# Role of core polarization in inelastic electron scattering from even Ni isotopes

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It is shown that surface-peaked transition charge densities can be obtained for the first excited $2^+$ states of even-mass Ni isotopes by considering core-polarization effects based on a first-order perturbation theory in the standard shell model of the $(p_{3/2},p_{1/2},f_{5/2})^n$ configurations.

Electron scattering experiments were recently done for several Ni isotopes, and Coulomb form factors were obtained for some of the low-lying states up to the high-momentum-transfer region.[1,2,3] Transition charge densities $\rho^{\rm tr}(r)$ can be extracted reliably in a model-independent way by taking the Fourier-Bessel transform of the form factors.[1] It has been shown that the $\rho^{\rm tr}(r)$ for the first excited $2^+$ states in $^{58,60,62,64}$Ni are characterized by their prominent peaks at the nuclear surface.

A macroscopic model, e.g., the Tassie model, always gives a transition charge density with a surface peak for any collective state, because the $\rho^{\rm tr}(r)$ is defined by a first derivative of the ground-state charge density.[4] On the other hand, from a microscopic point of view, there is no *a priori* reason why the $\rho^{\rm tr}(r)$ should have a surface-peaked shape. By using the shell-model wave functions obtained with the neutron $(p_{3/2},p_{1/2},f_{5/2})^n$ configurations[5] and the constant effective charge deduced phenomenologically for the active neutrons, where $n$ is the number of neutrons outside an inert $^{56}$Ni core, we can calculate the $\rho^{\rm tr}(r)$ for the first excited $2^+$ states of even Ni isotopes. It turns out that there appear two peaks in the $\rho^{\rm tr}(r)$ as shown in Fig. 1, and the one inside the nucleus is significantly larger than the other at the nuclear surface.

The purpose of this Brief Report is to consider the particle-hole(p-h) excitations of the $^{56}$Ni core, referred to as core polarization, and to see how this effect can remedy the unsatisfactory situation in the standard shell-model calculations. A first-order perturbation theory is adopted in order to take account of core polarization, which was considered earlier by Horie and Arima[6] for the electric quadrupole ($E2$) effective charges of odd-mass nuclei. For Ni isotopes, Federman and Zamick[7] and Rimini[8] performed configuration-mixing calculations, and they found that there exists considerable state dependence in the $E2$ effective charges at a photon point.

A correlated transition operator $\tilde{f}^{(L)}$ can be defined by using a perturbation expansion of Rayleigh-Schrödinger type up to first order,

$$\tilde{f}^{(L)}=f^{(L)}+V\frac{Q}{E-H_0}f^{(L)}+f^{(L)}\frac{Q}{E-H_0}V\ . \tag{1}$$

Here, $f^{(L)}$ is the one-particle transition operator of tensorial rank $L$, $V$ is the mixing interaction, and $Q$ is the projection operator which takes account of the Pauli principle. For the charge scattering of electrons, $f^{(L)}$ is given by

$$f^{(L)}=\sum_{k=1}^{A} e_k j_L(qr_k)Y^{(L)}(\Omega_k)\ , \tag{2}$$

where $e_k$ is the charge of the $k$th nucleon, $j_L$ is the spherical Bessel function, $Y^{(L)}$ is the spherical harmonic of order $L$, $q$ is the momentum transfer, and $r_k$ and $\Omega_k$ denote, respectively, the radial and angular coordinates for the $k$th nucleon. We assume the charges of a proton and a neutron to be $1.0e$ and 0, respectively. Therefore, any neutron transition matrix element vanishes, and thus we have no contribution from the first term of Eq. (1). The basis functions involving the proton excitations only are used as the intermediate state in this perturbation calculation. This means that the basis functions do not possess good isospin, and thus the isobaric symmetry is not conserved. It is, however, expected that an error caused by breaking the isobaric symmetry might be small for qualitative discussions on the $C2$ transitions of Ni isotopes. The following form is assumed for the mixing interaction $V$,

$$V=V_C(r)+V_{LS}(r)\mathbf{L}\cdot\mathbf{S}+V_T(r)S_{12}\ , \tag{3}$$

where

$$\mathbf{L}=\tfrac{1}{2}(\mathbf{r}_1-\mathbf{r}_2)\times(\mathbf{p}_1-\mathbf{p}_2)\ ,$$

$$\mathbf{S}=\tfrac{1}{2}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\ ,$$

and

$$S_{12}=3(\boldsymbol{\sigma}_1\cdot\mathbf{r})(\boldsymbol{\sigma}_2\cdot\mathbf{r})/r^2-(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\ .$$

Here, $\boldsymbol{\sigma}_i$, $\mathbf{p}_i$, and $\mathbf{r}_i$ are the Pauli spin, the momentum, and the coordinate operators for the $i$th particle, respectively, and $\mathbf{r}$ denotes the relative coordinate. The terms denoted by $C$, $LS$, and $T$ are the central, the two-body spin-orbit, and the tensor forces, respectively. Each component is given in the even-odd triplet-singlet representation by

$$V_C(r)=(V_C^{\rm SO}P^{\rm SO}+V_C^{\rm TE}P^{\rm TE}+V_C^{\rm SE}P^{\rm SE}+V_C^{\rm TO}P^{\rm TO})f_C(r)\ , \tag{4}$$

 

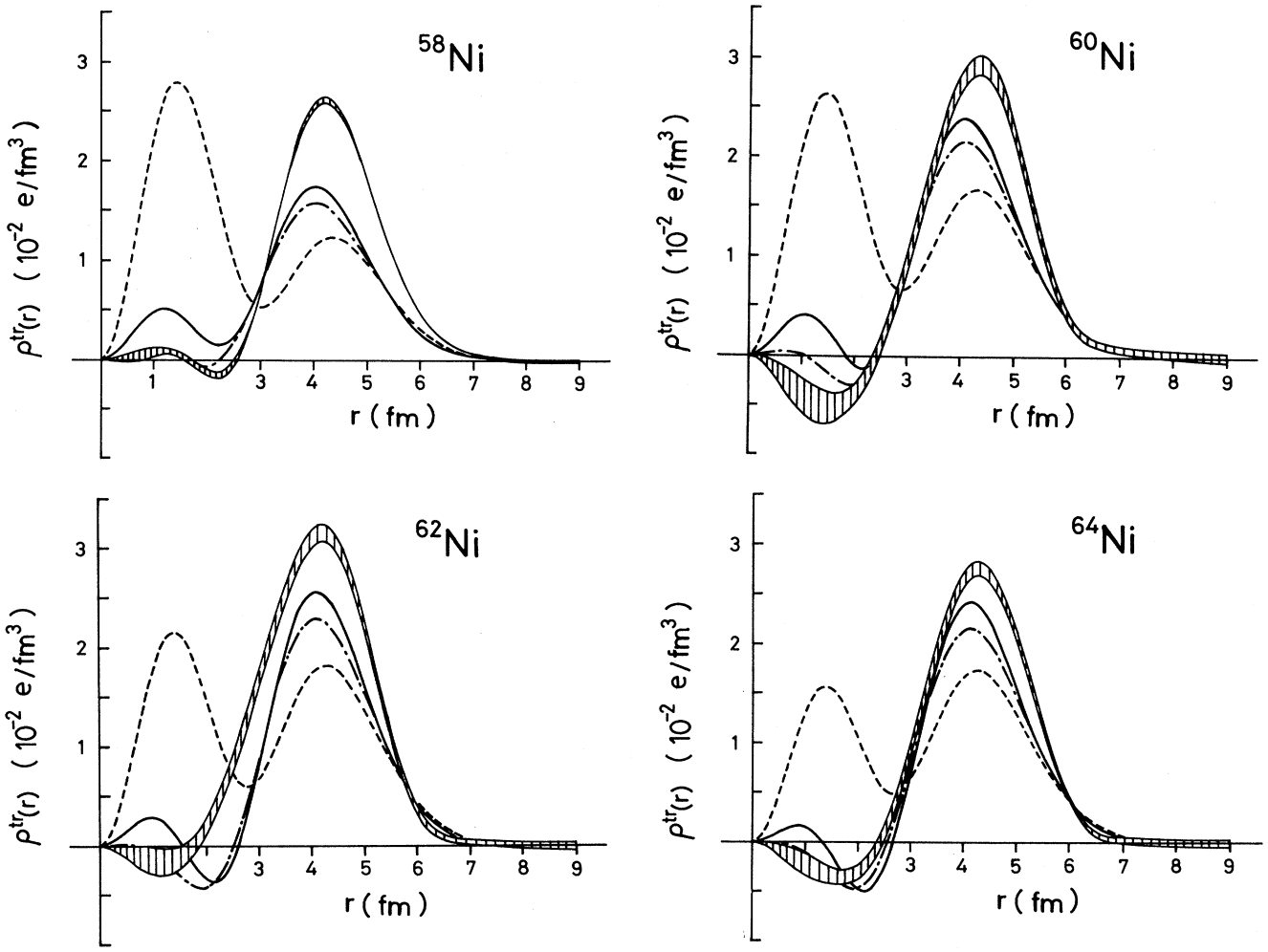


FIG. 1. Calculated and experimental (Ref. 1) transition charge densities $\rho^{\mathrm{tr}}(r)$ for the first excited $2^+$ states of even-mass Ni isotopes. The broken curve denotes the calculations with the constant neutron effective charge of 1.0$e$. The dotted-dashed and solid curves are obtained from the configuration-mixing calculations up to $2\hbar\omega$ and $12\hbar\omega$ p-h excitations, respectively. The experimental transition charge density is shown by the shaded area.

$$V_{LS}(r)=(V_{LS}^{\mathrm{TE}}P^{\mathrm{TE}}+V_{LS}^{\mathrm{TO}}P^{\mathrm{TO}})f_{LS}(r)\ , \tag{5}$$

$$V_{T}(r)=(V_{T}^{\mathrm{TE}}P^{\mathrm{TE}}+V_{T}^{\mathrm{TO}}P^{\mathrm{TO}})f_{T}(r)\ , \tag{6}$$

where $P^{\mathrm{SO}}$, $P^{\mathrm{TE}}$, $P^{\mathrm{SE}}$, and $P^{\mathrm{TO}}$ are projection operators for the singlet-odd (SO), triplet-even (TE), singlet-even (SE), and triplet-odd (TO) states, respectively. The radial dependence is denoted by $f(r)$. We assume the following effective interactions in the present calculations: (1) Gaussian central (CAL),[9] (2) Yukawa central (Serber),[10] (3) Schiffer-True (full range),[11] and (4) $G$-matrix simulating Yukawa potentials M3Y ($r^2$ Yukawa).[12]

In the calculation of matrix elements of $f^{(L)}$ and $V$, we use the harmonic oscillator radial wave functions with the oscillator constant $\nu=0.96A^{-1/3}$ fm$^{-2}$, where $A$ is the mass number of a nucleus. The corrections arising from the center-of-mass motion and the nucleon finite size are taken into account by multipling the Gaussian form[13]

$$\exp\left[\left[\frac{1}{4A\nu}-\frac{0.43}{4}\right]q^2\right]\ . \tag{7}$$

Only the single-particle energy is assumed for the energy denominators in Eq. (1), being evaluated by using the empirical formula given by Noya *et al.*[14]

The radial matrix element of $j_2(qr)$ with the harmonic oscillator wave functions does not vanish, even when the difference of principal quantum number between the initial and final states exceeds $2\hbar\omega$, contrary to the matrix element of $r^2$ in the usual $E2$ gamma transitions. We have made configuration-mixing calculations up to $2\hbar\omega$ and $12\hbar\omega$ p-h excitations, which will be referred to as $2\hbar\omega$ and $12\hbar\omega$ calculations, respectively. In order to see how perturbative terms converge, contributions from each $k\hbar\omega$ p-h excitation at $q=0.7$ fm$^{-1}$ (around the first peak in the form factors) with the M3Y interaction are shown in Fig. 2. It is easily seen that the $2\hbar\omega$ p-h excitations give the leading terms in the perturbation calcula-

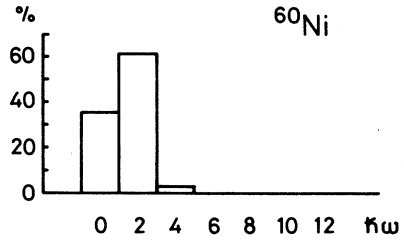


FIG. 2. Contribution from each $k\hbar\omega$ p-h excitation in the perturbed matrix elements at $q=0.7$ fm$^{-1}$, given in percent.

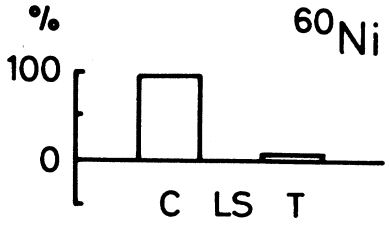


FIG. 3. Contribution from each component in the M3Y interaction at $q=0.7$ fm$^{-1}$, given in percent. $C$, $LS$, and $T$ denote the contribution from the central, two-body spin-orbit, and tensor forces, respectively.

tions, and the contributions from the $2\hbar\omega$ together with the $0\hbar\omega$ excitations dominate in this lower and intermediate $q$ region. Therefore, it seems that the $12\hbar\omega$ is large enough to get a sufficient convergence.

Figure 1 summarizes the calculated and experimental[1] transition charge densities for the first excited $2^+$ states of $^{58,60,62,64}$Ni. The calculations are made with the three different assumptions on the effective charges, although the same shell-model wave functions are used, which are obtained from the $(p_{3/2},p_{1/2},f_{5/2})^n$ configurations.[5] The crudest model assumes the constant effective charge for neutrons, and neither state nor $q$ dependence is considered there. We adopt $e_n=1.0e$ in Fig. 1 for the sake of comparison with the others. This prescription predicts a transition charge density with a large peak inside a nucleus, as mentioned earlier, for all the $2_1^+$ states of the Ni isotopes. The large inside peak in the $\rho^{\rm tr}(r)$, for example for $^{58}$Ni, corresponds to the broad peak ranging from 1.5 to 3.0 fm$^{-1}$ in the form factors. The radial matrix element $\langle p|j_2(qr)|p\rangle$ is responsible for this picture. Since this $p\rightarrow p$ contribution is the main ingredient in the $0^+_{\rm gnd}\rightarrow 2_1^+$ transition matrix element, the state dependence of the effective charges cannot be expected to cause any significant change in the $|F|^2$ and in the $\rho^{\rm tr}(r)$. It is thus clear from this phenomenological point of view that either $q$ dependence of $e_n$ or larger model space must be considered explicitly.

The $2\hbar\omega$ calculations with the M3Y interaction are shown in Fig. 1. The significant feature of these calculations is that there appears the second peak at $q\simeq 1.6$ fm$^{-1}$ in the $|F|^2$, which is responsible for giving the surface peak in the $\rho^{\rm tr}(r)$. The main components come from the $2\hbar\omega$ p-h excitations, and the $0\hbar\omega$ contributions (excitations from the $f_{7/2}$ to the remaining $fp$ orbits) are added constructively (see Fig. 2). High Fourier components of the matrix elements can be brought in by the mixing of the higher $\hbar\omega$ p-h excitations. The $12\hbar\omega$ calculations with the M3Y interaction are presented in Fig. 1. The higher $\hbar\omega$ p-h excitations with this interaction, however, do not give rise to any significant change in the matrix elements in the low and intermediate $q$ regions. Therefore, there remains the second peak in the $|F|^2$ at $q\simeq 1.6$ fm$^{-1}$, and thus the surface peak in the $\rho^{\rm tr}(r)$. An appreciable change occurs in the $|F|^2$ only at $q\gtrsim 2.0$ fm$^{-1}$ and correspondingly in the $\rho^{\rm tr}(r)$ inside the nucleus ($r\lesssim 2.0$ fm), having little influence on the feature around the nuclear surface.

When the other interactions are employed as the mixing interaction, the $|F|^2$ calculated up to $2\hbar\omega$ excitations are somewhat similar to those obtained with the M3Y interaction. It seems that higher $\hbar\omega$ contributions are slightly different and tend to destroy those calculated up to $2\hbar\omega$ excitations in the intermediate and higher $q$ regions. They are, however, not so large that the surface-peaked feature of the $\rho^{\rm tr}(r)$ is preserved.

Figure 3 shows each contribution from the central, $LS$, and tensor forces of the M3Y interaction. It is clear that the central force plays a dominant role, and noncentral forces are not important in any $q$ range. Similar results are obtained with the Schiffer-True interaction.

Shell-model calculations of Ni isotopes were carried out by Aalders *et al.*,[15] in which the one-particle excitations from the $f_{7/2}$ to the remaining $fp$ orbits were included in the model space. The $\rho^{\rm tr}(r)$ is calculated[3] for the $2_1^+$ state of $^{58}$Ni. The constant additional effective charges of $1.0e$ are assumed both for protons and neutrons. The defect of the $(p_{3/2},p_{1/2},f_{5/2})^n$ calculations is remedied appreciably due to the $f_{7/2}\rightarrow p_{3/2}$ contributions, but there still remains a significant peak in the $\rho^{\rm tr}(r)$ inside the nucleus. This means clearly that the $0\hbar\omega$ contributions only are not sufficient, and more than $2\hbar\omega$ p-h excitations should be considered explicitly. As demonstrated by our configuration-mixing calculations, it is the $2\hbar\omega$ p-h excitations that are responsible for giving the surface-peaked shape for the $\rho^{\rm tr}(r)$ for the $2_1^+$ states of even-mass Ni isotopes.

It should be pointed out finally that the absolute value of $|F|^2$ is underestimated, typically by a factor of 2 in $^{58}$Ni, part of which might be explained by using the random-phase approximation and by using good isospin basis functions.

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[1]J. Heisenberg, Adv. Nucl. Phys. **12**, 61 (1981); and private communication.
[2]B. Frois *et al.*, Phys. Lett. **122B**, 347 (1983).
[3]H. Blok, Ph.D. dissertation, Vrije Universiteit Amsterdam, 1986.
[4]H. Überall, in *Electron Scattering From Complex Nuclei* (Academic, New York, 1971).
[5]J. E. Koops and P. W. M. Glaudemans, Z. Phys. A **280**, 181 (1977).
[6]H. Horie and A. Arima, Phys. Rev. **99**, 778 (1955).
[7]P. Federman and L. Zamick, Phys. Rev. **177**, 1534 (1969).
[8]A. Rimini, in *Theory of Nuclear Structure: Trieste Lectures 1969* (International Atomic Energy Agency, Vienna, 1970), p. 713.
[9]V. Gillet, A. M. Green, and E. A. Sanderson, Nucl. Phys. **88**, 321 (1966).
[10]A. de-Shalit and H. Feshbach, in *Theoretical Nuclear Physics Vol. I: Nuclear Structure* (Wiley, New York, 1974).
[11]J. P. Schiffer and W. W. True, Rev. Mod. Phys. **38**, 191 (1976).
[12]W. G. Love, in *The (p,n) Reaction and the Nucleon-Nucleon Force,* edited by C. D. Goodman, S. M. Austin, S. D. Bloom, J. Rapaport, and G. R. Satchler (Plenum, New York, 1980), p. 23.
[13]R. S. Willey, Nucl. Phys. **40**, 529 (1963).
[14]H. Noya, A. Arima, and H. Horie, Prog. Theor. Phys. Suppl. **8**, 33 (1958).
[15]A. F. Aalders, A. G. M. van Hees, and P. W. M. Glaudemans, University of Utrecht Internal Report v(k) 81-9, 1981.