1p$ shell.[8,9] The effective interactions determined in this analysis contained tensor forces. The resulting wave functions of the ^{14}C ground state ($T=1,J=0$) and ^{14}N ground state ($T=0,J=1$) were used to calculate the Gamow-Teller matrix element. No complete cancellation was obtained[9] but the various amplitudes had the correct signs leading to attenuation. A very slight change in the ^{14}N ground-state wave function leads to the desired cancellation. This fact is one of the attractive features of the Cohen-Kurath wave functions.[9] These, however, do not seem to give a good fit for the electron scattering data as discussed in Ref. 1.

Simple shell-model wave functions, like those in the p^{-2} configuration cannot be the real wave functions of nuclei where strong short-range nuclear interactions lead to short-range correlations. The latter affect physical observables like matrix elements of two-nucleon interactions. The effect of correlations on the latter seems to be well approximated by a simple renormalization of the interaction between free nucleons. This enables the successful use of shell-model wave functions in calculations of nuclear energies. Such renormalization may take place also for other operators like electromagnetic moments.

Electron scattering is rapidly becoming a very important probe of nuclear structure and we should learn how to use it in extracting information on nuclear wave functions. If shell-model wave functions cannot reproduce results of electron scattering they must be modified. Such modification may well involve the admixture of higher configurations. The search made in Ref. 1 for phenomenological wave functions was limited, however, to simple configurations determined by the single-nucleon shell-model Hamiltonian. As is well known, that Hamiltonian correctly yields nuclear energies as well as many other properties of nuclear states. The aim of this comment is to show that the sets of wave functions, $H1$, $H2$, and $HF2$ of Ref. 1 are eigenstates of a shell-model Hamiltonian with very peculiar properties. It is in sharp contradiction to the basic ingredient of the Mayer-Jensen shell model—the strong spin-orbit interaction.

To demonstrate this point it is sufficient to consider the $T=1,J=0$ state of the p^{-2} configuration. This is the ^{14}C ground state and its analogue—the 2.3 MeV state in ^{14}N. Its wave function can be expressed as

$$m\,|p_{1/2}^{-2}T=1,J=0\rangle+n\,|p_{3/2}^{-2}T=1,J=0\rangle,\quad m^2+n^2=1\ . \tag{1}$$

The amplitudes m and n are determined by the lowest eigenvalue of the Hamiltonian 2×2 submatrix

$$\begin{vmatrix} 2\epsilon+\frac{2}{3}V(S)+V(P) & \sqrt{2}/3[V(S)-V(P)] \\ \sqrt{2}/3[V(S)-V(P)] & \frac{1}{3}V(S)+\frac{2}{3}V(P) \end{vmatrix}\ . \tag{2}$$

In (2) ϵ is the energy difference between the $1p_{3/2}^{-1}$ hole state and the $1p_{1/2}^{-1}$ hole state. It is equal to 6.3 MeV in ^{15}N and to 6.2 MeV in ^{15}O. We can safely use the value $2\epsilon=+12$ MeV. The $V(S)$ is the diagonal matrix element of the effective interaction in the $S=0,L=0$ state ($^1S_0$)

of the p^2 (or p^{-2}) configuration. Similarly, $V(P)$ is the matrix element in the $S=1, L=1$ ($^3P_0$) state.

The amplitudes in the sets $H1$, $H2$, and $HF2$ of Ref. 1, are such that $|m|<|n|$. This implies that in the matrix (2) the diagonal element in the $|p_{3/2}^{-2}T=1,J=0\rangle$ state should be *lower* than the one in the $|p_{1/2}^{-2}T=1,J=0\rangle$ state. Hence,

$$12+\tfrac{2}{3}V(S)+\tfrac{1}{3}V(P)<\tfrac{1}{3}V(S)+\tfrac{2}{3}V(P)\ ,$$

from which follows

$$V=\tfrac{1}{3}[V(S)-V(P)]<-12\text{ MeV}\ . \qquad (3)$$

This condition leads to rather strange values of the matrix elements of the effective interaction. Let us, however, proceed. For the lowest eigenstate of (2), the relative sign of the amplitudes m and n must be *opposite* to that of the nondiagonal matrix element. Since the latter is negative according to (3), m and n must have the *same* sign. This rules out the set $H1$ of Ref. 1.

Even if the condition (3) is satisfied, there is an upper limit to the value of n. The matrix (2) can be expressed as

$$V(S)-2V+\begin{bmatrix} 12+V & \sqrt{2}V \\ \sqrt{2}V & 0 \end{bmatrix}\ . \qquad (4)$$

In the limit $V\rightarrow-\infty$, the amplitudes m and n become equal to $1/\sqrt{3}$ and $\sqrt{2/3}$ which are their values in the pure LS-coupling state $^1S_0$. For any finite value of V, n cannot exceed the value $\sqrt{2/3}\sim 0.8166$.

The amplitude n in the set $HF2$ of Ref. 1 is larger than this limit (0.851). In the set $H2$ the value of n is lower but it is still higher than $\sqrt{2/3}$ (0.818). The only way to obtain such amplitudes is to reverse the sign of the spin-orbit interaction. The $H2$, $HF2$, and particularly, $H1$, imply that the $1p_{3/2}^{-1}$ hole state is *lower* than the $1p_{1/2}^{-1}$ hole state. Thus, the single-nucleon $1p_{3/2}$ orbit should be *higher* than the $1p_{1/2}$ orbit.

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