y scaling, binding effects, and the nucleon momentum distribution in ^{3}He

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The method, previously used to obtain the nucleon momentum distribution in ^{2}H from the y-scaling analysis of inclusive electron scattering data, is extended to ^{3}He. It is shown that the binding correction which has to be handled in this case does not hinder the extraction of the momentum distribution from the scaling function. The obtained momentum distribution satisfactorily agrees with the one extracted from exclusive $(e,e'p)$ reactions.

The nucleon momentum distribution $n(k)$ is a relevant quantity in understanding the structure of nuclei, for its behavior at large values of k ($k \geq 300$ MeV/c) does provide valuable information on nucleon correlations.[1] For this reason serious efforts are being done in order to figure out experiments from which $n(k)$ could be obtained with as little as possible theoretical bias. In fact, the momentum distribution is not a directly observable quantity, and several theoretical assumptions must always be made in order to extract it from measured cross sections. For example, the momentum distributions obtained from the *exclusive* $(e,e'p)$ experiments might reflect the theoretical treatment of final state interaction (FSI) and meson exchange currents (MEC's), whose effects have to be removed from the experimental cross sections.[2,3] It is therefore highly desirable to have complementary information on $n(k)$ stemming from different kinds of experimental data. In this regard, it has been argued[4-6] that *inclusive* quasielastic (qe) electron scattering *in the y-scaling limit* can be used to this end. In a previous paper[7] we have demonstrated that a y-scaling analysis of inclusive data for the deuteron for $y \leq 0$ does provide a nucleon momentum distribution which agrees with the same quantity extracted[2] from the exclusive process ^{2}H$(e,e'p)n$, provided that nucleon momentum, as well as the effect of FSI are properly considered in the theory of y-scaling.[8] The aim of this letter is to show how the analysis of Ref. 7 can be extended to heavier nuclei, and to this end the case of ^{3}He will be studied, since for this nucleus a large wealth of theoretical calculations and experimental data are available.

Following Ref. 7, the scaling function is defined as follows:

$$F_1(q,y)=\frac{\sigma_2}{(Z\overline{\sigma_{ep}}+N\overline{\sigma_{en}})}\left|\overline{\frac{\partial\omega}{k\,\partial\cos\alpha}}\right| , \qquad (1)$$

where σ_2 is the inclusive qe cross section, $\sigma_{ep(n)}$ the relativistic off-shell cross section for electron scattering by a proton (neutron) with momentum k (see, e.g., Ref. 9), $\omega(q)$ the energy (trimomentum) transfer and $|\partial\omega/k\,\partial\cos\alpha|^{-1}$ the proper kinematical phase-space factor.

In plane-wave impulse approximation (PWIA) F_1 represents the nuclear structure function

$$F(q,y)=2\pi\int_{E_{\rm min}}^{E_{\rm max}(q,y)} dE \int_{k_{\rm min}(q,y,E)}^{k_{\rm max}(q,y,E)} P(k,E)k\,dk , \qquad (2)$$

where $P(k,E)$ is the nucleon spectral function, k and E are the nucleon momentum and removal energy, and the limits of integration are determined from the relativistic energy conservation [$E_{\rm min}=|E_A|-|E_{A-1}|$, E_A and E_{A-1} being the (negative) ground-state energies of the A and $A-1$ nuclei, respectively]. For ease of presentation, we have considered in Eq. (2) $P_p(k,E)=P_n(k,E)=P(k,E)$, but in actual calculations the proper proton and neutron spectral functions have been used. In Eqs. (1) and (2) y is the scaling variable defined by the following equation:

$$\omega+M_A=[M^2+(q+y)^2]^{1/2}+[M_{A-1}^2+y^2]^{1/2} , \qquad (3)$$

where M_A (M_{A-1}) is the mass of the ground state of the A ($A-1$) system and y represents the minimal longitudinal (along q) momentum of a nucleon with the minimum value ($E_{\rm min}$) of the separation energy, i.e., $|y| = k_{\rm min}(E_{\rm min})$. The quantities with an overbar in Eq. (1) are evaluated at $E=E_{\rm min}$ and $k=k_{\rm min}(E_{\rm min})$. The spectral function appearing in Eq. (2) can be represented in the following general form:

$$P(k,E)=P_{\rm gr}(k,E)+P_{\rm ex}(k,E) , \qquad (4)$$

where

$$P_{\rm gr}(k,E)=n_{\rm gr}(k)\delta(E-E_{\rm min})$$

yields the probability distribution that the final ($A-1$) system is left in its ground state (corresponding to the excitation energy $E_{A-1}^*=0$ and $E=E_{\rm min}$), whereas $P_{\rm ex}(k,E)$ yields the probability distribution that the final ($A-1$) system is left in any of its excited states (with $E_{A-1}^*\neq 0$, $E=E_{\rm min}+E_{A-1}^*$.[8] Equation (4) separates out that part of the spectral function [$P_{\rm gr}(k,E)$] having a singularity at $E=E_{\rm min}$ from the rest [$P_{\rm ex}(k,E)$], which has singularities corresponding to the possible discrete excited states and continua associated to the breakup channels; it is clear therefore that $P_{ex}(k,E)$ vanishes for $E=E_{min}$ (in ^{3}He only the two-body breakup singularity at $E=E_{min}=|E_3|-|E_2|=5.5$ MeV, with probability $p_d=0.65$, and the three-body channel continuum with threshold removal energy $E=|E_3|=7.72$ MeV occur). Thus, the relation between the spectral function and the momentum distribution is

$$n(k)=\int_{E_{min}}^{\infty} P(k,E)dE=n_{gr}(k)+\int_{E_{min}}^{\infty} P_{ex}(k,E)dE$$
$$=n_{gr}(k)+n_{ex}(k) \ . \quad (5)$$

It will be clear from what follows that the separation of the spectral function given by Eq. (4) has been adopted in order to single out the effects of nucleon binding on the scaling function, coming exclusively from $P_{ex}(k,E)$.

Because of the decrease of the spectral function as a function of k and E, one can safely consider $k_{max}\approx E_{max}\approx\infty$ in Eq. (2), even at relatively low values of q; therefore, using Eq. (4), and remembering that $|y|=k_{min}(E_{min})$, the structure function [Eq. (2)] becomes

$$F(q,y)=2\pi\int_{|y|}^{\infty} n_{gr}(k)k\,dk + 2\pi\int_{E_{min}}^{\infty} dE\int_{k_{min}(q,y,E)}^{\infty} P_{ex}(k,E)k\,dk \ . \quad (6)$$

The first term of Eq. (6) trivially scales, whereas the second term explicitly depends upon the momentum transfer. Such a "scaling violation," which is due to the nucleon binding, is only present at finite q, since in the asymptotic limit one has[10]

$$\lim_{q\to\infty} k_{min}(q,y,E)=k_{min}^{\infty}(y,E)\approx|y-(E-E_{min})| \ .$$

(The latter equality holds when $M_{A-1}\gg|y|$ and $M_{A-1}\gg|E-E_{min}|$. In actual calculations the exact relation has been used.) Therefore, the asymptotic scaling function is

$$F(y)=2\pi\int_{|y|}^{\infty} n_{gr}(k)k\,dk + 2\pi\int_{E_{min}}^{\infty} dE\int_{|y-(E-E_{min})|}^{\infty} P_{ex}(k,E)k\,dk \ , \quad (7)$$

Placing Eq. (5) in Eq. (7), the latter can be rewritten in the following way:

$$F(y)=f(y)-B(y) \ , \quad (8)$$

where

$$f(y)=2\pi\int_{|y|}^{\infty} n(k)k\,dk \quad (9)$$

is the longitudinal momentum distribution and

$$B(y)=2\pi\int_{E_{min}}^{\infty} dE\int_{|y|}^{|y-(E-E_{min})|} P_{ex}(k,E)k\,dk \quad (10)$$

is the contribution arising from $P_{ex}(k,E)$. Taking the derivative of both sides of Eq. (8), one gets

$$n(k)=-\frac{1}{2\pi y}\left[\frac{dF(y)}{dy}+\frac{dB}{dy}\right], \quad k=|y| \ , \quad (11)$$

where [cf. Eq. (10)]

$$\frac{dB}{dy}=-2\pi y\int_{E_{min}}^{\infty} dE\left[P_{ex}(|y|,E)-\left[1-\frac{E-E_{min}}{y}\right]P_{ex}[|y-(E-E_{min})|,E]\right] \ . \quad (12)$$

Equation (11), which did not previously appear in the literature, is the basic equation relating the momentum distribution to the asymptotic scaling function within the underlying assumption of the validity of the PWIA; any attempt at obtaining the momentum distribution of a complex system from y scaling should face such an equation. The quantities $B(y)$ and dB/dy, which appear in Eqs. (8) and (11), will be called the *binding corrections* to the scaling function and to the momentum distribution, respectively. In absence of the binding correction, there is a direct link between $F(y)$ and $n(k)$, as it occurs in the deuteron, where $E=E_{min}$ so that $B(y)=0$; the theoretical bias in obtaining $n(k)$ from y scaling in deuteron is therefore a minimum one, since the effect of MEC is strongly suppressed at $y<0$ and the effect of FSI, which turns out to be of minor importance at very high momentum transfer,[7] can be treated "exactly"[2] [in the exclusive ^{2}H$(e,e'p)n$ reaction the corrections for FSI and MEC may be as large as 100% at $k\approx500$ MeV/c (Ref. 2)]. Unlike the deuteron case, the extraction of the momentum distribution of a complex nucleus suffers from an additional theoretical bias represented by the effect of nucleon binding, which hinders, in principle, the direct relation between $n(k)$ and $F(y)$. However, in a wide range of y, the binding correction in Eq. (11) is expected to be much smaller than the derivative of the scaling function. In fact, as is well known, when $y\approx0$ (i.e., $\omega\approx q_\mu^2/2M$) the qe cross section and, correspondingly, the scaling function are only slightly affected by correlations [which in the three-body system generate $P_{ex}(k,E)$], whereas, for arbitrary values of y, dB/dy vanishes if $\overline{E_{A-1}^*}\ll|y|$ [$\overline{E_{A-1}^*}$ denotes the average excitation energy of the spectator $(A-1)$ system].

In what follows the nucleon momentum distribution will be obtained from Eq. (11), using the experimental asymptotic scaling function $F_1^{expt}(y)$ and calculating the binding correction with various spectral functions. It turns out that the above general considerations on the smallness of the binding correction are indeed confirmed by direct calculations.

The experimental scaling function $F_1^{expt}(q,y)$ is presented in Fig. 1; it can be seen that for the lowest values of the momentum transfer and for $y<0$ it decreases with momentum transfer (see, e.g., the data for $y=-150$ and -500 MeV/c); as in the deuteron case,[7] such a decrease should be attributed to the effects of FSI, as demonstrated by the results of explicit calculations (dashed lines in Fig. 1). The data at $q^2\le40$ fm^{-2}, which we consider to be strongly affected by FSI, have therefore been discarded in the construction of the asymptotic scaling function, which has been obtained by taking the average of the values of $F_1^{\rm expt}(q,y)$ at the highest values of q, where the FSI should have only minor effects, as it has been shown in the deuteron case.[7] The asymptotic scaling function is presented in Fig. 2, where the binding correction $B(y)$ is also shown. The latter has been obtained using in Eq. (10): (i) the full spectral function of Ref. 14 (any spectral function resulting from realistic two-body forces is expected to yield similar results); (ii) a model for $P_{\rm ex}(k,E)$, where only the average excitation energy of spectator pair is considered, namely $P_{\rm ex}(k,E)=n_{\rm ex}(k)\delta(E-\bar{E})$, where $n_{\rm ex}(k)$ is the three-body channel momentum distribution and $\bar{E}=E_{\rm min}+\overline{E_{A-1}^*}$ [$n_{\rm ex}(k)$ and $\bar{E}$ have been taken from three-body calculations with two- (Ref. 14) and two- plus three-body (Ref. 16) interactions].

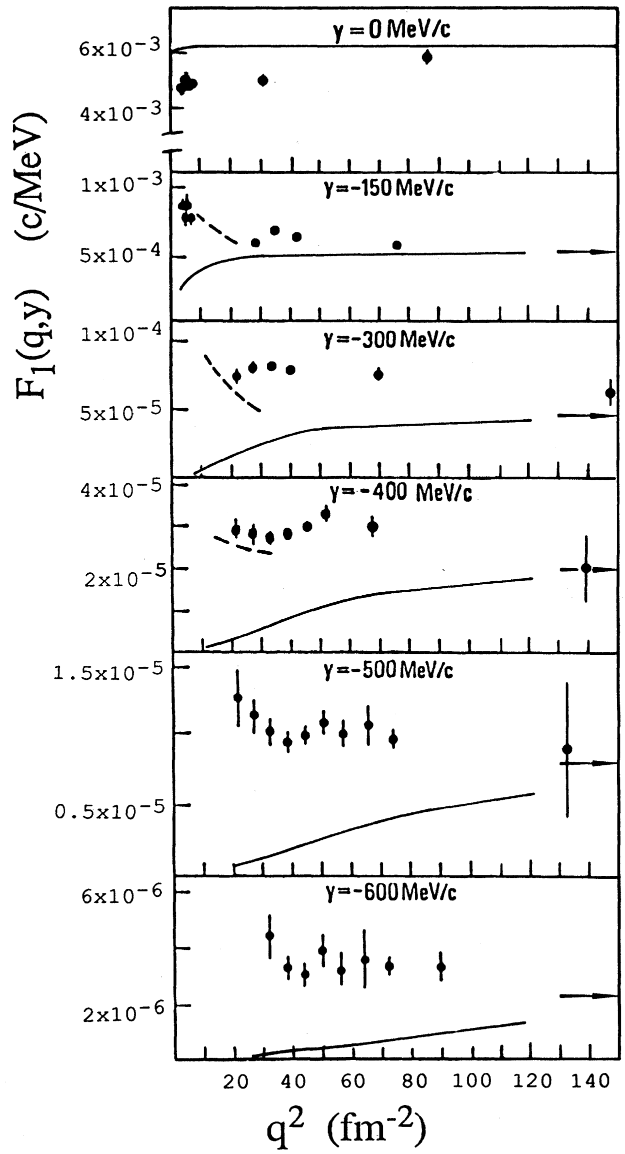

FIG. 1. The scaling function $F_1^{\rm expt}$ [Eq. (1)] of ^{3}He vs momentum transfer for fixed values of y, obtained from the experimental data of Ref. 11 using the relativistic cross section, $\sigma_{ep(n)}$ of Ref. 9 with the nucleon form factors of Ref. 12 [adopting the nucleon form factors of Ref. 13 the values of $F_1^{\rm expt}(q,y)$ at the highest values of q change less than 6%]. The solid lines represent the PWIA results [Eq. (2)] computed with the spectral function of Ref. 14 and the dashed lines include the FSI, evaluated according to Ref. 15. The arrows show the asymptotic values of $F_1^{\rm th}$ in PWIA [Eq. (7)].

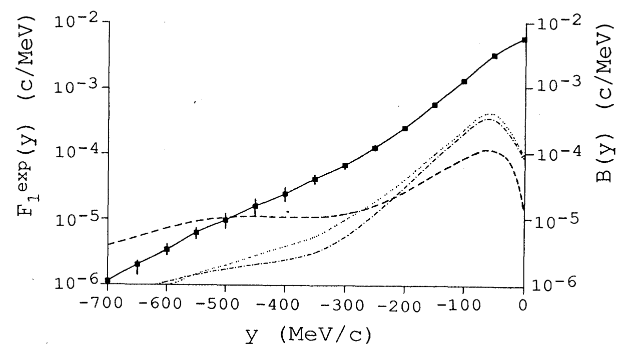

FIG. 2. The experimental asymptotic scaling function of ^{3}He, $F_1^{\rm expt}(y)$ (solid squares) obtained from the experimental data shown in Fig. 1 taking the average values of $F_1^{\rm expt}(q,y)$, for $q^2 \geq q_{\rm min}^2 = 65$ fm^{-2}; the error bars include the experimental errors on $\sigma_2^{\rm expt}$ and the variation of the average value of $F_1^{\rm expt}(q,y)$ obtained by considering several values of $q_{\rm min}^2$ in the range 45 fm$^{-2} \leq q_{\rm min}^2 \leq 70$ fm^{-2}. The solid line is a polynomial interpolation of the data. The dashed line is the quantity $B(y)$ [Eq. (10)], calculated with the spectral function of Ref. 14; the dot-dashed and dotted lines are the quantity $B(y)$ computed with the model spectral function $P_{\rm ex}(k,E)=n_{\rm ex}(k)\delta(E-\bar{E})$ using for $n_{\rm ex}(k)$ and $\bar{E}$ the values from Refs. 14 and 16, respectively.

The nucleon momentum distribution obtained from Eq. (11) is shown in Fig. 3. It can be seen that disregarding the binding correction [i.e., placing $B(y)=0$, which corresponds to the popular closure approximation] or taking it into account by the prescriptions (i) and (ii) changes $n(k)$ only by less than 20% up to $k \approx 500$ MeV/c and by $\approx 80\%$ at $k \approx 600$ MeV/c. The model dependence associated to nucleon binding in the momentum distribution extracted from y scaling is therefore negligible for $k \leq 500$ MeV/c and quite relevant at higher values of k [in $(e,e'p)$ reactions the correction to the PWIA associated with the treatment of MEC and FSI exceeds 40% at $k \approx 500$ MeV/c (Ref. 3)]. Using the full spectral function the binding correction to $n(k)$ is small because $B(y)$ itself is negligible at small $|y|$, whereas it turns out to be almost a constant in the region 300 MeV/$c \leq |y| \leq 500$ MeV/c. Adopting our model (ii) for the spectral function, the y dependence of dB/dy is mainly governed by the value of $\overline{E_{A-1}^*}$ [Eq. (12) reduces to $(2\pi|y|)^{-1} dB/dy \approx n_{\rm ex}(|y|) - n_{\rm ex}(|y|+\overline{E_{A-1}^*})$]: varying $\overline{E_{A-1}^*}$ from 0 (closure approximation) up to values (≈ 20 MeV) resulting from realistic calculations with two- (Refs. 14 and 17) and three-body (Ref. 16) forces pro-

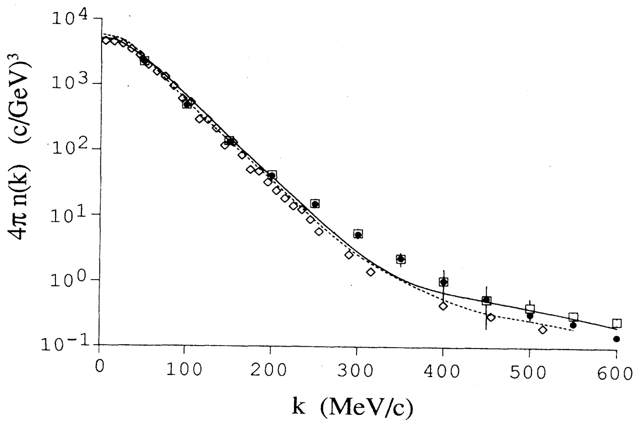

FIG. 3. The nucleon momentum distribution $n(k)$ in ^{3}He obtained from Eq. (11) by disregarding the binding correction (dots) and by calculating it using Eq. (12) with the full spectral function of Ref. 14 (open squares); the results corresponding to the model spectral functions (ii), described in the text, fall between the open squares and the dots and are not shown (the meaning of the error bars is the same as in Fig. 2). Diamonds: $n(k)$ obtained (Ref. 3) from the exclusive cross sections ^{3}He$(e,e'p)d$ and ^{3}He$(e,e'p)np$ including the FSI and MEC corrections, evaluated according to Ref. 15. Solid and dashed lines: the proton momentum distribution obtained from the Reid soft core (Ref. 14) and the Paris (Ref. 17) interactions, respectively. The normalization of $n(k)$ is $\int n(k)d^3k=1$.

duces small corrections (less than 15% up to $k=500$ MeV/c) and only large, unrealistic values of E^*_{A-1} raise the correction to high values.

The comparison of $n(k)$ with the proton momentum distribution extracted[3] from exclusive cross sections corrected for FSI and MEC according to Ref. 15, shows a satisfactory agreement except in the region 250 MeV/$c \leq k \leq 400$ MeV/c. Various arguments could be advocated to explain such a difference. From the experimental point of view, more inclusive data, covering the range 80 fm$^{-2} < q^2 < 140$ fm^{-2} would be highly necessary in order to improve our understanding of the onset of the scaling regime. From the theoretical point of view, the treatment of FSI should be improved (e.g., by the continuum Faddeev approach) both in exclusive[18] and inclusive[19] scattering. Although various model calculations of FSI predict rather different effects on y scaling,[20,21] our assumption that the FSI should have minor effects on the data at high momentum transfer relies on the exact calculations for the deuteron, which predict that the FSI practically vanishes at high but finite values of the momentum transfer. It should also be considered that in the ^{3}He experiment[11] the value of the energy of the nucleon-spectator-pair system in the center-of-mass frame is much higher than in the neutron-proton system in the deuteron experiment,[6] for the same values of q and y. Recent calculations in nuclear matter using realistic interactions[22] also yield a vanishing contribution from FSI at high but finite values of the momentum transfer ($q^2 \approx 60-70$ fm^{-2}). Nevertheless, a calculation of FSI effects in ^{3}He, in the whole range of momentum transfer and scaling variable covered by the experimental data, would be highly desirable. In this regard, we would like to point out that if the observed disagreement, shown in Fig. 1, between the PWIA and the experimental data in the region around $q^2 \approx 60-80$ fm^{-2} is due to the FSI, and corrections for the latter are introduced following the procedure already adopted for ^{2}H, the values of the asymptotic scaling function for $k \geq 300$ MeV/c will be lower. Then in the same region the momentum distribution extracted from y scaling will be in closer agreement with the exclusive data.

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